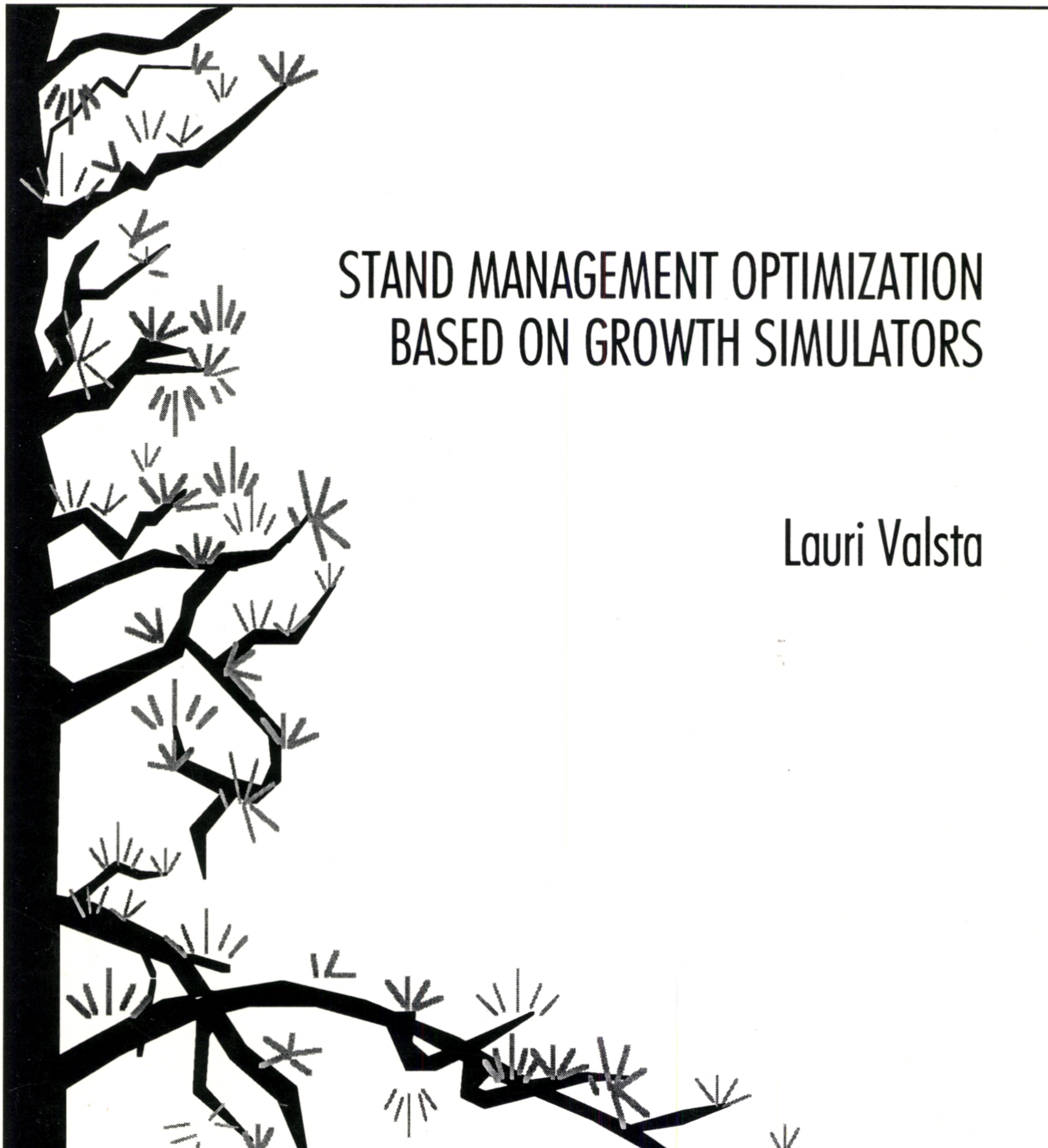

STAND MANAGEMENT OPTIMIZATION BASED ON GROWTH SIMULATORS

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Lauri Valsta

Academic dissertation

To be presented, with the permission of the Faculty of Agriculture and Forestry of the University of Helsinki, for public criticism in Auditorium XIV of the University Main Building, Aleksanterinkatu 5, on April 16th, 1993, at 12 o'clock noon

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The study reviews four publications on stand level optimization. They report on developing optimization approaches to be used in connection with the types of growth simulators utilized for forest planning purposes in Finland: one built on a whole-stand, diameter-free growth model and another consisting of individual-tree, distance-independent growth and mortality models.

Nonlinear programming using stand management control variables proved to be a successful approach to both deterministic and stochastic optimization, in the case of both whole-stand and individual-tree models. Other optimization methods applied are dynamic programming and random search. Questions concerning even-aged stand management are employed to illustrate the optimization methods. These include the optimum species composition (for a two-species stand); the optimum number, intensity, timing, and type of thinnings; and the effects of stochastic growth and catastrophes on optimum anticipatory solutions. The properties of the response surface generated by the stand simulator and the performance of optimization algorithms are also examined.

Key words: forest economics, optimum rotation, optimum thinning, dynamic programming, nonlinear programming, random search, stochastic optimization, *Pinus sylvestris*, *Picea abies*, *Betula pendula*.

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Hakapaino Oy, Helsinki 1993

Preface

This study was mostly carried out in the former Department of Forest Economics of the Finnish Forest Research Institute. I sincerely thank Professor Jouko Hämäläinen, then head of that department, for the necessary resources, the possibility to concentrate on my subject, and for a background in the economics of private forestry. After the reorganizing of the Finnish Forest Research Institute, I have been working in two units. I thank Professor Risto Seppälä, Research Director of the Department of Forest Resources, for supporting my work. I am thankful to Professor Hannu Saarenmaa and Mr. Erkki Kaila, Head of Information Processing, both in the Information Systems Unit, for encouragement in the final stages of this work.

I thank Professor Simo Poso for support in my graduate studies at the University of Helsinki. My inclination to research work was strengthened by course work under the late Associate Professor Pekka Kilkki. My one year period in the Department of Forest Mensuration and Management of the University of Helsinki, headed by Acting Professor Jouko Laasasenaho, also contributed to this work.

My M. Sc. studies at Oregon State University, Department of Forest Management, have strongly influenced my research approach. I am greatly indebted to my major professor, J. Douglas Brodie for personal encouragement and, above all, for teaching me stand-level optimization.

I thank Professor J. Douglas Brodie and Professor Esko Mikkonen for reading and commenting the manuscript of my dissertation.

In the course of the research, the different manuscripts have been read by several persons, mostly fellow researchers at the former Department of Forest Economics of the Finnish Forest Research Institute. Comments and ideas from Mr. Olli Salminen (M. For.) and Dr. Risto Sievänen have been especially valuable. I also thank Professor Pertti Hari, Mr. Markku Kuula (Lic. Econ.), Dr. Jari Kuuluvainen, Dr. Juha Lappi, Professor Kari Mielikäinen, Mr. Risto Ojansuu (M. For.), Dr. Pekka Ollonqvist, Mr. Ville Ovaskainen (Lic. For.), Dr. Markku Penttinen, Mr. Yrjö Sevola (Lic. For.), and Professor Yrjö Vuokila for comments on my work. I appreciate the English language editing by Dr. Ashley Selby.

I have been fortunate to share a researcher's life with my wife, Liisa. I am grateful for her support and views of life. Inspiration from my daughter, Suvi, has also contributed to my work.

Finally, I extend my thanks to all of the persons who have helped me in my work.

This research was supported in part by the Finnish Cultural Foundation, the Society of Forestry in Finland and the Academy of Finland.

Helsinki, March 1993

Lauri Valsta

In addition to the present report, the dissertation includes the following studies, which are referred to by roman numerals in the text:

I Valsta, L.T. 1986. Mänty-rauduskoivusekametsikön hakkuuohjelman optimointi. Summary: Optimizing thinnings and rotation for mixed, even-aged pine-birch stands. Folia Forestalia 666. 23 p.

II Valsta, L.T. 1990. A comparison of numerical methods for optimizing even-aged stand management. Canadian Journal of Forest Research 20:961-969.

III Valsta, L.T. 1992. An optimization model for Norway Spruce management based on individual-tree growth models. Tiivistelmä: Kuusikon käsittelyn optimointi puittaisiin kasvumalleihin pohjautuen. Acta Forestalia Fennica 232. 20 p.

IV Valsta, L.T. 1992. A scenario approach to stochastic anticipatory optimization in stand management. Forest Science 38(2):430-447.

Contents

1 Introduction

1.1 The management problem

The problem of managing a forest stand belongs to the larger concept of natural resource management. Natural resources may be renewable or non-renewable. Optimum utilization of both types of resources can be analyzed but the questions and answers differ (Fisher 1981, Johansson & Löfgren 1985). A characteristic of renewable resource management is that the resource is renewed or can be renewed after the harvest or other use of the resource. A renewable resource typically exhibits growth which may in turn be affected by the use of the resource. In most cases, forest management can be seen as renewable resource management, although virgin forests are practically non-renewable from the multiple use point of view. Resource stocks that are interesting from the planning point of view have either commercial or societal value.

Forest management decisions can be made at five different levels depending on the scope of the decisions and the level of available information (Fig. 1). The present study concerns the stand level.

Consider the different levels of decision making starting from that of the single tree. Treating the stand on the basis of a **tree level** analysis and disregarding the stand level is an unusable approach. The biology and economy of treating a tree is strongly dependent on the state and operations in the surrounding stand, which have to be taken into account.

The **stand level** offers the first meaningful level of decision making. Most forest economics theory is based on stand level analyses. The advantages of such an approach include its mathematical simplicity and the generality of results. Results obtained in this fashion are equal to results based on fully regulated forests, given that scale does not affect the returns or costs and that we may assume perfect markets for all things in question (the "linear forest" in Johansson and Löfgren 1985). In this case, the approach developed by Faustmann (1849) can be used as the basis of forest management decisions. It recognizes that both growing stock and forest land are investments to which returns are generated.

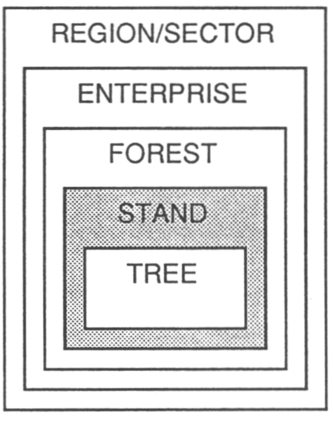

Figure 1. Different decision making levels in forest management.

Results from the stand level or the "linear forest" analysis are useful for several purposes. They can serve as guidelines for management practices if they are biologically and economically parameterized to be suitable for a large number of decision makers. Although not exactly correct for any individual forest owner, the stand level guidelines can form a basis which can be modified to satisfy each decision situation. Stand level results are also useful when examining the effects of changes in determinants of stand management, such as biological conditions, prices and costs, or the objective function. In a forest-level analysis, observed effects may be strongly dependent on the prevailing forest composition and may be misleading.

The stand level analysis is, naturally, unsatisfactory for many planning situations. The even-flow of harvests and stable investments in regeneration at the **forest level** may override stand level economic aspects. In addition, spatial harvest scheduling poses restrictions on stand management (Brodie & Sessions 1991).

To truly maximize the utility of the forest owner or the owners of an enterprise managing the forest, a complete financial analysis at the **enterprise level** should be carried out. This leads to the so called 'combined models' (Hämäläinen 1973). When forestry is not integrated into the rest of the owner's economy, misleading preferences may stem from, e.g., incorrect interest rate, or failure to incorporate changing economic circumstances over time.

Stand optimization studies have traditionally been divided into studies concerning even-aged (Hann & Brodie 1980) or uneven-aged (Hann & Bare 1979) management systems. Haight (1987) formulated and numerically solved a more general stand level optimization problem, in which even-aged and uneven-aged management are special cases. His approach allows for arbitrary planting and natural regeneration activities, which pose great demands on the stand simulation models.

Stand treatment optimization has been performed in Finland employing two basic approaches: stand level or forest level optimization. Kilkki and Väisänen (1969) as well as Siitonen (1972) solved the problem at stand level using dynamic programming. The growth models available limited their approach and they used only one state variable, stand volume. For example, the effect of thinnings on average tree size development could not be taken into to account explicitly. Stand level optimization studies by Valsta (1986 and thereon) are reviewed in the present publication. Recent applications of dynamic programming are by Ringbom (1992) and Salminen (1993).

Instead of developing stand level optimization for more evolved growth models, Kilkki turned to forest level optimization and co-build a linear programming (LP) -based planning system that initially utilized whole-stand growth models to generate the activities of the linear program (Kilkki and Pökälä 1975, Kilkki and Siitonen 1975). Individual-tree growth models were introduced into the LP planning system in the early 1980s (Siitonen 1983) and, lately, enhanced algorithmic capabilities (Lappi 1992). Other Finnish LP-based planning approaches are by Hämäläinen and Kuula (1993), Pukkala (1988a) and Hyttinen (1992). They have less detailed stand prescription analyses than in the system by Kilkki and others, but provide extensions for multiple use (Pukkala 1988a), decision theory (Kangas & Pukkala 1992), agriculture (Hyttinen 1992), and financial analyses within an integrated farm-forest model (Hämäläinen and Kuula 1993).

The temporal scale of forest management analysis must be considered. Economic comparisons have addressed time periods from, e.g., a single decade to perpetuity. Customarily, rotations of plantation management have been regarded as separable, and this can be done with little loss of biological realism in most situations. It is clear that the same condition is not true for natural regeneration.

The spatial distribution of trees has not been addressed in stand optimization studies. An exact solution requires solving a very large combinatorial problem (which trees of a stand to harvest in each cut) whereas an approximate solution could be obtained by relying on some spatial patterns of trees. So far, no optimization studies including the spatial distribution of individual trees are generally known to the international researcher community.

The purpose of using optimization is to improve decisions. A prerequisite for this is that the optimization system functions satisfactorily. Both the stand simulation model and the optimization algorithm play a role. In the present study, the question of the reliability of the stand simulator is not examined.

The performance of an optimization algorithm in solving a management problem is a result of the way the optimization problem is set and the properties of the algorithm itself. For example, the solution procedure may or may not take advantage of the fact that stand management is a dynamic decision problem (see Ch. 5). Factors to be considered when comparing optimization algorithms concern the reliability, or robustness of the algorithm, the precision of the optimum solution, sensitivity to algorithm parameters and the preparational and computational effort (Bazaraa & Shetty 1979). The computational efficiency can be measured by the number of function evaluations needed to achieve a given objective function value. In many situations, the computational efficiency and the ability to ensure a global optimum solution are conflicting characteristics. This question, for example, falls into the realm of the art of optimization.

1.2 Objectives of the studies

The general objective of the studies reviewed in the present paper is to develop and compare optimization methods for different, commonly used stand simulators. Because many of the models used in stand simulators are typical of the country in question, methods originally developed in other countries must be examined with Finnish models. The planned application of the optimization programs developed in the reviewed studies would be stand management optimization made by an expert in a forestry organization or company.

The two most widely used growth model types in Finland are whole-stand models and individual-tree, distance-independent models. Two stand types, one for each of the two growth model groups, are studied: even-aged mixtures of pine and birch (whole-stand model), and pure spruce stand (individual-tree model).

The specific objectives of the four studies of the dissertation are:

Study I
- develop an optimization approach for a two-species, even-aged pine-birch stand based on a whole-stand growth model
- determine the optimum species composition, thinnings and rotation
- study the dynamics of pine and birch from both biological and economical standpoints and examine the 'mixture effect'
- examine the conditions under which a mixed stand is superior to a pure pine stand

Study II
- compare three optimization methods in solving a two-species, even-aged stand management problem based on a whole-stand growth model

Study III - study the management options of an even-aged spruce stand in southern Finland including the number, intensity, type and timing of thinnings; rotation; and initial density.
- compare the results to present guidelines in Finland
- contrast the results obtained using two different growth models
- examine the performance of the optimization algorithm and the properties of the response surface

Study IV - develop a method for stochastic anticipatory optimization using individual-tree growth models
- investigate the effects of yearly growth variations and catastrophic mortality on optimum even-aged stand management

2 Models of stand development

2.1 Classification criteria

Stand management optimization requires a projection model to compute the effects of chosen treatments. Such models are customarily called (stand) simulators. They can be classified by the following properties: (1) the unit of prediction, (2) employment of tree location data, i.e., distance-dependent vs. distance-independent, (3) deterministic vs. stochastic, or (4) statistical-empirical vs. process based. The unit of prediction can be a complete stand, all trees of a given species or crown class, an age or size class, or an individual tree. Some process models go into further detail, but a tree is almost always the smallest unit of management. The unit of prediction is taken as the primary grouping factor in the following review. The terminology followed is that used by Davis and Johnson (1987).

2.2 Whole-stand models

2.2.1 Density-free whole-stand models

Density-free models assume a predetermined stand density development over the rotation. For a given species, site, and location, stand development is a function of time only, and follows a predefined trajectory. These models are usually reported in tabular form and they are termed growth and yield tables. For a given stand, the only decision that can be optimized is rotation. Sometimes there are different tables for alternative treatment policies or for unmanaged and managed stands. Graphical smoothing was an important method of constructing the models of this type. Finnish examples are by Blomqvist (1891), Ilvessalo (1920), Nyyssönen (1954), Vuokila (1956) and Koivisto (1959).

2.2.2 Variable-density whole-stand models

For a considerable period of time, these models were the prevailing basis of stand projection and they still are used in practice. The properties of a forest stand typically recognized are age, competition, tree size, and site quality. The corresponding driving variables of the model are stand age, basal are or volume, number of trees or average diameter, and site index or site type. Finnish models have been derived, for example, by Kuusela and Kilkki (1963), Gustavsen (1977), Nyyssönen and Mielikäinen (1978), Mielikäinen (1980), and Vuokila and Väliaho (1980)

2.2.3 Diameter distribution models

Diameter distribution models predict the development of diameter distribution over time based on stand level variables (Hyink & Moser 1983). Two types of models can be distinguished: (1) The stand level variables used are averaged characteristics, such as stand age, basal area, average diameter or the number of trees. When a prediction is made, the stand level variables are updated based on their own growth models. Then, a diameter distribution is predicted using regression models with the new stand characteristics as independent variables and distribution parameters as dependent variables (e.g., Cao et al. 1982). The diameter distribution can be seen as a template over the stand characteristics. (2) Distribution function models directly use the parameters of diameter distributions and the total number of trees as independent variables in the dynamic models. Additional stand level variables may be utilized. Diameter distribution percentiles are sometimes used in place of parameters (Bailey et al. 1981).

2.3 Age/stage-structured models

Age/stage-structured models of biological populations are widely used in fisheries, zoology, and wildlife management. These models are based on grouping individuals into cohorts, characterized by the age, size, or developmental state of an individual. An early application in forestry is by Usher (1966). Tree growth is described as a transition from one stage to another. The proportion of trees moving from stage i to stage i+1 is given by a transition rate (or a transition probability in a stochastic model). Stage-structured models are defined as linear or nonlinear depending on whether the transition rates/probabilities are determined by a linear or nonlinear function. The models of this group are also called matrix models because the transition probabilities between two stages form a matrix. A thorough examination of this modelling approach and the respective management applications can be found in Getz and Haight (1989).

In Finland, as well as other Nordic countries, stage-structured models have not been widely applied, so far. However, uneven-aged management has recently received more attention and studies have been published in Finland and Norway (Pukkala & Kolström 1988, Solberg & Haight 1991, Kolström 1992).

2.4 Individual-tree models

Whole-stand models are more applicable to stands with average tree size structure and less applicable to stands with irregular structure. The first individual-tree growth models were reported in the 1950s and 1960s (Petterson 1954, Buckman 1962, Lemmon & Schumacher 1962) but their use was initially infrequent, compared with whole-stand models. Individual-tree models are usually grouped into two classes: distance-dependent and distance-independent, based on whether or not they utilize information about the locations of other trees close to the subject tree. Distance-dependent growth models were presented in the mid 1960s by Newnham and Smith (1964) and Vuokila (1965).

As implied by their name, the unit of prediction of individual-tree growth models is the single tree. In distance-independent models, the information used in prediction includes characteristics of the subject tree (diameter, height, a crown measure, age), site (site index, latitude, altitude, topography), and other trees (stand basal area, dominant height, crown competition). Finnish examples are by Nyyssönen and Mielikäinen 1978, Mielikäinen 1980 and 1985, and Ojansuu et al. 1991. Distance-dependent models take advantage of additional information concerning the location and characteristics of close-by trees (Vuokila 1965, Pukkala 1988b).

2.5 Process based models

Process based models operate on a representation of the physiological processes of the tree. The structure and resolution varies but typical processes included are photosynthesis, respiration, allocation, and decomposition (Botkin et al. 1972, Hari et al. 1982, Mäkelä & Hari 1986). Many models include the environment of the tree and can be called *ecosystem models* (Pastor & Post 1985, Kellomäki et al. 1988, 1992). Stand structure may be described by just one average tree or a tree list. *Gap models* (Shugart 1984) predict succession (birth, growth and death of trees) on a given area, defined as the crown projection of a mature tree or some larger area.

3 A review of prior research on stand level optimization

An exposition to previous research is given in Table 1 based on the management question treated and the growth model type used. Because of the objectives of the present investigation, only studies that have produced numerical results have been included. This does, of course, not imply that nonnumerical studies would be less important.

3.1 Deterministic methods

3.1.1 Dynamic programming

Dynamic programming was developed in the early 1950s (Bellmann 1954) as a method for solving sequential decision problems. The first generally known application to forestry problems is by Arimizu (1958a). The stand growth model of Arimizu (1958a) resembles a stand table projection system but optimization is done based on uneven-aged management. The variable optimized is the harvested volume of trees larger than 22 inches. Only a ten-year period is analyzed and it is not clear whether a long time horizon, discounted returns problem could be solved. The stand level study is paired by an input-output model (Arimizu 1958b) with the aim that regional activity analysis could be connected to stand level decision making. Another early application is by Brun Madsen (1964, ref. Kilkki & Väisänen 1969)

Deterministic dynamic programming applications in stand management were continued by three studies solving the optimization of growing stock level and rotation, published almost simultaneously at the end of the 1960s (Amidon & Akin 1968, Kilkki & Väisänen 1969, and Risvand 1969). The investigations by Risvand (1969) and Kilkki & Väisänen (1969) (with improvements and further analyses in Kilkki 1972 and Siitonen 1972) appear more oriented to solving practical problems. Arimizu (1958a) and Risvand (1969) used forward recursion whereas Amidon & Akin (1968) and Kilkki & Väisänen (1969) used backward recursion. An application is also reported by Nesterov and Avtukhovich (1974).

A new period of active research was started by Brodie and others in the USA (Brodie et al. 1978, Brodie & Kao 1979, Kao 1979, Chen et al. 1980, Kluyver et al. 1980, Martin & Ek 1981, Riitters et al. 1982). This new wave of research was encouraged by the more usable formulation by Brodie et al. (1978) as well as by developments in comping. In the 1980s, dynamic programming applications started to increasingly emerge outside the USA and the Nordic countries (Manabe 1984, Meneses & Olivares 1986, Klocek 1988, Medina de Munoz & Gardich Briseno 1989, Torres-Rojo & Brodie 1990, Zadnik Stirn 1990, and Filius & Dul 1992).

Dynamic programming analyses may suffer from the "curse of dimensionality", which increases the computational task beyond available resources. In stand management optimization this often becomes a problem when the state space is defined by more than, say, three variables. Dynamic programming has been made computationally more efficient by employing LaGrangean relaxation (Paredes & Brodie 1987) and region limiting strategies (Yoshimoto et al. 1990).

Table 1. Stand-level optimization studies classified according to the growth model type and the decision to be optimized. Stochastic analyses are in italics. Studies which form part of the present dissertation are in bold face. Uneven-aged management studies are in one common decision group.

	Whole stand models	Distribution template models	Age/stage-structured models	Individual-tree models
Rotation	Chapelle & Nelson 1964 Amidon & Akin 1968 Kilkki & Väisänen 1969 Risvand 1969, *76* Schreuder 1971 *Norstrøm 1975* Brodie et al. 1978 Brodie & Kao 1979 Chen et al. 1980 Kao & Brodie 1980 *Martell 1980* *Routledge 1980, 1987* *Kao* 1979, *82, 84* Cawrse et al. 1984 Manabe 1984 *Reed & Errico 1985* Roise 1986a **Valsta 1986**, 88, **90** *Lohmander 1987* Paredes & Brodie 1987 *Brazee & Mendelsohn 1988* *Caulfield 1988* Betters et al. 1991 *Haight & Holmes 1991* *Haight & Smith 1991* *Reed & Apaloo 1991* *Ringbom 1992*	Valsta & Brodie 1987 *Teeter & Caulfield 1991* *Teeter & Somers 1991*	*Hool 1966* *Lembersky & Johnson 1975* Riitters et al. 1982 Bullard et al. 1985 *Haight 1991* Solberg & Haight 1991	Martin & Ek 1981 Haight et al. 1985b Roise 1986b Roise et al. 88a, 88b Valsta 1987, 91, ***92***, **93** Arthaud & Klemperer 1988 Carlsson 1990 Haight & Monserud 1990a,b Yoshimoto et al. 1990
Growing stock level	Chapelle & Nelson 1964 Amidon & Akin 1968 Kilkki & Väisänen 1969 Risvand 1969 Schreuder 1971 Brodie et al. 1978 Brodie & Kao 1979 Chen et al. 1980 Kao & Brodie 1980 *Kao* 1979, *82, 84* Cawrse et al. 1984 Manabe 1984 Roise 1986a **Valsta 1986**, 88, **90** Paredes & Brodie 1987 Betters et al. 1991 *Haight & Smith 1991* *Reed & Apaloo 1991* *Ringbom 1992*	Valsta & Brodie 1987 *Teeter & Caulfield 1991* *Teeter & Somers 1991*	*Hool 1966* *Lembersky & Johnson 1975* Riitters et al. 1982 Bullard et al. 1985 *Haight 1991* Solberg & Haight 1991	Martin & Ek 1981 Haight et al. 1985b Roise 1986b Roise et al. 88a, 88b Valsta 1987, 91, ***92***, **93** Arthaud & Klemperer 1988 Carlsson 1990 Haight & Monserud 1990a,b Yoshimoto et al. 1990
Thinning type	Kilkki & Väisänen 1969		Bullard et al. 1985 Solberg & Haight 1991	Haight et al. 1985b Roise 1986b Valsta 1987, 91, ***92***, **93** Arthaud & Klemperer 1988 Haight & Monserud 1990a,b Yoshimoto et al. 1990
Planting density	*Haight & Smith 1991*	*Teeter & Caulfield 1991* *Teeter & Somers 1991*	*Lembersky & Johnson 1975* Hann et al. 1983 Solberg & Haight 1991	**Valsta 1993**
Vegetation management	*Haight & Smith 1991*	Valsta & Brodie 1987		
Multi-species	**Valsta 1986**, 88		Bullard et al. 1985	Haight & Monserud 1990a,b Carlsson 1990 Yoshimoto et al. 1990
Fertilization	Kao 1979			
Uneven-aged management	Arimizu 1958a		*Hool 1966* Adams & Ek 1974 *Buongiorno & Michie 1980* Rapera 1980 Haight 1985, 87, *1990, 1991* Haight et al. 1985a *Kaya & Buongiorno 1987* *Gove & Fairweather 1992*	Buongiorno & Michie 1980 Bare & Opalach 1987

Forestry questions other than stand management, treated with dynamic programming, include deer management (Davis 1967), allocation of research funds (Bethune & Clutter 1969), forest fire protection (Kourtz 1973 and 1989, O'Regan et al. 1975), tree bucking (Strand 1967, Pnevmaticos & Mann 1972, Luo 1983, Busby & Ward 1989), locating forest roads (Zhang & Xu 1987), and sampling design (Omule & Williams 1982).

Dynamic programming has proved to be very successful in optimizing whole-stand growth models. The version of dynamic programming used has almost exclusively been the discrete-time, discrete-state formulation. Dynamic programming is well suited for computerized analysis. It guarantees the global optimum solution within a specified accuracy, the node interval. It allows the use of any form of model, including tabular and discontinuous relationships. The main shortcoming is that dynamic programming can be used with individual-tree growth models only if these are seriously reduced to sufficiently restrict the number of state variables . Ways exist to overcome this problem (Brodie & Haight 1985, Paredes & Brodie 1987, Yoshimoto et al. 1990) but their potential remains to be seen.

3.1.2 Optimal control theory

Optimal control theory is used to control a dynamical system in a way that maximizes the performance criterion and, at the same time, satisfies the state equations (Kirk 1970). Stand management applications of optimal control theory can be divided into two groups based on whether they obtain numerical results or not. The first theoretically oriented study was by Näslund (1969), followed by Anderson (1976) and Clark (1976). Other studies without numerical results include Hellman (1982, 1986), and Snyder and Bhattacharyya (1990). Simple numerical models have been applied by Clark (1976), Cawrse et al. (1984), Lohmander (1988), and Betters et al. (1991), whereas models aimed at stand management decision making are given by Rapera (1980), Haight (1987), and Solberg and Haight (1991). Successful formulations of optimal control theory for stand management (Haight 1987, Solberg & Haight 1991) have been in the discrete-time form. This is because of the difficulty of determining optimum solutions in continuous time for stand management problems which are multi-dimensional. These studies have been based on stage-structured growth models.

3.1.3 Nonlinear programming

Nonlinear programming is used for finding an optimum solution to a problem where the objective function or constraints or both have nonlinear components. If only quadratic and linear relations exist, quadratic programming can be used. For an account for nonlinear programming, see, e.g., Himmelblau (1972), or Bazaraa and Shetty (1979).

Nonlinear programming has been used in stand management research for two principal problems: selection harvests for uneven-aged management based on stage-structured growth models, and even-aged (or all-aged) management based on individual-tree models. The first line of research was started by an innovative study of uneven-aged management of hardwood stands in the U.S.A. (Adams & Ek 1974). Further elaborations of the optimization problem are given in Adams and Ek (1975). The uneven-aged management problem was not completely solved until Haight et al. (1985a). The strength

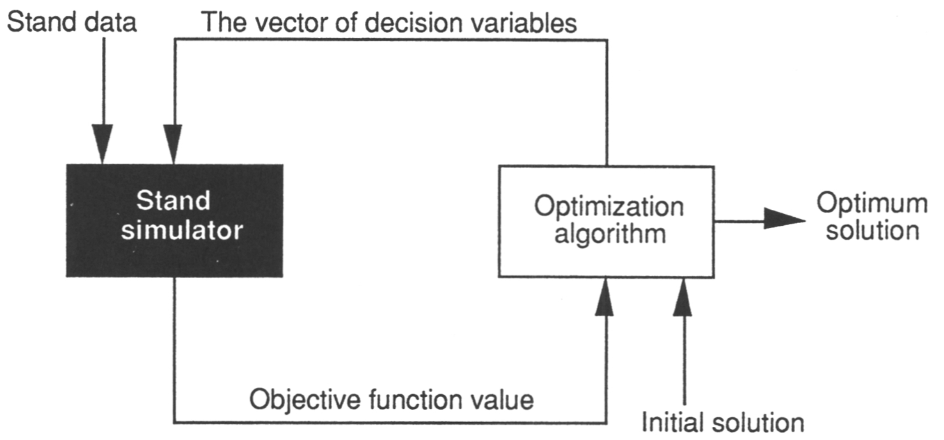

Figure 2. The structure of the simulation optimization system for nonlinear programming.

of the method was further demonstrated by its application to the central stand management problem of comparing even-aged and uneven-aged management economically (Haight 1987). These problems have the form of discrete-time optimal control but the numerical solution method used is an application of nonlinear programming.

Nondifferentiable nonlinear programming algorithms have gained considerable popularity in stand management optimization studies. Their advantage is that they do not require the use of derivatives which are either tedious or impossible to derive for stand simulators.

Two problem structures have been used. In the first one, treatments can occur at predefined points of time, e.g., cuttings are made possible at 5 or 10-year intervals (first used by Adams & Ek 1974). The variables to be optimized are the intensities of the operations. If they are zero, the operation is postponed. The second formulation adds the times between operations to the decision variables (first used by Kao & Brodie 1980).

The former approach has been used for optimizing uneven-aged management (Adams & Ek 1974, Rapera 1980, Haight et al. 1985a, Bare & Opalach 1987, Haight & Monserud 1990a and 1990b) and the latter for optimizing even-aged management (Kao & Brodie 1980, Roise 1986a and 1986b, Valsta 1987, 1988, **II, III, IV**, Roise et al. 1988a, Yoshimoto et al. 1990). The reason for this distinction is the number of cuttings, in particular, that need to be optimized within a meaningful planning horizon. This number is smaller for even-aged management where the timing of thinnings and final harvest is also more important than in the case of uneven-aged management.

Nonlinear programming was linked with individual-tree growth models first by Roise (1986b). The structure of the simulation-optimization system was the same as in Roise (1986a) and it has gained popularity thereafter (Valsta 1987, 1988, **II, III, IV**, Roise et al. 1988a and b, Haight & Monserud 1990a and 1990b, Monserud & Haight 1990, Yoshimoto et al. 1990). Figure 2 shows the overall structure of the simulation-optimization system. This optimization formulation completely ignores the dynamic structure of stand management. This appears potentially inefficient (solving a dynamical problem without recognizing dynamics) but because of the complex nature of individual-tree based stand dynamics it has been a successful approach, compared to dynamic optimization. It also allows complete flexibility of the structure of the stand simulator which is inside a "black box" (Roise 1986a). The optimization problem appears to the algorithm as a standard nonlinear programming problem.

Nonlinear programming has been used for solving problems that are set as discrete-time optimal control problems (Haight et al. 1985a, Haight 1987, Solberg & Haight 1991). The discrete-time formulation permits numerical solutions to large-dimensional nonlinear control problems that would not be solvable in continuous-time form (Dreyfus & Law 1977, pp 102-106).

3.1.4 Random search

Random search is an alternative to optimization algorithms when assumptions of continuity, differentiability or unimodality of the objective function cannot be met. Random search also serves as a baseline method: to be considered viable, any optimization algorithm should perform better than random search.

Pure random search is seldom used in optimization. Rather, modifications are made which increase the efficiency of the search. For example, Bullard et al. (1985) used multistage random search where search regions are reduced after each stage based on accrued information about objective function values. Valsta (**II**) compared random search to dynamic programming and nonlinear programming in optimizing stand management based on a whole-stand growth model.

3.2 Stochastic methods

3.2.1 Adaptation and anticipation

Stochastic optimization methods can be characterized basically by the same arguments as for deterministic optimization. Additional attributes are the structure of the stochastic process involved (where there are obviously numerous alternatives) and the role of information obtained in the course of the planning period. Based on the latter characteristic, two basic forms of stochastic optimization are distinguished. Depending on the discipline, this pair has been termed in at least three different ways: adaptive vs. anticipatory (nonadaptive) optimization; closed-loop vs. open-loop control; optimization with recourse vs. without recourse. The essential difference is whether the system is observed and the decision adjusted during the planning period (hence the term 'adaptive') or the decisions are made before hand for the whole planning period ('anticipative'). Stochastic optimization studies in stand management have employed both forms and they seem to be justifiable by the various decision problems encountered.

Apart from optimization approaches, stochastic investment analyses have been used forest economics, including Monte Carlo simulation (Anderson et al. 1987, Taylor & Fortson 1991), Decision Tree method (Martell & Fullerton 1988), and Capital Asset Pricing Model (Cathcart & Klemperer 1988, Wagner & Rideout 1991).

3.2.2 Stochastic dynamic programming and optimal stopping

Two large groups of probabilistic analyses of stand management are: the problems of optimum rotation based on simple yield models utilizing the optimum stopping approach; and the more complex problems of rotation and/or thinnings applying stochastic dynamic programming, some of which are in the form of a Markovian decision model.

The usefulness of stochastic dynamic programming has been demonstrated by the large amount of research relying on it (see Table 1). In stochastic dynamic programming applications, one faces the same limitations as with the deterministic version, namely the restrictions on state space size. With the advent of individual-tree growth models, stochastic analysis has mostly been restricted to wood prices, as the state space for stand development far surpasses the computational possibilities. On the other hand, whole-stand growth models or whole-stand state space defined over an individual-tree simulator may be sufficient for short term (say, 10-20 years) planning periods. Therefore, decision support systems based on stochastic dynamic programming may also have practical importance in the future.

Stochastic optimal control applications to stand management are not known to the author, other than those that customarily are classified under stochastic dynamic programming or optimal stopping (Williams 1982). This is to be expected, as continuous-time stochastic control problems are not likely to have practical importance in stand management and the discrete-time problems are best solved by stochastic dynamic programming. However, Haight (1990, 1991) used gridded Monte Carlo simulation to solve for the parameters of a feedback function which was time-constant. Although not stated by the author, the studies can be seen as applications of stochastic optimal control. The stand simulator was a stage-structured model and both uneven-aged (Haight 1990) and any-aged stand structures (Haight 1991) were analyzed.

3.2.3 Stochastic nonlinear programming

Judged by deterministic nonlinear programming applications to stand management, stochastic nonlinear programming has great potential as a tool for stand optimization. While capable of solving large problems and offering a range of solution methods (e.g., Ermoliev & Wets 1988), stochastic nonlinear programming can also be used with individual-tree growth models.

Stochastic nonlinear programming has anticipatory and adaptive variants. Adaptation in stochastic programming is usually called recourse. It may take place one or several times, giving rise to the terms 'two-stage' or 'multi-stage' recourse problem, respectively.

The multi-stage recourse problem appears to be fairly similar to stochastic dynamic programming (or discrete-time stochastic optimal control). The solution to a multi-stage recourse problem includes decision variables for each decision time, whereas the solution to the optimal control problem is a feedback law which may or may not be a function of time (Varaiya & Wets 1988). The tradeoff is between finding the time paths of a smaller number of control variables of stochastic optimal control and finding point values for a greater number of decision variables of stochastic nonlinear programming. Decomposition can be used in the latter case to improve solution possibilities (Ermoliev & Wets 1988).

Valsta (**IV**) uses an individual-tree simulator and solves an anticipatory stochastic nonlinear programming problem as a scenario formulation utilizing a deterministic optimization algorithm (Wets 1989). A corresponding adaptive problem is stated in section 56 (Eqn 5.6.1).

4 Determination of a decision maker's utility

4.1 Deterministic models

The decision maker is assumed to possess some basic properties: he or she behaves rationally, maximizes utility, and has a quantitative utility function. There exists a wide selection of literature on utility theory and decision making. References regarding forest management on the topic include Duerr et al. (1982), Kilkki (1985), Davis and Johnson (1987), and Kangas (1992).

As is customary forest management decision making, the objective of stand management is assumed to be a maximum utility. It is computed via a utility function which maps the time paths of the inputs to the stand, the outputs from the stand, and the state of the stand to a quantitative measure of utility. The utility is computed over a given period of time, most commonly from the present to perpetuity.

Because forest management deals with intertemporal decisions, a time preference must be established in the utility function. Stand management is seen as an investment because forest land and standing trees are realizable capital and both costs and revenues are associated with decisions. The present value criterion (Fisher 1930) is used as the deterministic measure of the decision maker's utility. When computing present values of management regimes, it is assumed that rotations with equal treatments are repeated to perpetuity. In other words, the decision criterion is the Faustmannian soil expectation value (Faustmann 1849). The land occupied by the stand is assumed to be continuously used for timber management.

It is presumed that stand development can be predicted with certainty and that the prices and costs, and the interest rate are known and constant. Other well-known assumptions inherent in the present value criterion imply that lending and borrowing interest rates are the same and that the same rate of interest applies to any amount of money produced or required by stand management.

4.2 Stochastic models

Introducing stochasticity into stand management brings two new aspects: the return from stand management actions is uncertain and the decision maker's attitude towards risk may affect the decisions. Given the various sources and forms of stochasticity, there exist many alternative formulations for utility functions in a probabilistic setting.

A basic observation is that $E[\phi(x)] \neq \phi(E[x])$, i.e., the stochastic optimization problem gives a different answer than what is obtained when replacing the stochastic variables with their expected values and performing a deterministic optimization. This substitution is made when a stochastic system is modelled as a deterministic system, which is common practise in forest research.

Modelling approaches used in forest economics literature (with sample references) have been based on

- risk-adjusted discount rates (Reed 1984, Cathcart & Klemperer 1988)
- expected present value (Brazee & Mendelsohn 1988, Haight 1990, **IV**)

- expected present value with an attitude to risk
  - decreasing marginal utility of revenues (Koskela 1989)
  - mean-variance analysis (Taylor & Fortson 1991, **IV**)
  - stochastic dominance analysis (Caulfield 1988)
  - capital asset pricing (Binkley & Washburn 1990, Wagner & Rideout 1991)

The use of risk-adjusted discount rates has been shown to be theoretically correct in, e.g., a fire risk case with a simplified forest production function (Reed 1984). In many other situations, adjusting the discount rate may be an improper solution to account for risk (Cathcart & Klemperer 1988).

A widely accepted goal of utility maximization in the stochastic world is the maximization of expected utility (von Neumann & Morgenstern 1947). Let $\phi(\mathbf{x},\xi)$ be a function for net present value based on deterministic variables $\mathbf{x}$ and stochastic variables ξ. Let $\mathcal{U}$ be the utility function which translates the (stochastic) net present values $\phi(\mathbf{x},\xi)$ into the utility of the decision maker. When the decision maker is not risk neutral, the solution for maximizing expected utility does not equal the solution for maximizing expected returns, i.e.:

$$E[\mathcal{U}(\phi(\mathbf{x},\xi))] \neq \mathcal{U}(E[\phi(\mathbf{x},\xi)]) \qquad (4.1)$$

A well known reason for risk aversion is the decreasing marginal utility of money (or net returns) (e.g., Arrow 1971). An increment in wealth by a given amount of money increases utility less than a decrement by the same amount of money decreases utility.

5 Mathematical formulations of stand management optimization

5.1 The resource model

Suppose the changes in the state variables, $\mathbf{x}(t) \in \mathbb{R}^m$, of the biological system due to growth, renewal and mortality are given by the function $\mathbf{f}(\mathbf{x}(t)):\mathbb{R}^m \rightarrow \mathbb{R}^m$. Define the changes in the state variables by human actions $\mathbf{u}(t) \in \mathbb{R}^n$ as $\mathbf{h}(\mathbf{u}(t)):\mathbb{R}^n \rightarrow \mathbb{R}^m$. The frequently used form (e.g., Clark 1976, Johansson & Löfgren 1985) of the differential state equation is

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{f}(\mathbf{x}(t)) - \mathbf{h}(\mathbf{u}(t)) \tag{5.1.1}$$

Note that the "harvest" function, $\mathbf{h}$, can be continuous over time or, typical of forestry, of impulse type. To facilitate numerical analysis, discrete time formulations are usually employed in stand management studies. Defining $\bar{\mathbf{f}}:\mathbb{R}^m \rightarrow \mathbb{R}^m$ the periodic growth equation we obtain a corresponding difference state equation

$$\mathbf{x}(t+1) - \mathbf{x}(t) = \bar{\mathbf{f}}(\mathbf{x}(t) - \mathbf{h}(\mathbf{u}(t))) \tag{5.1.2}$$

Note that the growth function $\bar{\mathbf{f}}$ is evaluated after the stand has been harvested.

Both the continuous-time and the discrete-time formulation above share one important property: future development depends only on the present state of the system. We do not need to know what has taken place before time t. The model is said to be memoryless. This property is required by dynamic optimization methods.

The formulations of stand level optimization problems used in the forest management literature can be grouped depending on whether state or control variables are actually observed and manipulated by the optimization algorithm. Three approaches for deterministic and two for stochastic optimization are presented in the remainder of this chapter. The following ideas underlie the formulations that are given: (i) the decision maker wishes to maximize the utility of managing a forest stand; (ii) it is assumed that the utility is maximized when an objective function of stand level optimization problem is maximized; and (iii) it is supposed that the objective is to maximize the present net value of future revenues and costs.

5.2 Deterministic optimization using state variables $\mathbf{x}(t)$ and control variables $\mathbf{u}(t)$

The stand management problem for a rotation can be formulated in discrete time using state variables $\mathbf{x}(t)$ and control variables $\mathbf{u}(t)$. Let t take take values 0, 1, 2, ..., T. The optimization problem becomes:

$$\max_{\mathbf{u}(t),\, T} \sum_{t=0}^{T} g(\mathbf{x}(t), \mathbf{u}(t)) \frac{(1+r)^{T-t}}{(1+r)^{T}-1} \tag{5.2.1}$$

subject to:

$$\mathbf{x}(t+1) - \mathbf{x}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)), \quad t = 0, 1, \ldots, T-1 \tag{5.2.2}$$

$$\mathbf{x}(0) = \mathbf{x}_0 \tag{5.2.3}$$

$$\mathbf{u}(t) \in \mathrm{U}_t, \quad t = 0, 1, \ldots, T \tag{5.2.4}$$

where:

$g(\mathbf{x}(t), \mathbf{u}(t))$ = return function (revenue or cost when performing control $\mathbf{u}(t)$ on state $\mathbf{x}(t)$, $g:\mathbb{R}^m \times \mathbb{R}^n \rightarrow \mathbb{R}$)

r = (decimal) interest rate

$\mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))$ = state equation

$\mathbf{x}_0$ = the given state of the stand at the beginning of the rotation

U_t = the feasible controls for time t

T = rotation

Other symbols are as before.

This formulation is known as the discrete-time optimal control problem. The numerical solution uses gradient techniques and the partial derivatives $\partial g/\partial x$, $\partial g/\partial u$, $\partial f/\partial x$, and $\partial f/\partial u$. Because many of the partial derivatives are difficult or impossible to determine analytically for many stand growth simulators, the approach has had limited use (for examples, see Haight 1985, 1987, Haight and Getz 1987, Getz and Haight 1989, Solberg & Haight 1991).

5.3 Deterministic optimization using state variables $\mathbf{x}(t)$

The stand management optimization problem can also be solved without explicit control variables. Nonlinear programming or dynamic programming can be used for this purpose. Redefine function g so that it uses the stand state before ($\mathbf{x}(t)$) and after ($\tilde{\mathbf{x}}(t)$) harvest, $g:\mathbb{R}^m \times \mathbb{R}^m \rightarrow \mathbb{R}$. As the growth dynamics are defined by the state equations, the sequence of values of $\tilde{\mathbf{x}}(t)$ complete defines the management regime. In discrete time, the optimization problem for even-aged stand management can be stated as

$$\max_{\tilde{\mathbf{x}}(t),\, T} \quad \sum_{t=0}^{T-1} g(\mathbf{x}(t),\tilde{\mathbf{x}}(t)) \frac{(1+r)^{T-t}}{(1+r)^{T}-1} \;+\; g(\mathbf{x}(T),\mathbf{x}_0)\frac{1}{(1+r)^{T}-1} \qquad (5.3.1)$$

subject to:

$$\mathbf{x}(t+1) - \tilde{\mathbf{x}}(t) = \mathbf{f}(\tilde{\mathbf{x}}(t)), \quad t = 0,\, 1,\, \ldots,\, T\text{-}1 \qquad (5.3.2)$$

$$\tilde{\mathbf{x}}(0) = \mathbf{x}_0 \qquad (5.3.3)$$

$$1 \leq T \leq T_{max} \qquad (5.3.4)$$

where:

$\tilde{\mathbf{x}}(t)$ = state vector after a treatment at time t

$g(\mathbf{x}(t), \tilde{\mathbf{x}}(t))$ = return function (revenue or cost when performing state transition from stand $\mathbf{x}(t)$ to stand $\tilde{\mathbf{x}}(t)$, $g{:}\mathbb{R}^m{\times}\mathbb{R}^m{\to}\mathbb{R}$)

$\mathbf{f}(\tilde{\mathbf{x}}(t))$ = state equation

T_{max} = maximum rotation

Other symbols are as before.

The first part of equation (5.3.1) refers to the period of thinnings and other silvicultural operations in the stand. $\mathbf{x}_0$ is the initial stand condition which is also established after the final harvest. The return function g and the "growth" function **f** do not have time t as an argument. Tree or stand age can be defined a state variable if the functions are time/age dependent.

The many deterministic *dynamic programming* applications in stand management belong to this class of formulations. The corresponding discrete-time forward recursion equation becomes (used in **I** and **II**):

$$R(\mathbf{x}_{t+1}) = \max_{\{\mathbf{x}_t\}} \left[\bar{g}(\mathbf{x}_{t+1},\mathbf{x}_t) + R(\mathbf{x}_t)\right] \qquad (5.3.5)$$

$$t = 1,\ldots,T$$

where $\bar{g}$ is the discounted return associated with state transition from stand $\mathbf{x}_t$ to stand $\mathbf{x}_{t+1}$, $\{\mathbf{x}_t\}$ is the set of stands that can lead to stand $\mathbf{x}_{t+1}$ after growth and a possible thinning, and $R{:}\mathbb{R}^m{\to}\mathbb{R}$.

A method related to dynamic programming is the calculus of variations. It solves the so called variational problem where a time path is determined for $\mathbf{x}(t)$ such that the objective functional

$$J(\mathbf{x}) = \int_{t_0}^{T} \bar{\bar{g}}(\mathbf{x}(t), \dot{\mathbf{x}}(t), t)\, dt \tag{5.3.6}$$

is maximized (discounting and growth function are embedded in $\bar{\bar{g}}$ and $\dot{\mathbf{x}}$ denotes the time derivative of $\mathbf{x}$). Calculus of variations uses a continuous-time formulation that is not well-suited for stand management problems. The linear control problem (Cawrse et al. 1984, Betters et al. 1991) can be solved using the calculus of variations.

5.4 Deterministic optimization using control variables u

In this formulation, the state variable values are not used by the optimization algorithm. They are computed inside the stand simulator, based on a given initial state $\mathbf{x}_0$ and the control variables $\mathbf{u}$ (Fig. 2). It is important to note that $\mathbf{u}$ is no longer a function of t, i.e., the dynamical structure of the system is not seen by the optimization algorithm. The stand simulator is denoted by the redefined function $g{:}\mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}$ and U is the set of feasible controls.

$$\max_{\mathbf{u}} \quad g(\mathbf{u} \mid \mathbf{x}_0) \tag{5.4.1}$$

subject to:

$$\mathbf{u} \in \mathrm{U} \tag{5.4.2}$$

$$\mathbf{x}_0 \ \text{given} \tag{5.4.3}$$

This problem can be solved using nonlinear programming. The method was introduced by Kao and Brodie (1980) for whole-stand models and, after being applied to individual-tree models by Roise (1986b), it has gained wider use (e.g., **II** and **III**).

The stochastic versions of problems (5.4.1-3) and (5.3.1-4) create two additional formulations.

5.5 Stochastic optimization using control variables u

The starting point is a general stochastic programming problem without recourse, i.e., an anticipatory optimization problem (Ermoliev and Wets 1988). Define $\mathbf{x}$ a vector of variables with a feasible region X, Ξ the set of possible realizations, $\xi \in \Xi$ the vector of stochastic variables, and ϕ: $\mathbb{R}^m \times \Xi \rightarrow \mathbb{R}$ the stochastic objective function. Assume we wish to maximize the expected value of ϕ:

$$\max_{\{\mathbf{x} \in X \subset \mathbb{R}^m\}} \quad \mathrm{E}[\phi(\mathbf{x}, \xi)] \tag{5.5.1}$$

Because of the complexity of forest growth models, this maximization is difficult to solve (for methods, see Ermoliev and Wets 1988). An alternative formulation is based on scenario analysis.

A set of scenarios, $S = \{s^1, \ldots, s^L\}$, can be substituted for the vector ξ (Wets 1989, **IV**). The scenarios are generated prior to optimization and are treated as exogenous to optimization. They are independent of the variables **x**. Let p_s, $s \in S$, be the probability weights given for each scenario and redefine ϕ as $\phi:\mathbb{R}^m \times S \rightarrow \mathbb{R}$. The scenarios can be seen as discrete random variables. Because of the properties of the expected value of a function of discrete random variables, the maximization in (5.5.1) may be approximated by

$$\max_{\{\mathbf{x} \in X \subset \mathbb{R}^m\}} \sum_{s \in S} p_s \, \phi(\mathbf{x}, s^s) \tag{5.5.2}$$

Substitute **u** for **x** and redefine ϕ as $\phi:\mathbb{R}^n \times S \times \mathbb{R}^m \rightarrow \mathbb{R}$ Note the dependency of the return function on the initial state, $\mathbf{x}_0$, as well as the feasible set C for **u**, and we get the formulation in (**IV**):

$$\max_{\{\mathbf{u} \in C \subset \mathbb{R}^n\}} \sum_{s \in S} p_s \, \phi(\mathbf{u}, s^s \mid \mathbf{x}_0) \tag{5.5.3}$$

The important property of (5.5.3) is that it is a deterministic nonlinear programming problem, easier to solve numerically than the original stochastic programming problem (5.5.1).

5.6 Stochastic optimization using state variables x

The use of state variables **x** permits feedback rules for decisions - this form is termed adaptive optimization. The solution to an adaptive optimization problem is not directly an optimum management regime. Rather, it is a rule for making optimum decisions based on the observed state at any point in time.

Define the feedback rule as a function of system state, $\kappa(\mathbf{x}):\mathbb{R}^m \rightarrow \mathbb{R}^k$ where k is the dimension of treatment description. Modify the function g so that its argument is the decision rule instead of the decisions themselves (as in 5.4.1). Denote a stochastic version of the return function g by γ. Add the stochastic variables ξ to the formulation and obtain $\gamma(\kappa(\mathbf{x}),\xi):\mathbb{R}^k \times \Xi \rightarrow \mathbb{R}$. Let K be the class of admissible feedback functions. The problem can be set as:

$$\max_{\{\kappa \in \mathrm{K}\}} \mathrm{E}[\gamma(\kappa(\mathbf{x}),\xi)] \tag{5.6.1}$$

In stand optimization studies so far, the decision rule (function $\kappa(\mathbf{x})$) has been simple: either an age dependent price, called the *reservation price*, in the case of even-aged management with stochastic price (Lohmander 1987, Brazee & Mendelsohn 1988), or a harvest intensity function based on observed stand value in the case of uneven-aged and all-aged management (Haight 1990, 1991). The scenario technique can be used for adaptive optimization, as well, and a formulation parallel to (5.5.3) can be derived for (5.6.1).

6 Optimization based on whole-stand models (I and II)

6.1 Background

The primary motivation for study **I** was the development of a growth model for mixed, even-aged pine-birch stands in southern Finland (Mielikäinen 1980). Managing a two-species stand is a complicated task, for which optimization can provide valuable insights. Simultaneously controlling the growing stock level and species composition, and taking the different stumpage prices into account would require a considerable amount of work when using plain simulation.

Compared to single-species management, multi-species stand management poses additional questions: first, the stability of the ecosystem against environmental factors and pests; secondly, the economic viability when facing uncertain stumpage value, by species; thirdly, the multiple-use benefits of increasing ecosystem variability; and finally, the increased usage of information and management effort. Accounting for each of these as well as timber yields, requires a vast amount of knowledge and data, much of which is unavailable at present. To keep the study manageable, attention in study **I** was restricted to the yields, stumpage values, and discounted returns of a two-species, even-aged mixture.

The basic research task was to determine the optimal thinnings, rotation, and species composition through time. The yield model base was restricted to established stands that are at the stage of commercial thinnings. As important as regeneration and development of seedlings are from management point of view, it was not possible to include these stages into the optimization. Therefore, the results can be used to evaluate management options during the period of commercial thinnings and help the forest manager to decide how to treat already existing young mixtures.

Single-species management based on whole-stand growth models and dynamic programming was a well established method at the early 1980s when study **I** was initiated. Optimum species mix had not been dealt with, but Hann & Brodie (1980, p. 14) stated that it could be optimized using dynamic programming and that "an additional state descriptor would be required for each species in the problem". A two-species dynamic programming approach was adopted as the tool for economic analysis.

Although it has dominated the literature, dynamic programming is not the only method that can be used for deterministic stand-level optimization with whole-stand growth models. Nonlinear programming has also been successfully applied (Kao & Brodie 1980, Roise 1986a) and it has been competent with the more demanding individual-tree models. As we are dealing with nondifferentiable models, random search is another basic alternative (Bullard et al. 1985). Study **I** reports on a comparison of dynamic programming, nonlinear programming and simple random search in terms of accuracy of results, computing times, and stability of solutions. The growth and yield model is the same in both studies **I** and **II**.

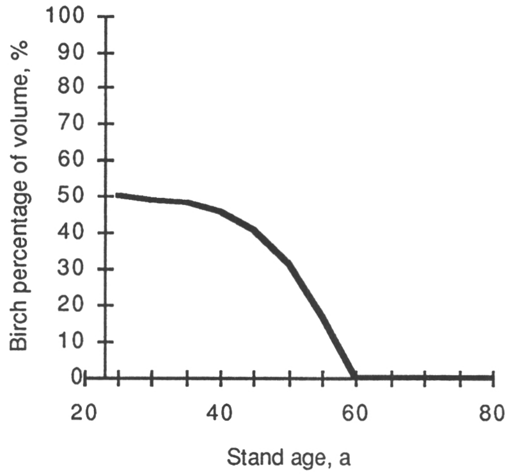

Fig. 3. The birch percentage with maximum volume growth as a function of stand age (Mielikäinen 1980).

6.2 Growth and yield model

The whole-stand growth model by Mielikäinen (1980) operates on three stand characteristics: stand age at breast height, stand volume, and birch percentage of volume. These variables are used for computing the total periodic (5-year) volume increment of a mixed stand. The proportional increment of each tree species is tabulated based on stand age and birch percentage. The relation of the periodic total volume increment to the birch percentage is such that, in young stands, maximum growth is obtained at about 50 % birch, 50 % pine, and at later ages, at successively lower birch percentages (Fig. 3). The dynamic programming algorithm poses no restrictions on the models' function forms.

The number of trees is not used by the growth model but was required for determining the average stem volume of trees which, in turn, was the independent variable of models for computing sawtimber and pulpwood proportions. Equations, by species, for sawtimber and wastewood percentages were estimated from the data in Mielikäinen (1980) using nonlinear regression and maximum likelihood estimation. The following equation form was used:

$$P = \frac{1}{1 + e^{b_0 + b_1 \left(\frac{V}{N}\right)^{b_2}}} \tag{6.1}$$

6.3 Economic considerations

Managing the stand is viewed as an investment, where the capital invested consists of the management costs, the merchantable growing stock, and the value of the land occupied. The returns are comprised of stumpage received from pine and birch pulpwood and sawtimber. The soil expectation value of an infinite series of equal rotations was used to compare management alternatives. The analysis was subject to the usual assumptions of deterministic growth models, costs and prices, and a perfect capital market with known and time invariant interest rates.

The ratio of stumpage prices of the two tree species is an important determinant of optimum species mix. This ratio has both temporal and geographical variation. The sensitivity of results to price ratio was tested by applying a recent price ratio and a deviation in both directions. Because stumpage prices have been subject to considerable variation due to economic cycles, recent prices were taken as trend values for the cutting year 1983/84, based on series of 16 years.

6.4 The dynamic programming algorithm

The optimization problem was set as a discrete-time, discrete-state, deterministic dynamic programming problem. The three-descriptor model of Brodie and Kao (1979) was expanded to account for two species. The state variables are stand volume, birch percentage (of volume), and number of trees. Stand age is obtained from the time steps of optimization. The 'principle of optimality' of dynamic programming (Bellman 1957) requires that stand development is can be uniquely determined based on the information contained in the values of the state variables and time.

The forward recursion version of dynamic programming was used in conjunction with the neighborhood concept of Brodie and Kao (1979), in which stands with state variable values within a given tolerance are regarded as occupying the same network node. Thus, it was not necessary to define the exact network nodes prior to optimization. Growth model changes were simple to implement and the network interval, the accuracy, (and computing load) could be made a parameter of the optimization.

Stand treatments included in the optimization are thinnings and final cut. Thinnings are defined as changes in stand volume and birch percentage, subject to the availability of each species. The number of trees after thinning is computed assuming systematic thinnings, i.e., the proportional change in the number of trees is the same as in stand volume.

Cuttings can be made at five year intervals or their multiples. Thinnings are specified to discrete steps, being multiples of 10 m^3/ha for stand volume, and 10 % for birch percentage. The same intervals were used to discretize the state space with the addition of the number of trees at an interval of 75 trees/ha. These state variable step sizes also indicate the accuracy of the optimization.

6.5 Optimum species composition in even-aged pine-birch stands (I)

The optimum species composition is determined as part of an optimum two-species thinning regime. Moreover, there is a unique optimum solution for each initial stand condition.

Assuming that we can arrive at different young stand species compositions by the same regeneration effort, we may optimize the initial species composition, as well. The optimum solution for a 25-year-old initial stand begins with 20 % birch (Figs. 4 and 5). The first thinning at age 40 does not change species composition. At ages 50 and 60, the birch percentage is strongly reduced leading to a practically pure pine stand at the end of the rotation. The increase of the birch percentage between ages 25 and 50 (Fig. 5) is due to the more rapid growth of birch in the early stages of stand development.

The results of Figs. 4 and 5 are subject to assumptions on stumpage prices and interest rate, among other things. Increasing or reducing birch stumpage prices shifted the optimum birch percentage up and down but a pure pine stand at the end of the rotation was always the aim (**I**). The optimum species composition was not sensitive to changes in discount rate, although the thinning program and rotation were strongly affected (**I**).

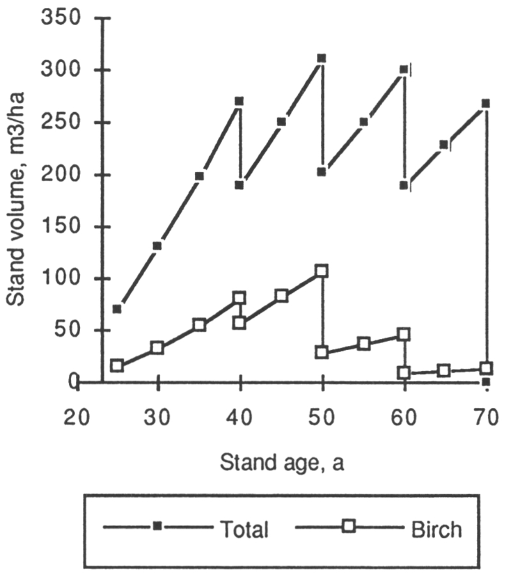

Fig. 4. The optimum thinning schedule by species, final harvest at age 70.

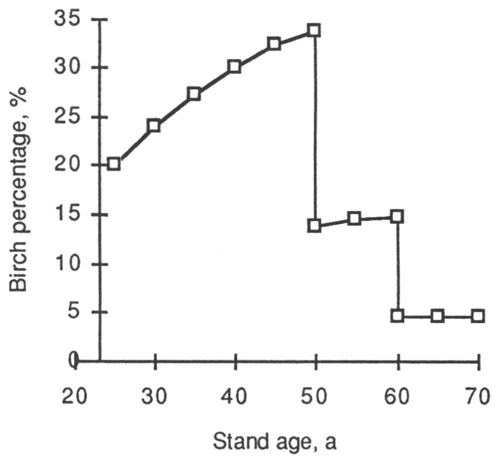

Fig. 5. The optimum birch percentage of volume.

One of the primary questions of mixtures management is the existence of the so called 'mixture effect'. Mielikäinen (1985, p. 48) reports that the periodic volume growth of mixtures of Scots pine and silver birch, as well as Norway spruce and silver birch, exhibit a mixture effect, i.e., the growth of a mixture is greater than a weighted average of pure stands. From a managerial point of view, we may define that a mixture effect exists when, say, the total growth during rotation or the discounted net revenues are greater for a mixed stand than for a weighted average of pure stands. This may also take place when the tree species have differing growth rhythms, even though at no point in time does the biologically defined mixture effect (based on periodic growth) exist. In a two species case, we would utilize the periods of fastest growth of both of the species by accordingly allocating growing space for them during the rotation.

The 'managerial mixture effect' can be tested by comparing optimum objective function values stemming from different species compositions. Because species composition of the optimum regimes changes through time, one has to decide upon the measure of species composition. The ones used here are the birch percentage of the initial stand and the birch percentage of total volume production over the rotation.

Let us first consider even-aged pine-birch stands (Fig. 6 A). When the objective is to maximize net present value, the largest managerial mixture effect is about 17 %, obtained with 40-60 % birch in the initial stand and by using optimum thinning regimes (**I**). The results are given in Fig. 6 in terms of birch percentage of volume production. Based on somewhat different thinning programs, the mixture effect (measured as mean annual increment, M.A.I.) reported by Mielikäinen (1980) was about 13 %. Naturally, these re-

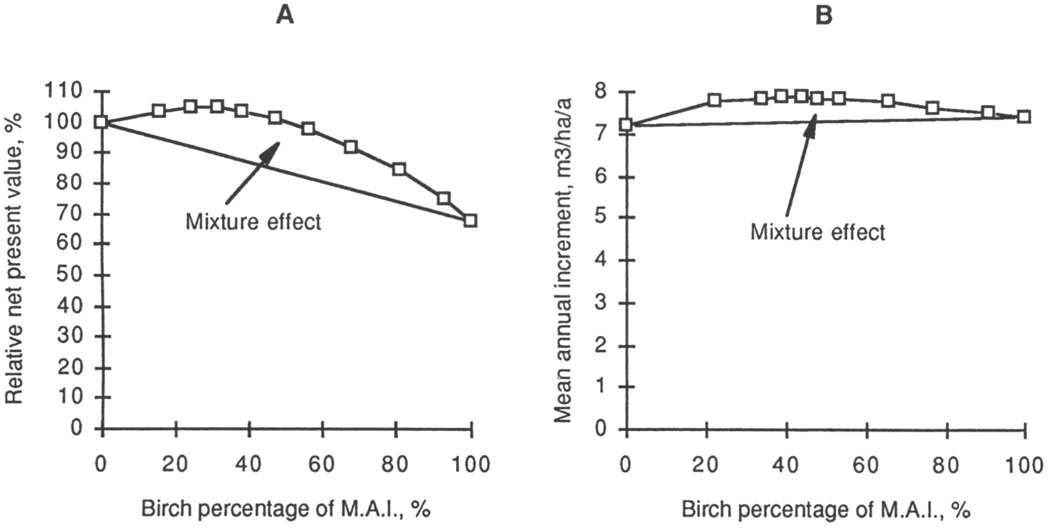

Figure 6. The managerial mixture effect. A: pine-birch stand with economic objective; B: spruce-birch stand with M.A.I objective.

sults are subject to the biological conditions behind the growth models and are not universally applicable.

The mixture effect is unique for each set of species and biological conditions. As a second example (Fig. 6 B), the even-aged mixture of Norway spruce and silver birch is a case where the productivities of the two species are closer to each other than in the case of Scots pine and silver birch. The maximum mixture effect on M.A.I. is 8 % and on soil expectation value 17 % (Valsta 1988).

Carlsson (1990) derives optimum treatment regimes for mixed Norway spruce - silver birch stands using dynamic programming. The state space consisted of two variables, the numbers of spruce and birch trees per hectare. A predefined network of alternative states was generated by using growth functions by Tham (1988) and Eriksson (1976) for Swedish forests. The network for each time step was only 4 times 5 (=20) nodes, representing discrete values for spruce and birch number of trees, respectively. Birches had to be removed at age 50 whereafter only a pure spruce stand was possible. Because of the sparse state space, the results obtained are merely approximations. With 4 % discount rate, an admixture of 300 to 600 birches per hectare (about 20 to 30 % of total growing stock) maximized present values. Compared to pure spruce stand, all of the birch admixture amounts (300 ... 1200) increased present values. Although limited, the results are in accordance with those of Valsta (1988) which also deals with Norway spruce-silver birch mixtures.

Hägg (1988) calculated the economic effects of a birch admixture to Scots pine and Norway spruce using a set of simulations based on growth functions by Agestam (1985), also for Sweden. The simulations contained admixtures of up to 1400 trees per hectare. Admixtures were profitable at all levels examined, but a slightly superior result was obtained with increased amounts of conifers. No optimization was used when selecting the alternatives simulated and their treatment regimes.

6.6 A comparison of numerical methods for stand optimization based on whole-stand growth models (II)

The methods used for deterministic stand level optimization based on whole-stand growth models can be grouped into dynamic programming, nonlinear programming, and random search. The relative merits of the three methods have rarely been discussed. Kao and Brodie (1980) compared discrete dynamic programming and nonlinear programming (flexible polyhedron method, a variant of the simplex method by Nelder and Mead (1965)). They report a 1 to 3 % loss in the objective function value of dynamic programming when compared to a continuous variable solution. Roise (1986a) studied three different algorithms of derivative-free nonlinear programming, and dynamic programming. Objective function values based on dynamic programming were less than 80 % of those by nonlinear programming, somewhat contradicting results by Kao and Brodie (1980).

The performance of optimization algorithms is dependent on both the algorithm parameters used and the type of problem being solved. The optimization results given by Roise (1986a) show considerable variation between runs starting from different initial points, whereas Kao and Brodie (1980) only mention that convergence times of repeated runs varied but the solutions were apparently the same. This suggests that the set of models used by Roise (1986a) forms a more difficult optimization problem.

In the two studies compared above, there is a substantial difference in the performance of dynamic programming with respect to nonlinear programming. The dynamic programming network used in the two studies differs considerably in terms of the time interval: Kao and Brodie (1980) - 1 year; Roise (1986a) - 10 years. It seems possible that the large time intervals used by Roise (1986a) worsened the dynamic programming performance.

Study **II** compares dynamic programming, nonlinear programming with Hooke and Jeeves' direct search algorithm (Hooke & Jeeves 1961), and simple random search. The decision problem, the growth and yield models, and the dynamic programming algorithm are the same as in **I**. The nonlinear programming formulation is similar to (5.4.1-3), augmented by constraints imposed by species composition, which ensure that neither tree species is cut in excess of that which exists in the stand. Random search is performed by generating uniformly distributed pseudorandom variates subject to the same bounds and constraints as in the case of nonlinear programming.

The decision variables optimized by nonlinear programming and random search are the times between cuts, thinned proportions of volume and of birch, and the birch proportion in the initial stand. Hence, the number of variables to be optimized is $3m + 2$, where m is the number of thinnings. The state space of dynamic programming is the same as in **I**.

Two problems were analyzed: maximization of soil expectation value with a 3 % discount rate and maximizing forest rent (average annual net return). The optimum solutions to these two problems differ in the number of thinnings, which is 2 or 3, or about 8 to 10, respectively, for the 3 % or forest rent cases.

The 3 % problem can be considered a small one and the forest rent problem a reasonably large one in the context of whole-stand models. When the number of thinnings is small, dynamic programming comes last in objective function values (Fig. 7). Its restriction to 5-year intervals reduces available solutions. In the forest rent case, the rotation is longer and thinnings can be scheduled at 5-year increments without a great deal of loss in objective function.

The computational efficiency of the optimization methods can be evaluated by comparing the execution times of computer runs. Dynamic programming consumed about 10 times more time than the other two methods. More efficient forms of dynamic programming have been developed with tens of times smaller computing load (Paredes & Brodie 1987, Yoshimoto et al. 1988, 1990).

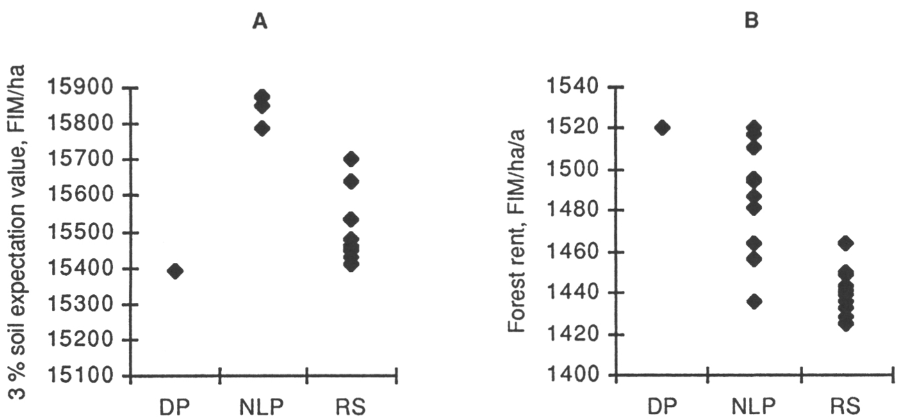

Fig. 7. The performance of dynamic programming (DP), nonlinear programming (NLP), and random search (RS) when maximizing (A) 3 % soil expectation value and (B) forest rent, in various optimization runs. The ranges of the number of decision variables for NLP and RS are 8 or 11 in A, and 11 to 29 in B. Ten repeated runs for NLP and RS.

7 Optimization based on individual-tree models

7.1 Background

Compared to whole-stand models, individual-tree models offer a much wider selection of silvicultural questions for analysis, such as thinning type, stand structure, and initial density. Costs and revenues can be computed in more detail, as well.

The richer description of stand processes or decisions signifies that the optimization problem is more complex - more state or decision variables will be used. This causes severe problems for applying dynamic programming, as the number of state variables increases. Dynamic programming can be used with individual-tree growth models, but only for restricted problems (Brodie & Haight 1985, Arthaud & Klemperer 1988).

Optimization based on control variables only was selected as the problem structure. In this approach, the stand simulator is connected to the optimization algorithm just by a subroutine or function call passing a control variable vector and obtaining an objective function value (the "black box" system of chapter 3.1.3). Hence, the elements of the stand simulator can be freely changed as long as the interface to optimization remains unchanged. This improves the efficiency of maintaining computer programs.

The optimization algorithm is a combination of the Hooke and Jeeves direct search method and random search, adopted from Osyczka (1984). Random search is used in two stages; first, when selecting an initial solution to repetitions of optimization; and second, when inspecting the search space around a local maximum reported by the algorithm. Both phases are used to reduce the problems caused by response surfaces that, viewed by the algorithm, are nonconcave or not strictly concave. The same optimization algorithm is used in both **III** and **IV**.

7.2 Growth and yield models

Stand management optimization under various economic conditions is a challenging task for growth models. Although existing models are not perfect, they are satisfactory for the purpose of illustration of the optimization methods developed. Two different growth models were used in **III** (Mielikäinen 1985, Ojansuu et al. 1991) and a third one in **IV** (Mielikäinen 1985). In the latter case, stand growth was transformed stochastic by multiplying predicted growth by lognormally distributed pseudorandom variates. Study **IV** also included randomly occurring catastrophes as an option. They destroy a random proportion of trees which are immediately harvested with increased logging costs and decreased stumpage values. A pure Norway spruce stand was the subject of analysis.

The set of models used in **III** consist of diameter and height growth models as well as mortality models. In addition, **IV** uses a live crown ratio model. Tree-wise information consists of diameter, height, breast-height age, and the number of trees represented per hectare. Simulation begins from an initial tree list, typically one representative tree for each one-centimeter diameter class.

Wood yields by assortment are computed using models by Laasasenaho and Snellman (1983). They use tree species, diameter, and height as independent variables. Log-

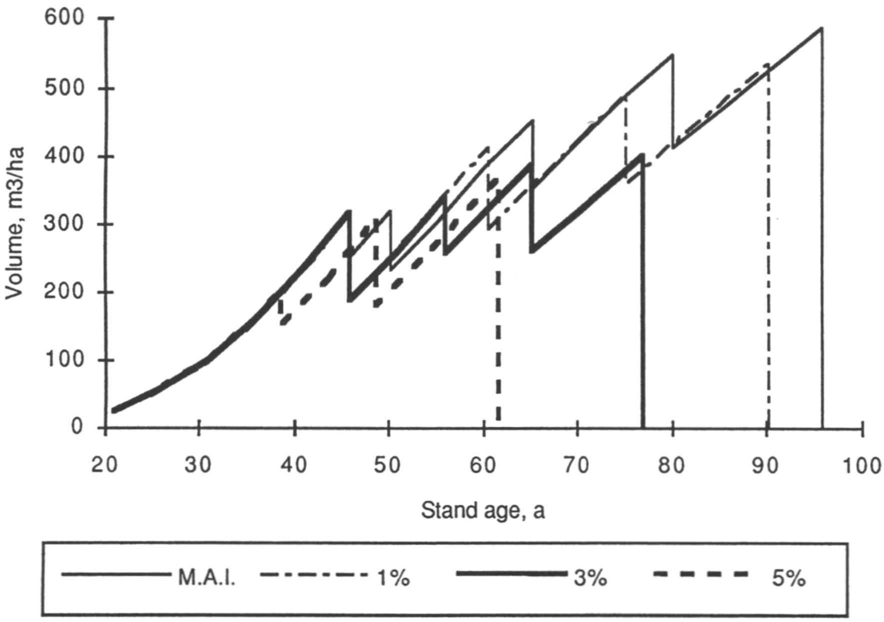

Fig. 8. Optimum cutting regimes for alternative interest rates. A maximum of four thinnings is available.

ging cost models are based on the average tree size, the total volume removed, and type of cut (thinning/final cut).

7.3 An optimization approach for Norway spruce management based on individual-tree growth models (III)

Interest rate is one of the most important variables affecting the optimum stand management regimes. As is well known, interest rate changes alter the growing stock level and the optimum rotation. An example of this is seen in Fig. 8 (from **III**) where the number, timing and intensity of cuttings depends on discount rate. The initial stand was the same in all cases. In theory, the mean annual increment (M.A.I.) is maximized by infinitely many thinnings. For computing resources reason, a maximum of four thinnings was considered.

Given the growth and yield, and economic models of **III**, the time of the first thinning is little influenced by the number of thinnings (Fig. 9). The result reflects the principle of using the alternative rate of return as the control criterion: the stand is left unthinned as long as the marginal value growth is high enough. The optimization approach used takes into account the whole rotation a consequence of which is that the marginal rate of return criterion is not strictly followed due to long term intertemporal effects. Because the thinnings are from above, Fig. 9 shows that dominant height is slightly reduced by thinning.

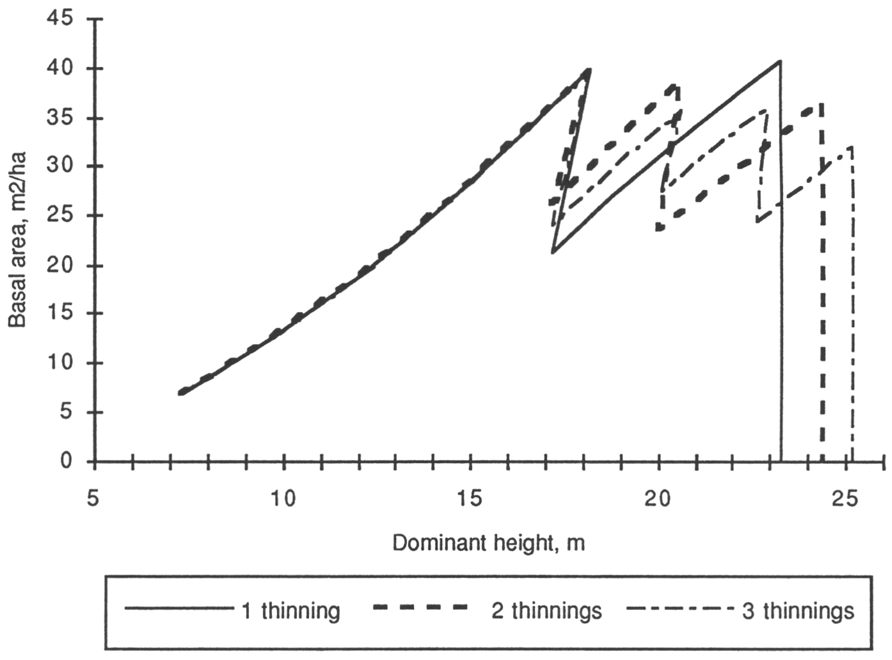

Fig. 9. Optimum cuts for regimes with one, two, or three thinnings.

Another example of analyses available by the present optimization approach concerns thinning type (thinning from below, thinning from above) and its dependence on the discount rate, for instance. A flexible and efficient way of defining thinning types was developed for **III** and **IV**. A commonly used definition of thinning type is the ratio average diameter of trees thinned divided by average diameter before thinning, known as the 'd/D-ratio'. The objective of stand management clearly affects the thinning type (Fig. 10). In the growth models used, removing the largest trees of the stand improves the growth of the remaining trees. Thinning from above was optimal in most of the cases examined.

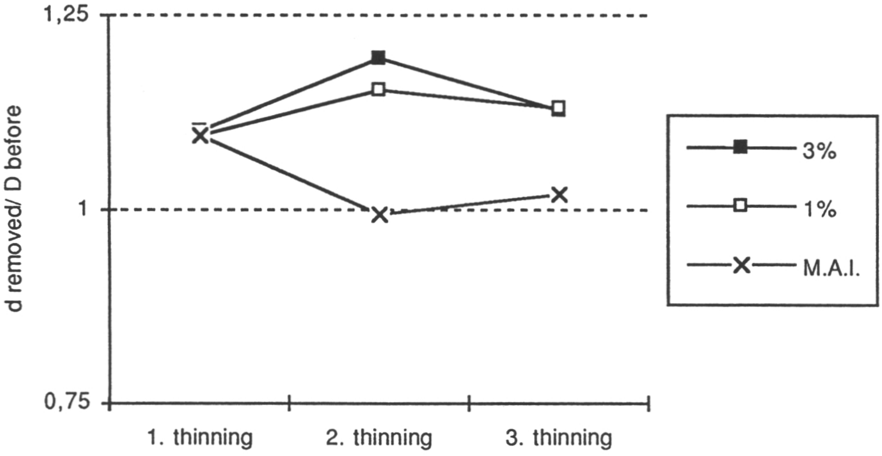

Fig. 10. Thinning type (d/D-ratio) in three-thinning regimes with different objectives.

7.4 A scenario approach to stochastic anticipatory optimization in stand management (IV)

Tree growth predictions differ from actual growth because of inaccurate input data, errors in models, and unpredicted environmental conditions as well as unaccounted competition from other trees. How do these factors affect the optimum management regimes? Stochastic optimization can be used to provide answers to these questions.

Anticipative optimization was chosen as the methodology for **IV**. It produces optimal once-and-for-all decisions that take into account the uncertainty during the planning period, the rotation in this case. Decisions are not adjusted based on observations, which is the case in adaptive optimization.

The study of this kind requires a stochastic stand simulator. Stochasticity was added into a deterministic simulator built on growth models by Mielikäinen (1985) and mortality models by Ojansuu et al. (1991). Items in the computer program that were made stochastic were yearly diameter and height growth and their long-term trend, catastrophes that destroy part of standing trees, and the initial growing stock level. Lognormal distribution was used instead of normal distribution to compute growth level variation, because growth level can never be a negative number.

The present model formulation offers great flexibility for defining the objective function of stochastic optimization. For the purpose of illustration, three variants were used: maximization of expected discounted net revenues (risk-neutral preferences), the maximization of the probability of attaining a specified soil expectation value, and risk-adjusted expected return.

The random growth level was thought to represent climatic growth variation, for example. The effects of this form of stochasticity on optimum solutions were mixed and minor. Variations seemed largely to cancel out in the risk neutral case. Risk aversion brought up some nonsymmetric relations: increasing risk aversion shortened the optimum rotation if thinnings were disabled. However, thinnings enabled, the effect vanished.

The model for growth level variation was derived for demonstration and it has only a partial empirical basis. The effect of growth variation on the expected soil expectation value showed interesting behavior. In a no-thinning case, increasing growth variation slightly reduced the optimum expected soil expectation value (Fig. 3 in **IV**). With a thinning included, the effect was the opposite (Table 1 in **IV**). Presumably, a thinning made it possible to take greater advantage of growth variation. When comparing a deterministic and a stochastic version of the Stand Prognosis model, stochasticity of height, crown ratio, and basal area increment resulted in a 6.8 % increase of volume yield at age 100, without thinning (Hamilton 1991). It should be noted that in the latter analysis rotation length was not optimized and was constant, regardless of whether stochasticity was included or not.

The effects of catastrophic effects on optimum forest rotation have been studied extensively with theoretical models concerning fire risk, e.g. (Martell 1980, Routledge 1980, Reed & Errico 1985). The results show that the optimum rotation decreases with increasing risk of catastrophe. The same result was obtained by Caulfield (1988) for a risk averse case and an empirical yield relationship. An optimization approach is developed in **IV** in connection with an individual-tree growth model allowing detailed biological and economic models of catastrophes.

In study **IV**, the decrease of optimum rotation due to increasing risk of a catastrophe was clear and applied to both unthinned and thinned stands. This result serves as one test of the conclusions drawn from theoretical analyses, and it supports them. Because of the simplicity of the biological and economic models relating to catastrophes, the results obtained do not have much relevance to practical forestry. The optimization methodology is, however, suitable for more refined simulators which provide more useful results.

8 Conclusions

8.1 Compatibility of growth models and optimization methods

An important question when building an optimization system for stand management is the cooperation between the stand simulation model and the optimization algorithm. The compatibility of the three most widely used deterministic stand optimization methods, namely, dynamic programming (DP), discrete-time optimal control (DTOC), and non-linear programming (NLP, using a direct search method), with some model and solution characteristics may be summarized as in Table 2 (Valsta 1988). The different characteristics are:

Non-differentiable model -- This refers to the partial derivatives of the return function (objective function) with respect to the state or control variables of the optimization problem. Non-differentiability or non-continuous first order derivatives may arise, e.g., from detailed stem value models taking into account dimensions required by different wood assortments, or from 5-year time steps used in stand projection models.

Many state variables -- In approximate characterization, whole-stand models have few state variables whereas individual-tree models have many.

Global optimum found -- Some optimization algorithms have the property of guaranteeing a global optimum of the search space while others only find a local optimum (which may also be a global optimum).

Accurate optimum defined -- This is a subjective definition and arises from the practical performance of an optimization algorithm under present computing possibilities. Methods relying on discretized state space cannot achieve great accuracy.

It can be seen that none of the optimization methods fulfills all of the requirements stated. The art of optimization is to determine a combination that best meets the requirements for the optimization system.

Table 2. A comparison of model characteristics and optimization methods.

	Compatilibility of optimization method		
	DP	DTOC	NLP (DS)
Model characteristic:			
non-differentiable	yes	no	yes
many state variables	no	yes	yes
Solution characteristic:			
global optimum found	yes	no	no
accurate optimum determined	no	yes/no	yes

8.2 Areas of further development in stand level optimization

The development of simulators for stand management will challenge optimization algorithms and computers - more complicated problems should be solved without loss of speed and reliability. As environmental change analyses are undertaken, considerably more complicated models will be used (process models, ecosystem models). The new growth models may provide a speed problem (optimization takes too long) or a structure problem (algorithms do not find correct solutions). In principle, however, the nonlinear programming approach based on control variables (Eqns. 5.4.1-5.4.3) is suitable for any stand simulator.

The derivative-free nonlinear programming algorithms used in connection with nondifferentiable models are inherently less efficient optimization algorithms than those using derivatives. Derivative information might be available in some cases because some stand projection systems are differentiable, or they can be made so without considerable loss of accuracy. It might be worthwhile to modify simulation models in extensive use to comply with differentiability.

The problem of local optima has been reported in several stand optimization studies (Martin & Ek 1981, Roise 1986a, Valsta 1988 and **II**, Haight & Monserud 1990a). Global optimization algorithms (e.g., Törn & Zilinskas 1989) are specifically developed for multimodal functions (having several maxima or minima). These algorithms tend to be slow compared with direct search algorithms and have not been used in forest management. The problem of multiple extrema has been handled by multistart optimizations (Haight & Monserud 1990a) and augmentation by random search (**II**). Finding both more reliable and efficient methods merits additional research.

A natural development of the stochastic method of **IV** is the adaptive optimization approach. The long time horizon of stand management makes adaptation beneficial. For example, making the management decision conditional on the regeneration result offers valuable flexibility. A problem that has been repeatedly studied concerns unknown future wood prices (Norstrøm 1975, Lohmander 1987, Haight & Smith 1991), but in a 50 year time horizon almost every aspect of stand management is uncertain and could justify stochastic modelling.

When the rotation of an even-aged stand is optimized in an adaptive setting, the optimal reservation price is determined and it is compared against the observed current price. If the current price is higher than the reservation price, the stand is cut. In uneven-aged management, stand value can be used as the basis of the decision: if the observed stand volume times the observed price is higher than the reference value, the stand is cut to an optimal level. In even-aged management, the decision on intermediate cuttings is more difficult: stand value *per se* is an unacceptable criterion, contrary to uneven-aged management. When only prices are stochastic and a whole-stand growth model is used, the problem has been solved using stochastic dynamic programming (Haight & Smith 1991, Ringbom 1992). Useful decision rules for individual-tree growth models are more difficult to establish and applicable studies can only be looked forward to.

The stands considered in **III** and **IV** have been of single species. Multi-species, individual-tree optimization problems have been solved using similar methodology (Haight & Monserud 1990a and b). Additional control variables need to be assigned to new species, which increases the optimization problem. Experience from single-species optimization suggests that at least a two-species case should be manageable with the algorithms of **III** and **IV**.

8.3 Applicability of optimization systems

The four publications reviewed in this paper concern optimization methods and solutions to several frequent stand management problems. The purpose of using an optimization system is to improve decisions. Improvement requires, first, that the decision making procedure is flawless and, secondly, that the optimization results are correct. To choose the correct tool (the optimization system) for a given decision problem is seen as the decision maker's responsibility. However, stand optimization researchers may have important insights into the properties of the tools that best serve the decision maker.

The correctness of optimization results relates to the stand simulator, the utility/objective function, and the optimization algorithm. Naturally, each of the elements is essential for the usefulness of results. The present study deals mainly with selecting and implementing an appropriate optimization algorithm to be used with a given utility function and stand simulator. Questions relating to the developing and testing of stand simulators and the decision making process fall outside the scope of the present study.

When developing stand optimization systems, the optimization algorithms were tested in several ways. Because deterministic dynamic programming determines a global optimum solution (subject to the accuracy due to discretization of the state space), only formulation and programming errors had to be eliminated. Different from dynamic programming, the Hooke and Jeeves algorithm for nonlinear programming has parameter values to be decided upon and it only determines a local optimum solution. The algorithm was examined by running test problems found in textbooks (Himmelblau 1972) and by repeating optimization runs with different parameter values. The response surface generated by a stand simulator was analyzed by multiple optimization runs (**II**), as well as two and three dimensional graphs (**III** and **IV**). Also important were observations on the behavior of optimum solutions when changing the economic or biological parameters of the system.

When thinking of the applicability of present, state of the art stand optimization systems, one must differentiate between deterministic and probabilistic systems. Concerning deterministic systems, and judging on the basis of the current literature, the optimization part is well developed and does not limit the usefulness of optimization results (only the case of optimization based on distance-dependent growth models is not adequately advanced). The stand simulators, on the other hand, still seem to have serious limitations, and their predictive ability is further challenged by environmental change. Regarding probabilistic systems, there is yet substantial room for optimization method development. Successful ways of defining decision rules for adaptive optimization have remained largely unexplored, especially in the case of individual-tree growth models. Further more, probabilistic stand simulators need much development. Modelling the stochastic processes that predict the economic conditions of stand management is also deficiently developed.

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FOLIA FORESTALIA 666

Metsäntutkimuslaitos. Institutum Forestale Fenniae. Helsinki 1986

Lauri Valsta

MÄNTY-RAUDUSKOIVUSEKAMETSIKÖN HAKKUUOHJELMAN OPTIMOINTI

Optimizing thinnings and rotation for mixed, even-aged pine-birch stands

Approved on 29.8.1986

SISÄLLYS

VALSTA, L. 1986. Mänty-rauduskoivusekametsikön hakkuuohjelman optimointi. Summary: Optimizing thinnings and rotation for mixed, even-aged pine-birch stands. Folia Forestalia 666. 23 p.

Dynaamista ohjelmointia käyttäen on laadittu optimointimalli rinnankorkeudelta tasaikäisen, hoidetun mänty-rauduskoivusekametsikön harvennuksille ja kiertoajan pituudelle. Malli määrittää harvennusten optimaaliset ajankohdat, voimakkuudet ja puulajisuhteet. Optimoinnin kriteerinä on hakkuuohjelman antama nykyarvo.

Mallin syöttötietoina tarvitaan optimoitavan metsikön ikä, runkotilavuus ja runkoluku sekä puutavaralajeittaiset yksikköhinnat, metsikön perustamiskustannukset ja laskentakorkokanta. Koivun osuus puuston tilavuudesta voidaan antaa syöttötietona tai vaihtoehtoisesti malli etsii optimaalisen koivuosuuden myös alkupuustolle. Harvennusvoimakkuudelle voidaan lisäksi asettaa yläraja ja koivun osuudelle vähimmäisvaatimus. Malli soveltuu 30—80 -vuotiaille tuoreen ja lehtomaisen kankaan metsiköille maan keski- ja itäosissa. Metsänomistajan oletetaan myyvän puut pystykaupoin.

Tyypillisten hintasuhteiden vallitessa edullisin rauduskoivun osuus on kiertoajan alkupuolella 20—40 prosenttia. Kiertoajan loppua kohden on optimaalista poistaa koivut kokonaan. Hoidetuissa sekametsiköissä laskelmia vastaavissa olosuhteissa on mahdollista ylläpitää 50 prosentin rauduskoivuosuus läpi kiertoajan vain vähäisin taloudellisin tappioin puhtaaseen männikköön verrattuna.

Discrete-time, discrete-state dynamic programming is used to optimize thinnings and rotation for mixed, even-aged stands of Scots pine (*Pinus sylvestris* L.) and birch (*Betula pendula* Roth). The optimal timing, intensity and species composition of thinnings is also determined. The optimality criterion is soil expectation value, based on an infinite series of equal rotations. The state variables of the model are stand volume, birch percentage of volume and the total number of trees.

Pulpwood and sawtimber are priced individually for each tree species. Stumpage prices are dependent on the average stem size and the volume removed in any one cut. Thinning intensity may be constrained and a lower bound may be imposed for the birch percentage of the growing stock.

The results are applicable to properly managed stands with neither tree species overtopping in the canopy. Typical optimal management regimes include 20—40 percent birch in the growing stock during the first half of the rotation. Later, the birches are removed and a pure pine stand is final harvested at stand age 60—80, depending on the interest rate.

Keywords: forest economics, mixed-species stand, optimal thinning, dynamic programming
ODC 624.3 + 228.3/.5 + 174.7 *Pinus sylvestris* + 176.1 *Betula pendula*

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1. JOHDANTO

11. Sekametsikön kasvatus liiketaloudellisena ongelmana

Havumetsiköiden koivusekoitus vaikuttaa sekä puuston kasvuun että raakapuumarkkinoiden kautta syntyneeseen metsikön puuston arvoon puulajien hintasuhteiden välityksellä. Jos puulajisuhteet vaikuttavat metsien moninaiskäytölliseen arvoon, metsänomistajan saama hyöty tai metsän arvo kiinteistömarkkinoilla riippuu koivusekoituksen määrästä puuston rahallisen arvon ohella. Yleisesti oletetaan koivusekoituksen vaikuttavan myös metsikön tuhon- ja saasteidenkestävyyteen, joilla on luonnollisesti taloudellisia seurannaisvaikutuksia. Edellä mainittujen epäsuorien tai ei-rahallisten tekijöiden sisällyttäminen liiketaloudellisiin laskelmiin ei ole mahdollista toistaiseksi käytettävissä olevan tiedon pohjalta. Päätöksentekijä voinee silti hyödyntää kvalitatiivisten kriteerien ohella laskelmia, jotka ilmaisevat eriasteisen koivusekoituksen vaikutuksen hakkuutulojen perusteella määritettyyn metsikön käsittelyohjelman arvoon.

Vaikka mänty ja koivu ovat molemmat ns. pioneeripuulajeja ja siten nuorena nopeakasvuisia, on koivun kasvu kuitenkin selvästi nopeampaa ensimmäisinä vuosikymmeninä (Mielikäinen 1980, s. 26). Kahden kasvurytmiltään jossain määrin poikkeavan puulajin kasvatus sekametsikkönä tarjoaa mahdollisuuden hyödyntää molempien puulajien parhaan kasvun ajanjaksoja. Ongelmaksi muodostuu tällöin, kuinka hyvin puulajien rinnakkaiselo voidaan toteuttaa. Mänty-rauduskoivusekametsikön tapauksessa tärkeä kysymys on, voidaanko koivun nopeaa kasvua hyödyntää siten, että nuorella iällä metsikön tilavuudesta on merkittävä osa koivua, ilman että mäntyjen kehitys hidastuu liiaksi. Metsikön myöhemmällä iällä kasvutilaa voitaisiin järjestellä vastaavasti mäntyjen eduksi.

Mielikäisen (1980) tutkimuksen yksi päätulos on, että sopivan suuruinen rauduskoivusekoitus lisää metsikön kiertoajan kokonaiskasvua puhtaaseen männikköön verrattuna. Sekametsikön parempi kasvu koituu tällöin sekä männyn että koivun hyväksi. Suurin kasvu saadaan nuorella iällä n. 50 prosentin koivusekoituksella ja metsikön varttuessa vähenevällä koivusekoituksella. Sekametsiköiden kasvu- ja tuotoskysymyksiä on käsitelty laajasti Mielikäisen (1980, 1985) tutkimuksissa.

Sekametsikön kehitys perustamisvaiheesta nuoreksi harvennusmetsäksi on edelleen puutteellisesti tunnettu. Metsänuudistamisen ja taimikonhoidon yhteydessä tehtyä puulajisuhteiden järjestelyä on vaikea ottaa huomioon metsikön kehitysennusteissa ja metsänhoidon kustannuksia tarkasteltaessa.

Metsikön liiketaloudellisesti edullisimpaan puulajisuhteeseen vaikuttaa puumäärien ohella puulajien kantohintojen suhde. Männyn ja koivun kantohintojen suhde vaihtelee huomattavasti maamme eri osien välillä. Taloudelliselta kannalta keskeinen on tukkipuun kantohintojen suhde, johon puolestaan vaikuttaa etenkin koivun kysyntä, ja siis koivua käyttävän teollisuuden sijainti. Pelkästään hintasuhdetta tarkastelemalla voisi arvioida, että runsas koivusekoitus männikössä on taloudellisesti mielekäs lähinnä Järvi-Suomen piirimetsälautakuntien alueella (Itä-Hämeen, Etelä-Savon, Etelä-Karjalan, Itä-Savon, Pohjois-Savon ja Pohjois-Karjalan piirimetsälautakunnat). Männyn ja koivun kantohintojen ajallista ja alueellista vaihtelua tarkastellaan luvussa 22.

Metsikön kasvatus on ajan mukana etenevää pitkäaikaista tuotannon ohjausta, jossa prosessia koskevia päätöksiä tehdään tuotannon eri vaiheissa. Metsikön kasvatuksen optimointi on siten luonteeltaan dynaamista optimointia (vrt. Jääskeläinen ja Kuusi 1974, s. 202—209).

Sekametsikön kasvatuksen optimointi on liiketaloudellisena ongelmana pääpiirteissään samanlainen kuin puhtaan metsikön kohdalla. Jälkimmäistä ovat käsitelleet esim. Einola (1964, s. 45—53), Gregory (1972) ja Hämäläinen (1973a). Yhteinen tärkeä ongelma on, kuinka paljon puustoa sidotaan metsikön kasvun ylläpitämiseen ja kuinka paljon sitä realisoidaan kulutusta tai sijoituskohteita varten. Sekametsikön tapauksessa on lisäksi tarkasteltava puulajien osuuksia kiertoajan eri vaiheissa ottaen huomioon puulajien kasvunopeudet ja keskinäiset vaikutukset sekä toisaalta erilaiset arvot.

12. Aiemmat tutkimukset

Ensimmäinen yleisesti tiedossa oleva dynaamisen optimoinnin sovellutus harvennusten optimointiin on Arimizun (1958, viitt. Hool 1966) tutkimus. Siinä käytettiin dynaamista ohjelmointia, jonka Richard Bellman kehitti 1950-luvulla (Bellman 1954, 1957). Chappelle ja Nelson (1964) määrittivät marginaalianalyysin avulla optimaalisen puustopääomatason ja kiertoajan loblollymännylle (*Pinus taeda* L.). Tutkimuksessa ei sen sijaan selvitetty, mikä on optimaalinen hakkuuohjelma metsikölle, jonka puusto lähtötilanteessa ei ole optimaalisella tasolla. Hool (1966) yhdisti dynaamisen ohjelmoinnin Markovin ketjuja käyttävään kasvumalliin. Metsikön mahdolliset tilat määritettiin joukkona erillisiä tiloja ja kasvun sekä hakkuiden vaikutus metsikön kehitykseen ilmaistiin todennäköisyyksinä transiitiomatriiseissa. Myöhemmin Lembersky ja Johnson (1975) laajensivat analyysiä kattamaan ikuisuuteen ulottuvan aikahorisontin, todellisuutta paremmin vastaavat hinta- ja kustannustekijät sekä stokastisen kantohintatason.

Joukko dynaamista ohjelmointia harvennusten optimointiin soveltavia tutkimuksia ilmestyi 1960-luvun lopussa (Amidon ja Akin 1968, Kilkki ja Väisänen 1969, Risvand 1969). Kahdessa ensin mainitussa metsikköä kuvattiin pelkästään puuston hehtaarikohtaisella tilavuudella, kun taas Risvandin (1969) tutkimuksessa tilamuuttujat olivat puuston tilavuus ja keskiläpimitta. Suomessa julkaistiin 1970-luvun alussa Kilkin (1972) ja Siitosen (1972) tutkimukset, joissa tilamuuttujina olivat metsikön ikä, puuston tilavuus ja keskirungon koko.

Uusi aalto dynaamista ohjelmointia käyttäviä tutkimuksia alkoi 1970-luvun lopulta lähtien Brodien ym. (1978) aloittamana. Dynaamisen ohjelmoinnin käyttöä erilaisissa metsikön käsittelyn optimointitehtävissä on esitelty Brodien ja Haightin (1985) tutkimuksessa. Dynaamista ohjelmointia on sovellettu tasaikäisiin metsiköihin käyttäen erityyppisiä kasvumalleja: metsikön kasvumalli ilman läpimittajakaumaa (esim. Brodie ja Kao 1979), metsikön kasvumalli läpimittajakauman kanssa (Riitters ym. 1982), puun kasvumalli (esim. Haight ym. 1985b) ja puun kasvumalli, jossa puun sijainti vaikuttaa kasvuun (Reich ja Dippon 1986). Kiertoajan ja puustopääoman ohella on optimoitu metsikön istutustiheyttä ja taimikon harvennusta (esim. Hann ym. 1983), kasvua haittaavan lehtipuuston käsittelyä (Valsta ja Brodie 1986), harvennustapaa (Haight ym. 1985b), lannoitusta (Kao 1979), sienitautien torjuntaa (Reich ja Dippon 1986) sekä yhdistettyä puuntuotantoa ja karjan laiduntamista (Riitters ym. 1982).

Taloustieteessä on sovellettu optimiohjausteoriaa laajasti dynaamisen optimoinnin ongelmiin. Näslund (1969) esitti ohjausteoreettisen formuloinnin tasaikäisen metsikön kiertoajan ja harvennusten samanaikaiseksi optimoimiseksi. Probleema on laadittu jatkuvan ajan muotoon eikä menetelmän soveltamisesta tai numeerisista tuloksista anneta esimerkkejä. Schreuder (1971) on todennut, että Näslundin käyttämä hakkuumuuttujan jako harvennusmuuttujaan ja päätehakkuumuuttujaan oli tarpeeton, sillä kummassakin hakkuussa oli kysymys puustopääoman määrän säätelystä.

Sittemmin on esitetty myös numeerisia tuloksia jatkuva-aikaisille optimiohjaustehtäville tasaikäisissä metsiköissä (Clark 1976, Cawrse ym. 1984). Metsikköä on tällöin kuvattu vain tilavuuden ja iän avulla, koska tehtävän ratkaisu analyyttisesti on vaikeaa.

Epäjatkuvan ajan tehtäville soveltuva diskreetti maksimiperiaate näyttäisi tarjoavan mahdollisuuden laajempien ongelmien ratkaisuun. Haight ym. (1985a) ovat optimoineet harsintarakenteisen metsikön hakkuita ja Haight (1986) on verrannut tasaikäisen ja harsintarakenteisen metsikön edullisuutta useita kymmeniä tilamuuttujia käsittävällä mallilla.

Epälineaarista ohjelmointia voidaan käyttää samantapaisissa tehtävissä kuin optimiohjausteoriaakin. Niinpä samaa kasvumallia on optimoitu epälineaarisella ohjelmoinnilla (Adams ja Ek 1974) ja optimiohjausteorialla (Rapera 1980, Haight ym. 1985a). Epälineaarisen ohjelmoinnin käyttökelpoisuus on kuitenkin ollut vaihteleva. Tehtävää numeerisesti ratkaistaessa on toisinaan jouduttu turvautumaan heuristisiin menetelmiin melko suppeidenkin optimointitehtävien kohdalla (Bullard ym. 1985), kun taas laajahkojakin ongelmia on ratkaistu (Roise 1986).

Sekametsiköitä koskevia taloudellisia tutkimuksia on tämän kirjoittajan tietoon tullut vähän. Esimerkkejä mäntysekametsiköiden perustamisen kustannuksista Saksassa on julkaissut Liebeneiner (1958). Esitetyistä irrallisista laskelmista ei kuitenkaan saa aineksia puhtaiden ja sekametsien vertailuun. Darrah ja Dodds (1967) havainnoivat erityyppisiä sekametsiköitä Englannissa. Aineisto vaihteli laajasti puulajien ja metsikkörakenteen suhteen eikä yksittäisten metsikkötyyppien kasvusuhteista tai edullisimmista määräsuhteista saatu tuloksia.

Bullard ym. (1985) ovat esittäneet menetelmän kahden puulajin sekametsikön optimaalisen harvennusohjelman etsimiseksi (heuristic random search). Menetelmässä simuloidaan vaiheittain suuri joukko kasvatusvaihtoehtoja siten, että uutta simuloinnin vaihetta aloitettaessa käytetään hyväksi aikaisemmista simuloinneista saatu tieto eri kasvatusohjelmien edullisuuksista. Simulointien antamien nykyarvojen jakaumaa seurataan samalla ja arvioidaan, kuinka kaukana paras löydetty ratkaisu on todellisesta optimista.

13. Tutkimusongelma

Tämän tutkimuksen tarkoituksena on määrittää mänty-rauduskoivusekametsikön optimaalinen hakkuuohjelma. Metsänomistajan taloudellisena tavoitteena pidetään metsikön käsittelyohjelman antaman nykyarvon maksimointia.

Metsikön kasvatusta tarkastellaan tavalliseen tapaan investointina, jossa investointiin sidottu pääoma koostuu uudistamiskustannuksista, markkinakelpoisen puuston arvosta ja metsän kasvuun käytetyn maan arvosta seuraajametsiköiden tuottoarvona. Analyysissä optimoidaan metsikön kiertoajan ohella myös puustopääomaa ja puulajisuhteita kaikilla iänkohdilla. Kasvava puusto joutuu harvennettavaksi, mikäli puuston jokin osa tuottaa enemmän markkinahinnan mukaan realisoituna ja laskentakorkokannalla sijoitettuna kuin kasvamaan jätettynä. Harvennuksen edullisuutta määritettäessä otetaan huomioon harvennuksen vaikutukset sen hetkisiin ja myöhempiin korjuukustannuksiin sekä metsikön kehitykseen kiertoajan loppuun saakka.

Tuotot muodostuvat pystymyyntien antamista tuloista. Pääomamarkkinoiden oletetaan olevan täydelliset siten, että rahaa voidaan ottaa ja antaa lainaksi rajattomasti samalla, sekä ajan että määrän suhteen kiinteällä korkokannalla. Korkokanta on reaalinen, inflaatiosta vapaa. Puun markkinahinnat ovat kiinteitä ajan ja määrän suhteen sekä tunnettuja. Korjuukustannukset ovat ajan suhteen kiinteitä ja tunnettuja.

Tutkimusongelmaa rajattaessa on keskeinen kysymys, kuinka suuri osa metsänomistajan päätöksentekoon vaikuttavista tekijöistä otetaan huomioon. Jo aluksi rajataan tarkastelu koskemaan vain taloudellisia näkökohtia ja niistäkin vain metsäomaisuuteen liittyviä. Puunkorjuun sisällyttäminen optimointimalliin metsänomistajan tekemänä edellyttää mallin laajentamista metsätalouden ulkopuolelle, sillä metsänomistajan oman työpanoksen ja koneiden käytön arvottaminen vaatii metsänomistajan muun talouden tarkastelua. Tulokset ovat lisäksi tällöin sidottuja kulloiseenkin metsälön kokoon ja rakenteeseen sekä metsänomistajan taloustilanteeseen (ks. esim. Hämäläinen 1973a ja 1973b). Samasta syystä tutkimuksessa tarkastellaan metsikköä metsälökokonaisuudesta erotettuna.

Metsänomistajan oletetaan myyvän puutavaran pystykaupoin ja korjuukustannukset sisällytetään tarkasteluun vain siltä osin, kuin ne heijastuvat pystymyyntien hinnoittelussa. Laskelmissa lähdetään harvennusvaiheeseen kehittyneestä metsiköstä, joten nykymetsikön perustaminen ja taimikonhoito jäävät optimoinnin ulkopuolelle tekijöinä, jotka on määrätty ennalta. Jotta tulevien puusukupolvien vaikutus optimaaliseen kiertoaikaan olisi oikeata suuruusluokkaa, uudistamiskustannukset sisältyvät laskelmiin kiinteänä, metsikön käsittelystä riippumattomana eränä.

Männyn ja rauduskoivun sekametsikön voidaan ajatella edellyttävän intensiivisempää hoitoa kuin puhtaiden männiköiden tai koivikoiden, mikä johtaisi lisäkustannuksiin. Ilmiön tapauskohtaisuuden ja tutkimustulosten puutteen vuoksi tätä tekijää ei otettu huomioon taloudellisessa tarkastelussa. Laskelmien ulkopuolella ovat myös muut kuin edellä mainitut metsänkasvatuksen kustannukset ja verot. Niiden vaikutusta metsikön optimaaliseen puulajisuhteeseen pidettiin vähäisenä.

Tässä julkaisussa tarkastellaan aluksi käytettävää kasvumallia ja esitellään laaditut metsikön tukki- ja hukkapuuosuusyhtälöt. Sen jälkeen kuvaillaan männyn ja rauduskoivun kantohintasuhteen ajallista ja alueellista vaihtelua ja esitetään laskelmissa käytetyt hinnat ja kustannukset sekä optimointimalli. Tuloksia raportoidaan peruslaskelmien lisäksi kantohintasuhdetta ja eräitä muita analyysin oletuksia vaihdellen. Päätulokset koskevat olemassa olevan 40-vuotiaan sekametsikön käsittelyä. Lisäksi tarkastellaan puulajisuhteen valintaa koko kiertoajalle.

Metsäntutkimuslaitoksen sekametsikköprojekti on usean tutkimusosaston ja -suunnan yhteistutkimus, jonka tarkoituksena on selvittää koivusekoituksen merkitystä havupuumetsissä biologiselta, puuntuotannolliselta ja taloudelliselta kannalta. Nyt julkaistava projektin ensimmäinen liiketaloudellinen tutkimus koskettelee männyn ja rauduskoivun sekametsiköitä.

Lausun kiitokseni professori Jouko Hämäläiselle, KTT Pekka Ollonqvistille ja KTM Markku Kuulalle sekä muille työtovereilleni liiketaloudellisen metsäekonomian tutkimussuunnalla arvokkaista huomautuksista ja parannusesityksistä työni kuluessa. Professori Yrjö Vuokila ja MML Yrjö Sevola ovat myös lukeneet käsikirjoituksen ja tehneet hyödyllisiä huomautuksia. MMT Kari Mielikäinen on tukenut työtäni opastamalla kasvumallien soveltamisessa ja antamalla käyttööni tutkimusaineistoaan. Kiitän häntä samoin kuin MMK Risto Ojansuuta myös käsikirjoitusta koskeneista kommenteista. Lisäksi kiitän fil. yo. Pekka Ripattia osallistumisesta ohjelmointityöhön sekä professori Pekka Kilkkiä dynaamista ohjelmointia koskeneista keskusteluista.

2. OPTIMOINTIMALLI

21. Metsikön kehitysmalli

Sekametsikön kasvatuksen optimointia varten tarvitaan kyseisten metsiköiden kehitystä kuvaava malli. Lappi-Seppälän (1930) tutkimus koski käsittelemättömiä mänty-koivusekametsiä ja siinä esitettiin tuloksia vain kolmelle kiinteälle puulajisuhteelle. Se ei siten tarjoa riittävää aineistoa talousmetsien käsittelyn optimointiin. Mielikäinen (1980) laati tutkimuksessaan mänty-koivusekametsiköiden tilavuuskasvuyhtälöt talousmetsistä kerätyllä aineistolla. Yhtälöillä voidaan ennustaa suhteellista tilavuuskasvua metsikön iän, puuston tilavuuden ja koivuosuuden funktiona. Yhtälöitä voidaan käyttää joustavasti vaihtoehtoisten käsittelyohjelmien muodostamiseen. Tutkimuksen aineisto kattaa 40—80-vuotiaat metsiköt.

Mielikäisen (mt.) kasvumallia sovellettaessa on oletettava, että taimikonhoito ja ensiharvennus on tarvittaessa tehty, ja että metsikön runkoluku on 40 vuoden iällä 700—1500 kpl hehtaarilla. Mäntyjen ja rauduskoivujen keskipituuksien edellytetään lisäksi olevan likimain samat siten, että kumpikaan puulaji ei ole etukasvuinen. Kasvupaikan mäntyjen valtapituusbonitetti (H_{100}) on noin 28, mikä vastaa rehevähköä tuoreen kankaan kasvupaikkaa. Nämä rajaukset on johdettu kasvumallin pohjana olevasta aineistosta.

Mänty-koivusekametsien kehitystä taimikkovaiheesta nuoreksi harvennusmetsäksi ei ole maassamme varsinaisesti tutkittu. Mielikäinen (mt.) analysoi tutkimuksessaan luontaisesti syntyneiden mäntyjen ja koivujen pituuskehitystä ja totesi varttuneiden mäntyjen olevan keskimäärin 5—6 vuotta vanhempia kuin samanpituisten rauduskoivujen. Harin ym. (1982) simulointien mukaan 6 vuoden ikäero olisi riittävä turvaamaan mäntyjen eloonjäämisen ja 8 vuoden ikäero johtaisi mäntyjen valta-asemaan metsikössä. Simulointien pohjana oleva kasvupaikka, lehtomainen kangas, oli rehevämpi kuin Mielikäisen (mt.) aineiston boniteetti, mikä heikensi männyn kilpailukykyä. Sekametsiköiksi kasvatettavien mänty-koivu-taimikoiden perustaminen ja käsittely on edelleen puutteellisesti tunnettu.

Mielikäisen (mt.) laatima tilavuuskasvukasvuyhtälö mänty-rauduskoivusekametsiköille on seuraava:

$$\ln(P_v) = 7.050 - 0.8732 \ln(T_{1.3}) - 0.4187 \ln(V) - 0.5154 \cdot 10^{-6} B \cdot T_{1.3}^2 - 1.403 \frac{(50 - B)^2}{T_{1.3}^3}$$

jossa

P_v = tulevan 5-vuotiskauden tilavuuskasvu, prosenttia nykytilavuudesta,
$T_{1.3}$ = puuston rinnankorkeusikä, a,
V = puuston tilavuus, m^3/ha ja
B = koivun osuus puuston tilavuudesta, %.

Yhtälön kaksi viimeistä termiä osoittavat koivuosuuden vaikutuksen tilavuuskasvuprosenttiin. Niiden yhteisvaikutus on, että suurin tilavuuskasvu saadaan nuorella iällä hieman alle 50 %:n koivuosuudella ja iän lisääntyessä jatkuvasti alenevalla koivuosuudella (ks. Mielikäinen mt., kuva 20 ja taulukko 9).

Jotta puulajeittaiset tilavuudet kasvujakson lopussa voidaan laskea, on tunnettava kasvun jakaantuminen puulajien kesken. Tätä varten Mielikäinen (mt.) on esittänyt taulukon, josta osuudet saadaan metsikön rinnankorkeusiän ja männyn tilavuusosuuden perusteella. Taulukko on myös tämän tutkimuksen liitteenä.

Kasvuyhtälön muoto ja parametrien arvot ovat lähtöaineistosta johtuen sellaiset, että samalla iänkohdalla kasvu on aina sitä suurempaa, mitä suurempi on metsikön puustopääoma. Siten esimerkiksi täysin harventamattoman, runsaspuustoisen metsikön kasvu muodostuu epärealistisen suureksi eikä luonnonpoistumaa esiinny. Tämän vuoksi metsikön kehitysmalliin liitettiin tässä tutkimuksessa osamalli, joka rajoittaa kasvua hyvin suurilla puustopääomilla.

Aluksi kokeiltiin Ilvessalon (1920) luonnonnormaalien metsien tilavuuksia eräänlaisina biologisina maksimipuustoina puulajeittain. Osoittautui kuitenkin, että sekä Mielikäisen (mt.) että Lappi-Seppälän (1930) aineistoissa oli runsaasti metsiköitä, joiden puuston tilavuus oli suurempi kuin luonnonnormaaleissa puhtaissa metsiköissä vastaavilla kasvupaikoilla ja valtapituuden arvoilla.

Tässä tutkimuksessa päädyttiin ratkaisuun, jossa puuston liikatiheyden rajoina pidetään Mielikäisen (mt.) tutkimuksen aineiston suurimpia tilavuuksia kullakin iänkohdalla. Kyseiset tilavuudet eivät olleet vielä johtaneet itseharvenemiseen koemetsiköissä. Tilavuusraja ilmaistaan metsikön biologisen iän (T) funktiona seuraavalla yhtälöllä:

$$V_{max} = 273{,}42 + 0{,}9308\ T.$$

Tukkipuun määrän selvittämiseksi Mielikäinen (mt.) laati yhtälöt, jotka antavat sekametsikön tukkipuuosuudet puulajeittain metsikön iän funktiona. Lähinnä puutavaran hinnoitteluperusteiden vuoksi tämän tutkimuksen kannalta olivat tarkoituksenmukaisempia tukkiosuusyhtälöt, jotka perustuivat puuston keskirungon kokoon. Käyttäen Mielikäisen (mt.) tutkimuksen metsikköaineistoa laadittiin epälineaarisella regressioanalyysillä maximum likelihood -estimointia käyttäen yhtälöt puulajeittain tukkipuun ja hukkapuun osuuksille metsikössä. Kuitupuun osuus saatiin edellisten perusteella.

Metsikköaineistosta (215 havaintoa) karsittiin harkinnan jälkeen ne metsiköt, joissa jommankumman puulajin tilavuus oli alle 50 m^3/ha. Metsikölle määritetyissä tukki- ja kuitupuun määrissä havaittiin tuntuvaa satunnaisvaihtelua silloin, kun puulajin tilavuus oli pieni. Käytetty aineisto oli sama kaikille yhtälöille ja sen kooksi muodostui 133 metsikköä. Rungon keskikoon ohella selittäjänä kokeiltiin myös metsikön ikää, mutta sillä ei ollut riittävää selitysvoimaa, jotta se olisi voitu sisällyttää malleihin.

Tukki- ja hukkapuuosuuksien yhtälöille valittiin seuraava muoto, jotta ne voisivat saada arvoja vain nollan ja yhden väliltä:

$$\text{Osuus} = \frac{1}{1 + e^{\left\{b_0 + b_1 \left(\frac{V_t}{N_t}\right)^{b_2}\right\}}}$$

jossa

V_t = puulajin tilavuus iällä t,
N_t = puulajin runkoluku iällä t ja
b_o, b_1, b_2 = yhtälön parametreja.

Mallin perusmuoto on perinteinen logistinen yhtälö. Parametri b_2 lisättiin, jotta malli saisi enemmän joustavuutta. Mallit estimoitiin suoraan esitetyssä muodossa linearisoimatta niitä. Epälineaarista regressioanalyysiä käytettiin sen vuoksi, että em. yhtälömuotoa ei voida saattaa kokonaan lineaariseksi muunnoksilla. Samalla vältettiin linearisointiin liittyvästä logaritmimuunnoksesta aiheutuvat ongelmat vakiotermin korjauksen määrittämisessä.

Käytetyn mallin ominaisuus on, että varianssi ei ole homogeeninen selitettävän muuttujan suhteen. Varianssin homogenoimiseksi laskettiin havaintokohtaiset painot (w_i) iteratiivisesti selitettävän muuttujan ennustetun arvon (y_i) perusteella ($w_i = 1/(y_i(1 - y_i))$) (ks. esim. Draper ja Smith 1981, s. 238). Yhtälöiden kertointen ja keskivirheiden estimaattien arvot ilmenevät taulukosta 1. Yhtälöitä vastaavat kuvaajat on esitetty kuvissa 1 ja 2.

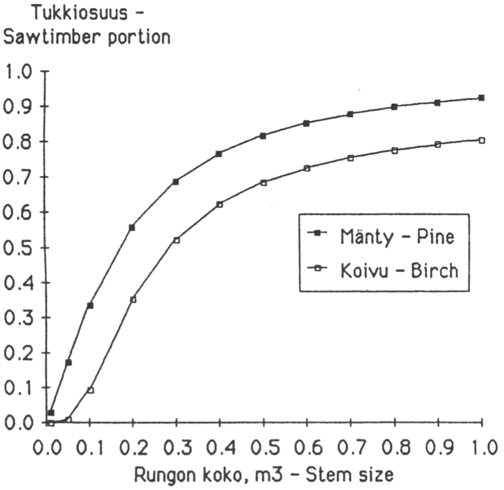
22. Hinta- ja kustannustiedot

Tämän tutkimuksen päätavoitteena on määrittää mänty-rauduskoivusekametsikön optimaalinen puulajisuhde. Tutkimus ei pyri

Taulukko 1. Tukki- ja hukkaosuusyhtälöiden kertointen (b_i) ja keskivirheiden (MSE) estimaatit.

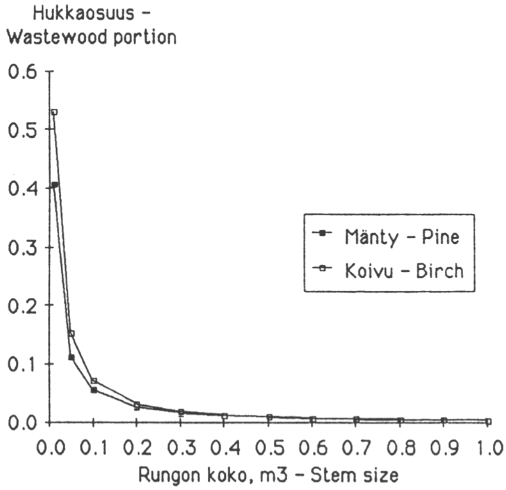
Table 1. Estimates of the coefficients (b_i) and mean squared errors (MSE) for the sawtimber and wastewood equations.

	Puulaji — *species*	b_0	b_1	b_2	MSE
Tukkiosuus *Sawtimber portion*	mänty *pine*	26,5087	−29,0039	0,050616	0,01453
	koivu *birch*	−2,76338	1,34138	−0,573641	0,06009
Hukkaosuus *Wastewood portion*	mänty *pine*	−15,6872	21,3952	0,062216	0,000156
	koivu *birch*	−17,8019	23,4687	0,061549	0,000304



Kuva 1. Mänty-rauduskoivusekametsikön tukkipuun osuudet puulajeittain puulajin rungon keskikoon funktiona.

Fig. 1. Sawtimber portion as a function of average stem size for mixed pine birch stands, by tree species.



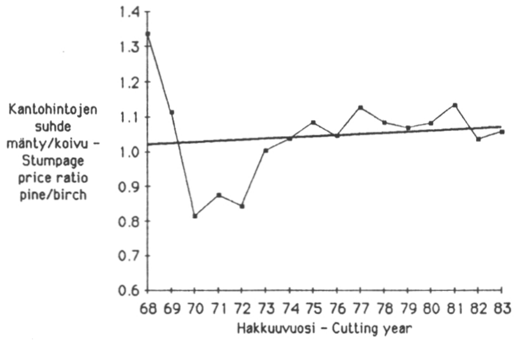
Kuva 2. Mänty-rauduskoivusekametsikön hukkapuun osuudet puulajeittain puulajin rungon keskikoon funktiona.

Fig. 2. Wastewood portion as a function of average stem size for mixed pine birch stands, by tree species.

selvittämään ko. metsikön kasvatuksen kannattavuuden tasoa absoluuttisesti. Käytetyt hinta- ja kustannustiedot eivät siten ole kaikin osin riittäviä laajempaa kannattavuustarkastelua ajatellen. Puulajien välinen kantohintojen suhde on keskeisin hintatekijä.

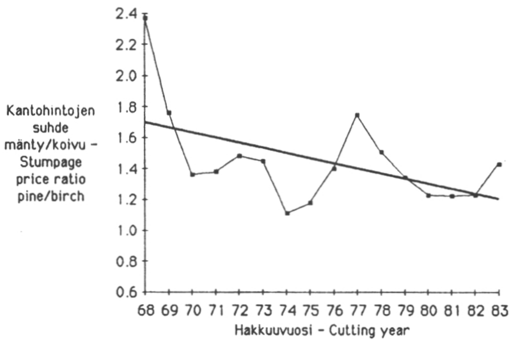
Analyysejä varten laskettiin lineaariset trendiyhtälöt männyn ja koivun kantohintojen suhteelle, tukki- ja kuitupuulle erikseen. Vuotuishavaintoina käytettiin Järvi-Suomen piirimetsälautakuntien alueiden kantohintojen aritmeettisten keskiarvojen suhteita. Järvi-Suomen piirimetsälautakunnilla tarkoitetaan tässä Itä-Hämeen, Etelä-Savon, Etelä-Karjalan, Itä-Savon, Pohjois-Savon ja Pohjois-Karjalan piirimetsälautakuntia. Aineisto käsitti hakkuuvuosittaiset havainnot vuosilta 1968/69—1983/84 (Uusitalo 1985). Aineiston alkuvuoden myöhäisyys johtui siitä, että koivutukkipuulle ei ollut käytettävissä aiempia havaintoja.

Mäntytukkipuun hinnat tilastoitiin hakkuuvuoteen 1977/78 saakka yhdessä kuusitukkipuun kanssa havutukkipuuna. Mäntytukkipuun ja koivutukkipuun hintasuhteen kehittymistä varten aineiston ensimmäisten 10 hakkuuvuoden kohdalla oli määritettävä mäntytukkipuun kantohinnan suhde havutukkipuun kantohintaan. Tämä suhde oli edellä mainittujen piirimetsälautakuntien alueella keskimäärin 1,1 hakkuuvuosien 1978/79—1983/84 aikana. Tätä keskimää-



Kuva 3. Mäntytukkipuun ja koivutukkipuun kantohintojen suhteen kehitys hakkuuvuosina 1968/69—1983/84 Järvi-Suomen piirimetsälautakuntien alueen keskiarvoina. Lisäksi on esitetty hakkuuvuosittaisista havainnoista laskettu lineaarinen trendiyhtälö.

Fig. 3. The stumpage price ratio between pine and birch sawtimber in cutting years 1968/69–1983/84 in Eastern Finland. A linear trend equation, based on yearly observations, is included.



Kuva 4. Mäntykuitupuun ja koivukuitupuun kantohintojen suhteen kehitys hakkuuvuosina 1968/69—1983/84 Järvi-Suomen piirimetsälautakuntien alueen keskiarvoina. Lisäksi on esitetty hakkuuvuosittaisista havainnoista laskettu lineaarinen trendiyhtälö.

Fig. 4. The stumpage price ratio between pine and birch pulpwood in cutting years 1968/69–1983/84 in Eastern Finland. A linear trend equation, based on yearly observations, is included.

räislukua käytettiin muunnettaessa piirimetsälautakunnittaiset havutukin kantohinnat mäntytukin kantohinnoiksi hakkuuvuosille 1968/69—1977/78. Puulajien kantohintasuhteiden kehitys on esitetty kuvissa 3 ja 4. Lisäksi laskettiin samalle alueelle ja vastaavalle ajanjaksolle mäntytukkipuun ja -kuitupuun lineaariset trendiyhtälöt käyttäen tukkuhintojen kokonaisindeksin avulla reaalisiksi muunnettuja kantohintoja. Trendiyhtälöiden antamista arvoista hakkuuvuodelle 1983/84 laskettiin kantohinnat koivun puutavaralajeille puulajien hintasuhteiden trendiarvoja käyttäen.

Tukkipuun hintasuhde muuttui kuvatun periodin kahtena ensimmäisenä vuotena voimakkaasti koivun eduksi. Kyseisinä vuosina mäntytukkipuunkin reaalihinta nousi mutta koivutukkipuun reaalihinnan nousu oli poikkeuksellisen nopeaa. Hakkuuvuodesta 1970/71 lähtien ajanjakson loppuun asti hintasuhde on muuttunut vähitellen männyn eduksi. Suurin muutos tapahtui hakkuuvuodesta 1972/73 hakkuuvuoteen 1973/74. Tässä korkeasuhdanteessa mäntytukkipuun hinta nousi selvästi voimakkaammin kuin koivutukkipuun hinta.

Suhdannehuipun jälkeen mäntytukkipuun reaalihinta laski puoleen entisestä arvostaan hakkuuvuoteen 1977/78 mennessä. Samana jaksona koivutukkipuun reaalihinta laski suhteellisesti vielä enemmän. Hakkuuvuodesta 1977/78 aina kuvatun jakson loppuun hintasuhde on muuttunut hitaasti edelleen männyn eduksi. Koivukuitupuun hinta nousi kuvatun jakson alusta hakkuuvuoteen 1974/75 asti keskimäärin nopeammin kuin mäntykuitupuun hinta. Koivukuitupuun kysyntä vahvistui selluloosateollisuuden prosessien muuttumisen myötä.

Piirimetsälautakunnittaisten keskihintojen mukaan koivutukkipuun hinta on tarkastellulla alueella ollut korkeampi kuin mäntytukkipuun hinta viimeksi hakkuuvuonna 1976/77, tuolloin Etelä- ja Itä-Savon piirimetsälautakunnissa. Viime vuosien kehityksen mukaan on todennäköistä, että männyn kantohinnat säilyvät koivun kantohintoja korkeampina ainakin lähivuosina.

Puulajien hintasuhteiden (mänty/koivu) trendiarvot hakkuuvuodelle 1983/84 olivat tukkipuulle 1,072 ja kuitupuulle 1,203. Puutavaralajeittaisiksi kantohinnoiksi hakkuuvuodelle 1983/84 saatiin siten seuraavat:

Puutavaralaji	mk/m^3	%
mäntytukkipuu	203,59	100
koivutukkipuu	189,92	93
mäntykuitupuu	91,71	45
koivukuitupuu	76,23	37

Edellä olevia puulajien hintoja ja hintasuhteita nimitetään normaaliarvoiksi (”No”). Tuloksia esitetään myös kahdelle muulle hintasuhteelle, joilla yksikköhinnat ovat seuraavat:

Puutavaralaji	Hintasuhde "Ta" mk/m³	%	Hintasuhde "Mä" mk/m³	%
mäntytukkipuu	196,76	100	215,54	100
koivutukkipuu	196,76	100	179,62	83
mäntykuitupuu	83,97	43	95,74	44
koivukuitupuu	83,97	43	73,65	34

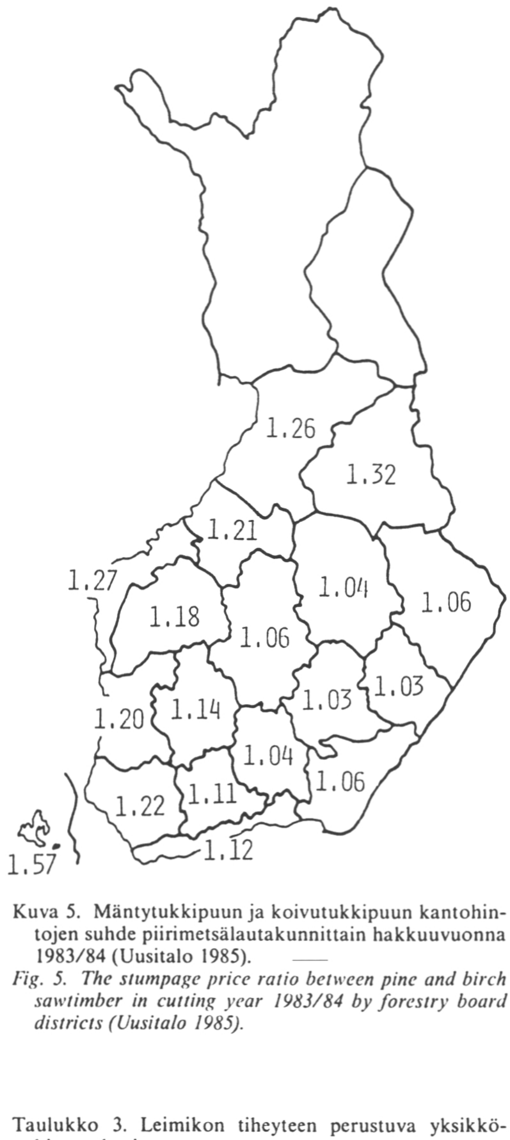
Kantohintasuhteen alueellisessa tarkastelussa (kuva 5) käytetään esimerkkinä hakkuuvuoden 1983/84 tilastoituja arvoja (Uusitalo 1985). Kyseisenä hakkuuvuotena Keski- ja Itä-Suomessa oli yhtenäinen alue, jossa koivutukkipuun kantohinta oli lähellä mäntytukkipuun kantohintaa (hintasuhde oli 1,06 tai pienempi). Vain maan länsirannikolla ja pohjoisosissa hintasuhde oli suurempi kuin 1,2. Trendiyhtälön antama tukkipuun hintasuhteen arvo (1,07) hakkuuvuodelle 1983/84 vastaa keskimääräistä tilannetta valtaosassa Etelä- ja Keski-Suomea tuona hakkuuvuotena. Männyn kannalta edullisempi hintavaihtoehto, "Mä" (hintasuhde 1,2), tarjoaa puolestaan esimerkin maan muissa osissa tavallisesta tilanteesta.

Käytetty hakkuupoistumien hinnoittelu perustuu raakapuun hintasuositussopimukseen hakkuuvuodelle 1983/84 (maan eteläpuolisko). Kyseisissä hintasuosituksissa oli runkojen järeyteen perustuva yksikköhinnan korjaus vain tukkipuulla. Korjaukset olivat samanlaiset männylle ja koivulle ja ne ilmenevät taulukosta 2.

Hehtaaria kohden poistettava puumäärä vaikuttaa yksikköhintaan kahdella tavalla: leimikon tiheyden ja koon välityksellä. Koska jälkimmäisellä tekijällä on huomattava vaikutus yksikköhintaan, se haluttiin ottaa huomioon, vaikka siitä johtuva korjauksen suuruus ei ole välittömästi johdettavissa hehtaarikohtaisesta poistuman määrästä. Olettamalla leimikon kooksi neljä hehtaaria ja yhdistämällä mainitut kaksi tekijää saatiin taulukossa 3 esitetyt, laskelmissa käytetyt leimikon tiheydestä riippuvat yksikköhinnan korjaukset.

Taulukko 2. Järeyteen perustuva yksikköhinnan korjaus.
Table 2. Stumpage price adjustment caused by average stem size.

Runkolajin käyttöosan keskijäreys, m³ — *Average stem size*	<0,3	<0,4	<0,5	<0,6	>0,6
Korjaus, mk/m³ — *Adjustment*	−6	−3	0	4	8



Kuva 5. Mäntytukkipuun ja koivutukkipuun kantohintojen suhde piirimetsälautakunnittain hakkuuvuonna 1983/84 (Uusitalo 1985).
Fig. 5. The stumpage price ratio between pine and birch sawtimber in cutting year 1983/84 by forestry board districts (Uusitalo 1985).

Taulukko 3. Leimikon tiheyteen perustuva yksikköhinnan korjaus.
Table 3. Stumpage price adjustment caused by the volume removed.

Leimikon tiheys, m³/ha - *Volume removed*	Korjaus, mk/m³ - *Adjustment*
— 12,5	−24
12,5— 25	−18
25 — 30	−11
30 — 50	− 7
50 — 60	− 5
60 — 75	− 2
75 —100	0
100 —125	+ 2
125 —150	+ 3
150 —175	+ 5
175 —250	+ 6
250 —	+ 8

Kun tulevat puusukupolvet otetaan huomioon, optimaalisen kiertoajan määrittäminen nykyarvomenetelmällä edellyttää, että uudistamisvaiheen kustannukset tunnetaan. Koska ei ollut käytettävissä tietoja uudistamiskustannusten ja syntyvän mänty-rauduskoivusekametsikön puulajisuhteiden välisistä riippuvuuksista, käytettiin samoja uudistamiskustannuksia kaikille puulajisuhteille. Yksikkökustannuksina käytettiin Metsätilastollisen vuosikirjan (Uusitalo 1985) vuoden 1983 arvoja yksityiset ym. metsänomistajaryhmälle. Kalenterivuoden 1983 arvot muunnettiin hakkuuvuoden 1983/84 arvoiksi käyttäen tukkuhintojen kokonaisindeksin kuukausiarvojen aritmeettista keskiarvoa kalenterivuodelle ja hakkuuvuodelle. Eri työlajien yksikkökustannuksiksi (mk/ha) saatiin seuraavat:

Hakkuualan raivaus	282
Maanmuokkaus	606
Istutus ja taimet	2240
Taimikonhoito	686

Koska analyysi perustui rehevähkön kasvupaikan metsiköihin, taimikonhoito oletettiin jouduttavan suorittamaan kahdesti, 2 ja 5 vuoden kuluttua päätehakkuusta. Istutuksen oletettiin tapahtuvan vuoden kuluttua päätehakkuusta. Kokonaiskustannukset metsänhoitotöistä olivat siten 4500 mk hehtaaria kohden kiertoajan kuluessa.

23. Optimointimenetelmän valinta

Hann ja Brodie (1980) ovat esittäneet vertailevan katsauksen metsikön ja metsälön päätöksenteko-ongelmista ja niihin soveltuvista optimointimenetelmistä. Katsaus antaa myös läpileikkauksen Pohjois-Amerikassa aiheesta tehdyistä tutkimuksista. Dykstran (1984) oppikirjassa esitellään matemaattisen ohjelmoinnin menetelmiä ja niiden käyttöä luonnonvarojen hoidon suunnittelussa, erityisesti metsätaloudessa.

Eräissä aiemmissa optimointitutkimuksissa on havaittu, että metsiköstä saatavien nettotulojen nykyarvo ei ole kovin herkkä puustopääoman tai kiertoajan vaihteluille (Kilkki ja Väisänen 1969, Hann ym. 1983). Toisin sanoen optimit ovat olleet laakeita. Edelliseen perustuen simulointi saattaisi olla käyttökelpoinen menetelmä sellaisten käsittelyohjelmien löytämiseen, jotka antavat lähes saman taloudellisen tuloksen kuin optimikäsittely. Epävarmuus optimihakkuuohjelman luonteesta on kuitenkin tavanomaista suurempi käsillä olevassa tutkimustehtävässä, jossa tarkastellaan kahta toisiinsa vaikuttavaa puulajia. Simuloinnilla katettavien vaihtoehtojen joukko on myös monin verroin suurempi kuin yhden puulajin tapauksessa. Simulointia pidettiin näistä syistä riittämättömänä asetetun tehtävän ratkaisuun.

Optimointimenetelmän valinnassa huomioon otettavia tekijöitä ovat mm.

- ajan ja optimointimallin tilamuuttujien jatkuvuus,
- kasvumallien sekä hinta- ja kustannusriippuvuuksien muoto ja
- tulosten muunnettavuus toimenpideohjeiksi.

Metsä ekosysteeminä muuttuu ja kehittyy jatkuvasti. Taloudellisen toiminnan kannalta on kuitenkin tarkoituksenmukaista pitää metsän kehitystä meidän ilmasto-olosuhteissamme epäjatkuvana puiden kasvun vuotuisen rytmin mukaan. Koska yksittäisten vuosien kasvut vaihtelevat sääeroista johtuen huomattavasti ja lisäksi yhden vuoden kasvun mittaaminen on epätarkkaa, metsikön kehityksen mallit laaditaan yleensä 5 vuoden jaksoille. Nämä mallit ovat siten luonteeltaan epäjatkuvia. Myös kasvatusprosessin ohjaus eli hakkuut tapahtuvat epäjatkuvasti tiettyinä ajankohtina.

Kun käytetään jatkuva-aikaista optimointia, harventaminen määritellään yleensä jatkuvana ajan funktiona. On vaikea soveltaa käytäntöön tuloksia, jotka perustuvat metsikön jatkuvaan harventamiseen. Jatkuvassa ajassa on myös mahdollista tehdä pulssimaisia harvennuksia, jolloin puumäärä vähenee yhtäkkisesti (kalastukseen liittyvistä sovellutuksista ks. Clark 1976).

Optimiohjausteoria ja epälineaarinen ohjelmointi edellyttävät, että kohdefunktio ja tilayhtälöt ovat jatkuvasti derivoituvia tilamuuttujien suhteen. Tätä rajoitusta ei ole dynaamista ohjelmointia käytettäessä, vaan muuttujien väliset riippuvuudet voivat olla esim. taulukkomuodossa. Joissain tapauksissa epäjatkuvuuden ongelma voidaan tosin kiertää approksimoimalla epäjatkuvia riippuvuuksia jatkuvilla funktioilla.

Käytettävänä olevan kasvumallin keskeinen ominaisuus optimointimenetelmän valinnan kannalta on, että puulajien osuudet kasvusta määritetään annetun taulukon perusteella. Kasvumalli ei siten täytä edellä mainittua derivoituvuuden vaatimusta. Käyttökelpoiseksi optimointimenetelmäksi jäi esillä olleiden menetelmien joukosta ainoastaan dynaaminen ohjelmointi (ks. esim. Valsta 1986). Tässä tutkimuksessa kasvumalli voidaan lisäksi määritellä vain muutamien muuttujien avulla ja siten välttää dynaamisessa ohjelmoinnissa toisinaan ongelmana oleva laskentatehtävän kasvu mahdollisuuksien ulkopuolelle.

24. Dynaaminen ohjelmointi

Dykstra (1984) esittää neljä ominaisuutta tehtävälle, joka voidaan ratkaista dynaamisella ohjelmoinnilla:

1. Tehtävä voidaan jakaa vaiheisiin, joista jokaisen kohdalla tehdään valinta vaihtoehtojen välillä.
2. Jokaisessa vaiheessa on joukko vaihtoehtoisia tutkittavan systeemin tiloja.
3. Jokainen valinta kulloisessakin vaiheessa siirtää systeemin uuteen tilaan seuraavassa vaiheessa.
4. Tehtävä noudattaa optimaalisuuden periaatetta.

Käsillä olevassa ongelmassa metsikön kasvatuksen 5-vuotisjaksot muodostavat edellä määritellyt vaiheet ja jokaisessa vaiheessa tehdään hakkuuta koskeva päätös. Hakkuiden poisto- ja koivuprosentit määrittävät vertailtavat vaihtoehdot. Kunkin valinnan jälkeen metsikköä kasvatetaan 5 vuotta, mikä siirtää metsikön uuteen tilaan seuraavassa vaiheessa.

Dynaamisen ohjelmoinnin perustana on optimaalisuuden periaate, jonka on esittänyt Bellman (ks. esim. Bellman ja Dreyfus 1962, s. 15). Sen merkitys on seuraavanlainen: Kun optimaalinen reitti (metsikön kasvatusohjelma) alkutilasta tiettyyn tilaan on määritetty, jäljellä olevan matkan optimaalinen reitti voidaan määrittää erillisenä tehtävänä ja riippumattomana jo kuljetun matkan reitistä. Toisin sanoen optimiratkaisu jäljellä olevalle metsikön käsittelyohjelman osalle on sama, olipa käsillä olevaan metsikön tilaan saavuttu minkä tahansa toimenpideketjun seurauksena. Tarvittavien laskelmien määrä vähenee ratkaisevasti optimaalisuuden periaatteen ansiosta.

Jotta dynaamista ohjelmointia voidaan soveltaa, optimaalisuuden periaatteen tulee päteä tutkittavassa ilmiössä tai sitä kuvaavassa mallissa. Käytettävässä kasvuyhtälössä metsikön kehitys riippuu metsikön iästä, tilavuudesta ja koivuosuudesta. Kasvu riippuu vain näiden muuttujien hetkellisistä arvoista, joiden oletetaan sisältävän myös kaiken tarvittavan informaation metsikön historiasta. Kun nämä muuttujat sisällytetään optimointimalliin, se on optimaalisuuden periaatteen mukainen.

Hakkuupoistumien hinnoittelua varten otettiin metsikön runkoluku neljänneksi muuttujaksi. Jakamalla metsikön kokonaistilavuus runkoluvulla saadaan keskirungon tilavuus, jota käytetään puutavaralajiosuuksien laskentaan sekä leimikkokohtaisten hintakorjausten tekoon. On huomattava, että runkolukua pienennetään harvennuksissa samassa suhteessa kuin tilavuus vähenee, toisin sanoen systemaattisen harvennuksen periaatteen mukaan. Metsikköä kuvaavat muuttujat ilmaistaan vektorimuodossa seuraavasti:

$$x_t^T \equiv (V_t, B_t, N_t)$$

jossa

x_t = metsikön tilavektori hetkellä t,
V_t = metsikön runkotilavuus hetkellä t,
B_t = koivun osuus edellisestä hetkellä t ja
N_t = metsikön runkoluku hetkellä t.

Käytettävissä ollut tieto ei antanut mahdollisuutta sisällyttää malliin uudistamistoimenpiteiden vaikutuksia syntyneen metsikön ominaisuuksiin. Analyysi joudutaan siten perustamaan yhteen metsikön alkutilaan, jossa tosin koivun osuuden voidaan antaa vaihdella. Samalla joudutaan olettamaan, että uudistamiskustannus on sama riippumatta tuloksena olleesta koivun osuudesta. Edelliseen perustuen uudistuskustannukset eivät vaikuta eri käsittelyvaihtoehtojen edullisuuksiin yhden kiertoajan tarkastelussa, vaan optimointi voi aluksi perustua pelkästään hakkuutulojen nykyarvoon. Metsikön käsittelyohjelman optimointiin käytetään dynaamisessa ohjelmoinnissa ns. rekursioyhtälöä, jonka avulla karsitaan epäoptimaaliset vaihtoehdot. Koska päätehakkuutulot lasketaan samoin perustein kuin harvennustulot, päätehakkuu voidaan ajatella 100 prosentin harvennukseksi. Rekursioyhtälö saa siten seuraavan muodon:

$$R(x_t) = \max_{\{x_{t-1}\}} [H(x_t, x_{t-1}) + R(x_{t-1})] \quad t = 1,\ldots,T$$

jossa

$R(x_t)$ = tavoitefunktion arvo jakson t alussa metsikölle x_t,

$\{x_{t-1}\}$ = niiden metsiköiden joukko jakson t − 1 alussa, joista voi kehittyä metsikkö x_t kasvun ja mahdollisen hakkuun seurauksena,

$H(\cdot)$ = mahdollinen diskontattu hakkuutulo, kun siirrytään metsiköstä x_{t-1} optimaalisesti metsikköön x_t,

$R(x_{t-1})$ = tavoitefunktion arvo jakson t − 1 alussa metsikölle x_{t-1} ja

T = optimoinnin viimeinen periodi, suurin mahdollinen kiertoaika.

Kun käydään läpi jaksot 1:stä T:hen, tavoitefunktion arvoksi muodostuu diskontattujen hakkuutulojen summa kiertoajan kuluessa. Rekursioyhtälöstä nähdään, että se on optimaalisuuden periaatteen mukainen. Jakson t − 1 käsittely määräytyy riippumatta siitä, miten metsikön tilaan x_{t-1} on tultu.

Optimoinnin kuluessa tallennetaan optimaaliset hakkuuohjelmat kaikille kiertoaikavaihtoehdoille. Kun oletetaan samaa hakkuuohjelmaa noudatettavan perättäisinä kiertoaikoina, ikuisuuteen ulottuvan tulo-menosarjan nykyarvo (L) eri kiertoajoille saadaan yhden kiertoajan hakkuutulojen nykyarvon (NA) ja uudistuskustannusten (UK) perusteella seuraavalla kaavalla kiertoajalle (u) ja korkoprosentille (i):

$$L = \frac{(NA - UK)(1 + i)^u}{(1 + i)^u - 1}$$

Kaikki optimointimallin tilamuuttujat diskretoitiin analyysiä varten. Muuttujien luokkavälit olivat seuraavat:

Metsikön ikä (aika)	5 vuotta
Puuston tilavuus	10 m^3/ha
Metsikön runkoluku	75 kpl/ha
Koivun osuus tilavuudesta	10 %

Kullakin iänkohdalla metsiköllä oli 7 200 mahdollista tilaa, ts. tilavuuden, runkoluvun ja koivun osuuden yhdistelmää. Muuttujien luokkavälit ilmaisevat myös optimointitarkkuuden. Malli antaa globaalin optimiratkaisun niiden hakkuuohjelmien joukosta, jotka poikkeavat toisistaan vähintäin luokkavälin verran.

Tyypillisessä analyysissä tehtiin 350 000 kasvuennustetta ja harvennusta. Mikäli ei olisi käytetty dynaamista ohjelmointia, vaan optimiratkaisu olisi etsitty käymällä systemaattisesti läpi kaikki mahdolliset harvennusvaihtoehdot, olisi simuloinneilla täytynyt kattaa suuruusluokkaa 10^{16} kiertoajan mittaista harvennusohjelmaa.

3. TULOKSET

31. Yleistä

Optimoitaessa metsikön hakkuuohjelmaa on määriteltävä laskennan lähtötilanne, tässä tapauksessa puuston ikä, tilavuus, koivuosuus ja runkoluku. Ennustaessaan kiertoajan tuotoksia Mielikäinen (1980) lähti liikkeelle rinnankorkeudelta 30-vuotiaista metsiköistä, mikä vastasi mäntyjen 40 vuoden biologista ikää. Verraten myöhäinen lähtötilanne oli seurausta kasvumallin laadinta-aineiston ikäjakaumasta, jossa nuorimmatkin metsiköt olivat rinnankorkeudelta yli 25-vuotiaita. Arvioitaessa koivusekoituksen vaikutusta koko kiertoajan tuotokseen joudutaan tekemään oletuksia metsiköiden kehittymisestä laskelmien lähtötilanteeseen. Mielikäisen (mt.) laskelmissa oletettiin, että puhdas männikkö ja sekametsikkö olivat kasvaneet kiertoajan 40 ensimmäisen vuoden kuluessa yhtä paljon. Lähtökohta on sekametsiköiden tuotoskyvyn suhteen varovainen, sillä myös mainitussa tutkimuksessa rauduskoivujen todettiin kasvaneen nuorella iällä mäntyjä nopeammin.

Edellä mainitut näkökohdat huomioon ottaen päädyttiin kahteen eri näkökulmaan taloudellisessa tarkastelussa: Ensimmäisessä päätöksenteko koskee 40-vuotiaita metsiköitä (rinnankorkeusiältään 30-vuotiaita). Lähtien eri koivuosuuksista määritettiin kullekin niistä optimaaliset hakkuuohjelmat kiertoajan loppuosalle. Tarkastelu antaa perusteita olemassa olevien harvennusikäisten sekametsiköiden käsittelylle.

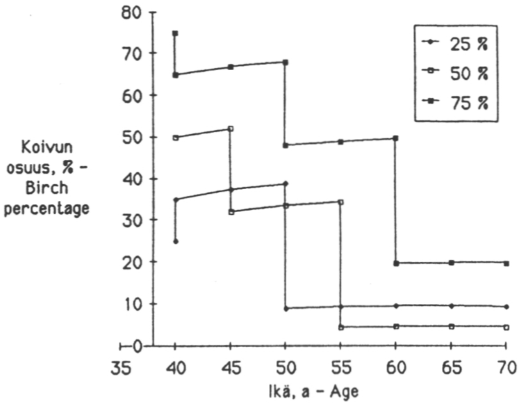
Toiseksi pyrittiin ennustamaan koko kiertoajan kehitystä ottaen mahdollisuuksien mukaan huomioon myös ennen 40 vuoden ikää syntyneet tuotoserot. Tätä tarkastelua on pidettävä olennaisesti epävarmempana kuin ensimmäistä ja sen tulokset ovat vain suuntaa antavia. Lähtökohdaksi otettiin 25-vuotias (rinnankorkeusiältään 15-vuotias) metsikkö. Samalla oletettiin, että puulajisuhteiltaan erilaiset metsiköt olivat kehittyneet tähän ikään mennessä samoin. Ennustettavana oli 15 vuoden jakso, 40 vuoden ikään asti, joka oli kasvumallin varsinaisen soveltamisalueen ulkopuolella. Käytetylle kasvumallille on ominaista, että nuoren sekametsikön kasvuetu puhtaaseen männikköön verrattuna on sitä suurempi mitä nuorempi metsikkö on. Varovaisuussyistä rajoitettiin mainittu kasvuetu alle 40-vuotiailla metsiköillä sille tasolle, jolla se on 40 vuoden iällä.

Eri tarkasteluissa seuraavat peruspiirteet ovat yhteneviä: Metsiköt ovat yksijaksoisia mänty-rauduskoivusekametsiköitä, joissa puulajien keskipituudet ovat suunnilleen samat, joten kumpikaan puulaji ei ole etukasvuinen. Tuloksissa esiintyvä metsikön ikä on uudistamisesta kulunut aika. Vaikka Mielikäisen (1980) tutkimus koskee luonnonmetsiköitä, tässä tutkimuksessa oletetaan, että tasapuustoiset sekametsiköt joudutaan perustamaan osittain viljelemällä ja että kokonaisuudistamiskustannukset vastaavat keskimäärin täyttä metsänviljelyä. Puuston runkoluku 40 vuoden iällä on 1200 kpl/ha. Laskennan taloudelliset parametrit vaihtelevat analyysin mukaan, mutta perusvaihtoehtona on 3 prosentin korkokanta ja hintasuhde "No".

32. Optimaaliset hakkuuohjelmat 40-vuotiaille metsiköille

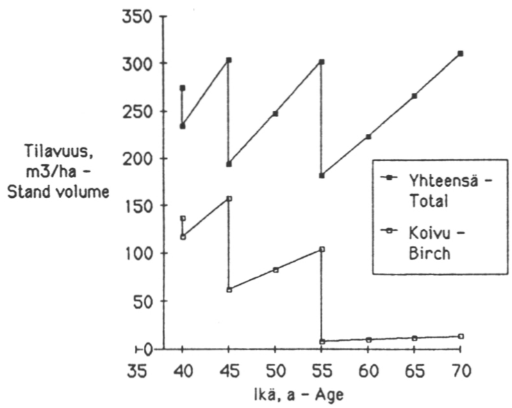
Optimaalisen koivuosuuden kehitys 40 ikävuodesta eteenpäin esimerkkinä oleville kolmelle metsiköille on esitetty kuvassa 6. Alkupuuston koivuosuus on 25, 50 tai 75 prosenttia. Yleispiirre on koivuosuuden pieneneminen kiertoajan loppua kohden. Tarkastelujakson alkuvuosina edullisin koivuosuus on 30—50 %, minkä jälkeen koivut tulee muutoinkin tehtävissä harvennuksissa poistaa. Kun koivua on runsaasti (75 %), sitä kannattaa poistaa vain sen verran kuin puustopääoman säätelyn kannalta on tarpeen. Puuston määrän vähentäminen taloudellisen optimitason alapuolelle ei ole tulosten mukaan perusteltua, vaikka se parantaisikin mäntyjen kasvua.

Harvennusohjelman vaikutus puuston tilavuuteen ilmenee esimerkinomaisesti kuvasta 7, jossa on esitetty kokonaistilavuuden ja koivun tilavuuden kehitys metsikössä, jossa oli 50 % koivua 40 vuoden iällä. Tällöin puuston tilavuus oli 275 m^3/ha ja runkoluku 1200 kpl/ha. Kiertoajan kuluessa tehtiin 3 harvennusta, joista ensimmäisen yhteydessä koivun osuutta metsikössä ei vielä vähennetty. Jäljellä olleet koivut poistettiin 55 vuoden iällä kolmannessa harvennuksessa käytännöllisesti katsoen kokonaan. Kiertoajan kokonaiskasvu oli 580 m^3/ha (8,3 m^3/ha/v), josta koivun osuus oli 39 %.



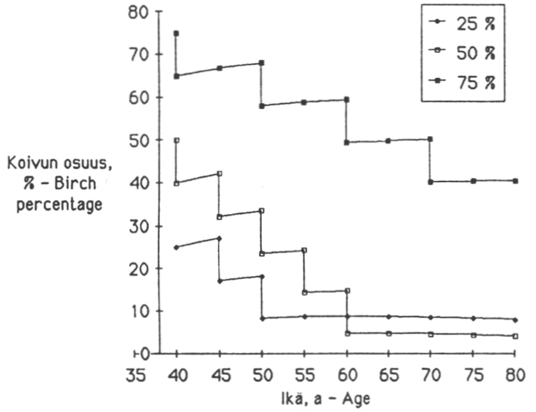
Kuva 6. Optimaalisen koivuosuuden kehitys lähtöarvoista 25, 50 ja 75 %. Hintasuhde "No", korkokanta 3 %.

Fig. 6. Optimal birch percentages, given initial values 25, 50 and 75 %. Price set "No", interest rate 3 %.



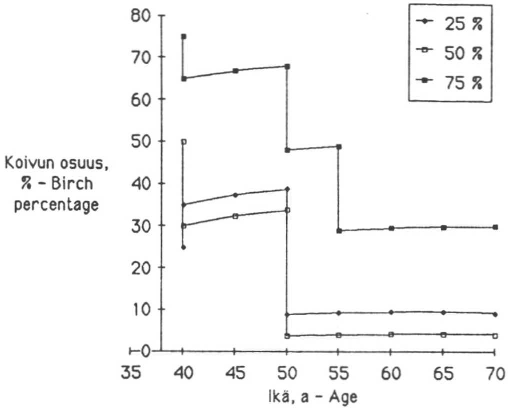
Kuva 7. Optimaalinen harvennusohjelma metsikölle, jossa on 50 %:n koivuosuus 40 vuoden iällä. Kiertoaika 70 vuotta, korkokanta 3 %.

Fig. 7. An optimal thinning schedule for a stand with 50 % birch at age 40. Rotation age 70, interest rate 3 %.



Kuva 8. Optimaaliset koivuosuudet metsänkorkoa maksimoitaessa, kiertoaika 80 vuotta. Alkuarvot ovat 25, 50 ja 75 % 40 vuoden iällä.

Fig. 8. Birch percentage in optimal forest rent thinning regimes, rotation is 80 years. Initial values are 25, 50 and 75 %, at age 40.



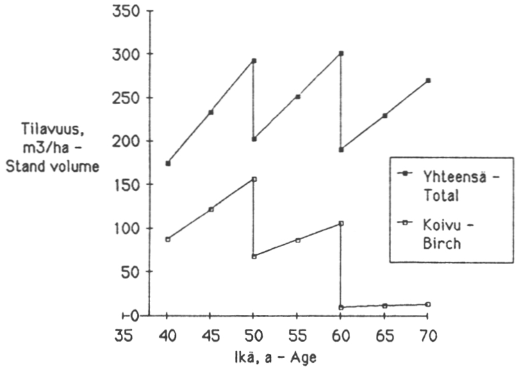
Kuva 9. Optimaalisen koivuosuuden kehitys lähtöarvoista 25, 50 ja 75 %. Hintasuhde "Mä", korkokanta 3 %.

Fig. 9. Optimal birch percentages, given initial values 25, 50 and 75 %. Price set "Mä", interest rate 3 %.

Laskentakorkokannan alentaminen nollaan prosenttiin johti kuvassa 8 esitettyyn tilanteeseen. Laskelma vastaa metsänkoron maksimointia, joskin optimaalinen kiertoaika olisi tällöin ollut noin 90 vuotta. Metsikköä harvennettiin 5 vuoden välein, mikä johti koivuosuuksien tasaiseen laskuun iän lisääntyessä. Harvennustuloja laskettaessa kantohintaan vaikutti kerrallaan poistettavan puumäärän suuruus. Vaikka lievät harvennukset johtivat alhaisempaan kantohintaan, niiden avulla oli kuitenkin mahdollista ylläpitää suurempaa kasvua metsikössä, mikä teki ne voimakkaampia harvennuksia edullisemmiksi. Kun kantohinnan riippuvuutta harvennuskertymästä koemielessä vahvistettiin, harvennusten välinen aika piteni ja kertapoistumat suurenivat.

Kun hintasuhdetta muutettiin männyn eduksi, hintasuhteesta "No" hintasuhteeseen "Mä" (kuva 9), koivun määrää vähennettiin 50 prosentin alkupuuston kohdalla hieman aiemmin. Muutoin käsittelyohjelmat säilyivät samanlaisina. Hintasuhteen muutos koivun eduksi siten, että molempien puulajien hinnat tulivat yhteneviksi (hintasuhteesta "No" hintasuhteeseen "Ta"), ei aiheuttanut muutoksia hakkuuohjelmiin.

Kun korkokanta oli 3 %, optimaaliset kiertoajat olivat yhden kiertoajan nykyarvon mukaan noin 90 vuotta. Ottaen huomioon tulevat, samanlaisiksi oletetut puusukupolvet optimaalinen kiertoaika lyheni 65 vuoteen.



Kuva 10. Optimaalinen harvennusohjelma metsikölle, jossa on 50 %:n koivuosuus 40 vuoden iällä. Alkutilavuus 175 m^3/ha, kiertoaika 70 vuotta, korkokanta 3 %.

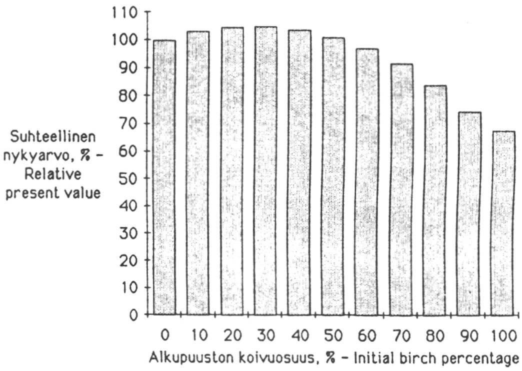
Fig. 10. An optimal thinning schedule for a stand with 50 % birch at age 40. Initial volume 175 m^3/ha, rotation age 70, interest rate 3 %.

Kiertoajan keskimääräinen tilavuuskasvu saavutti suurimman arvonsa niinikään 65 vuoden kohdalla. Liitteessä 2 on esitetty yksityiskohtaisemmin eri koivuosuuksien antamia tuloksia 40 vuoden iältä päätehakkuuseen.

Kun alkupuuston tilavuutta alennettiin tasolle 175 m^3/ha (kuva 10), harvennus 40 vuoden iällä jäi pois. Koivun osuutta vähennettiin tuntuvasti kummankin harvennuksen yhteydessä niin, että päätehakkuussa hakattiin puhdas männikkö.

Taulukko 4. Puuston kokonaistilavuuskasvu koivuosuuden eri alkuarvoille. Alkutilavuus 25 vuoden iällä on 69,17 m³/ha.
Table 4. Cumulative growth for various initial birch percentages. Initial stand volume is 69,17 m³/ha, at age 25.

Koivuosuus, % — *Birch percentage*	Metsikön ikä, a — *Stand age* 30 Puuston tilavuus — *Stand volume* m³/ha	40 m³/ha	%
0	125	250	100
10	127	260	104
30	130	271	108
50	130	274	110
70	129	266	106
90	125	251	100
100	123	240	96



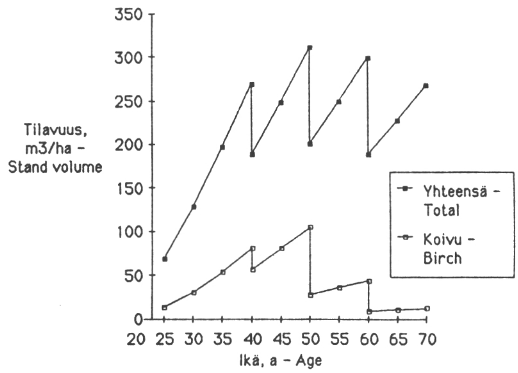
Kuva 11. Lähtöpuuston koivuosuuden vaikutus optimaalisiin nykyarvoihin, korkokanta 3 %.
Fig. 11. The effect of initial birch percentage on optimal present values, interest rate 3 %.

33. Koko kiertoaikaa koskevat hakkuuohjelmat

Kuten jo edellä korostettiin, koko kiertoaikaa koskeviin tuloksiin tulee suhtautua varauksella, koska ne perustuvat kasvumallin käyttöön osittain sen laadinta-aineiston ikäalueen ulkopuolella. Lähtien koivuosuuden eri arvoista 25 vuoden iällä ennustettiin metsiköiden kehitys kohdassa 31. kuvatuilla perusteilla. Puuston tilavuudeksi 25 vuoden iällä valittiin esimerkinomaisesti sellainen arvo, että puhtaan männikön tilavuus 40 vuoden iällä tulisi olemaan noin 250 m³/ha. Sopivaksi arvoksi osoittautui 69,17 m³/ha. Puuston kokonaistilavuuskasvut eri koivuosuuksilla muodostuivat taulukon 4 mukaisiksi.

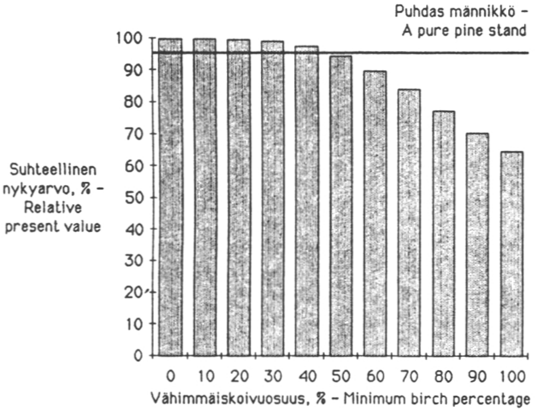
Ensimmäisten 15 vuoden aikana 50 prosentin koivuosuuden metsikkö oli saavuttanut 10 prosentin tilavuusedun puhtaaseen männikköön verrattuna. Esimerkiksi 80 vuoden kiertoajalla kokonaiskasvujen ero nykyarvoa maksimoivissa harvennusohjelmissa supistui 8 prosenttiin sekametsikön tilavuuskasvun heikennyttyä metsikön ikääntyessä.

Kun kullekin lähtöpuuston koivuosuudelle (25 vuoden iällä) määritettiin optimaalinen hakkuuohjelma, saatiin kuvassa 11 esitetyt tulokset. Suurimpaan nykyarvoon päädyttiin lähtöpuuston 20—30 prosentin koivuosuudella. Paras sekametsikkövaihtoehto johti puhtaaseen männikköön verrattuna 5 prosentin etuun nykyarvoilla mitaten (kuva 11). Puhdas koivikko sen sijaan merkitsi vastaavasti 32 prosentin tappiota. Merkille pantavaa on myös, että vielä lähtöpuuston 50 prosentin koivuosuudella saatiin nykyarvolla mitaten yhtä hyvä tulos kuin puhtaassa männikössä. Tulosten yleispiirre oli, että sekametsikkövaihtoehto oli puhdasta männikköä edullisempi silloin, kun mäntyjä oli metsikössä vähintäin niin paljon, että harvennusten avulla päästiin likimain puhtaaseen männikköön ennen päätehakkuuikää, ilman että jouduttiin alentamaan puustopääomaa alle optimaalisen tason.

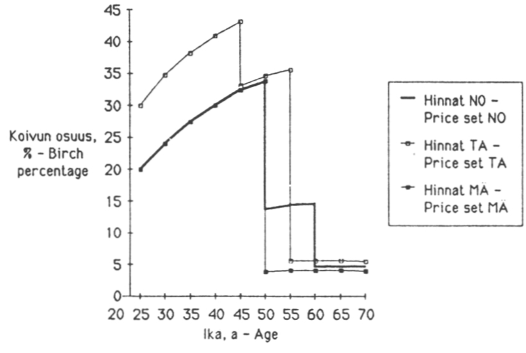


Kuva 12. Optimaalinen hakkuuohjelma 3 prosentin korkokannalla.
Fig. 12. The optimal harvest schedule for 3 percent interest rate.

Optimaalinen hakkuuohjelma 25 vuoden iältä 70 vuoden kiertoajan loppuun on esitetty kuvassa 12. Kyseessä on myös alkupuuston koivuosuuden suhteen optimaalinen hakkuuohjelma. Kiertoajan kuluessa oli optimaalista harventaa kolmesti — harvennusvoimakkuudet olivat 30, 35 ja 37 prosenttia



Kuva 13. Optimaaliset nykyarvot koko kiertoajan vallitseville vähimmäiskoivuosuuksille tilavuudesta, korkokanta 3 %. Vaakaviiva osoittaa puhtaan männikön antaman nykyarvon.
Fig. 13. Optimal present values for various levels of minimum birch percentage over the rotation, interest rate 3 percent. The horizontal line indicates the present value of a pure pine stand.



Kuva 14. Optimaalinen rauduskoivuosuus eri hintavaihtoehdoilla, korkokanta 3 %.
Fig. 14. Optimal birch percentages under various stumpage price sets, interest rate 3 %.

tilavuudesta laskettuna. Koivuosuuden annettiin lisääntyä rauduskoivun nopean kasvun myötä alkuarvostaan (20 %) 50 vuoden iälle asti, jolloin osuutta alennettiin 34 prosentista 14 prosenttiin ja 60 vuoden iällä edelleen 15 prosentista 5 prosenttiin. Puuston hehtaarikohtainen runkoluku aleni lähtöpuuston 1200 rungosta päätehakkuupuuston 345 runkoon.

Sekametsikön käsittelyä saatettaisiin moninaiskäytöllisistä tai ekologisista syistä rajoittaa siten, että tietty vähimmäisosuus puustosta tulee olla koivua läpi kiertoajan. Tällaisen rajoituksen vaikutus sekametsikön antamaan nykyarvoon ilmenee kuvasta 13. Rauduskoivun osuutta läpi kiertoajan voitiin nostaa 30 prosenttiin tappion ylittämättä yhtä prosenttia nykyarvosta. Kun koivuosuus nostettiin puoleen, tappio kasvoi 5,5 prosenttiin. Näissä luvuissa vertailukohtana on parhaan sekametsikön antama nykyarvo. Puhtaaseen männikköön verrattaessa koivuosuus oli nostettava 50 prosenttiin, ennen kuin menetyksiä alkoi syntyä.

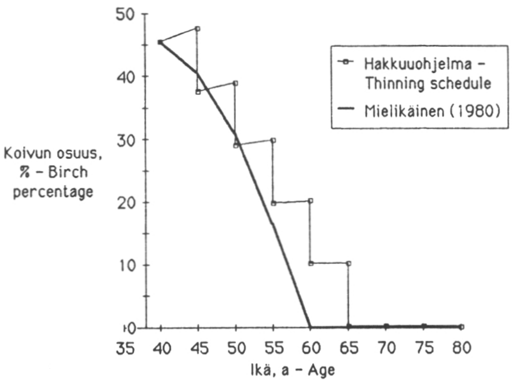
Puulajien kantohintasuhteen vaihtelun vaikutus optimaaliseen koivuosuuteen ilmenee kuvasta 14. Kantohintojen saattaminen puulajeittain samalle tasolle (Hinnat "Ta") nosti odotetusti optimaalista koivuosuutta. Koivut oli kuitenkin edullista poistaa 55 vuoden iällä mäntyjen myöhemmällä iällä voimakkaamman kasvun ja korkeamman tukkipuuprosentin vuoksi.

Kun tukkipuun hintasuhde nostettiin 1,2:een ja kuitupuun hintasuhde 1,3:een (hinnat "Mä"), optimaalinen koivuosuus säilyi pääosin muuttumattomana perusarvoihin verrattuna. Koivut poistettiin kokonaan 50 vuoden iällä, mutta muuten hakkuuohjelma vastasi hintasuhteella "No" saatua tulosta.

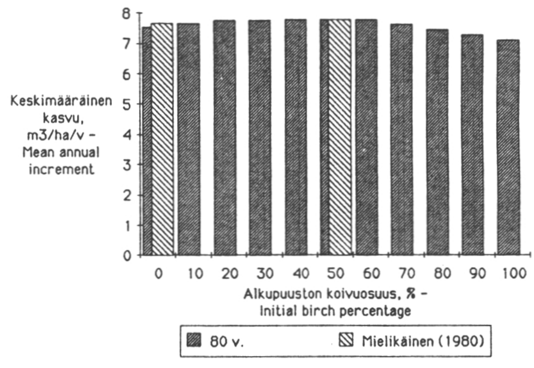
4. TARKASTELU

Saatujen tuloksien tarkastelua vaikeuttaa aiempien vertailukelpoisten tutkimusten vähyys. Tarkastelu painottuu sen vuoksi optimointimallin taustatekijöiden muutosten analysointiin. Siten saadaan viitteitä optimiratkaisun herkkyydestä eri lähtökohtaolettamusten muutoksille sekä sovellettavuudesta toisiin olosuhteisiin.

Tässä tutkimuksessa on käytetty Mielikäisen (1980) laatimaa kasvumallia. Tilavuuskasvua maksimoivan kasvatusohjelman koivuosuudet approksimoivat hyvin niitä Mielikäisen (mt.) esittämiä koivuosuuden arvoja, jotka antoivat suurimman tilavuuskasvun kullakin iänkohdalla (kuva 15). Koska koivuosuutta voitiin muuttaa harvennuksissa



Kuva 15. Suurimman tilavuuskasvun antavat koivuosuudet tilavuuskasvun suhteen optimaalisen hakkuuohjelman ja kasvuyhtälön mukaan.
Fig. 15. The birch percentage yielding maximum volume growth, by the optimal M.A.I. thinning schedule, and based on the growth equation.



Kuva 16. Kiertoajan keskimääräinen kasvu alkupuuston eri koivuosuuksilla nykyarvon suhteen optimaalisissa hakkuuohjelmissa 80 vuoden kiertoajalla.
Fig. 16. Mean annual increment in relation to initial birch percentage, based on optimal present value thinning schedules with 80 year rotation.

vain 10 prosenttiyksikön välein, hakkuuohjelma ei pysty tarkasti seuraamaan suurimman kasvun koivuosuuksia.

Olettaen tilavuuskasvuksi 40 vuoden ikään mennessä 275 m^3/ha saatiin kuvassa 16 esitetyt kiertoajan keskimääräisen kasvun arvot eri alkupuuston koivuosuuksille. Vertailuna kuvaan on sijoitettu myös Mielikäisen (1980) esittämät keskikasvun arvot puhtaalle männikölle ja yhdelle sekametsikön kasvatusohjelmalle. Tutkimuksessa saadut keskimääräisen kasvun arvot vastaavat hyvin Mielikäisen (mt.) esittämiä tuloksia. Suurimmat arvot 80 vuoden kiertoajalla (7,82 m^3/ha/v) saatiin 30—40 prosentin koivuosuudella alkupuustosta. Suotuisin sekametsikkövaihtoehto antoi 3,6 prosenttia suuremman keskimääräisen kasvun kuin puhdas männikkö 40 ja 80 ikävuoden välisenä aikana. Jos vertailun lähtökohtana on 25 vuoden ikäinen metsikkö, puhtaan männikön ja sekametsikön välille syntyy suurempi ero pidemmästä kasvujaksosta johtuen. Saatuja kasvuennusteita alle 40-vuotiaille sekametsiköille ei voida kuitenkaan pitää yhtä luotettavina kuin sitä vanhempien metsiköiden kohdalla.

Ruotsalaiset tilavuuskasvua koskeneet tutkimukset tukevat osittain Mielikäisen (1980) saamia tuloksia. Agestamin (1985) esimerkkisimuloinnissa mänty-koivusekametsikkö antoi 3 prosentin tuotoslisän puhtaaseen männikköön verrattuna 73 vuoden kiertoajalla. Kiertoaikaa pidennettäessä tuotosetu pieneni ja kääntyi tuotostappioksi 100 vuoden kiertoaikaan mennessä.

Ekön (1985) mukaan vastaavan sekametsikön kasvu oli tilanteesta riippuen joko pienempi tai suurempi kuin puhtaan männikön kasvu. Tulokset olivat sikäli odottamattomia, että kiertoajan pidentyessä sekametsikön kasvu lisääntyi suhteessa puhtaan männikön kasvuun päinvastoin kuin Mielikäisen (1980) ja Agestamin (1985) tulosten mukaan.

Tukkipuu- ja hukkapuuyhtälöt estimoitiin tätä tutkimusta varten Mielikäisen (1980) koeala-aineistoon perustuen. Puutavaralajiosuudet oli arvioitu pystypuista, mikä oli saattanut johtaa tukkipuun määrän yliarvioihin. Männyn kohdalla oli niukasti havaintoja sellaisista metsiköistä, joissa oli pieni rungon keskikoko ja alhainen tukkipuuosuus. Tältä osin yhtälöiden luotettavuus on heikompi. Toisaalta harvennuksia ei juurikaan vielä tehty kyseisillä rungon keskikoon arvoilla. Yhtälöitä ei ole tarkoitettu käytettäväksi puutavaralajirakenteen ennustamiseen yleisesti.

Männyn ja koivun kantohintojen suhde 1,2 (hintasuhde "Mä") oli vielä riittävän pieni rauduskoivun sekoituksen säilymisen kannalta. Hakkuuvuoden 1983/84 tilanteessa valtaosassa maan eteläpuoliskoa koivun kantohinta olisi keskimäärin puoltanut rauduskoivusekoitusta. Hintasuhteen ennustaminen kiertoajan mittaiselle ajanjaksolle on luonnollisesti mahdotonta, mutta käytetyillä kantohinnoilla saadut optimaaliset koivuosuudet olivat siksi pieniä, alle 50 prosenttia, että esim. siirtyminen jokseenkin puhtaaseen männikköön olisi mahdollista kesken kiertoaikaakin, jos hintasuhteiden muutos sitä edellyttäisi.

Tutkimuksen analyysi perustuu olettamukselle, että metsikköä kuvaavat muuttujat (ikä, tilavuus, koivuosuus ja runkoluku) sisältävät riittävästi informaatiota metsikön hakkuuohjelman optimoimiseksi. Käytettyjen muuttujien joukko on epäilemättä vajavainen. Subjektiivisesti valittuun koeala-aineistoon (Mielikäinen 1980) perustuvaan kasvuyhtälöön on suhtauduttava varauksin optimointimallin osana. Optimointimalli käy läpi monenlaisia hakkuuohjelmia, joista osa saattaa poiketa huomattavasti koemetsiköissä esiintyneestä vaihtelusta. Eri lähtökohtaolettamuksin tehdyt analyysit osoittavat, että koivuosuuden optimiratkaisu on vakaa lukuun ottamatta kantohintasuhteen aiheuttamaa selvää vaikutusta. Tähän perustuen tuloksia voidaan pitää suuntaa antavina myös olosuhteissa, joissa metsikön puustopääoma, harvennusten lukumäärä, kiertoaika, runkoluku, korkokanta tai korjuukustannukset poikkeavat tässä tutkimuksessa käytetyistä arvoista.

Muutettaessa puulajisuhdetta voimakkaasti yhdessä harvennuksessa metsikköön saattaa syntyä aukkoja, mikäli puut sijaitsevat puulajeittain ryhmittäisesti. Esitetyt tulokset perustuvat olettamukselle, että puulajit ovat sekoittuneet tasaisesti ja että toisen puulajin poisto ei johda aukkoisuuteen. Puulajien ryhmittäisyys voidaan ottaa huomioon optimointimallissa esimerkiksi rajoittamalla puulajisuhteen säätömahdollisuutta harvennuksen yhteydessä. Tällöin puulajisuhde voi muuttua vain vähitellen ja molempia puulajeja voidaan käyttää metsikön tasaisuuden säilyttämiseksi.

Käytetty kasvumalli ja puutavaran hinnat koskevat männyn ja rauduskoivun muodostamaa tasaikäistä metsikköä. Saatuja tuloksia ei voida suoraan soveltaa mänty-hieskoivusekametsiin. Mielikäisen (1980) esittämien tulosten mukaan ainakin kivennäismailla hieskoivun optimaaliset osuudet olisivat huomattavasti pienempiä kuin nyt esitetyt rauduskoivun osuudet.

Dynaamiseen ohjelmointiin perustuva optimointimalli tarjoaa joustavan analyysikehikon. Menetelmän etu on, että mikä tahansa mallin tilamuuttujiin (metsikön ikä, puuston tilavuus, koivuosuus ja runkoluku) perustuva tuotto- tai kustannustekijä voidaan sisällyttää optimointimalliin. Myös kasvumallia koskevat muutokset ja hakkuuohjelmiin mahdollisesti liitettävät rajoitukset voidaan ottaa huomioon.

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SUMMARY

Optimizing thinnings and rotation for mixed even-aged pine-birch stands

Introduction

The optimal species composition of a mixed conifer-hardwood stand is affected by several factors. It is informative to differentiate between the effects generated by the roundwood market and the effects due to nonmarket values. No attempt was made in this study to make the two groups commensurate and the analysis is limited to stumpage values. However, an analysis of the economic returns from various levels of species mixture is believed to be useful for the decision maker.

Regarding the management of mixed stands of Scots pine (*Pinus sylvestris* L.) and silver birch (*Betula pendula* Roth) the prevailing perception has been that pine is more valuable than birch. Hence, a pure pine stand has been preferable to a mixed species stand. For example, in precommercial thinnings the practice has been to favor pine over birch. A recent study by Mielikäinen (1980) suggests that the volume growth of a mixed pine-birch stand is superior to that of a pure pine stand during most of the rotation. The aim of this study is to determine the financially optimal species composition in mixed, even-aged pine-birch stands.

Managing the stand is viewed as an investment, where the capital invested consists of the management costs, the merchantable growing stock, and the value of land occupied. The returns are comprised of stumpage received from pine and birch pulpwood and sawtimber. The soil expectation value of an infinite series of equal rotations is used to compare management alternatives. The analysis is subject to the usual assumptions of deterministic growth models, costs and prices, and a perfect capital market with known and time invariant interest rates.

The growth model for pine-birch stands

This work was preceded by a yield study (Mielikäinen 1980) on the structure and development of even-aged mixed stands of Scots pine and silver birch. The following volume growth model had been estimated:

$$\ln(P_v) = 7.050 - 0.8732 \ln(T_{1.3}) - 0.4187 \ln(V) - 0.5154 \cdot 10^{-6}\, B \cdot T_{1.3}^2 - 1.403 \frac{(50 - B)^2}{T_{1.3}^3}$$

where

P_v = volume increment in the next 5-year period, percent of the present volume,
$T_{1.3}$ = stand age at breast height, a,
V = stand volume, m^3/ha, and
B = birch percentage of volume.

The allocation of the predicted growth between the two species is obtained from a table, dimensioned by species composition and stand age at breast height (see Appendix 1).

The last two terms in the equation above show the effect of birch percentage on growth. The birch percentage yielding the maximum volume growth is just below 50 percent in a young stand. For older stands the percentage decreases with increasing age.

Due to the form of the growth model and the values of the parameters, volume growth is a strictly increasing function of stand volume. If the stand is left unthinned its volume increases to unrealistic levels. Based on the data set in Mielikäinen (1980) a volume limit was constructed above which growth was terminated. This caused the optimal regimes to avoid too large stand volumes. The volume limit (V_{max}, m^3/ha) was defined as a function of stand age (T):

$$V_{max} = 273.42 + 0.9308\, T.$$

Sawtimber and pulpwood were priced individually and, hence, equations were needed to predict the sawtimber and wastewood percentages of stand volume. Using nonlinear regression, maximum likelihood estimates were computed based on the data in Mielikäinen (1980). The following function form was used:

$$\text{Percentage} = \frac{1}{1 + e^{\left\{b_0 + b_1 \left(\frac{V_t}{N_t}\right)^{b_2}\right\}}}$$

where

V_t = stand volume at age t,
N_t = number of trees at age t and
b_o, b_1, b_2 = parameters of the equation.

The values of the parameters and the estimated mean squared errors of the equations are given in Table 1. The equations are plotted in Figs. 1 and 2.

Price and cost data

The stumpage price ratio between pine and birch is a preeminent factor of the optimal species mix. Using historical data from central and eastern Finland (Uusitalo 1985) trend equations for the price ratio were derived for sawtimber (Fig. 3) and pulpwood (Fig. 4). In addition to the normal ("No") prices given by the trend equations, two other sets of stumpage prices are used in the analysis. The relative prices and the price of pine sawtimber in each price set are:

	Stumpage price set		
	No	Ta	Mä
Pine sawtimber	100	100	100
Birch sawtimber	93	100	83
Pine pulpwood	45	43	44
Birch pulpwood	37	43	34
Pine sawtimber (FIM/m^3)	203.59	196.76	215.54

Price set "Ta" refers to a circumstance where pine and birch have equal prices. Price set "Mä" implies that pine is 20 % more valuable than birch. The price ratio is shown to have considerable geographical variation (Fig. 5).

The stumpage prices of each removal were adjusted, based on average stem size (Table 2) and volume removed (Table 3). Final harvests had the same adjustments as thinnings. Regeneration costs were assumed fixed and independent of the resulting species composition. The discounted value of the total regeneration costs amounted to 4500, 4300, and 4180 FIM/ha for 0, 3, and 5 percent interest rate, respectively.

The dynamic programming algorithm

Discrete-time, discrete-state dynamic programming was used to derive optimal thinning, rotation, and species composition regimes. The computational method resembles that of Brodie and Kao (1979). The state variables are stand volume (V), birch percentage of volume (B), and number of trees (N). The state vector at time t (x_t) is thus

$$x_t^T \equiv (V_t, B_t, N_t)$$

Because the stumpage value of a final harvest is computed like that of a thinning, the dynamic programming recursion may be formulated as a single equation. In forward recursion the optimal value function $R(x_t)$ for stand x_t at age t with a discounted thinning revenue $H(x_t, x_{t-1})$ at age t is

$$R(x_t) = \max_{\{x_{t-1}\}} [H(x_t, x_{t-1}) + R(x_{t-1})] \quad t = 1,...,T$$

The thinning revenue is associated with moving along the optimal path between stand x_{t-1} and stand x_t. The set $\{x_{t-1}\}$ is the set of all possible stands at age t − 1 from which the stand x_t may be reached by first growing and then possibly thinning at age t. T denotes the last stage of optimization, the maximum rotation length. Once the recursion equation is applied to stages 1 to T the optimal value function equals the sum of discounted thinning revenues. A single rotation optimum solution is thus found for each rotation length. After the discounted regeneration cost is substracted from the single rotation discounted value, the transformation to soil expectation value is made. Soil expectation value is then used as the final optimality criterion.

The state space was discretized with the intervals V = 10 m^3/ha, B = 10 %, and N = 75 trees/ha. The time (stage) interval was 5 years. At each stage the stand had potentially 7200 alternative states. A single typical optimization analysis involved 350 000 thinnings. If simulation had been used instead of dynamic programming about 10^{16} whole rotation simulations would have had to be covered to achieve the same accuracy of analysis.

Results

The first set of results was derived for 40-year-old mixed stands with varying birch percentage. The initial volume was 275 m^3/ha and the number of trees 1200 per hectare. The underlying site index was 28 m, a dominant height at stand age 100. Pine and birch were assumed to be equal in height such that neither species was overtopping in the canopy.

The optimal time paths of birch percentage are seen in Fig. 6 for three different initial values (25, 50 and 75 %). At first, the optimal birch percentage is 30 to 50 percent. Later, the birches are removed in thinnings to produce a pure pine stand before the end of the rotation. However, maintaining a productive growing stock level turns out to be more important than achieving a desired species composition. A sample thinning schedule is seen in Fig. 7. More detailed results are presented in Appendix 2.

When the interest rate is lowered to zero percent several light thinnings are applied. The birch percentage is gradually adjusted (Fig. 8). The alternative stumpage price sets cause only minor changes in the optimal thinning schedules (Fig. 6 vs. Fig. 9). The effect of a decrease of the initial volume is seen in Fig. 10.

A second set of analysis is based on 25-year-old stands with varying birch percentage. The growth equation has the property that the effect of birch percentage is increased with decreasing age. Because the youngest stands in the data were more than 35 years old the growth equation would have been used outside the range of the data. For stands less than 40 years old, the influence of birch percentage on growth was restricted not to be larger than it is for 40-year-old stands. The early development of stands, using the modified growth equation, is seen in Table 4.

An optimal harvest schedule was determined for various levels of initial birch percentage. The results imply that 20—30 percent would be the most profitable starting values (Fig. 11). Both a pure pine and a pure birch stand give a lower present value than the optimal mixed stand. It is remarkable that even the 50 percent case results in a present value equal to that of a pure pine stand. A sample harvest schedule is seen in Fig. 12.

The manager might want to impose a lower limit for birch percentage because of scenic or ecological reasons. Compared to the optimal regime, up to 30 percent of volume could be birch without any noticeable loss (Fig. 13). If the comparison is made with a pure pine stand the minimum birch percentage could be raised to 50. The effect of stumpage price ratio on the optimal birch percentage is seen in Fig. 14.

The optimal birch percentage for maximizing volume production (Fig. 15) parallels the results by Mielikäinen (1980). The optimal birch percentage is initially 45, after which it gradually decreases almost to zero by the end of the rotation. Because the interval of birch percentage was 10 percent the exact zero was not reached in the thinning schedule. A comparison of mean annual increment to the results by Mielikäinen (1980) can be seen in Fig. 16.

Conclusions

Using the growth model by Mielikäinen (1980) for mixed even-aged pine-birch stands the optimal species composition, and thinning and rotation schedule was determined. The main result of this study is that a mixed stand of pine and birch is financially superior to a pure pine or birch stand. Volume maximization gave a similar result. The optimal species composition is dependent on the stumpage price ratio between pine and birch. However, the price variation inside the potential region of mixed stand management (the southern half of the country) is not likely to alter the conclusions. The results were relatively insensitive to changes in other parameters of the analysis.

Liite 1. Männyn osuus metsikön kasvusta (Mielikäinen 1980).
Appendix 1. Pine percentage of volume growth (Mielikäinen 1980).

Männyn osuus tilavuudesta, % — *Pine percentage of stand volume*	Metsikön ikä, a — *Stand age* 30	35	40	45	50	55	60	65	70	75	80
	Mäntyä kasvusta, % — *Pine out of growth*										
10	7	7	7	7	8	8	8	8	8	8	9
20	15	15	15	15	15	16	17	17	18	18	18
30	23	23	23	24	25	26	27	27	28	28	28
40	32	32	32	34	35	36	37	38	38	39	39
50	41	41	41	43	45	46	47	48	48	49	50
60	50	50	50	53	55	56	57	58	59	60	61
70	60	60	60	63	65	67	68	69	70	71	72
80	71	71	71	74	76	78	79	81	82	82	83
90	83	83	83	85	87	89	90	92	93	94	95

Liite 2. 40-vuotiaan alkupuuston koivuosuuden vaikutus kiertoajan kokonaisarvoihin nykyarvoa maksimoivien hakkuuohjelmien mukaan.
Appendix 2. The effect of 40-year initial birch percentage on the cumulative values of a rotation, based on thinning schedules that maximize present value.

Kiertoaika 65 v — *Rotation 65 yrs*

Koivuosuus *Birch*	Suhteellinen nykyarvo *Relative present value*	Kokonaistuotos *Total production*	Koivua edellisestä *Birch out of total production*		Keskikasvu *Mean ann. increment*
%	%	m^3/ha	m^3/ha	%	m^3/ha/v
0	100	531,59	0,00	0	8,18
10	99,2	535,26	64,06	11,97	8,23
20	98,2	535,28	120,12	22,44	8,24
30	96,9	538,88	165,91	30,79	8,29
40	95,5	538,89	191,09	35,46	8,29
50	93,9	538,95	224,38	41,63	8,29
60	92,0	537,72	277,18	51,55	8,27
70	89,5	535,06	341,33	63,79	8,23
80	86,3	523,27	399,07	76,26	8,05
90	82,4	518,29	474,29	91,51	7,97
100	78,9	504,55	504,55	100	7,76

Kiertoaika 80 v — *Rotation 80 yrs*

0	100	604,36	0,00	0	7,55
10	99,1	614,04	65,88	10,73	7,68
20	97,9	622,65	122,81	19,72	7,78
30	96,7	623,51	159,05	25,51	7,79
40	95,3	625,22	193,92	31,02	7,82
50	93,7	625,32	227,07	36,31	7,82
60	91,8	623,62	279,87	44,88	7,80
70	89,6	611,62	327,21	53,50	7,65
80	86,3	598,34	391,95	65,51	7,48
90	81,1	561,81	515,09	91,68	7,02
100	78,0	572,23	572,23	100	7,15

II

A comparison of numerical methods for optimizing even aged stand management

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A two-species, whole-stand, deterministic growth model was combined with three optimization methods to derive management regimes for species composition, thinnings, and rotation age, with the objective of maximizing soil expectation value. The methods compared were discrete time - discrete state dynamic programming, direct search using the Hooke and Jeeves algorithm, and random search. Optimum solutions for each of the methods varied considerably, required unequal amounts of computational time, and were not equally stable. Dynamic programming located global optimal solutions but did not determine them accurately, owing to discretized state space. Direct search yielded the largest objective function values with comparable computational effort, although the likelihood of finding a global optimum solution was high only for smaller problems with up to two or three thinnings during the rotation. Random search solutions varied considerably with regard to growing stock level and species composition and did not define a consistent management guideline. In general, direct search and dynamic programming appeared to be superior to random search.

VALSTA, L. T. 1990. A comparison of numerical methods for optimizing even aged stand management. Can. J. For. Res. **20** : 961-969.

Un modèle de croissance déterministe pour une table de peuplement de deux essences a été combiné avec trois méthodes d'optimisation pour déduire des régimes d'aménagement pour les compositions d'espèces, l'élagage et l'âge de rotation avec l'objectif de maximiser la valeur attendue du sol. Les méthodes comparées étaient la programmation dynamique de temps discret et d'état discret, la recherche systématique avec l'algorithme de Hooke et Jeeves et la recherche aléatoire. Les solutions optimums pour chacune des méthodes variaient considérablement, requéraient des temps inégaux d'ordinateur et n'étaient pas également stables. La programmation dynamique localisait les solutions optimums globales mais elles n'étaient pas déterminées de façon précise étant donné l'espace discret de l'état. La recherche systématique conduisait aux plus grandes valeurs de la fonction objective avec des temps comparables d'ordinateur même si la probabilité de trouver la solution optimum globale était grande seulement pour les petits problèmes comprenant deux ou trois élagages durant la rotation. Les solutions par la recherche aléatoire variaient considérablement pour les niveaux de croissance des volumes et la composition des espèces et ne définissaient pas des principes directeurs consistants d'aménagement. En général, la recherche systématique et la programmation dynamique étaient supérieures à la recherche aléatoire.

[Traduit par la revue]

Introduction

The emergence of simulators that predict forest stand development for the life-span of the forest has challenged forest economists to determine optimal ways to manage a stand. Dynamic optimization used in standard economic analysis usually obtains the optimum solution using derivative information (Clark 1976; Cawrse et al. 1984). Realistic forest growth models, and cost and revenue models for forest operations, generate complex function forms for which only derivative-free numerical solution methods can be used.

Optimization methods that have been used with nondifferentiable single-tree or whole-stand simulators include dynamic programming (starting from Hool 1966; Amidon and Akin 1968), random search (Bullard et al. 1985), and multidimensional search without derivatives (Roise 1986; Bare and Opalach 1987). Of these methods, dynamic programming has the longest history in forest management research and has been applied to a wide variety of management questions, tree species, growth model types, and geographical areas (for further references, see Hann and Brodie 1980; Brodie and Haight 1985; Paredes and Brodie 1987; Arthaud and Klemperer 1988).

Excluding dynamic programming analyses, most stand optimization studies have concentrated on examining the resulting objective function values produced by each method, and on testing whether or not the problem can be solved. However, an important use of optimization models is to study how changes in economic–biological factors of stand management can affect optimum solutions (Roise et al. 1988). To draw correct conclusions, the change in the optimum solution must be attributable to changes in circumstances and not be a result of problems of the solution method, as reported in some previous studies.

Roise (1986) compared three nonlinear programming methods and dynamic programming when deriving optimum thinning and rotation schedules for a Douglas-fir stand. All nonlinear programming methods were sensitive to the initial solution given as input to the algorithms, i.e., different objective function values and control variable values were reported for each run. Discrete dynamic programming yielded objective function values that were less than 80% of the values generated by nonlinear programming. On the other hand, Kao and Brodie (1980) reported only a 1-3% loss in the objective function value of dynamic programming when compared with a continuous variable solution obtained through nonlinear programming (using the flexible polyhedron method). Based on these studies, it seems premature to draw conclusions about the relative performance of the dynamic programming and nonlinear programming methods.

One approach to stand optimization is random search (Bullard et al. 1985). It is clearly the simplest method to apply and may also be considered a base-line method to evaluate the computational effort and the resulting objective function value of the optimum solution. A reasonable requirement is that any other optimization method should perform computationally more efficiently than random search in determining a comparable solution. Bullard et al. (1985) used two strategies of random search: simple and multistage. The authors recommend the latter strategy, in which a set of random solutions is generated at each stage. After each stage, the search ranges of the variables are reduced, based on the location of the solution with the greatest objective function value. As Bullard et al. (1985) pointed out, the inherent limitation of random search algorithms is that the optimum objective function value can not be determined with certainty. This shortcoming becomes more severe as the size of the optimization problem (the number of variables) increases.

The decision problem examined in the present study is that of determining the optimum combination of thinning, rotation, and species mix for a two-species even-aged stand. The thinning specification includes intensity by species and timing of entries. A comparison is presented of dynamic programming, nonlinear programming using the Hooke and Jeeves direct search method (see e.g., Hooke and Jeeves 1961; Bazaraa and Shetty 1979), and random search. Attention is paid to the objective function value, to the stability and other properties of the solution, and to the central processing unit (CPU) time used. It is assumed that the objective is to maximize the soil expectation value of a management regime; 3 and 0% real, before tax interest rates are used. The analysis is based on a forest stand simulator that describes the development of an even-aged mixture of Scots pine (*Pinus sylvestris* L.) and silver birch (*Betula pendula* Roth) (Mielikäinen 1980). Results concerning the dynamic programming approach have been reported previously in Valsta (1986).

Optimization methods and problem formulations

Dynamic programming

Dynamic programming is a widely used method to solve problems of multistage decision processes. When stand management is optimized, the stages are typically defined as time steps, e.g., 5-year periods. The process to be optimized is defined by state variables. The state variable vector that identifies a stand in a dynamic programming analysis is defined as

$$[1] \quad x_t^{\mathrm{T}} \equiv (V_t, B_t, N_t)$$

where

x_t^{T} = transpose of state vector at time t
V_t = stand volume at time t
B_t = birch percentage of volume at time t
N_t = number of trees at time t

All other variables used in the analysis are computed on the basis of these state variables, the time t, and a number of fixed parameter values in the equations. For computational reasons, only a few state variables can be handled. In most cases, the use of dynamic programming is therefore restricted to whole-stand models; single-tree models involve too large a number of state variables.

Net revenues from thinnings and final harvests are computed with the same model. Thus, final harvests may be viewed as 100% thinnings, and the dynamic programming recursion equation may be defined as

$$[2.1] \quad R(x_t) = \max_{\{x_{t-1}\}}[H(x_t\, x_{t-1}) + R(x_{t-1})] \qquad t = 1,\ldots,T$$

with the initial condition

$$[2.2] \quad x_0 \in X_0$$

where

$R(x_t)$ = the optimum objective function value for stand x_t at the beginning of period t
$\{x_{t-1}\}$ = the set of stands at time $t - 1$ from which it is feasible to reach stand x_t after growth and a possible thinning
$H(x_t, x_{t-1})$ = the discounted thinning revenue, if any, associated with moving along the optimum path from stand x_{t-1} to stand x_t
$R(x_{t-1})$ = the objective function value for stand x_{t-1} at the beginning of period $t - 1$
T = the last period of optimization, the maximum rotation length
X_0 = the set of initial stands with varying B_0, and fixed V_0 and N_0

The recursion equation gives the single rotation present net value for each stand x_t at $t = 1,\ldots,T$. Optimization is performed for each rotation length t by recording x_t^* for which $(V_t, N_t) = (0, 0)$, i.e., the last cut is a clear-cut.

With the rotation age defined by t for each stand x_t^*, the soil expectation value, SEV(x_t^*), is computed as an infinite series of equal rotations. Regeneration costs are taken into account to compute soil expectation values. The optimum solution to soil expectation value maximization is then found by selecting the maximum SEV(x_t^*) with respect to rotation length t, $t = 1,\ldots,T$.

In the present study, the discrete time - discrete state version of dynamic programming is used. The network that identifies the alternatives to be optimized is defined by state variable value intervals so that all stands that fall into a given interval (polyhedron) are considered to be at the same network node, according to the neighborhood concept of Brodie and Kao (1979). Using this problem structure, the thinnings may be defined freely, and the state variable values of residual stands need not be predefined.

Optimization is performed at each (5-year) time step by choosing the stand with the largest objective function value among the stands falling into the same state interval. The state intervals used in the optimization are 10 m^3/ha for stand volume, 5% for birch percentage, and 75 trees/ha.

Direct and random search

Instead of using state variables to define a stand management regime we can use control variables, which describe the treatments in the stand. Letting $B = B/100$ and $\tilde{x}_t^{\mathrm{T}} = (V_t, N_t)$, to simplify notation, the stand management problem may be defined in nonlinear programming form as

$$[3.1] \quad \max z = g(B_0, t_1, t_2, \ldots, t_m, u_{11}, u_{12}, \ldots, u_{1m}, u_{21}, u_{22}, \ldots, u_{2m}, t_{m+1} \mid \tilde{x}_0)$$

subject to

TABLE 1. Solutions generated by different optimization methods for maximizing soil expectation with 3% interest rate

Method and run number	Objective function value[a] Marks/ha	$/ha	%	Number of thinnings	Rotation age (years)
Dynamic programming	15 393	4160	97.0	3	65
Direct search 1	15 850	4284	99.9	2	63
Direct search 2	15 873	4290	100.0	3	63
Direct search 3	15 788	4267	99.5	4	62
Random search 1	15 461	4179	97.4	2	58
Random search 2	15 451	4176	97.3	2	64
Random search 3	15 427	4169	97.2	3	62
Random search 4	15 703	4244	98.9	4[b]	61
Random search 5	15 462	4179	97.4	3	60
Random search 6	15 641	4227	98.5	4	59
Random search 7	15 534	4198	97.9	3	54
Random search 8	15 455	4177	97.4	4	63
Random search 9	15 411	4165	97.1	3	60
Random search 10	15 478	4183	97.5	2	60

[a]Monetary units are Finnish marks and Canadian dollars; 1 Canadian dollar = 3.7 Finnish marks. Percent values are proportional to the largest value achieved.
[b]Two of the thinnings were extremely light, with 0.5 and 3% of the volume removed.

[3.2] $u_{1j} \leq h_{\max}, \quad \forall_j$

[3.3] $1 - \dfrac{1 - B_j}{u_{1j}} \leq u_{2j} \leq \dfrac{B_j}{u_{1j}}, \quad \forall_j$

[3.4] $u_{2j} \leq 1, \quad \forall_j$

[3.5] $t_j, u_{1j}, u_{2j} \geq 0, \quad \forall_j$

[3.6] $t_{m+1} \geq 0$

where

$g(\cdot)$ = function for the soil expectation value of a cutting regime
B_0 = birch proportion of volume in the initial stand
t_j = time between thinning $j - 1$ and j
t_{m+1} = time between the last thinning and the final harvest
u_{1j} = proportion of volume removed in thinning j
u_{2j} = birch proportion of the volume removed in thinning j
$h_{\max}$ = maximum thinning intensity, as a proportion of standing volume
B_j = birch proportion of volume before thinning j
$\tilde{x}_0$ = initial state (volume and number of trees at time 0)
j = the index of thinning
m = number of thinnings

The function $g(\ )$ is a nonlinear scalar function of the control variables B_0, t_j, u_{1j}, u_{2j}, and the initial state $\tilde{x}_0$. Constraint [3.2] imposes an upper limit for thinning intensity. This is necessary because only the growing stock level after thinning affects growth, not the amount thinned (see Appendix). Constraint [3.3] ensures that no more than 100% of either tree species is harvested in any thinning entry. Constraints [3.4–3.6] define upper and lower bounds for the variables in question.

The nonlinear programming problem given above may be solved using various methods. In the present study, the function $g(\ )$ is nondifferentiable, and a derivative-free multidimensional search method has to be used. Random search over the control variables is one alternative, although computationally more effective search methods are desirable. Direct search methods, such as the Hooke and Jeeves algorithm (Roise 1986) and the complex method of Box (Bare and Opalach 1987), have been found useful for optimization problems similar to the present one. The problem defined by [3.1–3.6] is a constrained nonlinear programming problem. Because of the simple form of the constraints and the optimization methods used, the problem, in practice, may be solved using methods for unconstrained nonlinear programming. In the present study, the objective function value is set to zero whenever one of the constraints is violated.

To perform direct search, a computer program given in Osyczka (1984) was adapted to the present problem. Each number of thinnings, m, forms a different optimization problem with a corresponding number of decision variables. Solutions to a smaller problem with fewer thinnings can be derived from a larger problem by setting some of the control variables to zero and thereby bypassing one or more thinnings. However, each number of thinnings tends to form a local minimum of the function $g(\ )$ because of increasing returns to scale in a thinning operation (see also Roise 1986). When the rate of interest is 3%, optimum solutions are computed for management regimes with 2, 3, and 4 thinnings.

The effect of thinning volume on logging costs was so weak that in the 0% case, a large number of thinnings were optimal. In the dynamic programming implementation used, thinnings cannot occur more frequently than 5 years apart. This resulted in the dynamic programming optimum solution of nine thinnings. Because the optimum number of thinnings was large in the 0% case and on the other hand, the number of thinnings has to be specified for both direct and random search, nine thinnings are assumed to make the search methods comparable to the dynamic programming implementation.

The optimization program calls the stand simulator repeatedly, employing the control variable values as arguments, and receives the resulting soil expectation value from the simulator. As in the dynamic programming analysis, the

TABLE 2. Solutions generated by different optimization methods for maximizing average yearly net cash flow (which corresponds to maximizing soil expectation value with 0% interest rate)

Method and run number	Objective function value[a]			Number of thinnings	Rotation age (years)
	Marks·ha^{-1}·year^{-1}	\$·ha^{-1}·year^{-1}	%		
Dynamic programming	1520	411	99.9	9	85
Direct search 1	1496	404	98.4	9	80
Direct search 2	1457	394	95.8	8	66
Direct search 3	1511	408	99.3	9	78
Direct search 4	1436	388	94.4	9	71
Direct search 5	1487	402	97.8	8	81
Direct search 6	1495	404	98.3	9	84
Direct search 7	1464	396	96.3	9	76
Direct search 8	1482	401	97.4	9	89
Direct search 9	1517	410	99.7	9	80
Direct search 10	1521	411	100.0	9	82
Random search 1	1425	385	93.8	4	72
Random search 2	1439	389	94.7	4	71
Random search 3	1449	392	95.3	5	80
Random search 4	1441	389	94.8	5	89
Random search 5	1450	392	95.4	5	82
Random search 6	1444	390	95.0	4	85
Random search 7	1433	387	94.3	3	77
Random search 8	1428	386	93.9	3	74
Random search 9	1464	396	96.3	4	86
Random search 10	1436	388	94.5	8	79

[a]Monetary units are Finnish marks and Canadian dollars; 1 Canadian dollar = 3.7 Finnish marks. Percent values are proportional to the largest value achieved.

total volume and the number of trees of the initial stand at age 25 years are given as inputs to the simulator and are constant throughout all analyses. The convergence criterion is that the difference of control variable values between successive moves must be less than 0.01 years for the time variables and 0.0001 for the thinning proportions.

Random search is implemented in the following way. First, an integer is generated that defines the number of thinnings during one rotation. The maximum number of thinnings is 4 for 3% interest rate, and 9 for 0% interest rate. A schedule with no thinnings is also an alternative. With the number thinnings as a pseudorandom integer, nonnegative pseudorandom numbers for the control variables are generated with the following upper bounds:

Control variable	Upper bound
B_0 (birch proportion of volume in the initial stand)	1.0
t_j (time between thinning j and j − 1; years)	20
u_{1j} (proportion of volume removed in thinning j)	0.4
u_{2j} (birch proportion of volume removed in thinning j)	1.0

One random search run consists of 25 000 simulation trials. If the set of random variables violates constraint [3.3], the whole random number set is rejected and a new set of variables is generated. For each analysis, random search was repeated 10 times with a different seed of the random number generator.

Results

Three optimization methods (dynamic programming, random search, and direct search) were used to solve the species composition, thinning, and rotation problem with the objective of maximizing soil expectation value, based on an infinite series of equal rotations. Results are reported for 3 and 0% interest rates. As usual, it was assumed that prices, costs, and interest rates are constant over time. The initial stand in the computations was 25 years old with 69.17 m^3/ha and 1200 trees/ha. The species composition in the initial stand was determined in the process of optimization.

Looking at the objective function values reported by different optimization methods it can be seen that with 3% interest rate, direct search gave the highest value (Table 1). Random search values were 97.1–98.9% of the direct search value and exceeded the dynamic programming result (97.0% of the direct search value). The optimum rotation age of random search solutions had a 10-year range.

When maximizing the average yearly net cash flow, i.e., the forest rent (which corresponds to the 0% interest rate case), dynamic programming performed much better, relatively speaking (Table 2). Direct search optimum solutions varied considerably depending on the randomly chosen solution given to the algorithm as a starting point. For example, the optimum rotation length varied from 66 to 89 years. In this case, 10 runs were needed to find a solution that would rival the dynamic programming solution in terms of objective function value. Of the methods tested, random search achieved the lowest objective function values in forest rent maximization and performed worse than in the 3% interest rate case. This was expected, because the same number of trials (25 000 per run) was used to optimize a larger number of variables.

The objective function defines a response surface, which shows the result of applying various combinations of the control variables of stand management. The differences in the best rotation lengths of the random search solutions in Tables 1 and 2 indicate that the response surface was

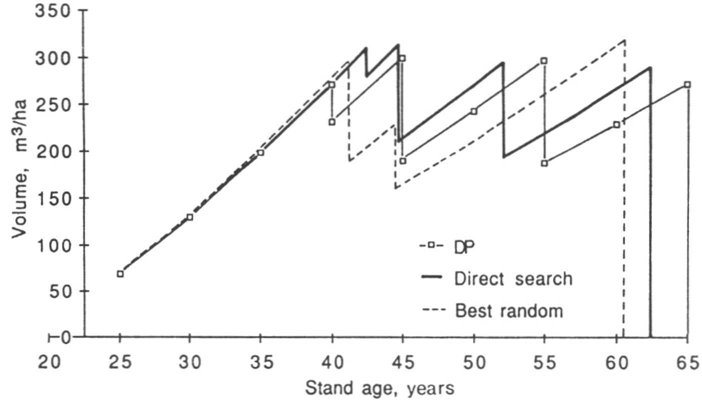

FIG. 1. Optimum volume development for regimes derived by dynamic programming (DP), direct search and random search. Interest rate = 3%.

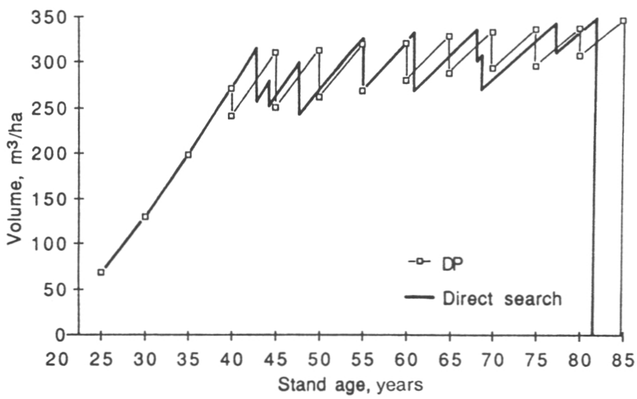

FIG. 2. Optimum volume development for regimes derived by dynamic programming (DP) and direct search. Interest rate = 0% (forest rent).

remarkably flat with regard to rotation length. The solutions reported in the present study indicate that when the thinning schedule is adjusted according to rotation length, equally profitable management regimes may be found for a fairly wide range of rotation ages. The weak significance of rotation length should not be generalized to other types of thinning schedules.

The various optimum solutions are viewed in terms of volume development in Fig. 1 for the 3% interest rate case and in Figs. 2 and 3 for the forest rent case. With a 3% interest rate, the dynamic programming solution was quite similar to the direct search solution (Fig. 1). The fact that it gave a lower objective function value than the random search solution is mostly due to the 5-year time steps used in dynamic programming to define the available points of time for cuttings. This reduced the largest attainable objective function value.

The random search optimum solution for forest rent maximization (Fig. 3) had only four thinnings, although solutions with up to nine thinnings were generated. The number of variables to be optimized in both direct and random search was $2 + 3m$, where m is the number of thinnings. In random search, the number of thinnings was obtained from a uniform distribution of integers between 0 and 9, and the search effort was equal for all numbers of thinnings. However, the probability of finding a high-valued solution is greater for smaller problems with less thinnings and variables. As a result, the random search optimum solution was biased towards a smaller number of thinnings.

When maximizing the soil expectation value with a 3% interest rate, the results concerning optimum species composition (Fig. 4) were in line with those regarding growing stock levels. Although dynamic programming had a lower objective function value than random search solutions, because of a lack of flexibility in thinning timing, the optimum species composition solution given by dynamic programming was closer to the highest optimum (given by direct search) than were solutions by random search. Dynamic programming approximated the direct search optimum solution in all dimensions involved in the optimization, contrary to random search.

The dynamic programming solution for species compo-

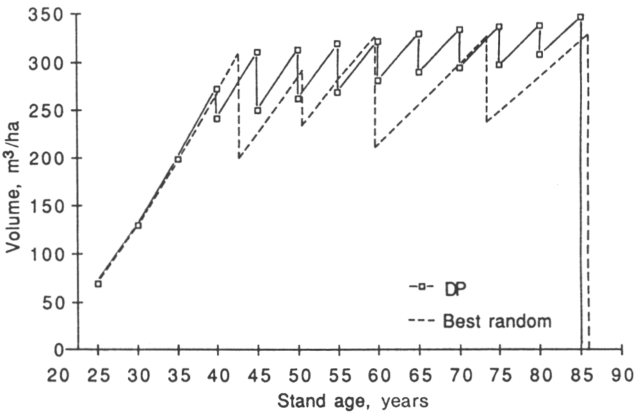

FIG. 3. Optimum volume development for regimes derived by dynamic programming (DP) and random search. Interest rate = 0% (forest rent).

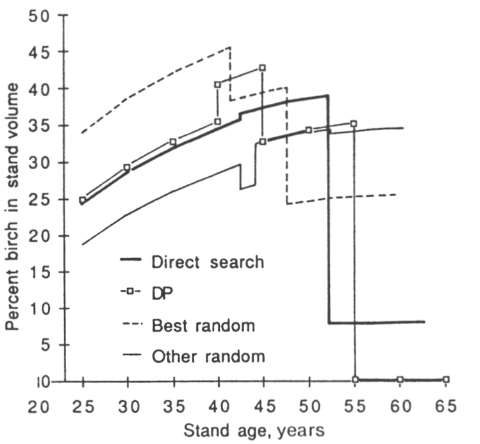

FIG. 4. Optimum species composition derived with direct search, dynamic programming (DP), and two different random search runs. Interest rate = 3%.

sition (Fig. 4) implied an unexpected jump of birch proportion at the age of 40 years. It seems to result from an economically justified attempt to end up with a pure pine stand at the end of the rotation. On the dynamic programming optimum path, the birch proportion was 35.3% at the age of 55 years. A value close to 0 (0.3%) can be reached from 35.3% through an allowable control that in the dynamic programming formulation used in this study, changes the birch proportion by multiples of 5% steps.

Dynamic programming optimization may be performed to various levels of accuracy, depending on the state variable intervals used. However, computational time is strongly affected by the choice of accuracy. The effect of the volume interval was examined while the intervals of all other variables were kept at their base values. Each run had ten 5-year time steps. Results from runs on a DEC VAX-11/785 computer showed that the volume interval had to be decreased significantly before the objective function value started to decrease markedly (Table 3). A comparison with results reported in Kao and Brodie (1980), Valsta (1985), and Roise (1986) suggests that small time intervals are important in achieving a dynamic programming solution with a high objective function value.

TABLE 3. The effect of volume interval on the objective function value and execution time in dynamic programming runs

Volume interval (m^3/ha)	Objective function value[a]			CPU (s)
	Marks/ha	$/ha	%	
10	15 393	4160	100	890
20	15 363	4152	99.8	333
50	15 227	4115	98.9	95
100	15 077	4075	97.9	12

[a]Monetary units are Finnish marks and Canadian dollars; 1 Canadian dollar = 3.7 Finnish marks. Percent values are proportional to the largest value achieved.

The random search runs used around 120 s of CPU time each. The direct search analysis used 30–100 s, and it required the input of an initial solution for the algorithm. The objective function value determined by each optimization method can be compared with the computing time used, and this ratio may be defined as the efficiency of the method. Based on the two optimization problems analyzed, the 3 and 0% cases, it can be noted that direct search was more efficient than random search. The efficiency of dynamic programming, compared with the other two methods, was variable. In the 3% case, the efficiency of dynamic programming was lower than that of the other methods.

In the 0% case, individual runs of the two search methods still consumed much less CPU time than dynamic programming runs. However, several direct search runs were required to arrive at a solution comparable to dynamic programming, in terms of objective function value. This reduced the efficiency of direct search to the same level as that of dynamic programming. The efficiency of random search was now lower than that of the other methods; the objective function value of the best random search solution, obtained among 10 runs of 25 000 trials, was 96% of the highest value (Table 2).

Concluding remarks

In this study, the management of a two-species, even-aged stand was optimized using three different optimization methods and a whole-stand growth model. The analysis required dynamic optimization in a three-dimensional state space (stand volume, number of trees, and species composition). The number of thinnings considered varied from 0 to 9, and the resulting number of control variables varied from 2 to 29.

The performance of the solution methods tested, relative to each other, was dependent on the problem size, i.e., the number of thinnings and variables to be optimized. In the present study and in the study by Roise (1986), direct search and random search solve the optimization problem in control variable space, whereas dynamic programming operates in state variable space. The dimension of the state space does not depend on the number of thinnings. On the other hand, the dimension of the control space increases with an increasing number of thinnings. Which of the variable spaces is more advantageous depends on the problem and the available optimization methods.

Dynamic programming was the most robust method among those tested. Its solutions were always close approximations of the accurate optimum in all dimensions of the state space. By definition, dynamic programming finds the global optimum solution, subject to the discretization of state space. The disadvantages associated with dynamic programming include relatively low computational efficiency when the number of thinnings is small and the fact that it is inapplicable to stand growth models with more than three or four state variables.

Direct search with the Hooke and Jeeves method performed computationally efficiently and determined the highest objective function values among the methods tested. However, when optimization involved several thinnings, the nonconvexity of the problem increased and results became dependent on the starting solution given as an input. Solution stability was weak, for example regarding the optimum rotation for forest rent maximization. Repeated runs were required to assume a globally optimal solution. Consequently, when employing this method, an analyst must be familiar with the method and the characteristics of the particular problem to be solved.

Random search gave larger or smaller objective function values than dynamic programming. For variables with a flat response surface, random search gave inconsistent optimum solutions that did not provide informative management guidelines. On the other hand, when the optimization problem was relatively small (one or two thinnings), the method performed competently in optimizing growing stock level and rotation length. This method provides an opportunity to substitute computing effort for analyst expertise.

The primary use of results from stand level optimization is to define profitable stand treatment schedules. A forest manager might not be too concerned whether the solution to be applied is exactly correct in terms of net revenues, or 2–3% away from the optimum. Another important application of optimization is to examine the effects of changes in costs, prices, and other parameters of stand management. This type of analysis demands greater accuracy and certainty of the optimum solution. It appears doubtful that methods with direct search strategy could be used for this type of analysis, except in problems with only a few decision variables (for an example, see Roise et al. 1988). Numerous local optima seem to be found in optimization problems with more than one thinning and the times between thinnings as decision variables. They may prove intractable for search methods utilizing a hill-climbing strategy. An important area for further research would be to explore the reasons for local optima and the possibilities to smooth the response surface of a stand simulator.

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Appendix: stand growth simulator

The simulator is based on the growth and yield study by Mielikäinen (1980). The growth model is classified as a whole-stand model and is for even-aged Scots pine and silver birch stands in southern Finland. It gives the total volume growth percentage (P,%) as a function of stand breast height age ($T_{1.3}$), total stand volume (V, m^3/ha), and birch percentage of volume (B, %):

$$\text{[A1]}\quad \ln(P) = 7.050 - 0.8732\ln(T_{1.3}) - 0.4187\ln(V) - (0.5154 \times 10^{-6}BT_{1.3}^2) - 1.403\,[(50 - B)^2/T_{1.3}^3]$$

The volume growth percentage is an annual average of a 5-year future growth period. When using random search or direct search as optimization methods, growth predictions are needed for time periods of varying length, i.e., not necessarily multiples of 5 years. In these cases, it is assumed that the growth of the last period, being shorter than 5 years, is proportional to the length of the period. To allocate the predicted total volume growth between the two species, a table from Mielikäinen (1980) is used (Table A1). Intermediate values are linearly interpolated.

The data used in the study by Mielikäinen (1980) were from managed stands only. A mortality model was not estimated in that study, nor is one available for the same population of stands from other studies. The growth model [A1] does not realistically predict the growth of fully stocked stands that have mortality due to high stand density. An *ad hoc* model, based on the data set in Mielikäinen (1980), was constructed as a simple volume limit, above which growth is terminated. This causes optimization to avoid the high stocking levels, because a failure to thin before the stand reaches the volume limit results in loss of growth. The volume limit (V_{max}, m^3/ha) is defined as a function of stand age (T, years)

$$\text{[A2]}\quad V_{max} = 273.42 + 0.9308T$$

During optimization, extremely heavy thinnings would occur, with more than 50% of the growing stock removed. The growth model has no variable that would carry on information from past thinnings and would reduce growth after extremely heavy thinnings. This type of information would, in fact, cause the growth simulator to violate the principle of optimality of dynamic programming, unless the variable containing this information would be chosen as one

TABLE A1. Pine growth as a percentage of the total volume growth of a mixed stand

Pine % of stand vol.	Stand age (years)										
	30	35	40	45	50	55	60	65	70	75	80
10	7	7	7	7	8	8	8	8	8	8	9
20	15	15	15	15	15	16	17	17	18	18	18
30	23	23	23	24	25	26	27	27	28	28	28
40	32	32	32	34	35	36	37	38	38	39	39
50	41	41	41	43	45	46	47	48	48	49	50
60	50	50	50	53	55	56	57	58	59	60	61
70	60	60	60	63	65	67	68	69	70	71	72
80	71	71	71	74	76	78	79	81	82	82	83
90	83	83	83	85	87	89	90	92	93	94	95

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TABLE A2. The parameter values of equations to estimate volumes by product classes

Dependent variable (proportion)	Parameter		
	b_0	b_1	b_2
Pine, sawtimber	26.509	−29.004	0.050 616
Birch, sawtimber	−2.7634	1.3414	−0.573 64
Pine, nonmerchantable	−15.687	21.395	0.062 216
Birch, nonmerchantable	−17.802	23.469	0.061 549

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TABLE A3. Stumpage price adjustment (Finnish marks) based on average tree size of a cut

Avg. stem size (m^3)	Adjustment ($marks/m^3$)
0–0.3	−6
0.301–0.4	−3
0.401–0.5	0
0.501–0.6	4
≥0.601	8

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TABLE A4. Stumpage price adjustment (Finnish marks) based on the volume cut

Vol. cut (m^3/ha)	Adjustment ($marks/m^3$)
0–12.5	−24
12.5–25	−18
25.1–30	−11
30.1–50	−7
50.1–60	−5
60.1–75	−2
75.1–100	0
100.1–125	2
125.5–150	3
150.1–175	5
175.1–250	6
≥250.1	8

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of the state variables in the dynamic programming model. Growth predictions after very heavy thinnings would most likely be erroneous with the current growth model. To avoid this problem, a maximum value for thinning intensity is set. This value is subjectively chosen to be 40%, and it is computed from total stand volume.

In a whole-stand growth simulator, the average tree size is a useful variable to determine stumpage values and logging costs. Because the simulator is restricted to operate in stands where, by presumption, no mortality occurs, the number of trees remains constant between thinnings. Assuming mechanical thinnings (average dbh after thinning equals that before thinning), it is possible to simply add the number of trees to the simulator as an additional state variable that does not affect stand development. The average tree size is assumed to be equal for both tree species and is obtained by dividing stand volume by the corresponding total number of trees.

The functions estimated in Valsta (1986) are used to compute the volumes of product classes (pulpwood, sawtimber). The function form for the proportions (P) of sawtimber and nonmerchantable wood is as follows:

$$[A3] \quad \frac{1}{1 + \exp[b_0 + b_1(V/N)^{b_2}]}$$

where V is stand volume, N is the number of trees, and b_0, b_1, and b_2 are the parameters of the regression equation. The parameter values for each equation are given in Table A2.

The stumpage prices are obtained from Valsta (1986) and are based on historical trends. The price for pine sawtimber is 203.59 Finnish marks/m^3 (approximately $58 (Canadian)/$m^3$, or $1.38 (U.S.)/$ft^3$). The prices for other wood assortments, relative to pine sawtimber, are 93% for birch veneer logs, 45% for pine pulpwood, and 37% for birch pulpwood. To reflect logging costs, the stumpage price is adjusted on the basis of average tree size and total volume removed in any one thinning or final harvest (Valsta 1986). These figures were obtained from the stumpage price agreement between Finnish forest industry and the farmers' association and are reproduced in Tables A3, and A4. The discounted total regeneration costs used in computing soil expectation value are 4500 and 4300 Finnish marks/ha for 0 and 3% real interest rates, respectively. It is assumed that any species composition at the initial age (25 years) can be achieved with the same amount of investment in regeneration.

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ACTA FORESTALIA FENNICA 232

AN OPTIMIZATION MODEL FOR NORWAY SPRUCE MANAGEMENT BASED ON INDIVIDUAL-TREE GROWTH MODELS

Kuusikon käsittelyn optimointi puittaisiin kasvumalleihin pohjautuen

Lauri Valsta

Approved on 28.10.1992

The Society of Forestry in Finland – The Finnish Forest Research Institute
Helsinki 1992

Valsta, L. 1992. An optimization model for Norway spruce management based on individual-tree growth models. Tiivistelmä: Kuusikon käsittelyn optimointi puittaisiin kasvumalleihin pohjautuen. Acta Forestalia Fennica 232. 20 p.

A nonlinear programming algorithm was combined with two individual-tree growth simulators consisting of distance-independent diameter and height growth models and mortality models. Management questions that can be addressed by the optimization model include the timing, intensity and type of thinning, rotation age, and initial density.

The optimum thinning programs were characterized by late first thinnings (at a dominant height of 15–17 m) and relatively high growing stock levels. It was optimal to thin from above, unless mean annual increment was maximized instead of an economic objective. In most cases, the optimum number of thinnings was two or three. Compared to a no-thinning alternative, thinnings increased revenues by 15–45 % depending on the objective of stand management. Optimum rotation was strongly dependent on the interest rate.

Hooke and Jeeves' direct search method was used for determining optimum solutions. The performance of the optimization algorithm was examined in terms of the number of functional evaluations and the equivalence of the objective function values of repeated optimizations.

Tutkimuksessa laadittiin metsikön käsittelyn optimointimalli yhdistämällä epälineaarisen ohjelmoinnin algoritmi kahteen simulaattoriin, jotka koostuivat puittaisista läpimitan ja pituuden kasvumalleista ja kuolemismalleista. Optimointimalli soveltuu harvennusten ajoituksen, voimakkuuden ja tavan sekä kiertoajan ja metsikön perustamistiheyden samanaikaiseen tarkasteluun.

Tuloksia laskettiin eteläsuomalaisille kuusikoille, lähinnä OMT:tä vastaaville kasvupaikoille, joissa puuston tiheys taimikonhoidon jälkeen oli n. 2000 kpl/ha. Laskelmissa käytettyjen taloudellisten tekijöiden arvojen vallitessa oli edullisinta harventaa puusto ensimmäisen kerran varsin myöhäisessä vaiheessa (15–17 metrin valtapituuden kohdalla). Suhteellisen korkeat puustopääomat (pohjapinta-ala 25–40 m²/ha) olivat optimaalisia. Harvennusvoimakkuus riippui harvennusten lukumäärästä. Yläharvennus oli edullisin harvennustapa muulloin paitsi tilavuuskasvua maksimoitaessa. Optimaalinen harvennusten lukumäärä oli kahdesta kolmeen ja harvennukset lisäsivät tuottoja puunkasvatuksen tavoitteesta riippuen 15–45% kiertoajan kuluessa harventamattomaan vaihtoehtoon verrattuna. Korkokanta vaikutti voimakkaasti optimikiertoaikaan, joka vaihteli 70 vuodesta yli 110 vuoteen.

Optimiratkaisut määritettiin Hooken ja Jeevesin suorahakumenetelmällä. Optimointialgoritmin toimivuutta arvioitiin funktioevaluointien lukumäärän ja toistettujen optimointien tavoitefunktion arvojen yhdenmukaisuuden perusteella.

Keywords: stand management, optimization, thinning, rotation, initial density, individual-tree simulator, Norway spruce, *Picea abies*.
FDC 174.7 *Picea abies* + 24

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Contents

1 Introduction

Controlling the amount and structure of the growing stock is one of the most important decisions in forestry. In practical decision-making, the true optimum treatment of a stand always depends on the rest of the forest property and the economic setting of the forest owner. Results concerning single, independent stands are, however, useful for management guidelines or comparative analyses where the effects of some economic or biological factors of stand management are studied.

At the present stage of forest modelling, regeneration optimization is deficient because of lack of usable models for stand regeneration and juvenile development. Considering individual stands, it is useful from optimization point of view to separate management questions regarding thinnings and final harvest from those dealing with regeneration.

The main decision problem in this study is the following: Assume that we have a young, established stand, free of competing vegetation. How should we treat the stand for the rest of the rotation? We are interested in the optimum combination of

- the number of thinnings
- the timing of thinnings
- the amounts thinned
- the types of thinnings (from below, from above)
- rotation
- pre-commercial thinning (tending)

Traditionally, these questions have been studied on the basis of field experiments incorporating sets of treatment regimes. At best, partial questions can be answered by these experiments consisting of a limited selection of treatment alternatives. To determine an optimum combination of many variables, we must resort to computer-based models. To accomplish this we need both a set of models that predict the development of any given stand subject to various treatments, a stand simulator, and a procedure for finding an optimum set of treatments, an optimization algorithm.

Thinnings have several economic effects. They affect the amount of capital invested in standing timber, the short and long term growth rates of standing trees, mortality due to different causes, the quality of remaining trees, future logging costs, and the optimum rotation. The complete effects of a thinning are not realized until the end of the rotation. Other decisions that may interact with thinnings include initial density, vegetation management, precommercial thinning, and fertilization.

One interesting management question is thinning type, i.e., whether one should thin the smaller or the larger trees of a stand. Thinning type has been a controversial issue in Finland, especially after the 1948 "Thinning Declaration" (Julkilausuma 1948). The thinning declaration banned thinnings from above because selection harvesting had led to devastation in Finnish forests and it was considered that the best way to improve thinning practices was to allow only thinning from below.

Finnish research on thinning types was not activated until the 1960s: the first results were published by Vuokila from temporary plots (1970) and remeasured plots (1977). Later measurements of the latter were reported by Mielikäinen & Valkonen (1991), who also computed monetary returns. Results from another set of experiments were reported by Hynynen & Kukkola (1989). Comparable studies in terms of biological conditions have been made in Sweden (Eriksson 1990). Regarding spruce stands, the Finnish and Swedish results suggest that thinning from above slightly reduces volume growth, compared to thinning from below. In present value computations, thinning from above was slightly superior (Mielikäinen & Valkonen 1991).

Finnish model-based analyses that include thinning type are limited to two. Both of them are based on whole-stand growth models and, as such, make simplifying assumptions about thinning returns and future growth relating to different thinning types. Kilkki & Väisänen (1969) used dynamic programming to determine the optimum thinning program for Scots pine stands and made separate analyses for thinning from below and above. Their results showed 2 to 12 % higher present values for thinning from above, when comparing optimum regimes of each thinning type. However, their analysis did not account for the increase in unit value of standing timber due to thinning from below, compared to thinning from above. Hämäläinen (1978) analyzed thinning types using a set of thinning alternatives based on the growth functions by Vuokila (1967) for Scots pine stands. In this study, as well, thinning type did not affect the growth of the stands and the results are clearly conditional. Thinning from below was more profitable when the interest rate was 2 % or less, whereas thinning from above was superior with interest rates of 3–5 %.

A Swedish study (Olsson 1986) utilizing individual-tree growth functions (Söderberg 1981 and Elfving 1982, ref. Olsson 1986) indicates that thinning from above is slightly superior under both a volume and a value criterion. However, the results for Norway spruce stands are based on only a few simulations.

The studies cited so far are based on roundwood market prices. If wood processing is taken into account in the computations, the results may be altered. This is because at least in Finland, roundwood market prices do not completely correspond to values derived from wood products, such as lumber. An example from Sweden is reported by Persson (1986) where log quality was considered as a pricing factor. This improved the profitability of thinning from above because most rapid diameter growth (resulting from thinning from below) was avoided and the tree rings did not become excessively wide.

Methods that have been used in other countries for optimizing thinning type include dynamic programming (Haight et al. 1985, Arthaud & Klemperer 1988, Torres-Rojo & Brodie 1988, Yoshimoto et al. 1990), nonlinear programming (Roise 1986b, Bare & Opalach 1987), and discrete time optimal control theory (Haight 1987, Solberg & Haight 1991). Given the large selection of species, growing conditions, and economic parameters, it is not reasonable to try to form an overall conclusion about the optimum thinning type. However, a pattern found in several of the studies is that thinning from below is optimal when maximizing the mean annual increment and thinning from above when maximizing discounted values. Most of the studies were made with various pine species. An optimum solution for Douglas fir, a species resembling Norway spruce, had a precommercial thinning from below and a commercial thinning from above (Roise 1986b).

The principal objective of the present investigation is to develop an optimization model for analyzing stand treatment options based on individual-tree, distance-independent growth models. A solution to this problem was first presented by Roise (1986a). (Kao & Brodie (1980) solved the same problem for a whole-stand growth model.) A model based on Finnish growth models (Mielikäinen 1985) was presented by Valsta (1987), but with limited numerical results. A more operational form of the optimization model was reported in Valsta (1992), but the main objective of that study was stochastic optimization. Using a deterministic version of the improved optimization model, the present study examines the effects of different elements of the optimization problem, such as the objectives of stand management, the number of available thinnings, the initial density, and the growth models. The numerical results concern examples of Norway spruce stands in southern Finland. The results are compared with previous research, as well as the present recommendations of a forestry extension organization in Finland. The performance of the optimization algorithm is also tested.

2 Stand simulators

2.1 Growth and mortality models

The growth models of the Finnish forestry planning system 'MELA' were chosen as the basic individual-tree, distance-independent growth simulator (Ojansuu et al. 1991). Optimization results may be strongly dependent on the growth models, and so for comparison, another set of models was chosen from the study by Mielikäinen (1985). The function forms of the models are presented in Appendix 1. Both sets of growth models have been estimated from temporary sample plot material. However, the MELA models have been extensively tested against repeated forest inventories and remeasured permanent plot data (Ojansuu et al. 1991).

The basic mortality model chosen (Haapala 1983, also used by Ojansuu et al. 1991) predicts individual tree mortality in managed stands free of large scale mortality. The model does not apply to stands at limiting densities. Another model (Hynynen 1991) identifies the self-thinning curves based on stand level variables. Because of lack of comprehensive data, the two models given above do not predict mortality reliably for older stands with less than 500 trees per hectare. These conditions are found by the optimization algorithm and taken advantage of. To handle these cases, live crown ratio is linked to mortality. This is achieved by combining a live crown ratio model (Mielikäinen 1985, Equation 17) with two ad hoc models. It should be noted that the crown ratio model is used for a purpose not intended by Mielikäinen (1985, p. 28–29). This is done because no other models are available. The mortality models are described in more detail in Appendix 1.

2.2 Thinning model

The thinning model defines the number of trees cut in each diameter class, and it is the same as reported in Valsta (1992). Being essential for the thinning type analysis, the model description is repeated here.

The present thinning type model was designed with the idea of being able to vary the accuracy and the number of variables required. To achieve reliable solutions with the available nondifferentiable optimization algorithms, the number of decision variables must be kept to the minimum. This can be done by specifying thinnings by groups of diameter classes (Haight & Monserud 1990, Yoshimoto et al. 1990), by removing trees from below or above (Haight et al. 1985, Haight 1991), or by using a diameter distribution function with its parameters as decision variables (Bare & Opalach 1987).

In the present study, a piecewise linear function defines thinning intensities as a function of tree diameter, relative to the smallest and the largest diameters of the stand at time of thinning. Thinning parameters define thinning rates (percentages of trees cut) at the corner points of the piecewise linear function. At one extreme, there may be only one thinning parameter, and the thinning rate is constant across all tree diameters. At the other extreme, there may be a thinning parameter for each diameter class. As an example, suppose that we wish to use three parameters, p_1, p_2, and p_3, to define a thinning. They denote thinning rates at the minimum, midpoint, and maximum tree diameter, d_{min}, d_{mid}, and d_{max}, respectively. Thinning rates for other diameters are computed using linear interpolation. An example of the thinning specification is shown in Fig. 1, with three parameters to define the thinning intensities for different diameters. The first and last parameter always refer to the smallest and largest diameter of the stand at time of thinning.

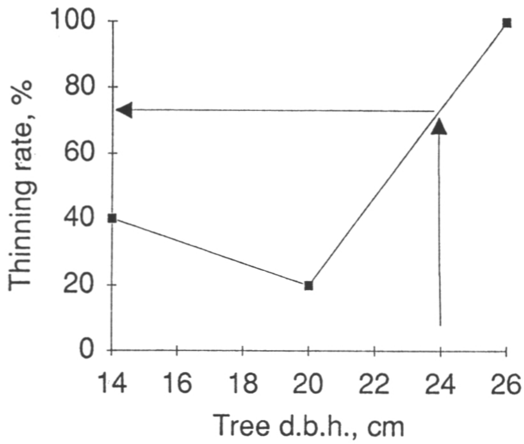

Fig. 1. Thinning specification based on three parameters for a stand with minimum and maximum breast height diameters of 14 and 26 cm, respectively. For example, the thinning rate for 24 cm trees is 73 %.

2.3 Yield models and economic data

Tree total volumes and volumes by wood assortment are computed based on the models by Laasasenaho (1982) using computer programs by Laasasenaho & Snellman (1983). Wood assortment volumes are based on tree dimensions only – no deductions due to defects are made. The amount of sawtimber produced is thus overestimated but the importance of this bias is reduced by the small difference between the road side prices of spruce sawlogs and pulpwood.

Logging costs are based on the Finnish logging and hauling work tariffs (Valsta 1992). The tabulated tariff values were smoothed to form equations of total logging cost as functions of average tree size and total volume harvested. The models are given in Appendix 1.

Road-side values for spruce sawlogs and pulpwood are 210 and 180 FIM/m^3, respectively. Regeneration costs are assumed to be 4300 FIM per hectare. When initial density (trees/ha) is a decision variable, regeneration cost (FIM/ha) is computed as:

$$\text{cost} = 1800 + 1.25\ \text{density} \quad (1)$$

Soil expectation value is computed based on a series of equal rotations. All costs and revenues are discounted to stand age 0, and then transformed to an infinite series using coefficient $(1 + i)^T / [(1 + i)^T - 1]$, where i is the decimal interest rate and T is rotation length.

2.4 Restrictions of the models

The model set employed causes some important factors to be ignored in the analysis. These include

- improvement of stand quality by thinnings
- spatial distribution of trees and skid roads
- logging damage to remaining trees

Also, the growth models are used for predicting growth in stands thinned from above, whereas the sample trees were taken from stands thinned mostly from below. Further, risk and uncertainty are not accounted for: growth and mortality of trees are deterministic, prices and costs are known and constant over time.

2.5 Plot data for simulation

Measurements of three experimental plots are used as starting points for stand simulation (Table 1). The plots were established and are managed by the Finnish Forest Research Institute, Department of Forest Production. All plots are of planted Norway spruce. The diameter distributions of the initial stands are shown in Fig. 2 and the complete listings by diameter class are given in Appendix 2. The diameter distribution of plot 11302 was expanded by duplicating diameter classes 1 to 4 cm in order to give more room for thinning type optimization. Tree height values are based on sample tree measurements.

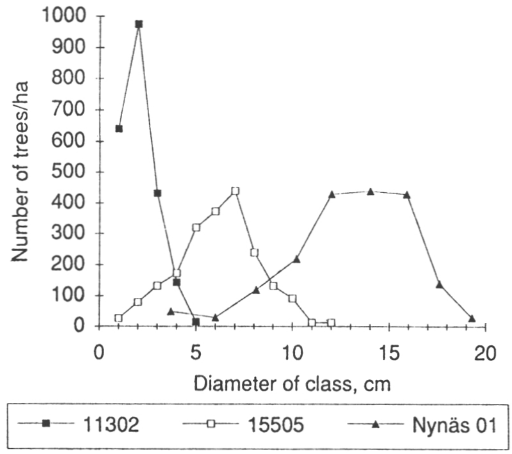

Fig. 2. The diameter distributions of plots 11302 (14 years old), 15505 (21 years old), and Nynäs 1/01 (40 years old).

Table 1. Characteristics of the three plots used as initial stands. For symbols, see App. 1.

Plot number	*H*100	Age	D_{ba}	*Hdom*	Basal area	Trees/ha
11302	28	14	2.8	3.4	0.8	2208
15505	29	21	7.4	7.3	6.8	2038
Nynäs 1/01	29	40	14.4	14.8	37.0	1890

3 Optimization method

The optimization method is a deterministic version of the one applied by Valsta (1987, 1992). The basic approach is after Kao & Brodie (1980) and Roise (1986a). Fig. 3 shows the overall structure of the simulation-optimization model. The stand growth simulator is depicted as a black box to reflect the fact that the optimization algorithm knows of the simulator only by the objective function values it obtains in return for decision variable vectors.

Hooke and Jeeves' (1961) direct search method is used as the optimization algorithm. It is classified a derivative free, multidimensional search method for unconstrained nonlinear programming (Bazaraa & Shetty 1979). Compared to other alternatives, it has performed well with whole-stand and individual-tree growth models (Roise 1986a, Linkosalo 1991).

Hooke and Jeeves' direct search algorithm operates using two search modes: exploratory search and pattern search. Given a base point, exploratory search examines points around the

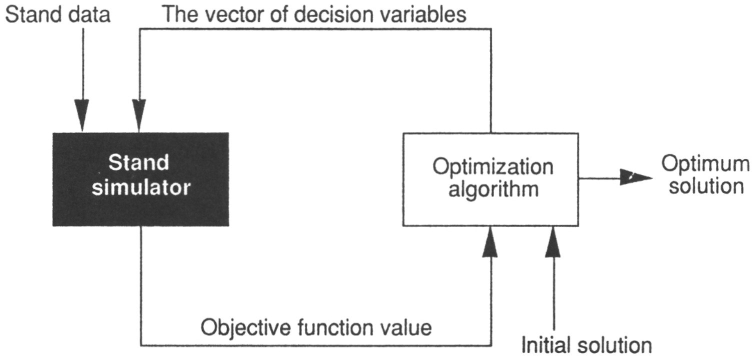

Fig. 3. The structure of the optimization-simulation model.

base point in the directions of coordinate axes. Pattern search moves the base point in the direction defined by the previous base point and the best point of exploratory search. An example of the algorithm operation is seen in Fig. 4, where the function y = $(x_1 - 2x_2)^2 + (x_1 - 2)^4$ is minimized. The points generated, joined with line segments, are labelled. The initial point is (10,10). The short horizontal and vertical moves are the exploratory searches and the longer jumps are the pattern searches. The first ten pattern moves lead to point (2.4,1.2) with a function value 0.0256, while the true optimum is at point (2,1) with a function value 0.

The vector of decision variables consists of times between forest operations and information on how the operation is executed, e.g., thinning percentages at different tree diameters, or the number of plants per hectare. When optimizing the rotation, two thinnings defined by three parameters (thinning percentage for the smallest, medium-sized, and the largest trees), and initial density, the vector of decision variables is, e.g.:

(14.2,	time from the last thinning to the final cut
29.9,	time from the start of the simulation to the first thinning
0.00,	first thinning, thinning percentage for the smallest trees
51.1,	first thinning, thinning percentage for medium-sized trees
100.0,	first thinning, thinning percentage for the largest trees
14.5,	time from the first thinning to the second (last) thinning
0.00,	second thinning, thinning percentage for the smallest trees
70.2,	second thinning, thinning percentage for medium-sized trees
100.0,	second thinning, thinning percentage for the largest trees
2200)	trees per hectare

This is the optimum solution with 3 % discount rate for plot 11302 with MELA growth models (two thinnings).

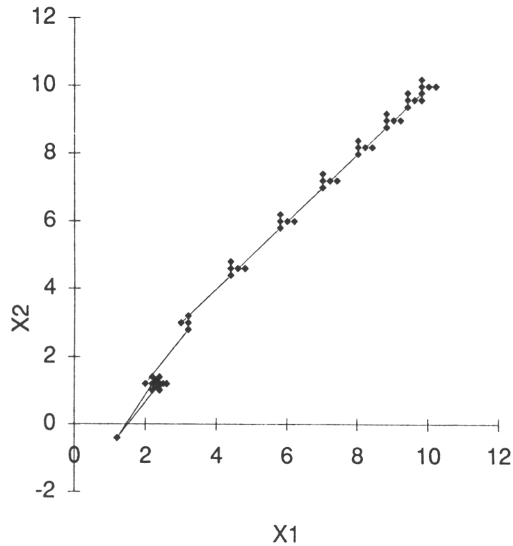

Fig. 4. Sequence of points on plane x_1x_2 generated by Hooke and Jeeves' algorithm when minimizing the function $y = (x_1 - 2x_2)^2 + (x_1 - 2)^4$ from initial point (10,10).

4 Results

4.1 Optimum thinning programs for different goals of stand management

Unless otherwise stated, the results presented are based on the MELA growth models (Ojansuu et al. 1991) and plot 15505 tree list as the initial stand. Stand volume development in the optimum thinning programs is shown in Fig. 5 for maximizing volume production, i.e., mean annual increment (M.A.I.), as well as soil expectation values at 1, 3, and 5 % interest rate. Thinnings were defined by three parameters and a maximum of four thinnings are considered.

As anticipated, the higher the interest rate, the shorter the rotation, and the earlier, the fewer and the heavier the thinnings. The solution for maximum M.A.I. with 4 thinnings produced 10.2 m^3/ha/yr in a 96-year rotation. Thinning intensity varied from 20 to 27 % of volume, and thinnings both captured mortality and removed larger trees. For an unthinned stand (not shown in Fig. 5), the optimum rotation was 71 years with an M.A.I. of 8.7 m^3/ha/yr. 21 % of the trees (35 m^3/ha) died during the rotation. Although thinnings can not increase periodic volume increment in a stand, they can increase the total volume harvested per year during the rotation. A part of the increase results from a longer optimum rotation in the thinning case. Then, a proportionally shorter part of rotation is used by the young stand period where volume growth is small.

4.2 Optimum thinnings and rotation for different numbers of thinnings

The optimization of thinnings and rotation for each number of thinnings constitutes a different problem with a corresponding number of variables. The number of thinnings is therefore a parameter given to the computer program at the outset. Optimum solutions for 1, 2, and 3 thinnings are shown in Fig. 6 in terms of stand basal area and dominant height. According to expectations, thinning intensity decreased with an increasing number of thinnings. The time of the first thinning was about the same in all cases. Dominant height was reduced at the time of thinning because the thinnings were from above.

The optimum rotation age increased with the

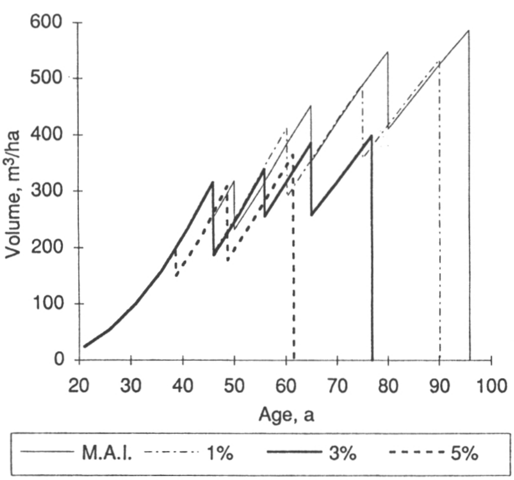

Fig. 5. Optimum thinnings and rotation for different objectives: maximum Mean Annual Increment (M.A.I.), and maximum soil expectation value at 1, 3, and 5 % discount rate. Thinnings are defined by three parameters.

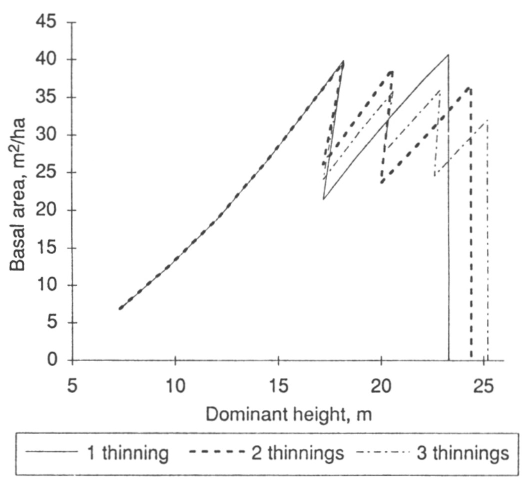

Fig. 6. Basal area development in the optimum thinning programs with 3 % interest rate for one, two, and three thinnings.

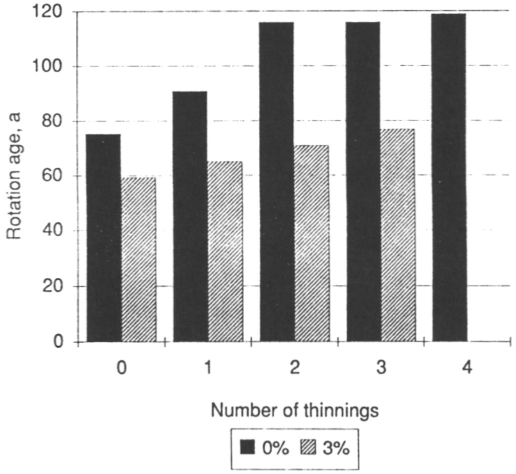

Fig. 7. Optimum rotation age for different numbers of thinnings for average annual net cash flow (0 %) and 3 % soil expectation value.

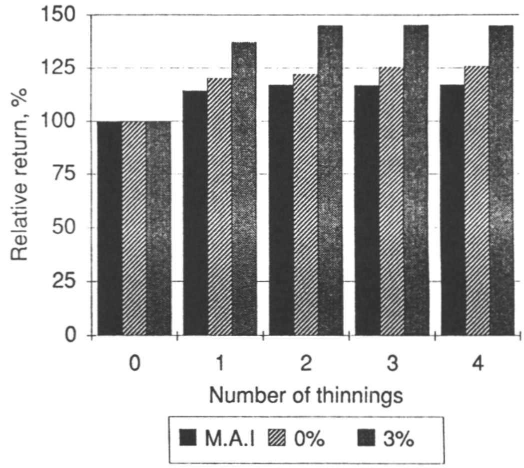

Fig. 8. Relative returns in relation to the number of thinnings for different objectives: maximum mean annual increment (harvested), average annual net cash flow (0 %), and soil expectation value with 3 % interest rate.

number of thinnings (Fig. 7). The effect was stronger with no discounting (0 %, i.e., forest rent case). The average diameter at final harvest was only slightly increased by thinnings: with a 3 % interest rate it amounted to 18, 21, 22, and 23 cm for 0, 1, 2, and 3 thinnings, respectively.

Returns to thinning were notable with all the criteria used and more so with a greater interest rate (Fig. 8). However, most of the gain was achieved with just one thinning. Optimum regimes with two to four thinnings produced about the same return. The fact that the returns were almost constant for regimes with two, three, or four thinnings may not hold for other (suboptimal) thinning regimes.

4.3 Optimum thinning type and initial density

The optimum thinning type (thinning from below/above, or low/high thinning) was affected by the objective of stand management and by stand density. In Fig. 9, thinning type is defined as the ratio of average diameter of trees thinned to average diameter of all trees (arithmetic averages), denoted by d/D. The ratio d/D < 1 implies thinning from below and d/D > 1 indicates thinning from above. With an economic objective, thinning from above was optimal in most situations.

When maximizing volume production, both the smallest and the largest trees were thinned. The first thinning was from above which was exceptional in view of other results for the same objective. The optimum regimes with one or two thinnings (not shown in Fig. 9) did not include thinnings from above but employed thinnings from both ends of the diameter distribution.

The optimum thinning program depends on the initial density. The basal area development in optimum thinning regimes with different initial densities is shown in Fig. 10. Soil expectation value was computed using density independent regeneration costs because initial density was not a decision variable in this analysis. Natural regeneration would be an appropriate assumption in this case.

Precommercial thinning was profitable only for the stand with 4400 trees per hectare. About 1000 trees (average d.b.h. 3.4 cm) were removed at a cost of 760 FIM/ha at 15 years. This reduced mortality during the following 25 years (the time up to the first commercial thinning) from 10 % to 3 %.

The time of the first commercial thinning was set in the optimum solution by the optimization algorithm so that adversely high growing stock levels were avoided. The optimum number of commercial thinnings was 2 for 1100 trees/ha and 3 for 2200 trees/ha, and 2 for 4400 trees/ha.

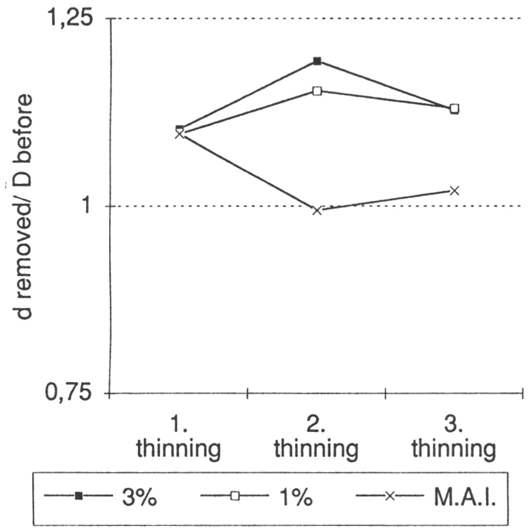

Fig. 9. Optimum thinning type (d/D -ratio) when maximizing soil expectation value with 1 % and 3 % interest rate, and M.A.I. in three-thinning regimes. Thinnings are defined by three parameters.

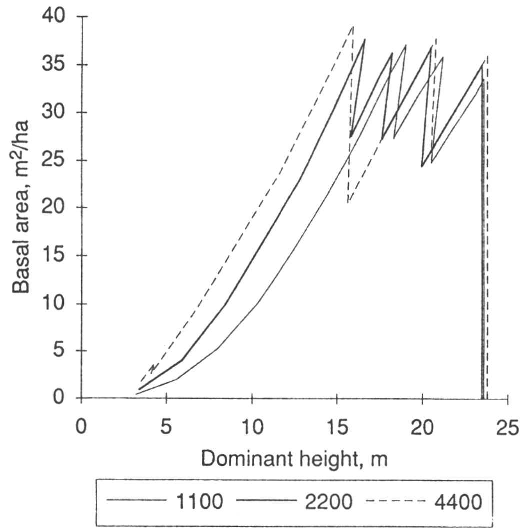

Fig. 10. Basal area development following different initial densities in optimum thinning regimes (3 % discount rate). Numbers of trees by diameter class of plot 11302 are multiplied by 0.5, 1.0, and 2.

The regime for the greatest density included one precommercial thinning, too. The thinning interval and intensity were also affected by the initial density. In most cases, the basal area before thinning was between 35 and 40 m²/ha.

Initial density was optimized by adding it to the vector of decision variables. Density optimization was made for stands based on the tree list of plot 11302 which is a 14 years old plantation at the beginning of simulation. Different initial densities were generated by multiplying the trees-per-hectare values of the tree list by the initial density decision variable. Establishment costs were computed according to Eqn. (1). Because of the lack of empirical basis of Eqn. (1), the purpose of the results is only to present the methodology of optimizing initial density (planting density) using the present algorithm.

An increasing interest rate decreased the optimum initial densities (Fig. 11). Rotation length was also clearly affected by the interest rate. The runs were made so that two thinnings were possible but optimization set one of them to zero in the 4 and 5 % interest rate cases. The soil expectation value was –46 FIM/ha at 5 % interest rate, which is the approximate maximum internal rate of return obtainable based on the present models and parameter values.

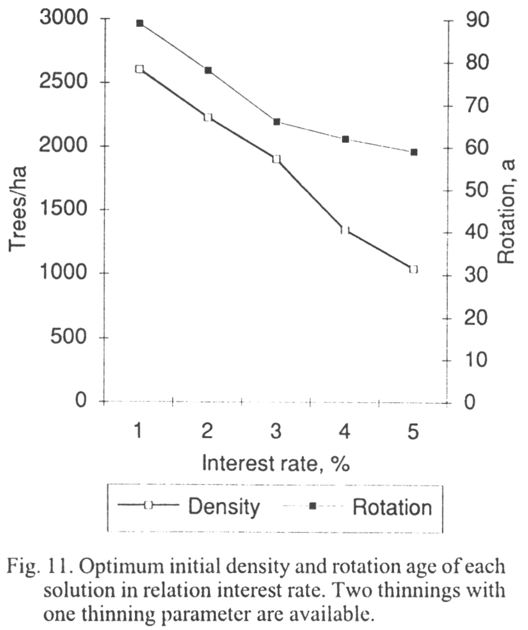

Fig. 11. Optimum initial density and rotation age of each solution in relation interest rate. Two thinnings with one thinning parameter are available.

4.4 Behavior of the optimization algorithm

The number of variables N in the problem of optimizing thinnings and rotation is given by $N = 1 + nt * (1 + np)$, where nt is the number of thinnings and np is the number of thinning parameters. The larger the number of variables, the slower the convergence to an optimum solution. The performance of the algorithm in optimizing stand management is shown in Fig. 12 for three different optimization problems: no thinnings (rotation length only), one thinning defined by three parameters, and two thinnings defined by three parameters. The number of variables to be optimized is 1, 5, and 9, respectively. Contrary to Fig. 4, Fig. 12 shows only the pattern search moves; the points generated by exploratory searches are not plotted.

The experience of the author has been that the time variables (times between cuttings) are much more time-consuming to optimize than the variables for thinning intensities. This can be seen in Figs. 13 and 14 which are based on the growth models by Mielikäinen (1985). The thinning parameters produce smooth response surfaces (Fig. 13) whereas the time variables (times between cuttings) create less regular surfaces (Fig 14). When the stand is subject to excess mortality, the surfaces become very uneven. This phe-

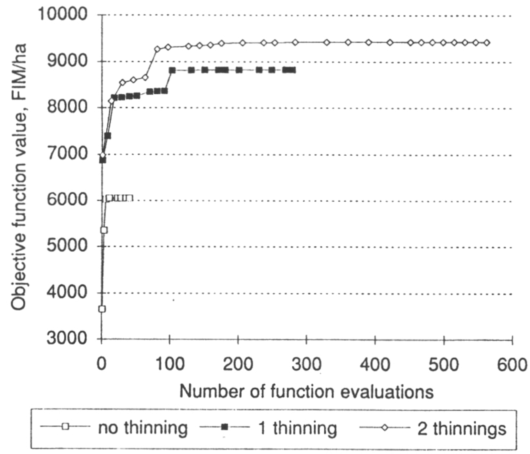

Fig. 12. Convergence of Hooke and Jeeves' algorithm in three optimization problems involving 1, 5, and 9 variables.

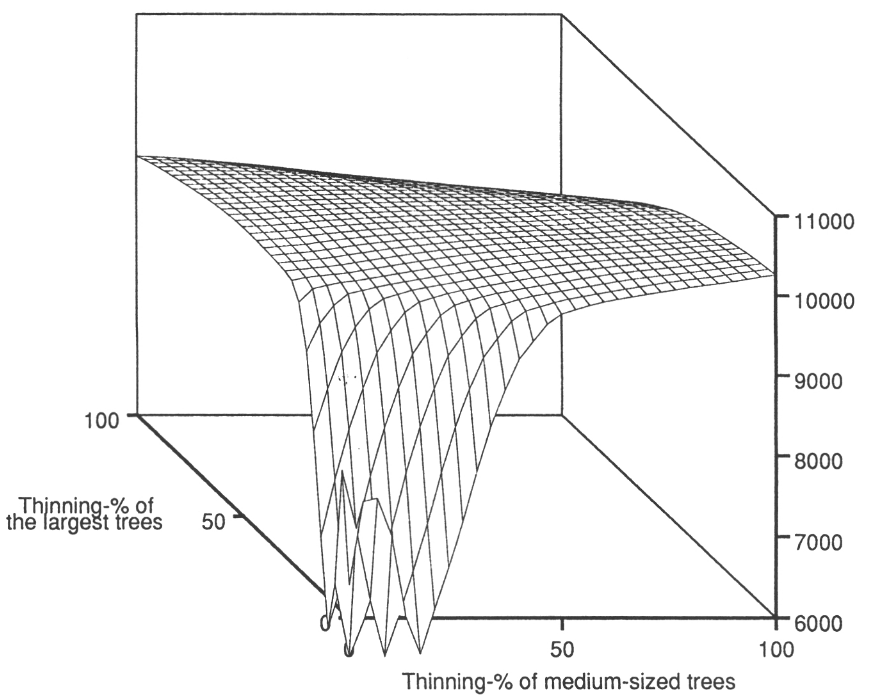

Fig. 13. Response surface generated by two thinning intensity variables around the optimum solution for maximizing 3 % soil expectation value; one thinning defined by 3 parameters in the regime; growth models by Mielikäinen (1985).

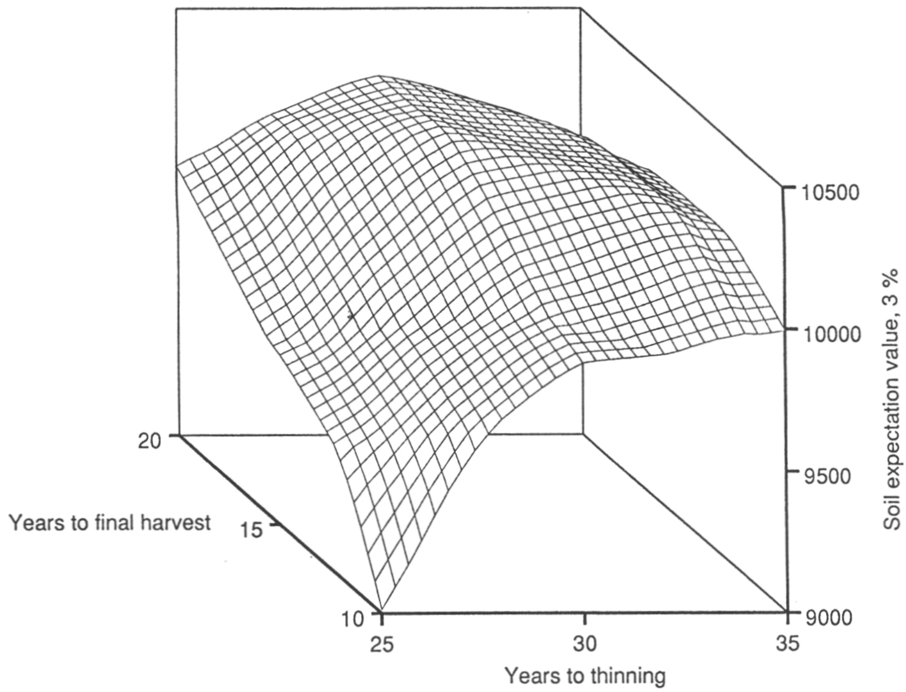

Fig. 14. Response surface generated by two time variables around the optimum solution for maximizing 3 % soil expectation value; one thinning defined by 3 parameters in the regime; growth models by Mielikäinen (1985).

nomenon is starting to show up in Fig. 13 when both thinning percentages are close to zero and the growing stock of the unthinned stand exceeds the self-thinning limit.

The most important idea behind the approach of defining thinnings for optimization is to allow flexibility in the number of parameters required to describe thinning type. This number should be large enough so that thinning type can vary adequately. On the other hand, computational load suggests a minimum number of parameters. A reasonable compromise was found by number three, which has been used in the analyses reported. Fig. 15 illustrates the effect of the number of thinning parameters in a one thinning regime when maximizing 3 % soil expectation value. The maximum value, 12, assigns a thinning parameter for each diameter class (when the number of parameters equals the number of diameter classes, thinning rates are no longer a function of diameter, instead, the model uses separate thinning rates for each diameter class in the order of increasing diameters). Basically, the smallest and largest diameters are harvested, except that the two smallest classes are left because they have no commercial value at the age of thinning. The thinning defined by two points is already from above, a fact that is seen clearly in the case of 5 or 12 points. In this example, three points gives a reasonable approximation of the more accurate solution. The objective function values for the solutions based on 12, 5, 3, and 2 thinnings are, 10384, 10201, 10009, and 9646 FIM/ha, respectively, and in percentage, 100, 98, 96, and 93.

The two growth model sets used in this study, based on Ojansuu et al. (1991) and Mielikäinen (1985), turned out to be unequal tasks for the optimization algorithm. The data for comparison were produced by five replications of optimization runs starting from different random initial solutions. Four problem set-ups, i.e., combinations of the numbers of thinnings and thinning parameters, corresponding to different numbers of variables to be optimized, were formed. The standard deviations of objective function values were computed for each problem set-up. The standard deviations for solutions obtained using the MELA model (Ojansuu et al. 1991) were larger than those based on growth models

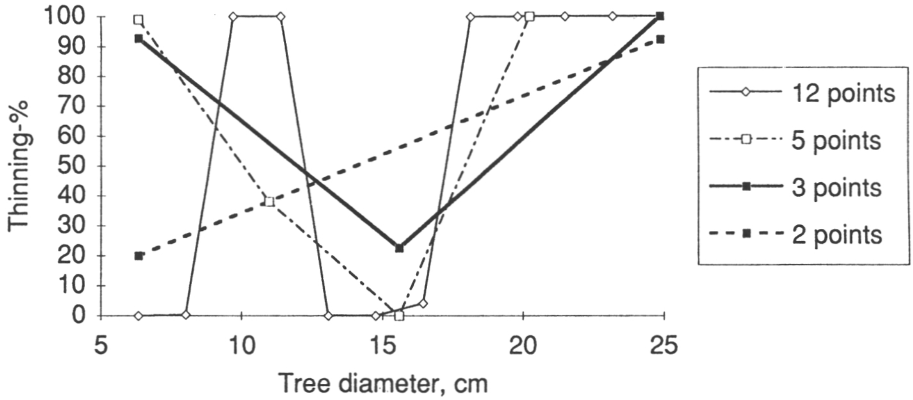

Fig. 15. The effect of thinning specification on the optimum thinning pattern in one-thinning regimes with the objective of maximizing 3 % soil expectation value.

by Mielikäinen (1985) (Fig. 16). The relations in the MELA growth models produced less smooth surfaces which caused more differences between the solutions that were reported as optimal in repeated optimization runs.

Fig. 17 shows the MELA model in a situation parallel to that of Fig. 14. For example, the sharp ridge located at time to thinning equal to 25 years is somewhat problematic for the optimization algorithm. Note that the thinning intensity variables change the location and shape of the ridge and they are optimized simultaneously.

Another feature of the MELA models that showed up in the response surfaces is that the diameter growth models have the maximum diameter of the stand as an independent variable. A decrease in that variable increases growth. When the diameter class with greatest trees is completely removed, the remaining trees get a growth boost. This creates a jump in the objective function value right at the edge of the feasible region of thinning intensity variables.

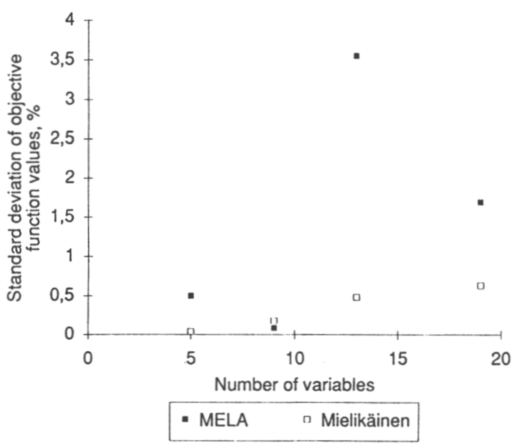

Fig. 16. Effect of the number of variables to be optimized on the variability of objective function values, based on repeated optimizations of two different growth simulators.

5 Discussion

The purpose of presenting numerical results on optimum thinning regimes in this report is to show the capabilities of the optimization model. Because many of the models for stand development in this study are not completely satisfactory and the biological data are limited in many respects, the results should be considered to be demonstrational rather than normative. The general trends shown may be taken as hypotheses for further studies or for consideration by practitioners. Most of the optimum results reported by this study are based on maximizing soil expectation value.

Finnish organizations for private forestry (Central Forestry Boards "Tapio" and "Skogskultur") have designed guidelines for profitable

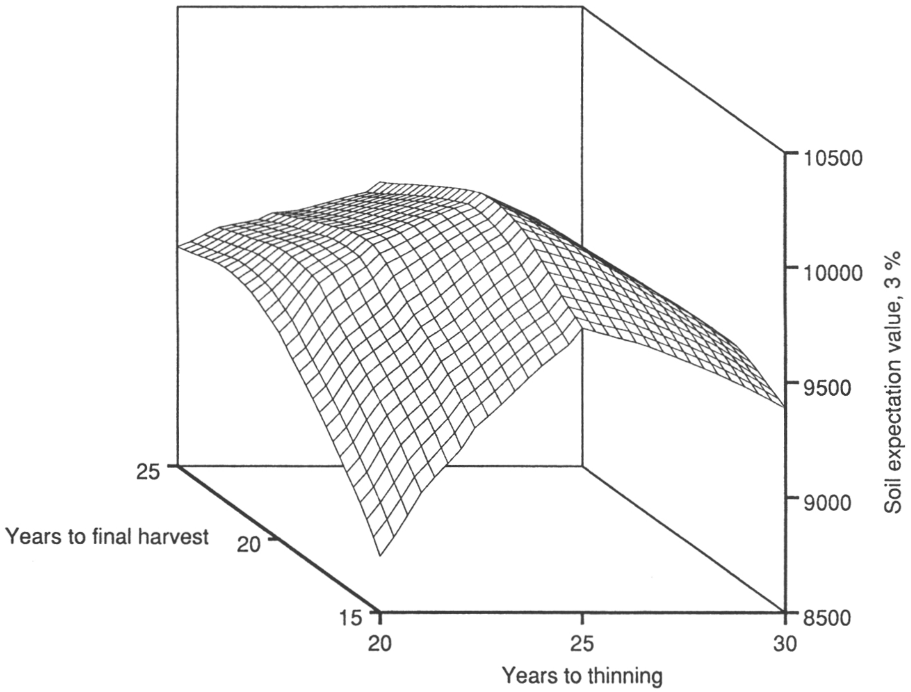

Fig. 17. Response surface generated by two time variables around the optimum solution for maximizing 3 % soil expectation value; one thinning defined by 3 parameters in the regime; growth models by Ojansuu et al. (1991).

thinning practices. The goals behind these recommendations have not been stated explicitly (Tapion taskukirja 1991). In a study based on permanent plot measurements (Valsta 1982), it was found that the treatment closest to guidelines was most profitable with 2–3 % interest rate. However, there were only four alternative treatments in the permanent plots, none of which correspons to the optimum solutions found in the present study. Compared to the guidelines (Tapion taskukirja 1991), the optimum thinning regimes of the present study involve higher growing stock levels, especially during the first two thirds of the rotation, and heavier thinnings (Fig. 18). The first thinning is scheduled more than 10 years later than in the guidelines. The 3 % soil expectation value resulting from the guidelines (8 142 FIM/ha) was 23 % lower than the optimum soil expectation value (10 575 FIM/ha).

Thinnings that are late, heavy, or from above may expose the stand to windfall or snow break. The simulation model used in the present study did not include this risk. If models are available, such factors could be taken into account, but

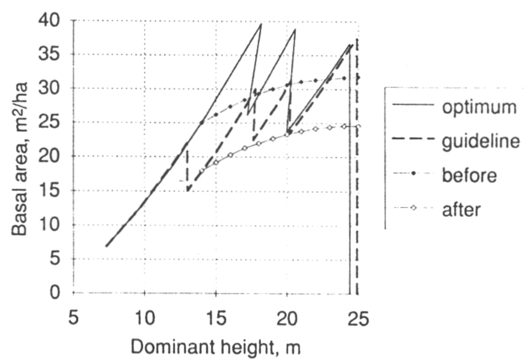

Fig. 18. Two-thinning optimum solution (3 % discount rate) compared to the solution based on the guidelines of Central Forestry Board Tapio. The curves "before" and "after" are the recommended basal areas before and after thinning, respectively.

best within a stochastic optimization model (e.g., Valsta 1992).

A recent Finnish project studied the economics of thinning at stand, forest, and national level (Harvennushakkuiden ... 1992). Based on inventory data and the MELA simulator (Ojan-

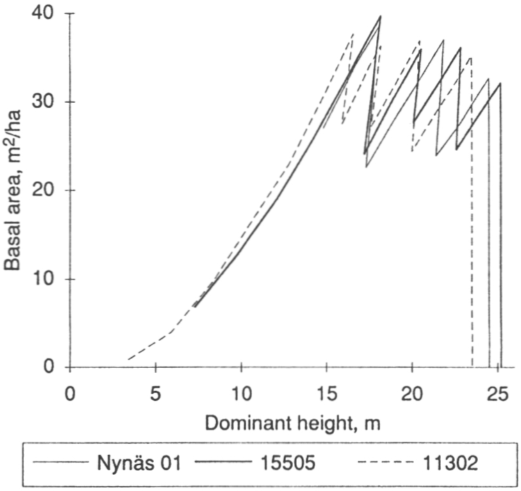

Fig. 19. Optimum thinning programs for three initial stands.

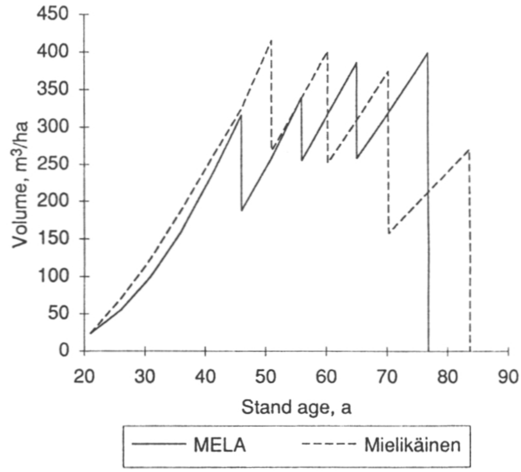

Fig. 20. Volume development in optimum solutions by two individual-tree growth simulators.

suu et al. 1991), the project reported thinning guidelines. The guidelines included in the 1992 report concern a less fertile site than the plot data of this study. The results are therefore only partially comparable.

The optimum thinning program of the present study had higher growing stock levels and the thinnings were scheduled later, compared to the 1992 report (Harvennushakkuiden... 1992). As a result, the average diameter at final harvest was only 20–22 cm in the present study. Also, there was no sawlog price premium on diameter and only logging costs depended on tree size. Thus, there was no incentive to enhance diameter growth. Tree volumes were already large enough from the logging cost point of view at 20–22 cm d.b.h., because the trees were about 22 m tall. The optimum rotation length was about the same in the two studies.

Optimum thinning programs stemming from three different plot data are shown in Fig. 19. The plots represent approximately the same site quality, H_{100} = 28–29. The undisturbed development of the stands (based on the MELA growth model) was quite similar in terms of basal area and dominant height. The optimum thinning programs exhibited a common ground: the first thinning is performed when the basal area is 37–40 m²/ha. Later, thinnings are done at somewhat lower basal areas. The thinnings were heavier for the plot "Nynäs 01" as the optimum program contains only two thinnings. Thinning was from above in all cases.

The optimum solutions produced by the two growth simulators of this study were similar in terms of the number of thinnings and thinning type (Fig. 20). On the other hand, the optimum rotation differed by 7 years and the growing stock levels after thinning behave dissimilarly. The last thinning in the solution for Mielikäinen's (1985) growth models was unrealistically heavy, 58 % of volume. This solution indicates the absence of history variables in the growth and mortality models.

Considering the same tree species and similar growing conditions, the optimization study by Solberg & Haight (1991) comes closest to the present study in terms of comparability. In their study, the optimum number of thinnings varied between 2 and 4. The optimum thinning type was always thinning from above and "the thinnings are made at a relatively high age". These results are very similar to those of the present study.

According to Solberg & Haight (1991), the optimum planting density decreased strongly with increasing interest rate (from 2900 to 1050 with interest rate from 2 to 4 %, respectively). The present study shows somewhat smaller change for the same interest rates, from 2230 to 1349. The optimum rotation decreased from 90 to 70 years while the change in the present study is from 78 to 62 years. Although the numerical values differ, the overall results are quite comparable.

There could be many desirable improvements to the present optimization model. The most important ones concern the stand growth simulator: its biological realism is deficient in several respects. Areas of major simulator development include: (1) a history variable, such as crown ratio, that would carry on the past growing conditions especially when dense stands are thinned relatively late; (2) more reliable growth predictions for trees in lower crown classes; (3) models for relating regeneration investment to early stand development. Also, the results presented here concern limited data in terms of costs and prices, site quality, and initial stand structure. Recommendations to practitioners should be based on a much wider set of analyses.

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Total of 39 references

Appendix 1. Forms of the equations of the simulators.

List of symbols:

e = base of the natural logarithm
k, m, β_i = parameters
BA = stand basal area
ba = tree basal area
BA_{above} = basal area sum of trees larger than the subject tree
ba_{max} = basal area of the largest tree in the plot
cr = live crown ratio, %
d = tree d.b.h.
D_{ba} = basal area weighted average stand d.b.h.
h = tree height
H_{100} = site index (dominant height at 100 years)
H_{dom} = dominant height of stand (average height of the 100 thickest trees/ha)
i_{ba} = tree basal area growth in the coming 5-year period
i_d = tree d.b.h. growth in the coming 5-year period
$i_{d_{ref}}$ = tree d.b.h. reference growth in the coming 5-year period (a tabulated function of d and BA)
i_h = tree height growth in the coming 5-year period
LC_{FH} = final harvest logging cost per cubic meter
LC_{TH} = thinning logging cost per cubic meter
ln = natural logarithm
N = number of trees per hectare
P = probability of death in the coming 5-year period, fraction
t = tree age at breast height
TS = temperature sum (day degrees)
v_{tot} = total volume cut per hectare
$\bar{v}$ = average tree volume in a cutting

1. The MELA growth models (Ojansuu et al. 1991)

Tree diameter increment (Eqn. 4.6.1a):

$$i_d = \beta_0 (1 + \beta_1 TS)^{\beta_2} (i_{d_{ref}} t^{\beta_3} + \beta_4 d) \qquad (A.1)$$

Tree height (Eqns. 4.7.1 and 4.7.2):

$$h = \beta_0 (\beta_1 + TS)^{\beta_2} (ba / ba_{max})^{\beta_3} (1 - e^{-kt})^{\frac{1}{1-m}} \qquad (A.2)$$

2. Growth models by Mielikäinen (1985)

Tree basal area increment (Eqn. 42):

$$i_{ba} = \beta_0 d^{\beta_1} t^{\beta_2} BA^{\beta_3} H_{100}^{\beta_4} (h / H_{dom})^{\beta_5} \qquad (A.3)$$

Tree height increment (Eqns. 55 and 56):

$$i_h = \beta_0 h^{\beta_1} H_{100}^{\beta_2} (h / H_{dom})^{\beta_3} \qquad (A.4)$$

Tree live crown ratio (Eqn. 17)[1]:

$$cr = \beta_0 d^{\beta_1} t^{\beta_2} e^{\beta_3 (h/d)} BA^{\beta_4} \qquad (A.5)$$

Ad hoc model for large basal areas:

$$cr = \begin{cases} cr & \text{if } BA < 40 \\ cr\left(1 - \left(\dfrac{BA-40}{20}\right)^2\right) & \text{if } 40 < BA < 60 \end{cases} \qquad (A.6)$$

3. Mortality models (used by both simulators)

Tree mortality (Ojansuu et al. 1991):

$$P = \frac{1}{1 + e^{\beta_0 + \beta_1 d + \beta_2 BA + \beta_3 BA_{above}}} \qquad (A.7)$$

Stand self-thinning curve (Hynynen 1991):

$$\ln D_{ba} = \beta_0 + \beta_1 \ln H_{100} + \beta_2 \ln H_{100} \ln N \qquad (A.8)$$

Ad hoc model for small crown ratios:

$$P = \begin{cases} P & \text{if } cr > 30 \\ \max\left[P, \left(\dfrac{30-cr}{10}\right)^2\right] & \text{if } 20 < cr < 30 \end{cases} \qquad (A.9)$$

4. Logging cost models (Valsta 1992):

Logging costs in thinning:

$$\ln LC_{TH} = 5.410 - .05217 \ln \bar{v} + .02429 (\ln \bar{v})^2 - .4451 \ln v_{tot} + .03969 (\ln v_{tot})^2 \qquad (A.10)$$

Logging costs in final harvest:

$$\ln LC_{FH} = 5.230 - .05976 \ln \bar{v} + .02076 (\ln \bar{v})^2 - .3840 \ln v_{tot} + .03273 (\ln v_{tot})^2 \qquad (A.11)$$

[1] The original equation includes the variable 'birch-% of basal area'. It was assumed constant (21.0, the minimum of the range of the original data) and its effect is included in parameter β_0.

Appendix 2. Plot data.

Table 1. Diameter distribution of plot 11302.

Diam. class no.	D.b.h. (cm)	Height (m)	Trees/ha
1	1.0	1.7	320
2	1.0	1.7	320
3	2.0	2.0	488
4	2.0	2.5	488
5	3.0	2.6	216
6	3.0	2.7	216
7	4.0	3.1	72
8	4.0	3.4	72
9	5.0	3.6	16

Table 2. Diameter distribution of plot 15505.

Diam. class no.	D.b.h. (cm)	Height (m)	Trees/ha
1	1.0	1.5	27
2	2.0	1.7	80
3	3.0	2.5	133
4	4.0	3.3	173
5	5.0	4.6	320
6	6.0	5.3	373
7	7.0	6.5	440
8	8.0	6.3	240
9	9.0	6.6	133
10	10.0	7.3	93
11	11.0	7.0	13
12	12.0	7.6	13

Table 3. Diameter distribution of plot Nynäs1/01.

Diam. class no.	D.b.h. (cm)	Height (m)	Trees/ha
1	3.7	4.3	50
2	6.0	6.7	30
3	8.1	8.8	120
4	10.2	10.6	220
5	12.0	11.9	430
6	14.0	13.0	440
7	15.9	13.9	430
8	17.6	14.6	140
9	19.3	15.2	30

IV

Forest Science, Vol. 38, No. 2, pp. 430–447

A Scenario Approach to Stochastic Anticipatory Optimization in Stand Management

LAURI T. VALSTA

ABSTRACT. A flexible model for stochastic optimization is developed that can be used with forest stand simulation models. Stochasticity is represented by a large set of scenarios, each of which is an outcome of stochastic processes. A stochastic environment is described by random yearly growth rate levels and catastrophes, such as wild fire or windthrow. The optimization model is defined in control variable space and includes the timing, intensity, and type of thinning, and rotation length for an even-aged stand. Single-tree growth and mortality models are used. Numerical results in a risk-neutral case show that the optimum rotation is shortened with an increasing probability of a catastrophe. Further, an increasing growth rate variation has mixed and weak effects that depend, in particular, on the tree mortality model. If a stand cannot be thinned, increasing risk-taking shortens the optimum rotation, given the model set used. FOR. SCI. 38(2):430–447.

ADDITIONAL KEY WORDS. Optimal harvesting, risk, even-aged stand, Norway spruce, *Picea abies*.

BECAUSE OF THE LONG TIME HORIZON IN FORESTRY, forest management planning depends on assumptions and predictions regarding the future. Most decision aids provided to foresters by forest management research are deterministic and, as such, do not address the uncertainties involved. Dealing with uncertainty is left to the professional judgment of the decision-maker. Facing an uncertain world, a decision-maker might react by ceasing long-term planning. This would easily lead to problems when to do nothing is not the best decision. Instead, based on a probabilistic decision analysis, the manager could choose an alternative that best hedges against the uncertain future. This type of analysis could also take into account the decision-maker's attitude toward risk, i.e., whether he or she is risk-neutral, risk-averse, or risk-taking.

Stochasticity enters forest management from several sources, e.g., incorrect or inaccurate information concerning the present forest, short- and long-term variations in the biological or economic environment, disastrous events, the actual outcomes of forest operations, and incomplete knowledge about the goals of forest management now and in the future. Understandably, only a few aspects of stochasticity may be included in any one analysis, even though important interactions between the different elements of stochasticity exist.

Stochastic optimization models may be divided into two groups—adaptive and anticipatory models (Ermoliev and Wets 1988). In adaptive optimization, the state of the system is observed intermittently, at regular or irregular intervals. The (optimal) decisions are made on the basis of observations. This approach is also

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called "feedback control" or "closed loop control." Studies on the subject have incorporated either stochastic stumpage prices (Norstrøm 1975, Risvand 1976, Brazee and Mendelsohn 1988, Haight 1990), stochastic stand dynamics (Hool 1966, Lembersky 1976, Kao 1984), or both (Lembersky and Johnson 1975, Kaya and Buongiorno 1987, Lohmander 1987).

Anticipatory models are used for deriving optimal decisions for the whole period of time under planning, in advance. An anticipatory solution takes into account the uncertainties over time, and it is optimal overall, according to selected criterion. This is relevant when the state of the system is not observable after a decision is made, perhaps because of high costs or inaccurate data. Anticipatory models are also preferable to adaptive ones when meaningful feedback rules are difficult to identify. Kao (1982) uses stochastic dynamic programming to analyze optimum thinning and rotation in an anticipatory setting when stand volume growth is probabilistic.

Several studies have concerned the optimum rotation of a forest stand under the risk of a catastrophe, summarized in, e.g., Caulfield (1988). Fire risk studies have usually included some adaptation, such as regeneration following fire, or sometimes the salvaging of timber from a burned stand. However, because only the rotation length has been endogenously determined, adaptation has played a minor role. Caulfield (1988) extends the analyses to cover the risk-aversity of the decision-maker. He discusses why risk-neutrality may very well be an improper assumption regarding many forest owners.

Apart from optimization studies, adaptive and anticipatory approaches are also used in forest investment analyses which consider the involved risks or uncertainties. An example of adaptive schemes is the study by Anderson et al. (1987) that combines Monte Carlo simulation with risk analysis for stand management under risk of beetle attack. An anticipatory problem formulation is used by Binkley and Washburn (1990) to study the magnitude of risk in private timberland investments and by Taylor and Fortson (1991) to investigate both the amount of risk and the effect of risk-aversion on optimum plantation management.

The results from studies based on adaptive stochastic optimization suggest some common patterns. The expected present values for adaptive policies were found to be considerably higher than present values based on deterministic models (Norstrøm 1975, Lohmander 1987, Brazee and Mendelsohn 1988, Haight 1990). For even-aged stands with stochastic timber prices, the expected optimum rotation was little different from the deterministic counterpart (Lohmander 1987, Brazee and Mendelsohn 1988). However, in the case of uneven-aged management, the difference between adaptive and deterministic optimum policies was substantial (Haight 1990). Using a nonadaptive model, Kao (1982) analyzed the effects of increasing risk on optimum stocking levels and rotation length with the objective of maximizing mean annual increment. He concluded that both optimum stocking level and rotation length decrease with increasing risk.

This paper presents a stochastic optimization model used with a single-tree simulator for even-aged stands. The optimization approach by Roise (1986b) is generalized to account for a probabilistic stand growth simulator. The optimization model is anticipatory (not adaptive), and stochasticity is introduced in the form of scenarios. This results in a flexible and approachable model construction. Numerical solutions are determined using the direct search method (Hooke and Jeeves 1961), as modified by Osyczka (1984). The optimization model is applied to a single-tree, distance-dependent growth model for Norway spruce (*Picea abies* Karsten), by Mielikäinen (1985) and Hynynen (1990) and the effects of stochasticity on the optimum thinning program and rotation age are examined.

STOCHASTIC OPTIMIZATION USING SCENARIOS

Previous approaches to stochastic stand level optimization problems include Markovian decision models (Hool 1966, Lembersky and Johnson 1975, Lembersky 1976, Kaya and Buongiorno 1987), other variants of stochastic dynamic programming (Norstrøm 1975, Risvand 1976, Kao 1982 and 1984, Lohmander 1987, Brazee and Mendelsohn 1988), and adaptive optimization based on stochastic simulation (Haight 1990). In Markovian models, the process to be optimized is described by a state vector, the elements of which are the possible alternative states of the process. The dynamics of the process are given by a state transition matrix that indicates the probabilities of being in one state, subject to being in another state one time step earlier. One or two stand characteristics, such as a stand density measure, are used to model stand development. Stochastic dynamic programming models are also limited to a few state variables. However, modern methods for predicting stand development utilize single-tree simulators that incorporate tens or even hundreds of state variables. The use of transition matrices is no longer possible: if there are 10 state variables and each of them has 10 possible states, a very modest amount, the size of the transition matrix would be $10^{10} \times 10^{10}$. Although the transition matrix would be sparse, or the problem might have a more efficient form, there would still be severe computational problems.

To overcome the dilemma between state description detail and computational burden, Kaya and Buongiorno (1987) use a stand simulator to estimate transition matrices, thereby enabling a more complex stand description. Their approach is viable when stand treatment is defined in broad terms, corresponding to state description. Haight (1990) develops a feedback thinning rule to be used in harvesting decisions in uneven-aged stand management when timber price is a stochastic process. The rule gives the total number of trees to be cut as a function of observed stand value. The thinning rule operates on stand level variables but it is estimated by using stochastic simulation based on a stage-structured growth model with 2-in. diameter classes.

A recent development in deterministic stand level optimization studies is the use of nonlinear programming and the stand management control variables (Roise 1986a, Bare and Opalach 1987, Haight and Monserud 1990a, Haight and Monserud 1990b, Valsta 1990, Yoshimoto et al. 1990). This nonlinear programming problem is

$$\max_{\{\mathbf{u} \in C \subset R^n\}} \quad g(\mathbf{u}|\mathbf{x}_0) \qquad (1)$$

where $g(\mathbf{u}|\mathbf{x}_0)$ is the (scalar) objective function generated by the stand simulator, $\mathbf{u}$ is the vector of control variables, $\mathbf{x}_0$ ($\mathbf{x} \in R^m$) is some initial condition for the stand simulator, and C is the set of feasible controls. The control variables may be defined as times between silvicultural operations, their intensities, or the time of the final harvest. Revenues, costs, and discounting are embedded in function $g(\cdot)$.

Problem (1) has a stochastic counterpart. Assume a probability space (Ξ,P), where Ξ is the set of possible realizations and P is the associated probability measure. The corresponding vector of random variables is ξ. Suppose that the decision-maker wishes to maximize the expected net return from stand management, given by a (scalar) return function, $f(\mathbf{u},\xi|\mathbf{x}_0)$. The problem is then to

$$\max_{\{\mathbf{u} \in C \subset R^n\}} \quad \mathbf{E}[f(\mathbf{u},\xi|\mathbf{x}_0)] = \int_{\Xi} f(\mathbf{u},\xi|\mathbf{x}_0)dP(\xi) \tag{2}$$

To solve Equation (2) is not a simple undertaking. Two possible approaches are: (1) to perform a multidimensional integration over Ξ, an extremely computing-intensive, if not impossible task, or (2) to use stochastic quasigradients or their approximations (Ermoliev and Wets 1988).

A different scheme for the problem is to model the stochastic phenomena using scenarios (Wets 1989). A scenario is here defined as one realization over time of the stochastic processes. Even though there may be several stochastic processes, they are all combined to a joint realization, a scenario. All scenarios are generated before optimization, and they are regarded as exogenous variables to stand simulation. For example, in the present study, a scenario is composed of yearly growth levels, and the times and severities of catastrophes for a 50-yr period. The present approach does not allow the random variables to depend on state variables. That is why a random variable is not basal area growth, for example, but a multiplier of basal area growth.

Define a set of scenarios, $S = \{s^1, \ldots, s^L\}$, where each s^s is a joint realization of the stochastic processes over the planning horizon. Suppose that p_s, $s \in S$, are the probability weights associated with each scenario. These weights may have empirical or subjective bases. The scenarios can be used in place of the random vectors, ξ, of Equation (2). Then, $E[f(\bullet,s^s]$ equals $\Sigma_{s \in S}\, p_s\, f(\bullet,s^s)$ and the maximization in (2) may be approximated by

$$\max_{\{\mathbf{u} \in C \subset R^n\}} \quad \sum_{s \in S} p_s\, f(\mathbf{u},s^s|\mathbf{x}_0) \tag{3}$$

The accuracy of the approximation depends on the number of scenarios and the probability weights used. The problem is now one of deterministic nonlinear programming, for which a variety of numerical solution procedures are available.

Formulas (2) and (3) maximize the expected net return, which is here considered the expected utility of the decision-maker. The scenario approach can also be used in the case of risk-aversion or risk-taking. The values of $f(\bullet)$ for each scenario provide the probability distribution of net returns. Various degrees of risk-taking can be expressed by using different values of z when maximizing the following probability over all $s \in S$:

$$\max_{\{\mathbf{u} \in C \subset R^n\}} \quad P[f(\mathbf{u},s^s|\mathbf{x}_0) \geqslant z] \tag{4}$$

A high value of z implies risk-taking, as the decision-maker wishes to maximize the probability of receiving a high net return. A low value of z corresponds to risk-aversion. The objective function is a step function, scaled between 0.0 and 1.0, and approaches a smooth function when the number of scenarios increases.

From a mathematical point of view, the optimization problem is not made more difficult when the objective function is transformed to reflect risk premiums.

In this modelling approach, the elements of **u** are quantitative variables that describe forest operations, such as planting density, intensity and timing of thinning, or rotation age. The constraints imposed on them are, typically, just lower and upper bounds to avoid stand simulations ignoring biological realities or valid ranges of models. Thus, the set C is convex, facilitating the maximization in (3) or (4). The function $f(\cdot)$, however, is likely to be nondifferentiable, or at least nonsmooth, and may be nonconcave, giving rise to local maxima that may mislead the optimization algorithm. These problems have been discussed in Roise (1986a, 1986b) and Haight and Monserud (1990a).

THE PROBABILISTIC STAND SIMULATOR

In probabilistic simulation models, some of the physical or economic variables, or both, are random. As most of the simulators are based on deterministic relationships, stochasticity may have to be introduced to the models afterward. In the present study, the starting point of the simulator is a deterministic, single-tree, distance-independent growth model developed by Mielikäinen (1985) and Hynynen (1990) for southern Finland. Only the models for Norway spruce are used. The biological part of the simulator consists of models for (1) dbh growth, (2) height growth, (3) crown ratio, and (4) mortality rate. The models, which are described in detail in the Appendix, represent a common modeling strategy used in many single-tree simulators. Although these models cover a fairly small geographical area, their behavior is likely to resemble that of several other models.

The present study deals with two types of risk. Firstly, stochastic yearly levels of growth are examined. The random variables for yearly growth levels are assumed to be independent, identically distributed lognormal(μ,σ^2) variates.[1] The subroutines provided by Numerical Recipes Scientific Subroutines (Press et al. 1986) are used to obtain uniform pseudorandom numbers (subroutine RAN2) and the deviates of normal distribution (subroutine GASDEV) required to compute lognormal deviates.

Different values of the standard deviation of the yearly growth level were used in the computer runs, the base value being 17.8%, which was obtained from the tree ring index values in Mielikäinen (1985). The same yearly growth levels are used for height growth and dbh growth. In reality, the variation of height and dbh growth may not necessarily coincide. More realistic representations of growth variations could be constructed by including, for example, autocorrelation between the growth levels of successive years (Jordan and Lockaby 1990). Such refinements would be simple to include in the scenario descriptions. Examples of the realization of the stochastic growth level for a period of 50 years are given in Figure 1. During simulation, growth is mostly predicted for a period of 5 years, for which the variation is smaller than for a single year.

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[1] The parameters μ and σ^2 are not the mean and variance of the lognormal(μ,σ^2) distribution, If X is log-normally distributed, $E[X] = e^{\mu+\sigma^2/2}$ and Var[X] is $e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)$. In the results section of this study, the standard deviation of yearly growth refers to the standard deviation of the variates, not to the parameter of the lognormal distribution.

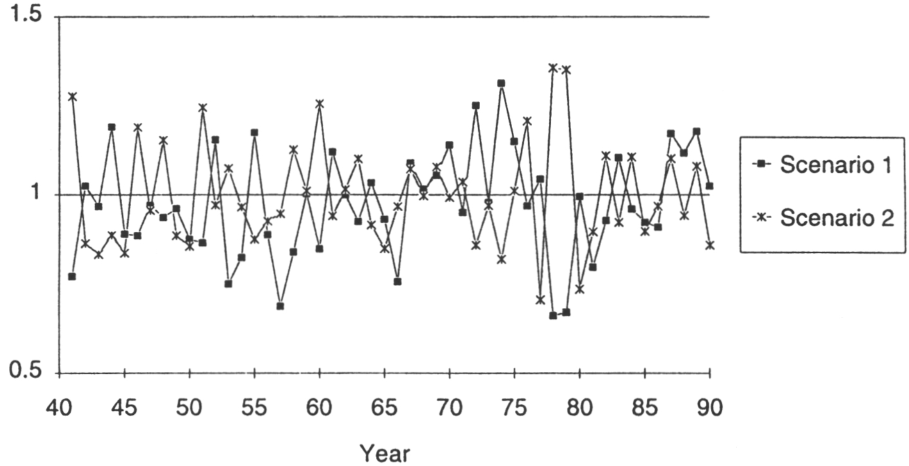

FIGURE 1. Yearly growth level values in two realizations of the stochastic growth process.

Secondly, the effects of catastrophes are studied. Catastrophes are modeled as random events that damage a part of the growing stock. A catastrophe is specified by two random variables. The occurrence of a catastrophe is modeled as a Bernoulli random variable with the parameter *pcat,* e.g., 0.01. If a catastrophe takes place, a uniformly distributed random variate, $U(0,1)$, gives the proportion of trees destroyed. It is assumed that the damaged trees have to be harvested immediately with a 25% reduction in their stumpage value and a doubling of the logging costs. These economic parameters were set subjectively as no data were available from applicable silvicultural conditions.

The scenarios used in this study have limited empirical background, which should be kept in mind when evaluating the results. The probability of the occurrence of a scenario is chosen to be $1/n$, where n is the number of scenarios.

To arrive at reliable solutions with the available nondifferentiable optimization algorithms, the number of decision variables is kept to the minimum. This can be done by assigning diameter class groups (Haight and Monserud 1990b, Yoshimoto et al. 1990), by removing trees from below or above (Haight et al. 1985, Haight 1990), or by using a diameter distribution function with its parameters as decision variables (Bare and Opalach 1987).

The grouping of diameter classes, or strict thinning from below or above may produce thinning patterns that are not applicable in real stands with a spatial distribution of trees. In this study, thinning types are specified by a set of thinning parameters, the number of which can be varied to achieve a desired degree of precision in thinning type specification. Thinning intensity is a piecewise linear function of tree diameter, relative to the smallest and the largest diameters in the stand. The thinning parameters define the thinning rates at the corner points of the piecewise linear function. At one extreme, there may be only one thinning parameter, and thinning rate is constant across tree diameters. At the other extreme, there may be a thinning parameter for each 1 in. or 1 cm diameter class. As an example, suppose that we wish to use three parameters, p_1, p_2, and p_3, to define a thinning. They denote thinning rates at minimum, midpoint, and maximum tree diameter, d_{min}, d_{mid}, and d_{max}, respectively. Thinning rates for other diameters are computed using linear interpolation. Thinning rate $p(d)$ of an arbitrary diameter d' is

$$p(d') = \begin{cases} p_1 & \text{if} \quad d' = d_{\min} \\ \dfrac{(d' - d_{\min})p_2 + (d_{\mathrm{mid}} - d')p_1}{d_{\mathrm{mid}} - d_{\min}} & \text{if} \quad d_{\min} < d' < d_{\mathrm{mid}} \\ p_2 & \text{if} \quad d' = d_{\mathrm{mid}} \\ \dfrac{(d' - d_{\mathrm{mid}})p_3 + (d_{\max} - d')p_2}{d_{\max} - d_{\mathrm{mid}}} & \text{if} \quad d_{\mathrm{mid}} < d' < d_{\max} \\ p_3 & \text{if} \quad d' = d_{\max} \end{cases} \qquad (5)$$

A thinning specification with three parameters is shown in Figure 2. The thinning definition can be adjusted to simulate different types of thinning with only a few parameters per thinning.

Tree volumes and stumpage values are computed by diameter class, and logging costs are derived by using an equation based on the tariff tables used by the Finnish forest industries and logging contractors. These models are described in the Appendix.

RESULTS

To illustrate the flexibility of defining the objective function in the present scenario-based stochastic optimization method, two objectives are used in the analysis. First, the net present value of stand management is maximized [Equation (3)], computed as the expected soil expectation value of an infinite series of equal rotations. This criterion corresponds to risk-neutral preferences. As a hypothetical example of risk-aversion, the second objective is to maximize the probability of attaining a given level of soil expectation value [Equation (4)]. The attitude to

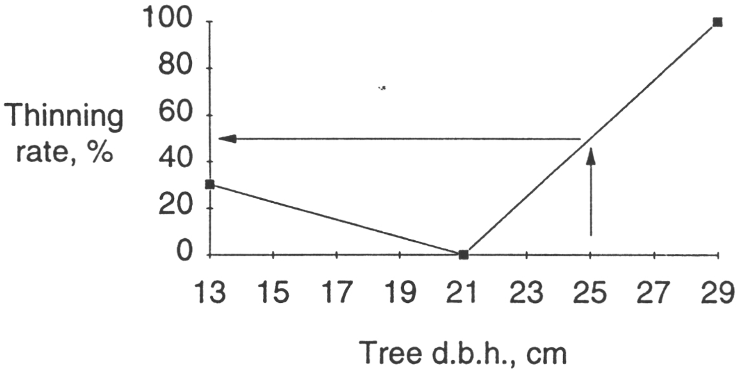

FIGURE 2. Thinning intensities defined by a piecewise linear function with three parameters. The minimum and maximum diameters of the stand are 13 and 29 cm, respectively. The three parameters define thinning rates at minimum, midpoint, and maximum diameters; 30, 0, and 100%, respectively. As an example, the thinning rate of the 25 cm diameter class is 50%.

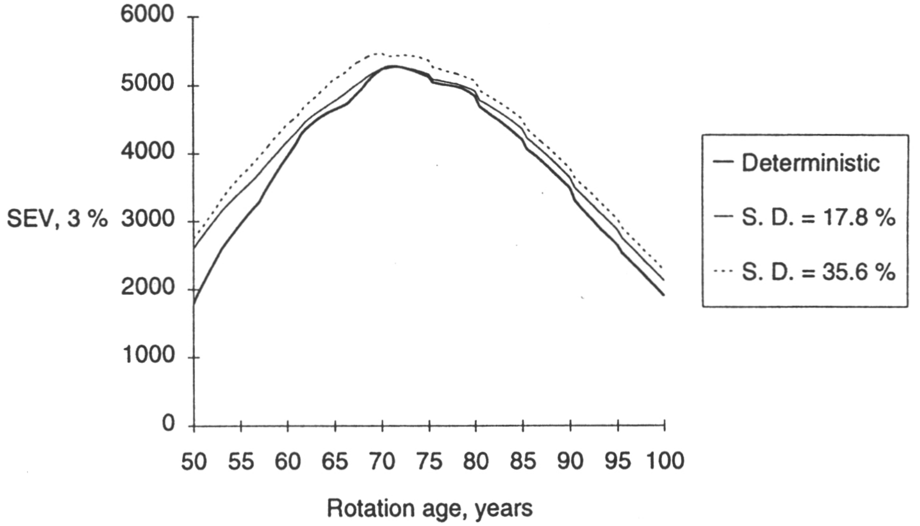

FIGURE 3. Soil expectation value in relation to rotation length for the deterministic case and two magnitudes of stochasticity. Legend numbers refer to the standard deviation of yearly growth, %. The discount rate is 3%.

risk can be expressed by varying this level, the higher levels relating to risk-taking. The control variables that define cuttings are adjusted so that the target soil expectation value is achieved under as many scenarios as possible.

A permanent plot in a long-term field experiment[2] in southern Finland (Heinola, Nynäs, 61°N, 26°E, elevation 90 m) was used as the initial stand for simulation. The stand is a pure Norway spruce stand growing on mineral soil with a predicted site index of 29 m (dominant height at 100 yr). The stand was established with 4-yr-old plants in 1926. The trees were 40 yr old at the start of the simulation (see Appendix for details).

A simple optimization problem is analyzed first: stand management with no thinnings and an objective function with no risk preferences. The only control variable is rotation length, and it is convenient to conduct an exhaustive search over a reasonable range of rotation ages. The effect of random yearly growth levels was marginal (Figure 3). Expected returns increased slightly with increasing stochasticity. The graph shows smooth curves on the large scale, combined with small-scale irregularities. The nonsmoothness results partly from using a growth simulator modeled for 5-yr growth periods to predict shorter time increments. The values of the independent variables are updated when a full 5-yr period is completed thereby creating small twists to the curves.

When the stand was subject to the risk of a catastrophe, increasing risk reduced the optimum rotation. This is demonstrated in Figure 4, where the deterministic case is compared with three levels of risk, represented by annual probabilities of a catastrophic event (e.g., wild fire or windthrow). No thinnings were made, but damaged trees are logged immediately after a catastrophe.

[2] The plots were established and are maintained by the Finnish Forest Research Institute, Helsinki, Finland.

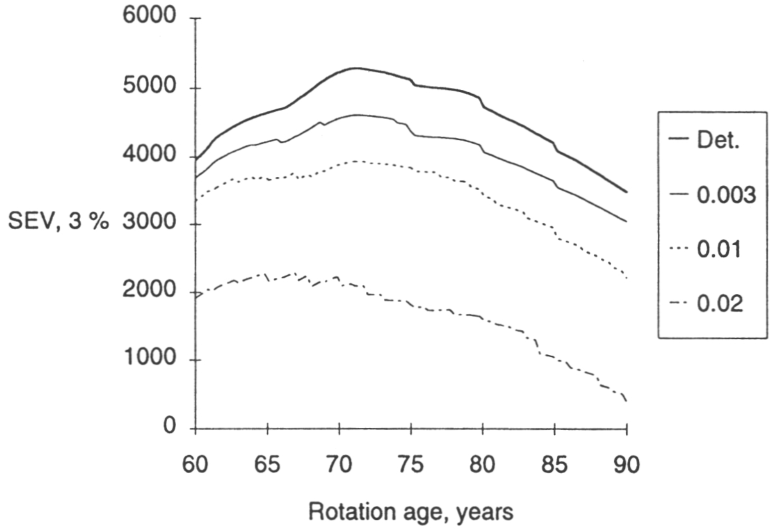

FIGURE 4. The effect of the increasing risk of a catastrophe on the soil expectation value for different rotations. Legend numbers refer to annual probability of disaster. The discount rate is 3%.

When there are several thinnings, the number of control variables is large and the results are difficult to illustrate. For this reason, only one thinning is considered in the following analysis. To permit the thinning type to vary, thinning intensities are defined by three parameters (as in Figure 2).

The optimum thinning schedule was derived for the deterministic case and four levels of yearly growth variation. With increasing stochasticity (Table 1), the rotation length was increased, thinning was scheduled sooner, and mortality from small trees was captured. The results in Table 1 were computed with the seed number of the random number generator equal to 1. Seed numbers 2 and 3 were also examined, and it was found that the numerical values varied by small amounts but the changes due to increasing stochasticity were similar.

The effect of an increasing probability of a catastrophe, decreasing rotation length, was also found when a thinning was included in stand management (Table 2). Given a constant thinning rate, the optimum rotation was shortened from 82 yr to 70 yr when the probability increased from 0 to 0.02. Thinning was brought

TABLE 1.

Optimum thinning schedule for different levels of stochasticity, standard deviation (S.D.) of yearly growth variation. P1, P2, and P3 refer to thinning parameters. The discount rate is 3%.

S.D. (%)	Rotation (yr)	Thinning age (yr)	P1 (%)	P2 (%)	P3 (%)	E[SEV] (Finn. marks/ha)
0.0	82.2	69.8	0	0	100	5694
8.9	82.8	69.7	0	0	100	5669
17.8	86.0	68.4	0	0	100	5757
26.7	86.0	67.5	3.9	0	100	5948
35.6	86.0	66.6	47.8	0	100	6172

TABLE 2.

Effect of the increasing probability of a catastrophe (*pcat*) on the optimum thinning schedule. The discount rate is 3%.

Pcat	Rotation (yr)	Thinning age (yr)	P1 (%)	P2 (%)	P3 (%)	E[SEV] (Finn. marks/ha)
0.0	82.2	69.8	0	0	100	5694
0.003	82.8	69.0	0	0	100	4936
0.01	78.0	63.8	0	0	100	4256
0.02†	70.0	62.9	0	0	100	2356
0.02†	82.0	64.6	0	11.4	100	2406

† Alternative optima.

forward in higher risk cases. With 0.02 probability of a catastrophe, two optimum solutions were found: one was based on the same thinning rate (0, 0, 100) as the lower risk cases and had a shorter rotation, while the other one indicated a higher thinning rate (0, 11.43, 100) and a longer rotation.

The other decision criterion examined in this study was conditioned by a specified soil expectation value. By adjusting rotation length, the forest manager wishes to maximize the probability of attaining a target soil expectation value. In this formulation, the decision-maker can increase willingness to take risks by raising the soil expectation value requirement.

When no thinnings were available, increased risk-taking caused the optimum rotation, i.e., the one with highest probability, to decrease (Figure 5). For example, the probability that a simulated soil expectation value is higher than 5000

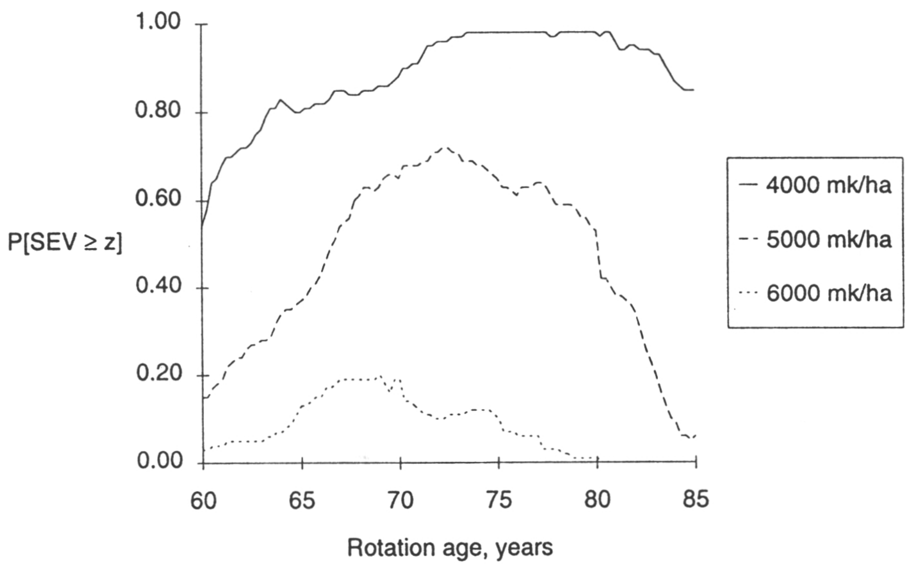

FIGURE 5. Probability that a computed soil expectation value, *SEV*, is greater than the specified soil expectation value target, *z*, for different rotations. The three targets are 4000, 5000, and 6000 Finnish marks/ha. The source of stochasticity is the yearly growth variation (S.D. = 26.7%). Interest rate is 3%.

Finnish marks/ha was 0.73 when the rotation was 72.8 yr. In other words, in 73 of the 100 simulations made, the soil expectation value was higher than 5000 marks. With a 4000 mark threshold, the probability was 0.98 when the rotation was 73.8–77.2 or 78.8–79.5 yr. A thinning enabled, the optimum rotation did not depend on the degree of risk-taking.

DISCUSSION

The main purpose of the present study is to demonstrate a stand level optimization approach capable of working with probabilistic and complex stand growth simulators. When the stand production function is defined as a well-behaved algebraic function, e.g., of the age of the stand, conclusive results concerning the effect of risk on optimal forest management can be inferred. Because the present study is based on a single set of growth models, definite consequences of stochasticity are not revealed. Results may depend on the growth models, site characteristics, initial stand structure, and economic parameters. Obviously many more analyses are required to establish the general importance and effects of stochasticity.

First consider the results derived under the assumption of risk neutrality. The effect of yearly growth variation is, in an interesting way, dependent on whether a thinning is available or not. Without a thinning, an increase in stochasticity results in a small decrease in the optimum rotation length. With a thinning included, the optimum rotation is increased by 2–4 yr, depending on the seed number of the random number generator. Thinning intensity, also, increases with amplified growth variation. Given the initial stand and simulation parameters, the growing stock reaches its self-thinning limit at about age 69. The self-thinning limit is a fixed equation and does not depend on growth level variation. Compared to other scenarios, those with many years of elevated growth lead to higher mortality at the end of the rotation. Such losses cause the optimum no-thinning rotation to shorten with increasing growth variation. When available, thinning is scheduled to coincide with the attainment of the limiting density. Limiting densities are not encountered after thinning, and increasing stochasticity leads to heavier thinnings and longer rotations. Overall, these observations point out that simulator parameter values and initial conditions may strongly influence the qualitative results of including stochasticity into stand level optimization.

The overall change in optimum rotation due to growth risk was small. This result is similar to the effects of price risk, as reported by Lohmander (1987), and Brazee and Mendelsohn (1988). Conversely, increasing risk of a catastrophe shortens the optimum rotation considerably, which has been observed also in fire risk studies (Martell 1980, Routledge 1980, Reed and Errico 1985, Caulfield 1988).

The case where the decision-maker is not risk-neutral was analyzed with a hypothetical model [Equation (4)]. For comparison, the formulation used by Taylor and Fortson (1991) was also tested. They define the utility of an investment at different risk attitudes as

$$\text{UTILITY} = \text{Return} * \text{ALPHA} - \text{Risk} * (1\text{-ALPHA}) \qquad (6)$$

An ALPHA value of 1 implies risk neutrality and 0 extreme risk-aversion. Values greater than 1 indicate risk-taking.

The analysis with objective function (4) was repeated for Equation (6), using expected present value in place of Return and the standard deviation of present values in place of Risk. ALPHA values 0.25, 0.5, 0.75, 1.0, 1.5, and 2.0 were tested. The results based on (6) were similar to results based on (4), i.e., for unthinned stands increasing risk-taking slightly shortens the optimum rotation, and risk-aversion has the contrary effect. The result is opposite to that of Taylor and Fortson (1991), which is not surprising because the studies concern biological and economic models and conditions which differ considerably.

The stochastic optimization method applied in the present study requires a fair amount of computing resources. Unfortunately, this may be unavoidable in numerical stochastic optimization. For example, Kao (1984) reports that the computational load was 100 times larger in probabilistic optimization compared with the deterministic case. Because the approach of the present study is based on scenarios, their number is the main determinant of the amount of computing required. The number of scenarios should be large enough, so that optimization results are not markedly dependent on the set of scenarios in the analysis.

The dependence of soil expectation value on the rotation age for an unthinned stand was computed with the number of scenarios ranging from 10 to 500 (Figure 6). The source of stochasticity was the probability of a catastrophe, which was set to 0.01. To determine the optimum solution with confidence, approximately 100 scenarios were required—a smaller number would have created an evident risk of reporting an incorrect (local) optimum solution. Increasing the number of scenarios to as much as 500 still left some nonsmoothness in the response curve. As a result, only algorithms suitable for nonsmooth optimization can be applied in the present approach.

The response surface generated by the stand simulator is both nonsmooth and nonconcave, so the optimization algorithm may converge to a local rather than global optimum solution. The performance of the currently used version of the Hooke and Jeeves direct search algorithm (Osyczka 1984) is illustrated in Table

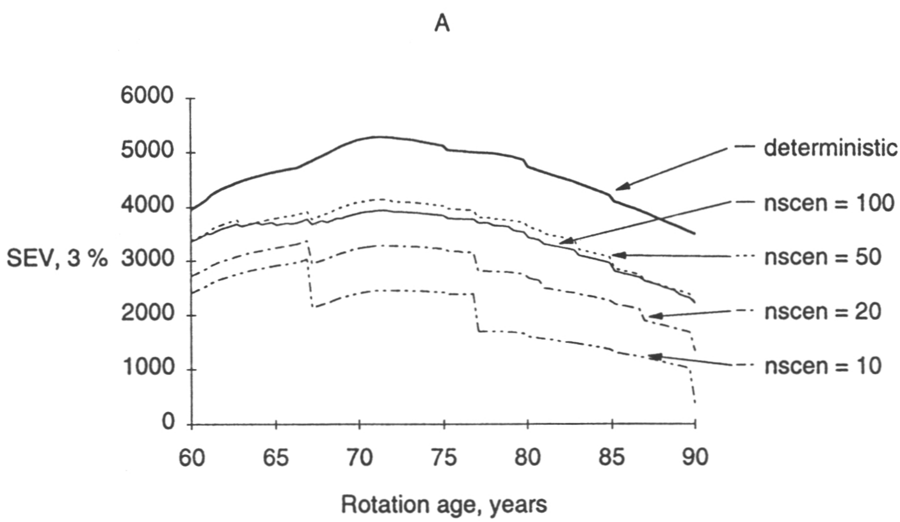

FIGURE 6. Soil expectation value as a function of rotation age for different numbers of scenarios and in the deterministic case. The discount rate is 3%.

B

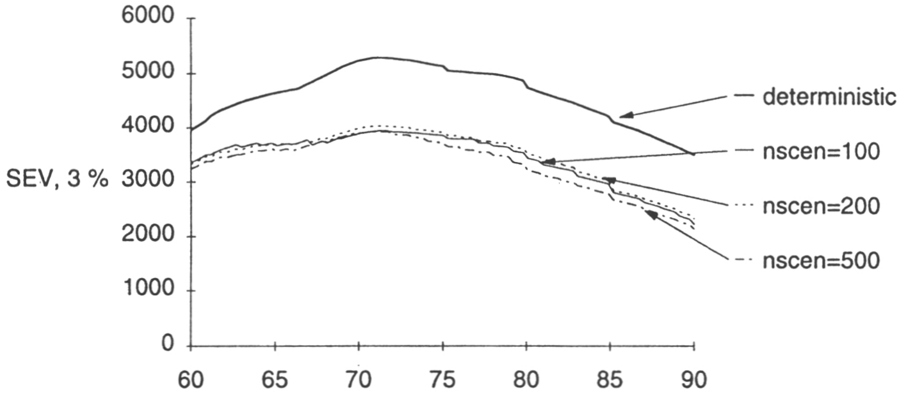

FIGURE 6 (Continued)

3, where the same problem was solved ten times, starting from randomly chosen initial solutions. Relying on a single solution would be risky. On the other hand, rerunning the optimization 3–5 times seems to provide adequate confidence. The results presented in Tables 1 and 2 are based on five replications of optimizations.

One of the strengths of the present scenario-based stochastic optimization method is that the distribution of the objective function values, by scenarios, is known and can be analyzed with respect to relevant variables. Furthermore, transformations of the distribution can be used to form other objectives based on the attractiveness of individual scenario outcomes. The scenario simulations produce large amounts of information all of which can be used to evaluate management strategies.

The scenario approach permits the simultaneous analysis of several stochastic phenomena. When including additional sources of risk, it is not necessary to increase the number of scenarios multiplicatively. For example, when computing

TABLE 3.

Ten optimum solutions for the same problem obtained from different random starting points of the optimization algorithm. The discount rate is 3%.

Number of run	Rotation (yr)	Thinning age (yr)	P1 (%)	P2 (%)	P3 (%)	E[SEV] (Finn. marks/ha)
1	86.0	66.6	47.8	0.00	100.0	6172
2	86.0	66.6	47.8	0.00	100.0	6172
3	86.0	66.6	47.8	0.00	100.0	6172
4	86.0	67.5	44.3	0.01	99.9	6168
5	86.0	66.6	47.8	0.01	99.9	6172
6	85.8	65.0	31.7	0.00	99.9	6165
7	86.0	67.6	43.3	0.08	100.0	6168
8	86.0	66.6	47.9	0.01	100.0	6172
9	86.0	66.6	47.9	0.01	100.0	6172
10	85.8	64.7	29.8	0.00	100.0	6165

results similar to those of Figure 4, the number of scenarios (100) would have been sufficient, had growth variation also been included. The approach has potential for other applications, such as the joint optimization of regeneration and other silvicultural operations, or analyzing the effects of stand inventory data inaccuracy.

In the present study, the scenarios are formed by generating random outcomes of the defined stochastic process. Another alternative would be to form scenarios subjectively, so that they would be likely realizations of the future. Scenarios have been used extensively in econometrics and management science (see, for example, Makridakis 1983, Godet 1987) where important questions have been the determination of the most reliable forecast or the aggregating of scenarios to form "the best estimate." Such an approach differs markedly from that of the present study, in which scenarios are used to model probabilistic processes and represent the real world stochasticity in a manageable form.

Nonlinear programming is here used to maximize the objective function—either the expected soil expectation value or the probability of attaining a specified soil expectation value. Most of the computing time is spent on evaluating the result of applying a decision variable vector to each of the scenarios. These scenario simulations are mutually independent. This approach is well suited for computers with parallel processing because the simulation computations are of the "single-instruction, multiple-data" type (Lootsma and Ragsdell 1988). Parallelism could be further extended by the use of scenario aggregation (Wets 1989) in which the optimization algorithm is also largely parallel.

An important extension to the present model would be to determine feedback rules for adaptive optimization. Haight (1990) developed feedback policies for uneven-aged stand management, using a stage-structured growth model. Finding practical and efficient forms of feedback rules for single-tree simulators with various stochastic components requires further work.

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APPENDIX

The Appendix describes in more detail the simulator constructed for the present study, based on Mielikäinen (1985) and Hynynen (1990). The growth models were estimated for periods of 5 yr. When shorter growth periods are needed, they are linearly interpolated from full 5-yr periods.

The simulation starts from a tree list containing data on tree dbh, height, age at breast height, and the number of trees/ha represented by each tree. In the present application, trees can only be of single species, namely Norway spruce (*Picea abies* Karsten). Before the growth projection, a set of stand variables is computed including stand basal area, basal area sum of trees larger than the subject tree, and dominant height. If not measured, the live crown ratio is computed for each tree. The models are listed below, and the respective coefficients are given in Table A1. Lower-case symbols refer to tree characteristics and upper-case symbols to stand characteristics.

Tree basal area growth:

$$i_{ba} = \beta_0 \, e^{\beta_1 d + \beta_2 d^2} \, cr^{\beta_3} \, BA^{\beta_4} \, H_{100}{}^{\beta_5} \, (21t_{1.3})^{\beta_6} \, (h/H_{\text{dom}})^{\beta_7} \cdot \epsilon \qquad \text{(A.1)}$$

Height growth:

$$i_h = \beta_0 \, h^{\beta_1} \, H_{100}{}^{\beta_2} \, (h/H_{\text{dom}})^{\beta_3} \cdot \epsilon \qquad \text{(A.2)}$$

Live crown ratio:

$$cr = \beta_0 \, d^{\beta_1} \, t_{1.3}{}^{\beta_2} \, e^{\beta_3 (h/d)} \, BA^{\beta_4} \cdot \epsilon \qquad \text{(A.3)}$$

To predict mortality in a stand, a two-stage procedure is used. In stands with less than full stocking, Equation (A.4) is applied. It is designed to predict the average mortality in managed stands when no major damages exist. In fully stocked stands, the number of trees is first checked against the stand level self-thinning curve [Equation (A.5)]. If it is above the curve, the number of trees is reduced to meet the limiting density. The excess number of trees is distributed into different diameter classes (or elements of the tree list) so that the probabilities given by Equation (A.4) for each diameter class are scaled to sum up to the excess number of trees. The initial diameter distribution used in all analyses is presented in Table A2.

Tree mortality:

$$P = \frac{1}{1 + e^{\beta_0 + \beta_1 d + \beta_2 BA + \beta_3 BA_{\text{above}}}} + \epsilon \qquad \text{(A.4)}$$

TABLE A1.

Coefficients of equations used to predict stand development.

Equation	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
A.1	2.583	0.1142	−0.00163	0.7742	−0.3772	0.03939	−0.2551	0.8393
A.2	−2.262	−0.7239	1.174	1.4405				
A.3	173.6	0.1254	−0.1811	−0.1344	−0.2192			
A.4	4.396	0.0956	0.2042	−0.199				
A.5	1.561	1.73	−0.175					

TABLE A2.

Diameter distribution of a pure Norway spruce stand used as the initial stand.

Diam. class no.	dbh (cm)	Height (m)	Live crown ratio (%)	Trees/ha
1	1.9	2.6	43.1	30
2	4.3	5.1	48.8	50
3	6.2	6.9	51.7	91
4	8.2	8.5	54.1	140
5	9.9	9.7	55.8	361
6	12.1	11	57.7	470
7	13.9	11.9	59.2	489
8	15.7	12.7	60.5	231
9	17.7	13.4	61.9	50

Self-thinning curve:

$$\ln D_{ba} = \beta_0 + \beta_1 \ln H_{100} + \beta_2 \ln H_{100} \ln N + \epsilon \qquad \text{(A.5)}$$

The symbols are:

i_{ba} = tree basal area growth in the coming 5-yr period, cm^2

i_h = tree height growth in the coming 5-yr period, m

cr = live crown ratio, %

P = probability of death in the coming 5-yr period

β_i = coefficients

d = tree dbh, cm

g = tree basal area, cm^2

h = tree height, m

$t_{1.3}$ = tree age at breast height, yr

BA = stand basal area, m^2/ha

BA_{above} = basal area sum of trees larger than the subject tree, m^2/ha

H_{dom} = dominant height of stand (average height of the 100 thickest trees/ha)

H_{100} = site index (dominant height at 100 yr)

D_{ba} = basal area weighted average dbh, cm

N = number of trees/ha

ϵ = error term of regression equation

Individual tree pulpwood and sawtimber volumes are based on the taper curve models by Laasasenaho (1982). Logging costs (*LC*) are computed as functions of the total volume removed, v_{tot}, and the average tree volume, $\bar{v}$. Separate functions for thinnings (A.6) and final harvests (A.7) were derived based on the Finnish logging and hauling work tariffs. Costs of logging after a disastrous event are doubled.

$$\ln LC_{TH} = 5.410 - 0.05217 \ln \bar{v} + 0.02429 (\ln \bar{v})^2 - 0.4451 \ln v_{tot} + 0.03969 (\ln v_{tot})^2 \quad (A.6)$$

$$\ln LC_{FH} = 5.230 - 0.05976 \ln \bar{v} + 0.02076 (\ln \bar{v})^2 - 0.3840 \ln v_{tot} + 0.03273 (\ln v_{tot})^2 \quad (A.7)$$

Roadside timber values were chosen as representative of southern Finland. They amounted to 210 Finnish marks/m^3 for spruce sawtimber and 180 Finnish marks/m^3 for spruce pulpwood. Total regeneration costs, including young stand tending, were taken to be 4300 Finnish marks/ha.

The soil expectation values for the deterministic analyses are computed as an infinite series of rotations equal to the first one. For stochastic analyses, a bare land value is added to the first rotation final harvest value. In cases where stochasticity does not considerably affect the profitability of timber growing, the optimum soil expectation value of a corresponding deterministic alternative is used as the bare land value. When stochasticity enters the model as the probability of a catastrophe, the level of the discounted net present value is reduced with increasing stochasticity. The bare land value of a deterministic analysis is, therefore, an overestimate. To obtain scaled down bare land values, repeated optimization runs were made iteratively for each combination of catastrophe risk levels and thinning specifications.

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