Forest Ecology and Management 563 (2024) 121998

Contents lists available at ScienceDirect

Forest Ecology and Management

journal homepage: www.elsevier.com/locate/foreco

Site index and stand characteristic models for silver birch plantations in southern and central Finland

Daesung Lee[a,*,1], Jouni Siipilehto[a,2], Jari Miina[b,3], Pentti Niemistö[c,4], Matti Haapanen[a,5], Jari Hynynen[d,6], Saija Huuskonen[a,7]

[a] *Natural Resources Institute Finland (Luke), Latokartanonkaari 9, Helsinki 00790, Finland*
[b] *Natural Resources Institute Finland (Luke), Yliopistokatu 6 B, Joensuu 80100, Finland*
[c] *Natural Resources Institute Finland (Luke), Kampusranta 9, Seinäjoki 60320, Finland*
[d] *Natural Resources Institute Finland (Luke), Vipusenkuja 5, Savonlinna 57200, Finland*

A R T I C L E I N F O

Keywords:
Betula pendula Roth
Site index
Chapman-Richards function
Seemingly unrelated regression
Nonlinear and linear mixed-effects models
Thinning
Site type

A B S T R A C T

Great interests in silver birch (*Betula pendula*) forests have increased in an attempt to handle biodiversity and resilient forest management with more broad-leaved forests. However, up-to-date silver birch growth and yield models are still needed to predict the stand development in the future and support operational forest management and planning. The objectives of this study were to develop site index and stand characteristic models for silver birch plantations. Data for modelling were from the thinning and spacing experiments and tree breeding trials on silver birch plantations in southern and central Finland. The dominant height and site index (SI) models were fitted using the nonlinear mixed effect regression approach based on the Chapman-Richards function with the genetic effect from improved seedlings. The modelling result indicated a logical growth performance over age and higher dominant height with genetically improved seedlings. The stand characteristic models for unimproved seedlings were developed using multivariate mixed-effects modelling approach. The targeted, response variables were the basal area-weighted mean height (HW, m), the basal area-weighted mean diameter (DW, cm) and stand basal area (G, m^2 ha^{-1}) of silver birch. Stand biological age (AGE, year), SI estimated based on the model of the current study with the base age of 50 years, and the number of silver birch trees (N, trees ha^{-1}) were all commonly applied as highly significant predictor variables for all of HW, DW, and G models. In addition to these predictors, thinning variables, comprised of the time since last thinning in year and the thinning intensity based on G, were highly significant in the DW and G models with logical behavior. Recent thinning affected negatively at first because trees in thinned stands were slimmer than those in unthinned stands that have grown from the early stage at the same level of N with thinned stands. After 13 years since last thinning, the effect turned positive indicating that the increments were expedited with thinning. Lastly, site type classified as either former agricultural land or forest site was included in the DW and G models. Consequently, all the models developed in this study were evaluated as practicable with easy-to-use predictors and desirable accuracy.

1. Introduction

An interest in silviculture of silver birch (*Betula pendula*) plantations has been increasing in Nordic and Baltic countries with respect to biodiversity and climate change (Hynynen et al., 2010; Dubois et al., 2020). In Finland, silver birch, the growing stock volume of which

* Corresponding author.
E-mail address: daesung.lee@luke.fi (D. Lee).
[1] orcid.org/0000-0003-1586-9385
[2] orcid.org/0000-0002-5661-8972
[3] orcid.org/0000-0002-8639-4383
[4] orcid.org/0000-0002-9152-2108
[5] orcid.org/0000-0003-3294-501X
[6] orcid.org/0000-0002-9132-8612
[7] orcid.org/0000-0001-8630-3982

https://doi.org/10.1016/j.foreco.2024.121998
Received 27 January 2024; Received in revised form 12 April 2024; Accepted 14 May 2024

occupies 418 million m^3 (16.5%) out of the total, is the most important deciduous tree species in commercial forests, which, however, are heavily dominated by Scots pine (*Pinus sylvestris*) and Norway spruce (*Picea abies*) (Kulju et al., 2023). Therefore, comprehensive research has been conducted addressing, e.g., tree breeding, seedling production, regeneration, and management of silver birch stands. There is a great potential for increasing silver birch role in forest management and so there is a need for updated research results (Liziniewicz et al., 2022).

The prevailing Finnish growth and yield models for silver birch were developed based on naturally regenerated stands or stands planted with bare-root seedlings representing an unidentified seed source (Oikarinen, 1983; Gustavsen and Mielikäinen, 1984) or based on mixed stands growing with conifers (Hynynen et al., 2002, 2014). These models set up the basis for current thinning guidelines for silver birch (Oikarinen, 1983; Gustavsen and Mielikäinen, 1984; Rantala, 2011; TAPIO, 2023). However, new silver birch stands in Finland are mostly established by planting container seedling material grown from genetically improved seed (Kulju et al., 2023). Thus, the existing models may be inaccurate when predicting the growth and yield of such stands.

Therefore, more accurate and up-to-date models for planted silver birch stands are required in practical forestry. Several studies have shown that stand-level models can adequately predict the growth and yield of managed, even-aged stands as an alternative to individual tree models (Pienaar and Rheney, 1995; Härkönen et al., 2010; Scolforo et al., 2019). Stand-level models describe age dependent development of the selected stand characteristics. They can be flexibly used together with diameter distribution models for converting stand-level information into tree-level information and predicting the timber assortments (Siipilehto, 2011; Lee et al., 2021b, 2023). For these reasons, the models for describing the stand structure have been developed, such as basal area-weighted mean height (HW), basal area-weighted mean diameter (DW), and stand basal area (G) (Næsset, 2002; Siipilehto, 2011; Lee et al., 2021b, 2023). Depending on the situation of target stands, the number of trees (N) was modelled (Siipilehto, 2011) or not modelled but instead used as a predictor variable (Lee et al., 2023). For accurate predictions of stem volume and biomass by timber assortment, N, HW, DW, and G are used as predictor variables in diameter distribution such as recovery of the 2-parameter Weibull function or individual tree height allometric equation, such as Näslund height curve (Siipilehto and Mehtätalo, 2013; Lee et al., 2021b). In addition to age dependent development of stand characteristics, the effect of stand management has to be also taken into account. Silver birch is a light-demanding species which benefits from quite heavy thinnings (Niemistö, 1997).

Site index (SI) is commonly used in growth and yield models to depict site productivity for single-species, even-aged stands. Site index models for silver birch have been developed in many countries in northern Europe (Eriksson et al., 1997; Donis, 2022; Maleki et al., 2022; Kiviste et al., 2022). In Finland, Oikarinen (1983) developed site index models for planted silver birch stands. However, since 1980s the growth rate of planted silver birch stands has noticeably improved due to tree breeding, development of seedling production in nurseries, and regeneration practices as well as climate change (Haapanen, 2024; Henttonen et al., 2024). Therefore, there is a need to develop more predictable models in practice.

The objectives of this study were to develop the new stand-level growth and yield models for silver birch plantations in southern and central Finland. The specific aims were 1) to develop the models for dominant height (H00) growth and site index, 2) to develop models for the stand characteristics (HW, DW, and G), 3) to assess the reliability of developed models, 4) and to examine the model performance by comparing them with the results of other silver birch studies in northern Europe.

2. Materials and methods

2.1. Data

Modelling data were collected from silver birch plantations on mineral soil sites located in southern and central Finland. The data comprised of two kinds of field trials: thinning and spacing trials (Luke 1) and tree breeding trials (Luke 2) to take into account both the effects of forest management and tree breeding in the analysis (Table 1).

2.1.1. Long-term thinning and spacing experiments

The first data source, the thinning and spacing experiments (Luke 1), consisted of repeatedly measured data from 21 stands in southern and central Finland (Niemistö, 1995a, 1995b, 1997) (Fig. 1). All the sites were situated in mineral soil and site type varied from former agricultural land to forest sites (MT or OMT) by plot. MT refers to fresh site (Myrtillus forest type) and OMT to herb-rich site (Oxalis–Myrtillus forest type) (Cajander, 1949; Tonteri et al., 1990). Soil types of the plots were mold, silt, fine sand, sand, gravel, fine sand moraine, or sand moraine, and it did not include clay soil type.

In the experimental stands, trees were planted in 1969–74 and the plots were established in 1980—1990 when biological age (AGE) of the stands was 11–32. At the last measurement the age was 39–53 years. However, one clear deviant site was in Punkaharju with four plots established 1976 at age of 27 years and last measurement at age of 69 years. All the experimental plots were planted silver birch stands, but the seedlings were unimproved materials and unknown in terms of the origin. Also, there were a few plots in which Norway spruce grew naturally from the understory, which is commonly occurred in silver birch stands of Finland. Thus, the plots were included in model development of this study considering the practical aspect in forestry fields. The plots of extremely rectangular spacing with row-to-row distance more than 3 times plant-to-plant distance in rows in the spacing experiments (cf., Niemistö, 1995a, 1995b) were excluded in this study.

The thinning experiments were firstly measured at the time of plot establishment, and thereafter regularly. Without few small exceptions the regular interval between consecutive measurements was 5 growing seasons. Depending on experimental design of plots in each site, one to three thinnings were carried out with various thinning intensity along with unthinned plots (Niemistö, 1997). Certain plots were additionally thinned once more nearly at age 40–45, also known as late heavy thinning. Thus, including the thinning before or at the plot establishment and late heavy thinning, the total number of thinning was up to four at maximum. The applied thinning method was thinning from below in the plots. As a result of thinning from below, thinning quotient, a quotient dividing the mean diameter of removed trees by the mean diameter of all trees before thinning, disclosed less than 1 in most plots. Only in some cases after late heavy thinning, this quotient exceeded 1 because of low stem number with even spacing targeted.

Rectangular plots were established in each stand, with an average plot size of 959 m^2 with buffer zones wider than 5 m. In the field data collection, all trees on the plots were mapped with their x and y coordinates and measured for diameter (mm) at breast height from two perpendicular directions. The species, retention class (living and to be retained, living and marked for removal in thinning, dead, or disappeared), and any damage were recorded for each tree on the plot. The height (dm) was measured from sample trees. The sample trees were selected with the aim of selecting around 40 trees on each plot before thinning treatments. Any trees with visible disease or damage were not included as sample trees. The probability of a tree being selected was proportional to its basal area at breast height, but the sample trees were randomly located on a plot. The same sample trees were measured in all measurements, and no new sample trees were added after thinning.

2.1.2. Tree breeding trials

The second data source, tree breeding trials (Luke 2), comprised of

Table 1
Characteristics and management of silver birch trials in southern and central Finland.

Variables	Thinning and spacing experiments (Luke 1)		Tree breeding trials (Luke 2)	
	Thinning experiments	Spacing experiments	Unimproved materials	Improved materials
Dataset structure				
No. of stands	18	3	22	19
No. of plots	180	60	57	72
No. of measurements by stand	1–9	1–7	1–3	1–3
Observation period by plot	28.9±7.7 (5–42)	24.3±3.9 (5–28)	5.5±6.1 (0–18)	1.9±4.4 (0–18)
Intervals between measurement in year	5±0.4 (4–8)	5±0.4 (4–10)	3±2.6 (0–7)	6±2.8 (2–11)
No. of observations	1494	411	143	95
Management				
Density[a] at the first measurement (trees ha^{-1})	880–3072	485–4467	832–4444	818–4365
Year of establishment	1977–1995	1984–1992	1989–2005	1989–2005
Seed information	unimproved	unimproved	unimproved	improved
Plot size (m^2)	490–1250	496–1710	300–4600	300–3310
No. of thinning	0–4	0–3	0–2	0–2
Thinning intensity[b] (%)	36.9±14.0 (1.8–66.3)	35.0±9.3 (18.2–60.5)	31.3±4.7 (24.5–39.5)	47.7±18.4 (23.7–75.1)
Time after last thinning (year)	8.7±7.9 (0–39)	6.8±6.3 (0–25)	4.3±3.8 (0–12)	8.8±3.5 (0–12)
Site characteristic				
Latitude	N 61°15′–65°36′	N 62°21′–65°51′	N 60°5′–62°49′	N 60°5′–62°48′
Longitude	E 24°7′–30°29′	E 26°9′–28°51′	E 24°47′–30°37′	E 24°47′–30°37′
Elevation above sea level (m)	80–204	70–105	40–150	40–150
Temperature sums above 5 °C (°C·days)	1070–1348	1124–1296	1282–1460	1287–1409
Lake index	0.3–63.9	0.7–21.7	0.8–44.5	2.9–44.5
Site type[c] with no. of stands	Former agricultural land (Field) 10 Forest site (OMT) 3 Forest site (MT) 5	Former agricultural land (Field) 3	Former agricultural land (Field) 15 Forest site (OMT) 1 Forest site (MT) 6	Former agricultural land (Field) 12 Forest site (OMT) 1 Forest site (MT) 6
Site index[d] (SI, m)	26.9±2.8 (18.6–32.9)	25.0±2.5 (19.9–30.4)	30.9±2.8 (23.8–38.2)	31.7±4.0 (23.0–41.9)

[a] The density is not always defined as initial planting density.
[b] Thinning intensity based on stand basal area.
[c] OMT is Oxalis-Myrtillus (a herb-rich heath forest) site type and MT is Myrtillus (a mesic heath forest) site type (Cajander, 1949).
[d] Site index with a base age of 50 years.

20 sites planted on former agricultural land and forest site in southern Finland (Fig. 1). The trials were established to determine genetic differences in growth and quality of various test materials comprising progenies of plus trees (classified as "improved") and seedlots from natural stands (classified as "unimproved"). A randomised complete block design was utilized in the trials. The experimental units (plots) in the trials of Luke 2 were smaller than those of Luke 1. We decided to pool all the subplots which were smaller than 250 m^2, so that it allowed us to retain an adequate number of observations while keeping the stand and tree characteristics sufficiently uniform.

The collection of field data in the Luke 2 trials was similar to those in Luke 1 group, involving recording of DBH, tree height, and mortality. Sometimes thinning history was not recorded sufficiently, but most of the trials had been thinned 0–2 times before the data collection. When thinning intensity based on basal area was not documented, we directly determined by record for few observations and the number of thinned trees was used to estimate the nearest former thinning intensity. Based on the thinning quotient which was below 1, these trials appeared to have been thinned from below.

2.1.3. Stand characteristics and additional variables for modelling

The calculated main stand variables for silver birch were N, HW, DW, G, H00, and SI. N was defined as the solely number of silver birch trees and H00 as the mean height of 100 thickest trees per ha (Table 2). Stand variables were calculated with KPL-software based on tree data (Heinonen, 1994). The response variables (HW, DW, and G) and predictor variables (AGE, N, and SI) were confined only to silver birch. Naturally grown Norway spruce trees existed occasionally in understory layer only in 21 plots from 5 sites (out of 369 plots in 62 sites total) with 66 trees ha^{-1} on average, representing 4.1% out of total stem number on average. SI was assessed based on the site index model developed in this study. The finally applied SI in the stand characteristic models was the mean SI values from all measurement instances to compromise any variations in each observation. For example, if a plot was measured 8 times with a 5-year interval from 20 to 55 years in AGE, the final SI was calculated by averaging all of the eight SI values.

Thinning effects were described by thinning intensity and time elapsed since last thinning. Thinning intensity based on proportion of removed stand basal area (THIN) was computed for each thinning scaled 0–1. Time after last thinning in years (Talty) was calculated by observations. Due to the relative lack of the observations and rather constant thinning effect when Talty ≥ 25, a modified variable was also tested to restrict a persistently intensified impact with a large Talty; $Talty_{max25}$ was considered to be the same value of Talty if Talty < 25 and be fixed to 25 if Talty ≥ 25. To examine any additional climatic and/or edaphic factors, the long-term average of the annual effective temperature sum (in degree-days, threshold value +5 °C, GDD5) for the period 1981–2010 (Venäläinen et al., 2005) was collected. Additionally, lake and sea index (scaled 0–100) referring to the proportional coverage of lakes or sea within a distance of 20 km radius were added (Ojansuu and Henttonen, 1983).

2.2. Modelling approach and statistical analysis

Model developments were proceeded with two set of model fitting stages: site index and stand characteristics. Additionally, genetic effects were considered in modelling. Therefore, dominant height and site index models were firstly developed considering genetic effects. Afterwards, stand characteristics models for HW, DW, and G of unimproved seedlings were developed considering SI estimated based on the model in the first stage (Appendices A, B, and C). Using only unimproved materials in modelling the silver birch stand characteristics, the level of

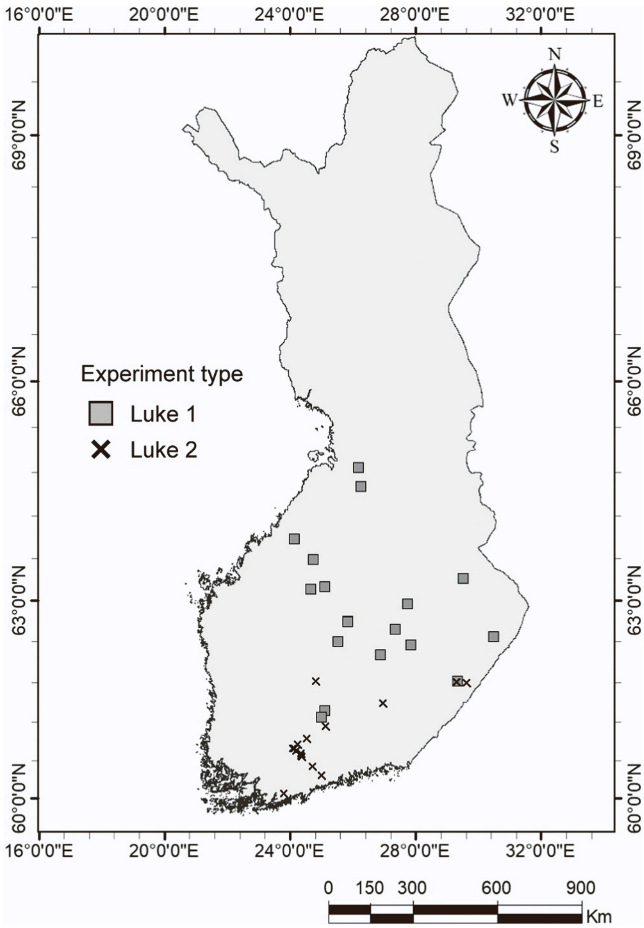

Fig. 1. Map of the experimental sites used for the models of silver birch plantations in southern and central Finland. Luke 1: Thinning and spacing experiments, Luke 2: Tree breeding trials.

genetic gain could be appropriately estimated and included in model predictions in the future as done earlier by incorporating genetic gains of Scots pine in the simulation software MOTTI (Haapanen et al., 2016).

2.2.1. *Site index*

To develop a site index model, a well-defined, conventional method was applied through the relationship between stand age and dominant height. Among many of growth functions used in the field of forest biometrics, Chapman-Richards function was selected as a base equation in this study (Richards, 1959). The function is known to be flexible with three parameters reflecting the asymptote, growth rate, and shape or inflection point of growth patterns (Goelz and Burk, 1992). Furthermore, the function has been widely studied in other countries and/or other tree species, so it's readily comparable to any other research using the same function (Prévosto et al., 1999; Lee et al., 2021a; Lee and Choi, 2022; Huuskonen et al., 2023; Zeltiņš et al., 2024).

For dominant height the Chapman-Richards model as a function of stand age was developed, and a genetic effect (GE) was tested as a categorical variable in all parameters (a, b and c) as well as all possible combinations of them as follows in Eq. (1):

$$\mathrm{H00} = (a + z \times \mathrm{GE} + u_1)\left(1 - e^{-(b+z\times\mathrm{GE})\mathrm{AGE}}\right)^{(c+z\times\mathrm{GE}+u_2)} + \varepsilon \quad (1)$$

where H00 is the dominant height (m), a, b, c, and z are the model parameters, AGE is the biological stand age (year), GE is the categorical variable for the genetic effect (1 if improved material, 0 otherwise), u_1 and u_2 are random effects at experiments (or sites) level, and ε is the random error term. In modelling, experiments (or sites) were considered as random effects, so the nonlinear mixed-effect model was fitted via the *nlme* function in the *nlme* package in R statistical software (R Core Team, 2019; Pinheiro et al., 2023) (Appendix C). Due to convergence problems when applying a random effect on parameter b, random effects were included in only parameters a and c (cf. Lappi and Bailey, 1988; Hall and Bailey, 2001; Huuskonen and Miina, 2007; Lee et al., 2021a).

Using the fitted dominant height model and the base age of 50 years, site index guide curves were presented based on anamorphic site index equations (Appendix B). The base age (or reference/index age) was determined as 50 years after referencing the former studies about silver

Table 2
Descriptive statistics of modelling dataset for silver birch plantations from southern and central Finland by first and last measurements.

Stand characteristics	Thinning and spacing experiments (Luke 1)				Tree breeding trials (Luke 2)			
	Thinning experiments		Spacing experiments		Unimproved materials		Improved materials	
Time of measurements	*First*	*Last*	*First*	*Last*	*First*	*Last*	*First*	*Last*
Biological stand age (AGE, year)	20±4 (13–27)	48±8 (24–69)	23±8 (11–32)	47±6 (32–52)	15±6 (7–24)	21±9 (12–35)	18±5 (7–24)	20±5 (12–35)
Stand density of silver birch (N, trees ha^{-1})	1816.0±486.9 (880–3072)	626.3 ±375.9 (171–2120)	1699.0 ±966.1 (485–4467)	850.6 ±481.2 (240–3090)	2106.1 ±1244.8 (832–4444)	1808.4 ±1259.9 (448–4524)	1647.3 ±691.7 (818–4365)	1617.8 ±696.6 (439–4167)
Dominant height (H00, m)	13.3±3.2 (5.3–21.4)	26.5±3.4 (16.7–33.0)	12.4±3.6 (5.8–19.5)	26.0±2.1 (17.0–29.6)	12.9±4.5 (5.3–19.7)	15.8±5.4 (8.2–24.9)	14.9±4.0 (5.6–21.5)	16.0±3.7 (9.7–25.1)
Height, basal area-weighted mean (HW, m)	12.1±3.1 (4.7–19.0)	25.4±3.4 (15.9–32.0)	11.2±3.3 (5.1–18.0)	24.5±2.1 (16.3–28.4)	11.9±4.4 (4.7–19.1)	13.8±6.7 (4.9–24.6)	13.6±4.3 (5.5–19.8)	14.3±4.8 (4.4–24.3)
Diameter, basal area-weighted mean (DW, cm)	10.8±2.5 (4.6–15.8)	22.6±3.7 (13.7–31.8)	10.6±2.7 (6.2–15.7)	20.9±2.9 (12.6–26.7)	10.1±3.9 (3.5–15.4)	13.1±5.0 (6.9–22.8)	11.7±3.2 (4.3–17.8)	12.8±3.3 (8.2–23.3)
Stand basal area (G, m^2 ha^{-1})	13.7±4.6 (2.6–22.7)	20.6±6.4 (9.1–39.9)	11.9±5.5 (2.0–24.7)	23.8±6.2 (10.2–35.6)	12.2±6.1 (3.5–24.2)	15.3±4.3 (6.2–24.2)	14.8±6.2 (5.5–29.8)	16.8±5.2 (9.6–29.8)
Cumulative mortality at last measurement[a] (trees ha^{-1})	-	641.9 ±495.0 (50–2536)	-	500.8 ±647.1 (38–2098)	-	-	-	-

[a] The cumulative mortality was calculated only from the unthinned plots based on the difference of the number of remaining silver birch trees between the first and last observation at which the measurement range is greater than 20 years.

birch in northern Europe (Oikarinen, 1983; Gustavsen and Mielikäinen, 1984; Donis, 2022; Maleki et al., 2022) and considering the silver birch growth and recommended final felling age in practical forestry (Rantala, 2011).

2.2.2. Stand characteristics

In the modelling process of stand characteristics, the correlations between the response variables, such as HW, DW, and G, can be taken into account by using the multivariate modelling approach (Zellner, 1962). One option is to apply the seemingly unrelated regression (SUR) in model fitting (e.g., Fang et al., 2001; Hall and Clutter, 2004; Siipilehto, 2011; Lee et al., 2023). SUR leads to efficient estimation that accounts for the between-model correlations (Goldstein, 1995). In addition to this, a mixed-effect model is preferred to handle hierarchical data such as repeated measurements (Laird and Ware, 1982). Thus, the SUR approach with random effects can be considered as the most adequate modelling technique, also known as multivariate mixed-effects modelling (Mehtätalo and Lappi, 2020).

To fit the parameters for the stand characteristics, we applied multivariate mixed-effects modelling techniques as HW, DW, and G could be interactively affected by one another. The data were restructured in a longitudinal format to fit the models of HW, DW, and G simultaneously with the SUR approach (Lee et al., 2023) (Appendix C). For each response variable, a multiplicative model structure was assumed (Eq. 2), so HW, DW, and G were transformed to logarithmic scale to linearize the equation (Eq. 3) (Lee et al., 2023). Logarithmic transformation in Eq. (3) adequately solved the scedasticity problem by homogenizing the original heteroscedastic residual error (Eid, 2001; Siipilehto, 2011).

$$Y = b_0 X_1^{b_1} X_2^{b_2} \cdots X_n^{b_n} \varepsilon \tag{2}$$

$$\ln Y = \ln b_0 + b_1 \ln X_1 + b_2 \ln X_2 + \cdots + b_n \ln X_n + \ln \varepsilon \tag{3}$$

where Y is the response variable, b_0–b_n are the model parameters, X_1–X_n are the predictor variables, and ε is the random error.

The major predictor variables were AGE, SI, and N. SI was the arithmetic mean of all observations by plot between AGE of 12–60 years to exclude extreme SI estimation from very young or too mature stands after the recommended final felling age. SI did not vary much between measurements, and the mean SI value provided more conservative or stable estimation as a finally applied predictor variable. THIN, Talty, and $\text{Talty}_{\text{max25}}$ were also examined to consider thinning effects on the stand development. Additionally, climatic and edaphic variables such as GDD5, lake and sea indices, site type, and soil type were tested.

In modelling stage, transformed variables, including reciprocal, squared, square root, natural logarithmic, etc., were tested to find the most significant and logical predictors. Also, a small constant k varying from 0 to 10 was tested in the reciprocal form of AGE, $1/(\text{AGE} + k)$, to solve the curvilinearity and figure out most suitable predictor variables.

Due to several observations from the same experiments, the random between-experiment effects were included in the intercept of the models for HW, DW, and G. The random effects and random errors had a mean of 0 and constant variances and were assumed to be correlated across the models. To take this into account, multivariate mixed-effects modelling was carried out through the *lme* function of the *nlme* package in R (R Core Team, 2019; Lee et al., 2023).

2.3. Simulation and model validity

All the models developed in this study were confirmed with statistical significance, goodness-of-fit statistics including AIC, BIC, and −2 log-likelihood, and simulations. Overall metrics of residuals (observed value − predicted value) were offered with root mean square error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE) for both mixed and fixed effects predictions. RMSE% was calculated in percentage scale by dividing RMSE by the observed mean of the stand characteristic in each model. The residuals not only over the predicted values but also over the predictor variables were examined to verify unbiased model performance. Supplementary figures via the *whiskers* function of the *lmfor* package were provided by adding vertical lines onto residual plots to show 95% confidence intervals for individual observations in the classes of the variable on the x-axis (Mehtätalo and Kansanen, 2022) (Appendix A).

Specifically, the dominant height model, which was applied to site index guide curves, was checked with both prediction types from mixed effects and fixed effects in residual plots. Thereafter, the general dominant height growth patterns by genetic effect over stand age were identified based on the developed model. The developed site index models were compared with corresponding models among the countries in northern Europe.

With regards to the stand characteristic models, the predictions in arithmetic scales were analyzed together with original logarithmic scale. The residual analyses and goodness-of-fit metrics for stand characteristics were similarly examined as in the site index model. Additionally, when the simulation was proceeded in the arithmetic scale, a half of the total residual variance was added in logarithmic scale for back-transformation to correct predictions and reduce unexpected underestimates (Baskerville, 1972; Burkhart and Tomé, 2012; Lee et al., 2023) (Appendix B).

Stand characteristics were simulated in a range of modelling data so that the model behaviors were analyzed in the stand condition with maximum and minimum values such as AGE, SI, and N. Furthermore, the level of the maximum number of trees and, by extension, the maximum basal area was checked during the simulation through the previously developed models (Hynynen, 1993; Hynynen et al., 2002) although any self-thinning model according to natural mortality was not developed in the present study.

3. Results

3.1. Dominant height and site index

3.1.1. Model development and validation

Using the relationship between AGE and H00, model parameters were fitted based on the Chapman-Richards function, and the most significant parameter for genetic effect was found in asymptote term (parameter a in Eq. 1). All the estimated parameters including the genetic effect were highly significant with the random effect of experiments (or sites) (Table 3), and thus, the finally selected equation was as below in Eq. (4).

$$\text{H00} = (a_0 + a_1 \times \text{GE} + u_1)\left(1 - e^{-b \times \text{AGE}}\right)^{(c+u_2)} + \varepsilon \tag{4}$$

where H00 is the dominant height (m), a_0, a_1, b, and c are the model parameters, AGE is the biological stand age (year), GE is the categorical variable (0 or 1) for the genetic effect, u_1 and u_2 are the random effects at experiment level, and ε is the random error term.

The dominant height model was unbiased over predicted values and stand age (Fig. 2). The model predicted dominant height accurately with circa RMSE 1 m and MAPE 4% with mixed effects and 2.4 m and 11%, respectively, with fixed effects. The genetic effect (GE) presented higher height (Fig. 3a) and height growth (Fig. 3b) at certain age. The height difference between improved and unimproved seedlings increased over age and was 1.0 m at age 20, 1.6 m at age 40, and 2.0 m at age 60, for example (Fig. 3a).

By comparing the dominant height curves and its periodic annual increment of our model with the prediction of Oikarinen (1983), it revealed a similar pattern at early age, but the line of the previous study was low at later stage (Fig. 3). The dominant height growth from Gustavsen and Mielikäinen (1984) was generally lower than that from our model indicating the difference between planted and naturally

Table 3
Parameter estimates and fit statistics of dominant height growth model for silver birch plantation in southern and central Finland using the nonlinear mixed-effect modelling approach based on the Chapman-Richards function as provided in Eq. (4).

Class	Parameter	Estimate	S.E.
Fixed effects	a_0	36.9164	0.8677
	a_1	2.4096	0.4791
	b	0.0320	0.0016
	c	1.2239	0.0499
Random effects	std(u_1)	2.3366	
	corr(u_1, u_2)	0.442	
	std(u_2)	0.2336	
Random error	std(ε)	1.0040	
Fit statistics	AIC	5402.874	
	BIC	5446.843	
	-2logLik	5386.874	
		Mixed effects	*Fixed effects*
Metrics of model accuracy	RMSE	0.9859	2.3800
	MAE	0.7634	1.8559
	MAPE	0.0404	0.1076

Note: all fixed-effect parameters were highly significant ($P < 0.0001$). a_0, a_1, b, and c are fixed-effect parameters. u_1 and $u_2$are random-effect parameters at experiment level. std refers to the standard deviation. corr refers to the correlation. std(ε) is the standard deviation of the residual in the model performance. AIC is the Akaike information criterion. BIC is the Bayesian information criterion. −2logLik is the −2 × log-likelihood value. RMSE is the root mean square error. MAE is the mean absolute error. MAPE is the mean absolute percentage error.

regenerated stands (Fig. 3). Note that the predicted values from Oikarinen (1983) and Gustavsen and Mielikäinen (1984) can be varied depending on the initial input value of age and dominant height as their models were developed targeting for dominant height increment or its percentage; the initial input values of dominant height and stand age in the simulation of the previous models were set similar to the observations in our data.

3.1.2. Site index examination

Based on the results of dominant height modelling, site index guide curves were provided using the algebraic difference equation approach (Lee et al., 2015, 2021a). By setting AGE=AGE_0 (or base age) in Eq. (4), H00 implied SI, and the asymptote term was chosen to be substituted for a type of anamorphic site index curve (Appendix B) (Lee et al., 2021a). The two set-up equations with the estimated parameters can be rearranged to either H00 for site index guide curves (Eq. 5) or SI for site index estimation (Eq. 6) (Lee et al., 2015) as follows:

$$\mathrm{H00} = \mathrm{SI} \times \left(\frac{1 - e^{-0.0320\mathrm{AGE}}}{1 - e^{-0.0320\mathrm{AGE}_0}}\right)^{1.2239} \quad (5)$$

$$\mathrm{SI} = \mathrm{H00} \times \left(\frac{1 - e^{-0.0320\mathrm{AGE}_0}}{1 - e^{-0.0320\mathrm{AGE}}}\right)^{1.2239} \quad (6)$$

where H00 is the dominant height (m); SI is the site index (m); AGE is the stand biological age (year); and AGE_0 is the base age of 50 years.

Using Eq. (5), site index guide curves of this study were produced with the base age of 50 years and checked comparing other previous studies (Fig. 4, Appendix B). Using Eq. (6), site index values were estimated as a predictor variable for each plot and used for the stand characteristic models in the latter part of analysis. Through this approach, one type of site index curves was formulated regardless of the GE parameter, but at the same time, the site index of improved seedling stands was estimated suitably with observations of the superior dominant height from stand measurement in practice (Fig. 4a).

The calculated site indices of the plots were dispersed mainly in the range of 18–33 m for unimproved seedlings. On the other hand, site indices of genetically improved seedlings were mostly 27, 30, or 33 m and even higher especially when the stand age was relatively young, near or less than 20 years (Fig. 4a). The site index curves of our study were compared with the previous studies on silver birch stands in northern Europe by setting up the same criteria with a base age of 50 years from biological stand age (Fig. 4). The site index curves in Oikarinen (1983), which provided 2-meter interval site indices, presented a lower line for a low SI level (22 m) and a higher line for a high SI level (28 or 30 m) until the base age compared to the present study, and it underestimated in a high SI level (28 or 30 m) after the base age. It seemed that the Oikarinen's (1983) model did not retain a convergence characteristic at young age and the asymptote was lower than the current study.

The models from Gustavsen and Mielikäinen (1984), which were developed based on natural silver birch stands, did not show a convergence characteristic in site index curves at young age. Overall, the site index curves of Gustavsen and Mielikäinen (1984) were lower than the present study. In the model of Eriksson et al. (1997) from Sweden, the asymptote was lower and higher growth rate appeared from early age, and thus, the curves at high SI level (27, 30, or 33 m) were higher than our curves before base age of 50 years and lower than our curves after base age. For example, an observation with H00=22 m and AGE=30 years is corresponded to SI 30 m from our model and SI 27 m based on Eriksson et al. (1997).

The models of Kiviste et al. (2022) in Estonia and Donis (2022) in Latvia were relatively comparable to our study, especially for the highest SI values. Specifically, their models showed less curvilinearity and it may indicate a higher growth rate and/or higher asymptote. Therefore, their models produced lower site index curves before base age of 50 years and higher site index curves after base age.

3.2. Stand characteristics

3.2.1. Multivariate model fitting

Multivariate mixed effects models for stand characteristics included AGE, SI, and N as common predictor variables (Table 4). The logarithmic forms of SI and N were used because of the assumed multiplicative structure (see Eqs. 2 and 3). The selected age-related variables, 1/(AGE+6) for HW and 1/(AGE+1) for DW and G, showed the best fit for the curvilinear relationship with age (Appendix A). The estimated parameters indicated that increasing N resulted in lower HW and DW and higher G (Table 4). All stand characteristics were positively correlated with SI.

Additional predictors regarding thinning information were found statistically significant in the models of DW and G (Table 4). The applied thinning variable was interaction between THIN and Talty. It was formulated as THIN×($\sqrt{\mathrm{Talty}_{max25}}$−3.5); here, Talty was restricted to have a maximum value of 25 (Talty_{max25}). The formulated thinning variable was capable to estimate the negative effect until 12 years of Talty and the positive effect since 13 years of Talty (Table 4, Fig. 5).

In searching for other climatic or edaphic factors, the categorical variable of former agricultural land (Agric) was found significant in the DW and G models (P=0.0395 for DW and P=0.0211 for G). It was not significant in the model for HW most probably because SI as a driving variable included its effect indirectly. The positive parameter signs of Agric in DW and G models stood for the superior/higher growth in a former agricultural land to the growth in a forest site. Meanwhile, other tested variables including temperature sum, soil type, and lake or sea index were found statistically insignificant in all models. The random experiment effect was associated to each intercept term. The estimated correlation of the random effects particularly between DW and G presented a highly correlated error (r=0.974), thus making calibration effective with best linear unbiased prediction (BLUP) if either one of the modelled variables is known (Siipilehto, 2011).

The models were validated through the residual plots over the

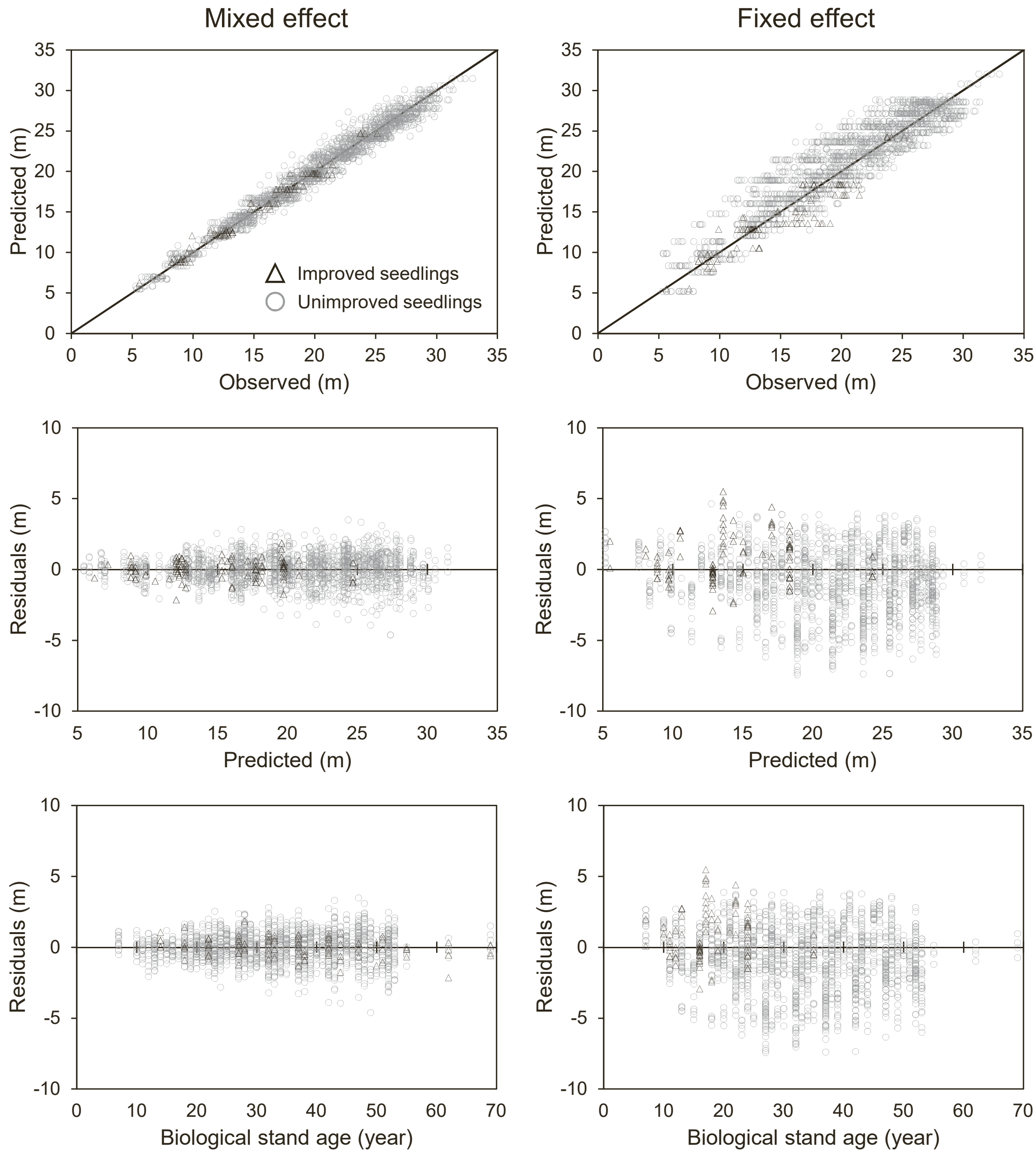

Fig. 2. Residual plots of dominant height development model via Eq. (4). The predicted values were calculated based on the fixed-effect or mixed-effect parameters provided in Table 3.

predicted values and predictor variables (Fig. 5, Appendix A). By comparing the contrast residual patterns from the models depending on thinning variables, the necessity of thinning variables and its suitability were obviously proved in Fig. 5. Moreover, the residual trend in the models without thinning variables demonstrated well that the constant of −3.5 (ca. 12 years in Talty class when squared) for Talty was adequate considering the transition of bias before and after this Talty class. The residuals did not show any obvious bias even after the logarithmic predictions were transformed back to arithmetic scale (Appendix A). Slight overestimation of HW could be seen when N was very low (Appendix A). In addition, a slight underestimation was found in the arithmetic residuals of HW model over stand age in very young and old age; still, it was the least biased prediction with the selected age-related term among all trials. Because of an impact of the random site effect, the residuals with mixed effects were dispersed narrowly compared to those with fixed effects (Appendix A).

Stand characteristic models demonstrated well-controlled predictions (Appendix B). In the residual analyses, log transformation homogenized the original heteroscedastic residual variation. All the residual plots using *mywhiskers* function can be checked from the supplementary figures (Appendix A). Considering the range or dispersion of stand characteristics in modelling data, all the metrics proved reasonable accuracy even in fixed-effect predictions with arithmetic scale: for example, RMSE of 1.03 m for HW, 1.12 cm for DW, 2.33 m^2 ha^{-1} for G, and RMSE% of 5.5%, 6.7%, and 13.0%, respectively (Table 5).

3.2.2. Simulation and comparison

Several model simulations of unthinned stands were performed to check model behaviors. Preassigned site index of 22, 26, and 30 m and the number of trees of 400, 1000, 1600 trees ha^{-1} were applied under the case of forest sites (Fig. 6). The predicted stand development of HW, DW, and G was mostly in the range of empirical modelling data. Among the predictions, some range in old and dense stands was found to be at the stand level undergoing self-thinning based on the previous model

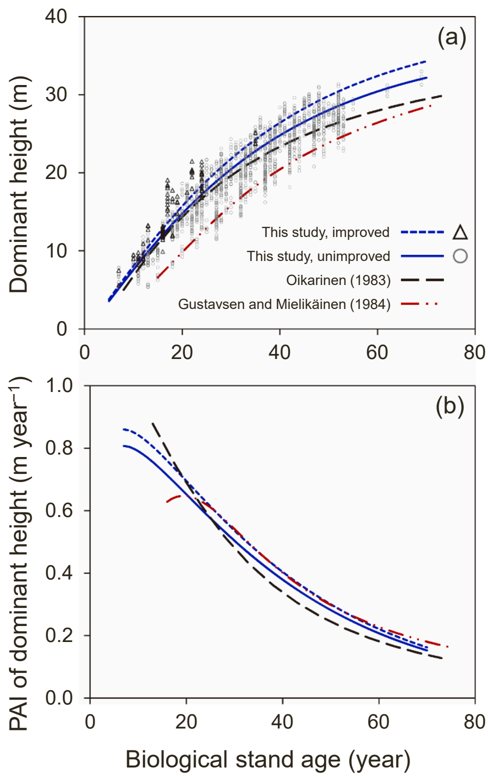

Fig. 3. Predicted dominant height curves (a) and periodic annual increment (PAI) (b) from the present study and previous studies with the observations of modelling data. The lines represent predicted values based on the models. The symbols represent the modelling data in this study: improved seedlings (GE) with black triangle (△) and unimproved seedlings with gray circle (○). The initial input values of dominant height and stand age were preset using the similar values from the observations in our data: 5.0 m at 8 years by Oikarinen (1983) from the bare root seedling plantations and 6.6 m at 15 years Gustavsen and Mielikäinen (1984) from the naturally regenerated stands as their model can be varied on a basis of dominant height increment or its percentage.

(Hynynen et al., 2002) (Fig. 6).

The predicted HW was affected intensively by site index (SI) and marginally by the number of trees (N). The HW model revealed a logical behavior as the predicted HW at age 50 in various N was close but lower than the given SI; for example, HW was 21.9–20.8 m in SI 22 and 30.6–29.1 m in SI 30 with N 400–1600 trees ha^{-1} (Fig. 6). While moderate impacts of SI and N were shown in DW model, G model was most obviously influenced by both SI and N and a certain prediction went far over the range of empirical data (Fig. 6). In general, the simulations demonstrated reasonable model performances in Fig. 6 except for the extrapolated combinations of N, SI, and AGE (e.g., N>3000, SI=30 at AGE ≥50).

In addition to the unthinned simulations, three stand conditions with varying the number of thinnings and thinning intensities were chosen from observations to verify the model performance on thinned stands as examples (Fig. 7). In general, the simulated development processes of stand characteristics proved logical model behavior and reasonable accuracy. Also, our models produced a similar increase of HW and DW and a decrease of G as a response of thinning compared with the observations. All the stand simulations of our data showed reasonable compatibility with the observed stand developments as demonstrated in Fig. 7.

4. Discussion

4.1. Evaluation of the materials and modelling rationale

Our modelling data of silver birch plantations covered the greater number of observations and longer measurement periods with more sites and plots compared to previous silver birch modelling studies (Oikarinen, 1983; Gustavsen and Mielikäinen, 1984; Eriksson et al., 1997). Our modelling data consisted of the plots in pure silver birch stands with thinning from below or unthinning treatments (Niemistö, 1995a, 1995b). Some of the plots comprised natural Norway spruce understory, which is common in planted silver birch stands in Finland. Data of breeding test trials from southern Finland (Luke 2) were included to enlarge geographical coverage and to take into account the genetic gain in growth due to deployment of improved regeneration material. Overall, the modelling data represented well planted silver birch stands in southern and central Finland.

Genetic origin of seedlings was considered when developing the site index model with the help of a categorical variable referring stands established with improved regeneration material. In the modelling of stand characteristics, data consisted of measurements from stands with unimproved seedling origin only because of the lack of representative measurement data from improved stands with varying thinning treatments. The applied multivariate mixed effects models showed practically unbiased predictions without heteroscedasticity over any of predictors (Appendices A, B, and C). Model simulations and comparisons verified reasonable and adequate model performances in both site index and stand characteristic models.

4.2. Assessment of dominant height and site index estimation

The dominant height model represented the observed development well and 2.4 m higher asymptote for improved seedlings seemed realistic (Fig. 3, Table 3). Unlike Oikarinen (1983) and Gustavsen and Mielikäinen (1984), the prediction by our model was not based on incremental growth, so that it is independent of initial input, e.g., preceding dominant height at a certain age. Regeneration method (natural vs planting) in addition to improved materials can be assessed as the main reasons why the predicted height by our model was generally higher than the previous models (Oikarinen, 1983; Gustavsen and Mielikäinen, 1984).

The site index curves were produced using the algebraic difference approach (ADA) regardless of breeding materials. Still, the application of ADA with the fitted parameters can be considered reasonable; site index curve itself was not affected by seed material, but it reflected site quality because the dominant height differed by seed material. Site indices of genetically improved seedlings were mostly 27, 30, or 33 m and even higher especially when the stand age was relatively small near or less than 20 years. The sample data of improved seedlings fitted for the genetic effect parameter were mostly distributed at young age compared to those of unimproved seedlings (Table 3, Figs. 3a and 4a). Therefore, a caution on applying the model at old stand and interpreting the prediction may be needed. In this perspective, subsequent measurements can be additionally required to estimate a more stable SI by including the observations of SI estimation near at final felling age or base age.

The estimated site index curves were comparable between previous studies and our study (Fig. 4). However, the models from Oikarinen (1983) and Gustavsen and Mielikäinen (1984) were not based on sigmoid growth function unlike the models from our and other studies (Eriksson et al., 1997; Donis, 2022; Kiviste et al., 2022), and thus, the inherent characteristic may cause the different convergence pattern at early stage (Fig. 4). The Swedish model by Eriksson et al. (1997) was not based on the silver birch plantations and did not distinguish between silver and downy birch. Moreover, the modelling data were not based on the repeatedly-measured observations but based on 5-year age and

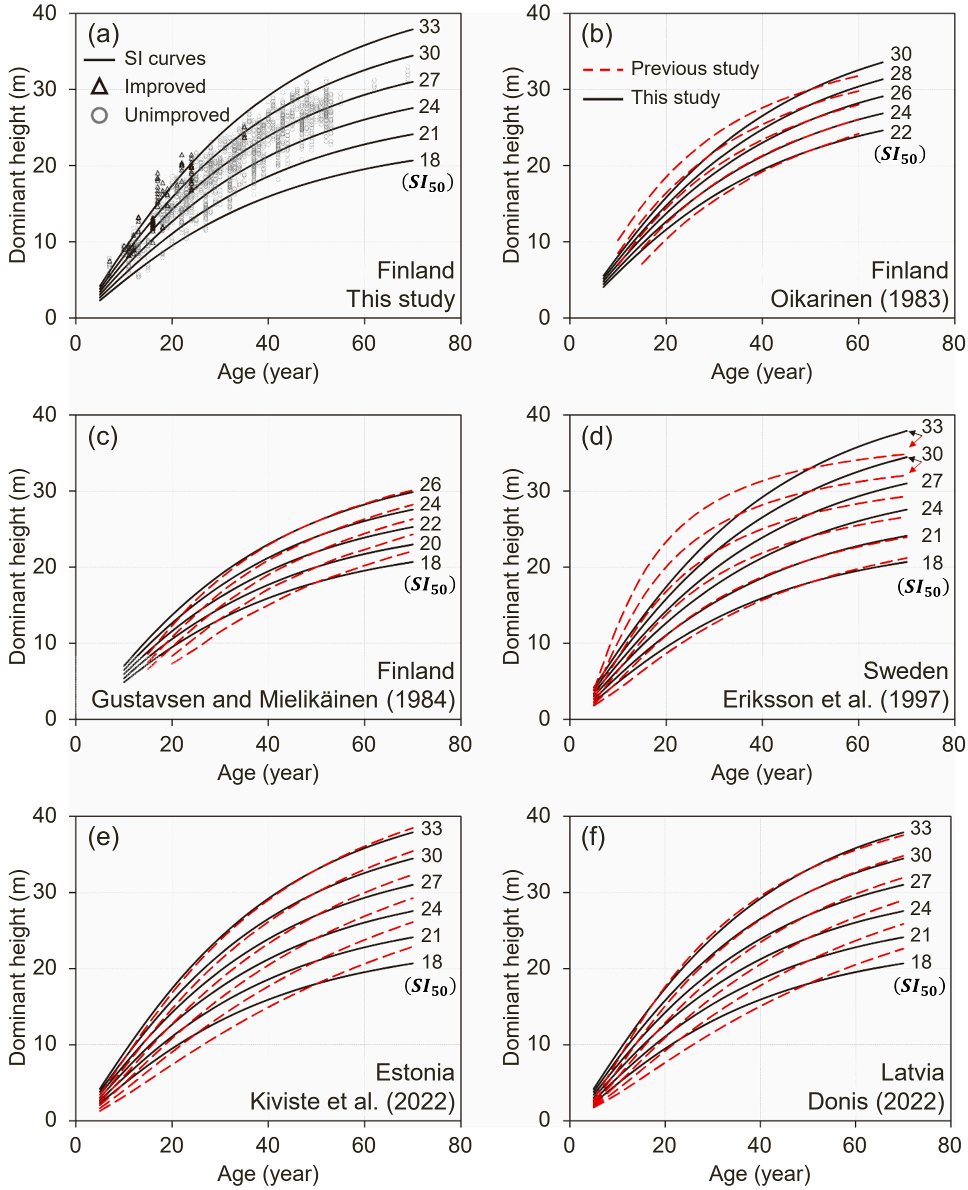

Fig. 4. Site index (SI) guide curves developed in this study (a) and compared to the previous studies (b–f) using the base age of 50 years (SI_{50}). The SI curves of this study (a) were presented along with observations: improved seedlings with black triangle (△) and unimproved seedlings with gray circle (○). The comparative studies were Oikarinen (1983) (b), Gustavsen and Mielikäinen (1984) (c), Eriksson et al. (1997) (d), Kiviste et al. (2022) (e), and Donis (2022) (f). To compare with the same criteria, the breast-height age was adjusted to biological stand age by applying 3 years, for example in Eriksson et al. (1997).

height series of each tree measured by stem analyzed data from temporary sample plots (Eriksson et al., 1997). Meanwhile, the modelling data by Kiviste et al. (2022) of Estonia and by Donis (2022) of Latvia were basically from natural birch stands, and accordingly, silver and downy birches were not distinguished. The plots were measured three to four times, but the inventory design was more similar to the national forest inventory and different from our study, which was measured intensively with experimental design. The SI values from recently developed models in Baltic countries, such as Kiviste et al. (2022) and Donis (2022), were more similar to our study than the other previous models (Fig. 4).

4.3. Interpretation of the explanatory variables and modelling structure in predicting stand characteristics

We developed the reliable and accurate stand characteristic models confirmed by fit statistics and simulation comparisons with observations. An effect of SI indicated the better growth with high site quality as expected (Figs. 6 and 7). In case of the stand age, the reciprocal form was applied as 1/(AGE+1) or 1/(AGE+6) showing curvilinearity by increasing stand characteristics with age while leveling off at a matured stage (Figs. 6 and 7). The effect of N was fitted using logarithmic transformation because of multiplicative assumption. The effect was positive in the G model as it is a function of N and squared tree dbh in a stand (Lee et al., 2023).

Regarding carrying capacity of a stand, the effect of N on HW and DW models was negative. Higher N implied smaller growing space and thus lower dimensions on average. However, the effect of N on the HW was minor compared with the effect on DW and G (Table 4, Fig. 6). Niemistö (1995b) reported decreased dominant height growth for both extremes, lowest and highest density. Therefore, slight overestimation of HW could be seen when N was heavily low (Appendix A). According to covariances by Siipilehto (2011), stand density affected negatively to all dimensions; specifically, more to diameter than height and also more to

Table 4
Parameter estimates and fit statistics of seemingly unrelated regression models for basal area-weighted mean height (HW, m), basal area-weighted mean diameter (DW, cm), and stand basal area (G, m^2 ha^{-1}) in silver birch plantations based on Eq. (3).

		ln(HW) Estimate	S.E.	ln(DW) Estimate	S.E.	ln(G) Estimate	S.E.
Fixed effects	Intercept	0.6100	0.0820	2.4888	0.0899	−4.2217	0.1873
	ln(SI)	1.0725	0.0239	0.7045	0.0257	1.4376	0.0536
	1/(AGE+1)			−16.3981	0.1297	−33.1204	0.2743
	1/(AGE+6)	−34.8503	0.1736				
	ln(N)	−0.0363	0.0024	−0.2221	0.0026	0.4901	0.0056
	$THIN\times(\sqrt{Talty_{max25}}-3.5)$			0.0710	0.0021	0.1290	0.0042
	Agric			0.0505	0.0245	0.1123	0.0487
Random effects	std(experiment)	0.0366		0.0709		0.1417	
	corr with DW	−0.176					
	corr with G	−0.247		0.974			
Random error	std(ε)	0.0518		0.0516		0.1105	
	corr with DW	0.556					
	corr with G	0.619		0.934			
Fit statistics	AIC	−19703.46					
	BIC	−19516.52					
	−2logLik	−19759.46					

Note: all fixed-effect parameters were highly significant ($P<0.0001$) except for Agric ($P=0.0395$ in DW model and $P=0.0211$ in G model). SI is the site index (m) at the base age of 50 years. N is the number of silver birch trees per hectare (trees ha^{-1}). AGE is the biological stand age (year). Agric is the categorical variable; code 1 if the site type is a former agricultural land or code 0 if it is a forest site. THIN is the proportion of thinning intensity based on stand basal area scaled 0–1. Talty is the time after last thinning in years. $Talty_{max25}$ is the time after last thinning restricted to have a maximum value of 25 (the same value as Talty if Talty < 25, a fixed value as 25 if Talty ≥ 25). corr refers to the correlation. std(ε) is the standard deviation of the residual in model performance. AIC is the Akaike information criterion. BIC is the Bayesian information criterion. −2logLik is the −2 × log-likelihood value.

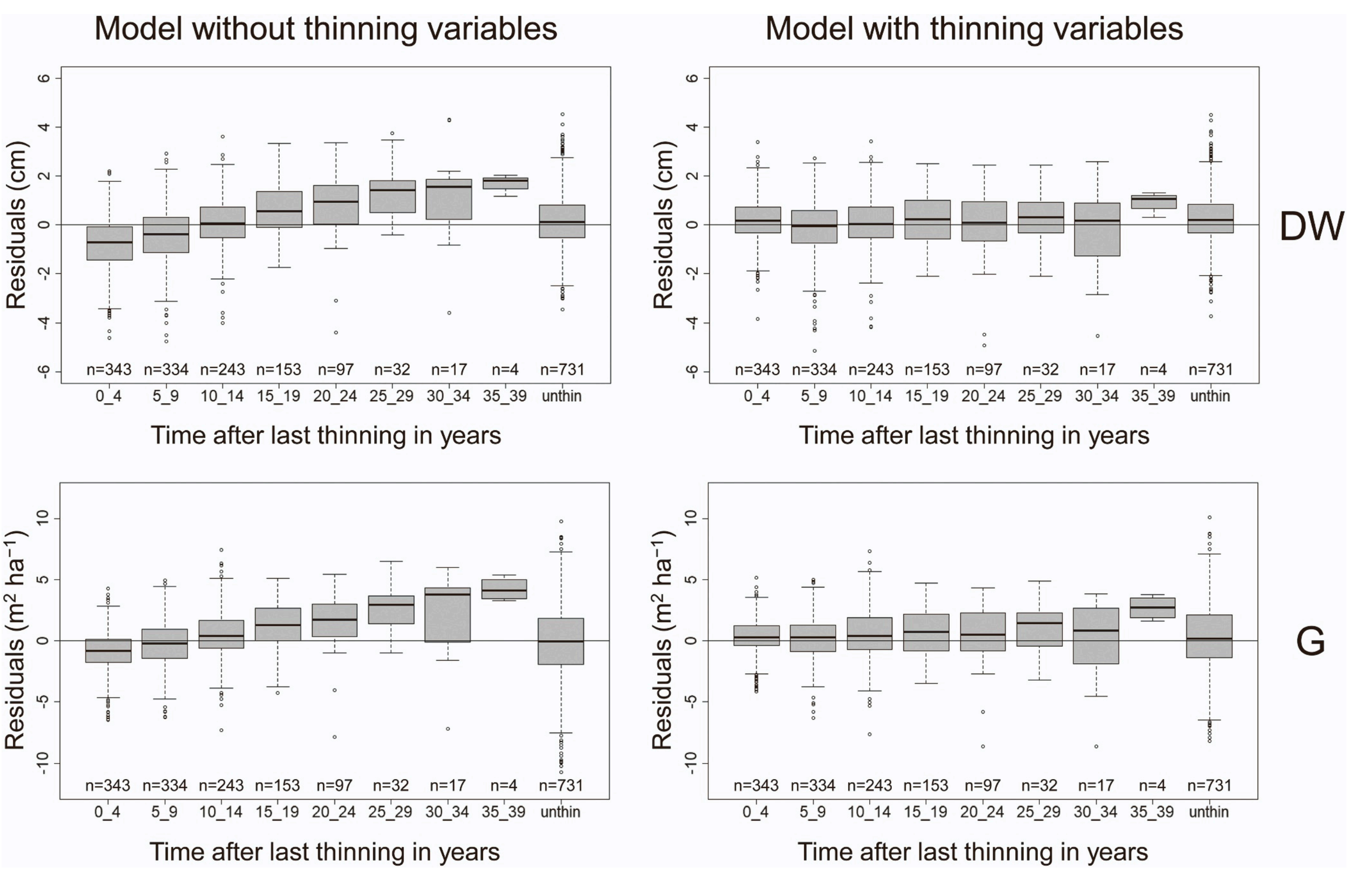

Fig. 5. Residual boxplots of the models for basal area-weighted mean diameter (DW, cm) and stand basal area per hectare (G, m^2 ha^{-1}) over the time after last thinning in years (Talty) in an arithmetic scale with and without thinning variables. The predictor variables of the models in the left column are identical except for the thinning variables; thinning variables are not included in the left column to demonstrate the trends of thinning effects. The predicted values were calculated based on the fixed-effect parameters. Each category on x-axis indicates the range of Talty, e.g., 0_4 for the years between 0 and 4. n: the number of observations in each Talty class.

Table 5
Metrics of model accuracy by mixed- and fixed-effects with root mean square error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE) for basal area-weighted mean height (HW, m), basal area-weighted mean diameter (DW, cm), and stand basal area (G, m^2 ha^{-1}) in silver birch plantations.

		Logarithmic scale			Arithmetic scale		
Effect	Model type	RMSE	MAE	MAPE	RMSE	MAE	MAPE
Mixed	HW	0.0514	0.0377	0.0136	0.9418	0.6860	0.0379
	DW	0.0511	0.0380	0.0142	0.8551	0.6242	0.0383
	G	0.1096	0.0803	0.0318	1.7525	1.2914	0.0822
Fixed	HW	0.0564	0.0414	0.0150	1.0306	0.7574	0.0423
	DW	0.0707	0.0530	0.0198	1.1237	0.8434	0.0525
	G	0.1431	0.1089	0.0416	2.3306	1.7396	0.1069

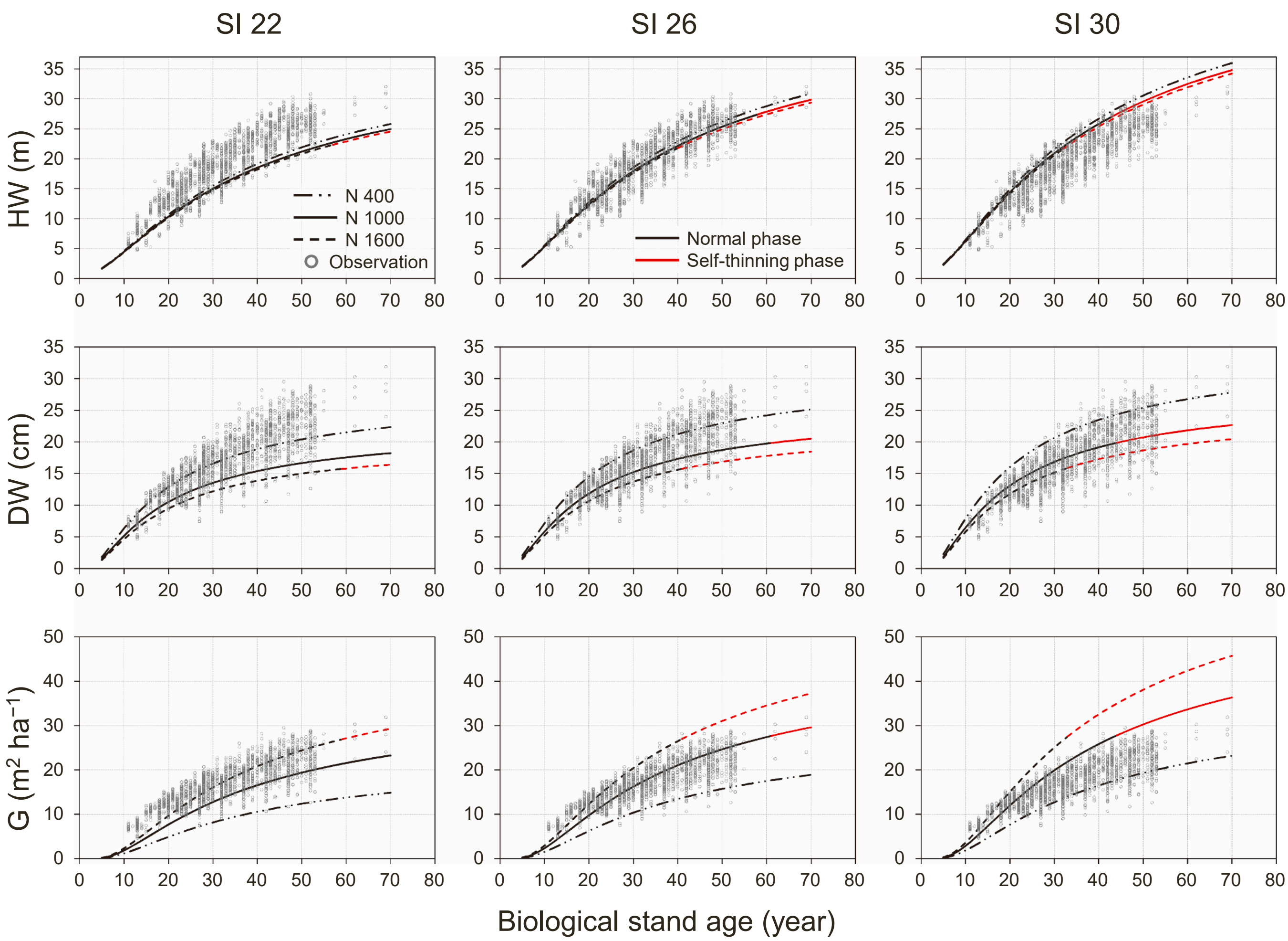

Fig. 6. Model simulations of unthinned stands under the selected site index (SI) and the number of trees per ha (N) in forest sites (Agric=0). HW: basal area-weighted mean height (m). DW: basal area-weighted mean diameter (cm). G: stand basal area per hectare (m^2 ha^{-1}). The red lines indicate the stand undergoing the phase of self-thinning based on the previous model (Hynynen et al., 2002).

mean height than to dominant height. Siipilehto et al. (2014) showed the effect of density on mean and dominant height for the early development of Scots pine stands. Also, there are other studies that found negative effect of stand density on tree height increment (Wykoff, 1986; Huang and Titus, 1999; Sharma et al., 2002; Antón-Fernández et al., 2011). However, the effect of stand density on height growth is not typically found and may be due to low density variation in the managed stands (Hynynen et al., 2002, 2011).

The predictor variables related to thinning were expressed as the multiplication between THIN and $Talty_{max25}$ in the DW and G models (Table 4). Basically, the parameter signs of the predictors were negative at first as it indicated the reasonable stand history. In this circumstance, a thinned stand had suffered denser between-tree competition in the past stand development process before thinning than an unthinned stand. Hence, the DW and G in a thinned stand (or dense stand in the past) possibly had remained smaller than those in an unthinned stand (or less dense stand in the past) (Hökkä et al., 1997; Hynynen et al., 2002; Siipilehto, 2011; Repola et al., 2018; Lee et al., 2023). The term of the thinning predictors in the DW and G models changed from negative to positive as $Talty_{max25}$ increased (Table 4). In the meantime, as the parameter implied a value of 0 nearly at 12 years after last thinning, it reached to the level of an unthinned stand retaining the same density. Later, the DW and G in a thinned stand were higher than in an unthinned stand after 13 years since last thinning. It revealed a transition and magnitude of thinning effect as time after last thinning elapsed more, indicating a positive thinning effect in a stand development stage.

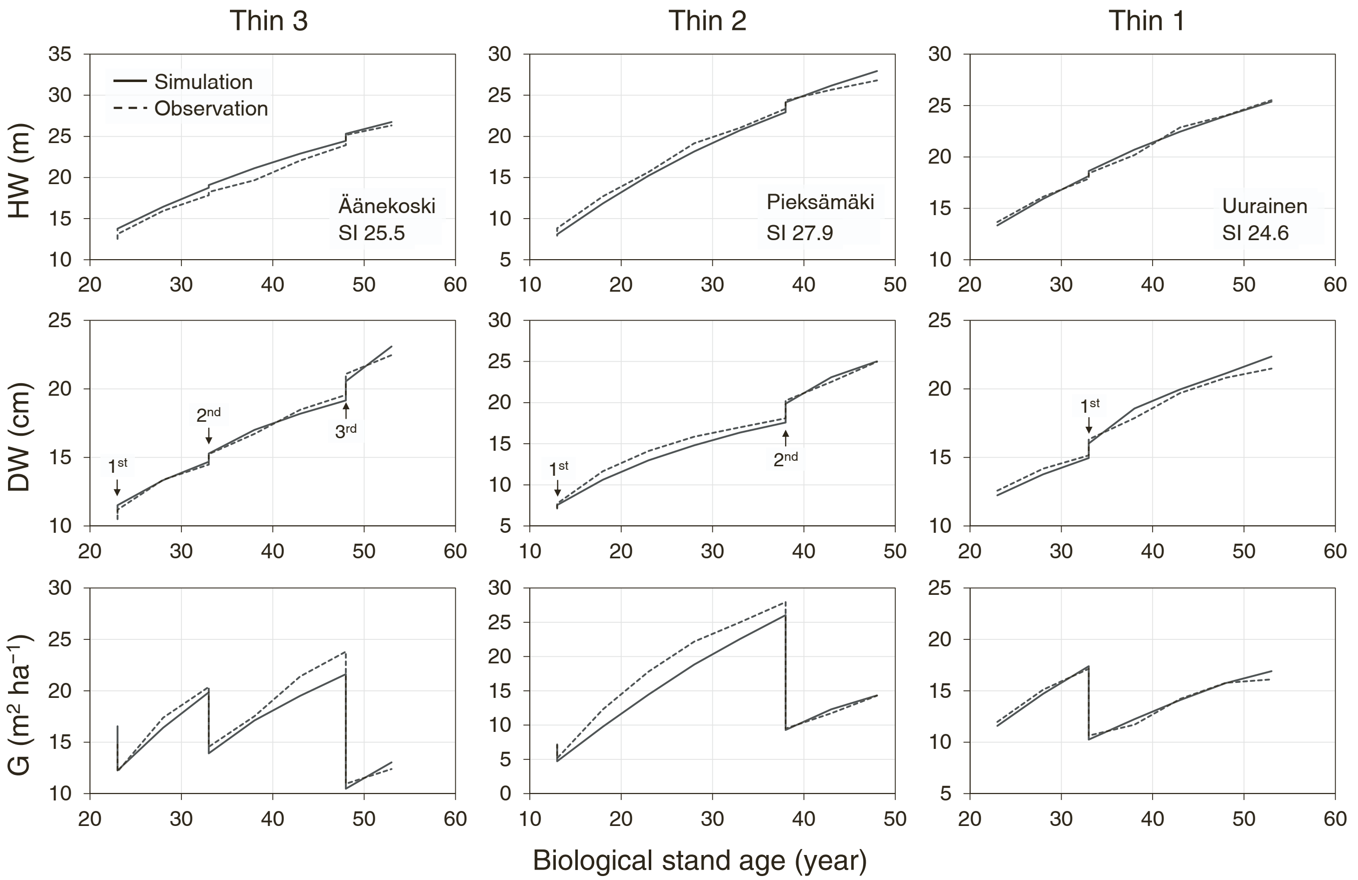

Fig. 7. Comparison of model simulations with observations from the selected thinned plots (Luke 1 data source, experiments at Äänekoski, Pieksämäki and Uurainen) with three, two or one thinnings and varying thinning intensities. HW: basal area-weighted mean height (m). DW: basal area-weighted mean diameter (cm). G: stand basal area per hectare (m^2 ha^{-1}). SI: site index (m) with a base age of 50 years.

Unlike the negative effect of recent thinning for stand characteristic models, the thinning effects were plainly positive in the models for individual tree growth (Hökkä et al., 1997; Hynynen et al., 2002; Siipilehto, 2011; Repola et al., 2018; Lee et al., 2023).

Silver birch uses an increased growing space effectively. According to Niemistö (1997), 5 years after thinning (remaining 50% or 65% of basal area or volume), the level of volume growth was already 72% and 87% of untinned stands, respectively. In our study, 12 years after last thinning, DW and G reached the level of unthinned stand of the same density (Fig. 5). Thereafter, the thinning resulted in DW and G above those of the unthinned stands. It could imply a changed stand circumstance regarding decomposition. For example, Mäkinen et al. (2006) reported that, after tree death, the wood density was rapidly decreased in silver birch compared to Scots pine and Norway spruce and the remaining stem mass already reached a half compared to that of living tree near at 10 years after death. Similar results about the decomposition and decay rate of silver birch were reported (Palviainen et al., 2010; Hynynen et al., 2014). Due to decomposition of logging residues, more abundant soil nutrients originated from stumps and small stems of the thinned birch trees (Palviainen et al. 2010). Finally, intensive thinnings benefited birch more than light thinnings.

The categorical variable referring the effect of former agricultural land (Agric) indicated higher growth of DW and G in former agricultural land than in forest site. This was consistent to the previous studies that reported a high growth potential of former agricultural land (Tullus et al., 2012; Lutter et al., 2015; Lutter, 2017; Rytter and Lutter, 2020). Climatic and hydrologic predictors at site level, such as temperature sums, lake index and sea index, were not significant. Also, other than the former agricultural site type dummy variable, specific soil types were not significant. Although those predictors (e.g., clay) were significant in the previous studies in Finland (Siipilehto, 2011; Lee et al., 2023), insignificance might be caused by a lack of observations in unfavorable fine-textured soils. Nevertheless, in terms of RMSE%, the models for silver birch showed slightly better accuracy for each of the stand characteristics than corresponding models for hybrid aspen (Lee et al., 2023).

4.4. Applicability of the final models in practical forestry

The developed site index model can be used in southern and central Finland for planted silver birch stands on mineral soils. The comparisons with previous studies demonstrated a possibility of applying our site index model to neighboring countries, so it could extend to the region with the caution about our materials. The site index model is evaluated to be most suitable for the seedling that originated from unimproved materials since 1970s and improved materials since 1990s. It is considered to be generally applicable regardless of breeding identifier. The range of applicability covers AGE from 15 to 60 years. The site index of young stands (AGE $<$ 15) should be carefully examined due to the sensitivity of an estimated value. It should be more cautious when interpreting SI of matured stands with the genetic effect (GE) because of the lack of observations of older stands (AGE $>$ 35).

For the stand characteristic models, spatial and temporal applicability ranges are similar to the site index model. However, the stand characteristic models were solely designed for silver birch plantations established using unimproved seedlings. To solve this limitation, correction factors or calibration models should be able to be developed in future studies as carried out for the other species' breeding traits

(Haapanen et al., 2016). The models are considered the most applicable to stands established with common levels of initial planting density in practice: 1600–2000 trees ha^{-1} (Rantala, 2011; TAPIO, 2023). Stand densities beyond this range were still included in our modelling data (Tables 1 and 2). Only few observations with Norway spruce understory were included in the modelling data without any noticeable effect on G. Therefore, it is recommendable in such stands to use only stand characteristics of silver birch as inputs in the models.

The development of unthinned stands undergoing self-thinning were simulated with high initial densities (Fig. 6). Several simulations especially with high N resulted in stand basal areas above the level of the existing self-thinning model by Hynynen (1993) (Figs. 6 and 7). Therefore, our models may not result in reliable prediction when applied beyond the observed stand densities of managed birch stands; such risk can be avoided using silvicultural treatments with thinning in practice. Our models included sufficient cases in terms of the number, timing, and intensity of thinning (Table 1), and thus, general thinning practices should be covered when a stand is thinned from below in the field.

Both former agricultural land and forest sites (OMT and MT) in mineral soil are applicable to use the models of HW, DW, and G all. However, predictions of stands growing on fine-textured soils should be avoided because the modelling data did not cover the soil types and because clay and silt soils are often too compact for silver birch to grow (Hynynen et al., 2010). Temperature sums and lake index were not significant as predictor variables in any of the stand characteristic models due to relatively narrow variation of these measures in the modelling data. Therefore, the current models can be applied without consideration of these factors in the forestry field of southern and central Finland, which are known to be suitable for the grow of silver birch. Overall, the developed models are evaluated to provide accurate predictions with easily observed predictors in practice (Appendix B).

5. Conclusion

Site index and stand characteristic models were developed for silver birch plantations in southern and central Finland to provide improved predictions on growth and yield of silver birch for forest management purposes. The dominant height and site index models were developed using nonlinear mixed-effect modelling based on the Chapman-Richards function. The effect of improved regeneration material of silver birch was included in the dominant height model resulting in a higher dominant height prediction. The predicted dominant height curves of our materials implied a superior development characteristic than the old materials of the earlier Finnish studies. The site index models of our study demonstrated similarity with the latest models for silver birch in northern Europe.

The models for stand characteristics were developed through the seemingly unrelated regression approach with mixed effects using the readily-accessible, driving variables: AGE, SI, and N. Thinning information applied via thinning intensity (THIN) and a variable based on time after last thinning in years with a maximum of 25 ($Talty_{max25}$) was disclosed as highly significant predictors in the DW and G models. It represented a logical stand development history with the predicted term changing from negative to positive value, which revealed the thinning effect with a transition and magnitude over Talty. The categorical variable of former agricultural land was significant for DW and G showing a higher level compared with forest sites. Unlike in the site index model, the effect of genetic gain was not included in the stand characteristic models due to lack of suitable modelling data from managed birch stands. Overall, the simulation comparisons with the observations confirmed the reliability and applicability of our models in predicting the development of planted silver birch stands in production forests.

Funding information

This research was funded by the Natural Resources Institute Finland (LUKE) strategic funded project "ProBirch - Possibilities and challenges of increasing birch with special attention to productivity and risk management".

CRediT authorship contribution statement

Daesung Lee: Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Formal analysis, Data curation, Conceptualization. **Jouni Siipilehto:** Writing – review & editing, Validation, Methodology, Formal analysis, Conceptualization. **Jari Miina:** Writing – review & editing, Validation, Methodology, Conceptualization. **Pentti Niemistö:** Writing – review & editing, Resources, Investigation, Data curation. **Matti Haapanen:** Writing – review & editing, Resources, Investigation, Data curation. **Jari Hynynen:** Writing – review & editing, Validation, Supervision, Methodology, Conceptualization. **Saija Huuskonen:** Writing – review & editing, Resources, Project administration, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no conflict of interest.

Data Availability

Data will be made available on request.

Acknowledgements

This study was carried out based on the empirical data provided by Natural Resources Institute Finland (LUKE) in Finland. The authors wish to thank all associated members for their invaluable efforts in fieldwork and data maintenance support.

Appendix A. Supplmentary data

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.foreco.2024.121998.

References

Antón-Fernández, C., Burkhart, H.E., Strub, M., Amateis, R.L., 2011. Effects of initial spacing on height development of loblolly pine. For. Sci. 57, 201–211.
Baskerville, G.L., 1972. Use of logarithmic regression in the estimation of plant biomass. Can. J. For. Res. 2, 49–53. https://doi.org/10.1139/x72-009.
Burkhart, H.E., Tomé, M., 2012. Modeling Forest Trees and Stands. Springer Science & Business Media, New York, NY.
Cajander, 1949. Forest types and their significance. Acta For. Fenn. 56, 1–72. https://doi.org/10.14214/aff.7396.
Donis, J., 2022. Algoritmu izstrāde mežsaimniecības plānošanai [Development of algorithms for forestry planning] (in Latvian only) (Overview of the study 2022 No. 5- 5.9.1_007h_101_21_71), PĀRSKATS PAR PĒTĪJUMA 2022. GADA REZULTĀTIEM [Annual results]. Latvian State Forest Research Institute (SILAVA), Salaspils, Latvia.
Dubois, H., Verkasalo, E., Claessens, H., 2020. Potential of birch (*Betula pendula* Roth and *B. pubescens* Ehrh.) for forestry and forest-based industry sector within the changing climatic and socio-economic context of Western Europe. Forests 11, 336. https://doi.org/10.3390/f11030336.
Eid, T., 2001. Models for prediction of basal area mean diameter and number of trees for forest stands in south-eastern Norway. Scand. J. For. Res. 16, 467–479. https://doi.org/10.1080/02827580152632865.
Eriksson, H., Johansson, U., Kiviste, A., 1997. A site-index model for pure and mixed stands of *Betula pendula* and *Betula pubescens* in Sweden. Scand. J. For. Res. 12, 149–156. https://doi.org/10.1080/02827589709355396.
Fang, Z., Bailey, R.L., Shiver, B.D., 2001. A multivariate simultaneous prediction system for stand growth and yield with fixed and random effects. For. Sci. 47, 550–562.
Goelz, J.C.G., Burk, T.E., 1992. Development of a well-behaved site index equation: jack pine in north central Ontario. Can. J. For. Res. 22, 776–784. https://doi.org/10.1139/x92-106.
Goldstein, H., 1995. Multilevel Statistical Models, Second Edition. ed. London: Edward Arnold; New York: Halsted Press.
Gustavsen, H.G., Mielikäinen, K., 1984. Luontaisesti syntyneiden koivikoiden kasvupaikkaluokittelu valtapituuden avulla [Site index curves for natural birch stands in Finland] (in Finnish with English abstract). Folia For. 597, 1–23 https://jukuri.luke.fi/handle/10024/522180.

Haapanen, M., 2024. Realised genetic gains from past Finnish silver birch seed orchards. Silva Fenn. 58, 23072. https://doi.org/10.14214/sf.23072.
Haapanen, M., Hynynen, J., Ruotsalainen, S., Siipilehto, J., Kilpeläinen, M.-L., 2016. Realised and projected gains in growth, quality and simulated yield of genetically improved Scots pine in southern Finland. Eur. J. For. Res. 135, 997–1009. https://doi.org/10.1007/s10342-016-0989-0.
Hall, D.B., Bailey, R.L., 2001. Modeling and prediction of forest growth variables based on multilevel nonlinear mixed models. For. Sci. 47, 311–321. https://doi.org/10.1093/forestscience/47.3.311.
Hall, D.B., Clutter, M., 2004. Multivariate multilevel nonlinear mixed effects models for timber yield predictions. Biometrics 60, 16–24. https://doi.org/10.1111/j.0006-341X.2004.00163.x.
Härkönen, S., Mäkinen, A., Tokola, T., Rasinmäki, J., Kalliovirta, J., 2010. Evaluation of forest growth simulators with NFI permanent sample plot data from Finland. For. Ecol. Manag. 259, 573–582. https://doi.org/10.1016/j.foreco.2009.11.015.
Heinonen, J., 1994. KPL - koealojen puu- ja puustotunnusten laskentaohjelman käyttöohje [A Computer Programme Package (KPL) for Computing Stand and Single Tree Characteristics from Sample Plot Measurements. User Manual], Metsäntutkimuslaitoksen tiedonantoja. Metsäntutkimuslaitos, Helsinki, Finland.
Henttonen, H.M., Nöjd, P., Mäkinen, H., 2024. Environment-induced growth changes in forests of Finland revisited - a follow-up using an extended data set from the 1960s to the 2020s. For. Ecol. Manag. 551, 121515 https://doi.org/10.1016/j.foreco.2023.121515.
Hökkä, H., Alenius, V., Penttilä, T., 1997. Individual-tree basal area growth models for Scots pine, pubescent birch and Norway spruce on drained peatlands in Finland. Silva Fenn. 31, 161–178.
Huang, S., Titus, S.J., 1999. An individual tree height increment model for mixed white spruce–aspen stands in Alberta, Canada. For. Ecol. Manag. 123, 41–53. https://doi.org/10.1016/S0378-1127(99)00015-8.
Huuskonen, S., Lahtinen, T., Miina, J., Uotila, K., Bianchi, S., Niemistö, P., 2023. Growth dynamics of young mixed norway spruce and birch stands in Finland. Forests 14, 56. https://doi.org/10.3390/f14010056.
Huuskonen, S., Miina, J., 2007. Stand-level growth models for young Scots pine stands in Finland. For. Ecol. Manag. 241, 49–61. https://doi.org/10.1016/j.foreco.2006.12.024.
Hynynen, J., 1993. Self-thinning models for even-aged stands of *Pinus sylvestris*, *Picea abies* and *Betula pendula*. Scand. J. For. Res. 8, 326–336. https://doi.org/10.1080/02827589309382781.
Hynynen, J., Niemisto, P., Vihera-Aarnio, A., Brunner, A., Hein, S., Velling, P., 2010. Silviculture of birch (*Betula pendula* Roth and *Betula pubescens* Ehrh.) in northern Europe. Forestry 83, 103–119. https://doi.org/10.1093/forestry/cpp035.
Hynynen, J., Ojansuu, R., Hökkä, H., Siipilehto, J., Salminen, H., Haapala, P., 2002. Models for predicting stand development in MELA System. Finnish Forest Research Institute. Res. Pap. 835, 116.
Hynynen, J., Repola, J., Mielikäinen, K., 2011. The effects of species mixture on the growth and yield of mid-rotation mixed stands of Scots pine and silver birch. For. Ecol. Manag. 262, 1174–1183. https://doi.org/10.1016/j.foreco.2011.06.006.
Hynynen, J., Salminen, H., Ahtikoski, A., Huuskonen, S., Ojansuu, R., Siipilehto, J., Lehtonen, M., Rummukainen, A., Kojola, S., Eerikäinen, K., 2014. Scenario analysis for the biomass supply potential and the future development of Finnish forest resources. Finnish Forest Research Institute. Res. Pap. 302, 106.
Kiviste, A., Tarmu, T., Padari, A., Laarmann, D., 2022. Modelling dominant height development on estonian forest permanent plot data, in: SNS Growth and Yield Nordic Biannual Conference. Presented at the Local solutions for regional and global forest management challenges, SNS Growth and Yield researchers Network, Riga, Latvia, pp. 8–9.
Kulju, I., Niinistö, T., Peltola, A., Räty, M., Sauvula-Seppälä, T., Torvelainen, J., Uotila, E., Vaahtera, E., 2023. Metsätilastollinen vuosikirja 2022 [Finnish Statistical Yearbook of Forestry 2022]. Luonnonvarakeskus, Helsinki.
Laird, N.M., Ware, J.H., 1982. Random-effects models for longitudinal data. Biometrics 38, 963–974. https://doi.org/10.2307/2529876.
Lappi, J., Bailey, R.L., 1988. A height prediction model with random stand and tree parameters: an alternative to traditional site index methods. For. Sci. 34, 907–927.
Lee, D., Beuker, E., Viherä-Aarnio, A., Hynynen, J., 2021a. Site index models with density effect for hybrid aspen (*Populus tremula* L. × *P. tremuloides* Michx.) plantations in southern Finland. For. Ecol. Manag. 480, 118669 https://doi.org/10.1016/j.foreco.2020.118669.
Lee, D., Choi, J., 2022. Development of variable-density yield models with site index estimation for Korean pines and Japanese larch. Forests 13, 1150. https://doi.org/10.3390/f13071150.
Lee, D., Seo, Y., Park, G., Choi, J., 2015. Estimation of site index for *Larix kaempferi* and *Pinus koraiensis* in Gangwon and North Gyeongsang Provinces. J. For. Environ. Sci. 31, 202–206. https://doi.org/10.7747/JFES.2015.31.3.202.
Lee, D., Siipilehto, J., Hynynen, J., 2021b. Models for diameter distribution and tree height in hybrid aspen plantations in southern Finland. Silva Fenn. 55, 10612. https://doi.org/10.14214/sf.10612.
Lee, D., Siipilehto, J., Beuker, E., Fahlvik, N., Liziniewicz, M., Hynynen, J., 2023. Multivariate mixed-effects models for stand characteristics of hybrid aspen plantations in southern Finland and southern Sweden. For. Ecol. Manag. 541, 121066 https://doi.org/10.1016/j.foreco.2023.121066.
Liziniewicz, M., Barbeito, I., Zvirgzdins, A., Stener, L.-G., Niemistö, P., Fahlvik, N., Johansson, U., Karlsson, B., Nilsson, U., 2022. Production of genetically improved silver birch plantations in southern and central Sweden. Silva Fenn. 56, 10512. https://doi.org/10.14214/sf.10512.
Lutter, R., 2017. Growth development and ecology of midterm hybrid aspen and silver birch plantations on former agricultural lands. Doctoral dissertation. Eesti Maaülikool [Estonian University of Life Sciences], Tartu, Estonia.
Lutter, R., Tullus, A., Kanal, A., Tullus, T., Vares, A., Tullus, H., 2015. Growth development and plant–soil relations in midterm silver birch (*Betula pendula* Roth) plantations on previous agricultural lands in hemiboreal Estonia. Eur. J. For. Res. 134, 653–667. https://doi.org/10.1007/s10342-015-0879-x.
Mäkinen, H., Hynynen, J., Siitonen, J., Sievänen, R., 2006. Predicting the decomposition of scots pine, Norway spruce, and birch stems in Finland. Ecol. Appl. 16, 1865–1879. https://doi.org/10.1890/1051-0761(2006)016[1865:PTDOSP]2.0.CO;2.
Maleki, K., Astrup, R., Kuehne, C., McLean, J.P., Antón-Fernández, C., 2022. Stand-level growth models for long-term projections of the main species groups in Norway. Scand. J. For. Res. 37, 130–143. https://doi.org/10.1080/02827581.2022.2056632.
Mehtätalo, L., Kansanen, K., 2022. Package 'lmfor', ver 1.6. Functions for forest biometrics.
Mehtätalo, L., Lappi, J., 2020. Biometry for forestry and environmental data with examples in R, Applied Environmental Series. Chapman Hall. /CRC, N. Y. https://doi.org/10.1201/9780429173462.
Næsset, E., 2002. Predicting forest stand characteristics with airborne scanning laser using a practical two-stage procedure and field data. Remote Sens. Environ. 80, 88–99. https://doi.org/10.1016/S0034-4257(01)00290-5.
Niemistö, P., 1995a. Influence of initial spacing and row-to-row distance on the crown and branch properties and taper of silver birch (*Betula pendula*). Scand. J. For. Res. 10, 235–244. https://doi.org/10.1080/02827589509382889.
Niemistö, P., 1995b. Influence of initial spacing and row-to-row distance on the growth and yield of silver birch (*Betula pendula*). Scand. J. For. Res. 10, 245–255. https://doi.org/10.1080/02827589509382890.
Niemistö, P., 1997. Ensiharvennuksen ajankohdan ja voimakkuuden vaikutus istutetun rauduskoivikon kasvuun ja tuotokseen [Effect of the timing and intensity of the first thinning on growth and yield of a *Betula pendula* stand (in Finnish only)]. Metsätieteen aikakauskirja 1997, 439–454. https://doi.org/10.14214/ma.6232.
Oikarinen, M., 1983. Etelä-Suomen viljeltyjen rauduskoivikoiden kasvatusmallit. Metsäntutkimuslaitos. [Growth and yield models for silver birch (*Betula pendula*) plantations in southern Finland]. Commun. Inst. For. Fenn. 113, 75 https://jukuri.luke.fi/handle/10024/522521.
Ojansuu, R., Henttonen, H., 1983. Kuukauden keskilämpötilan, lämpösumman ja sademäärän paikallisten arvojen johtaminen Ilmatieteen laitoksen mittaustiedoista. [Estimation of local values of monthly mean temperature, effective temperature sum and precipitation sum from the measurements made by the Finnish Meteorological Office.]. Silva Fenn. 17, 143–160.
Palviainen, M., Finér, L., Laiho, R., Shorohova, E., Kapitsa, E., Vanha-Majamaa, I., 2010. Carbon and nitrogen release from decomposing Scots pine, Norway spruce and silver birch stumps. For. Ecol. Manag. 259, 390–398. https://doi.org/10.1016/j.foreco.2009.10.034.
Pienaar, L.V., Rheney, J.W., 1995. Modeling stand level growth and yield response to silvicultural treatments. For. Sci. 41, 629–638. https://doi.org/10.1093/forestscience/41.3.629.
Pinheiro, J., Bates, D., DebRoy, S., Sarkar, D., Heisterkamp, S., Willigen, B.V., Ranke, J., Core Team, R, 2023. nlme: Linear and Nonlinear Mixed Effects Models.
Prévosto, B., Coquillard, P., Gueugnot, J., 1999. Growth models of silver birch (*Betula pendula* Roth.) on two volcanic mountains in the French Massif Central. Plant Ecol. 144, 231–242. https://doi.org/10.1023/A:1009858508312.
R Core Team, 2019. R: A Language and Environment for Statistical Computing; R Foundation for Statistical Computing. R Core Team, Vienna, Austria.
Rantala, S., 2011. Finnish forestry: practice and management. Metsäkustannus Oy, Helsinki.
Repola, J., Hökkä, H., Salminen, H., 2018. Models for diameter and height growth of Scots pine, Norway spruce and pubescent birch in drained peatland sites in Finland. Silva Fenn. 52, 10055. https://doi.org/10.14214/sf.10055.
Richards, F.J., 1959. A flexible growth function for empirical use. J. Exp. Bot. 10, 290–301.
Rytter, L., Lutter, R., 2020. Early growth of different tree species on agricultural land along a latitudinal transect in Sweden. Forestry 93, 376–388. https://doi.org/10.1093/forestry/cpz064.
Scolforo, H.F., McTague, J.P., Burkhart, H., Roise, J., McCarter, J., Alvares, C.A., Stape, J.L., 2019. Stand-level growth and yield model system for clonal eucalypt plantations in Brazil that accounts for water availability. For. Ecol. Manag. 448, 22–33. https://doi.org/10.1016/j.foreco.2019.06.006.
Sharma, M., Burkhart, H.E., Amateis, R.L., 2002. Modeling the effect of density on the growth of loblolly pine trees. South. J. Appl. For. 26, 124–133. https://doi.org/10.1093/sjaf/26.3.124.
Siipilehto, J., 2011. Local prediction of stand structure using linear prediction theory in Scots pine-dominated stands in Finland. Silva Fenn. 45, 669–692. https://doi.org/10.14214/sf.99.
Siipilehto, J., Mehtätalo, L., 2013. Parameter recovery vs. parameter prediction for the Weibull distribution validated for Scots pine stands in Finland. Silva Fenn 47, 1057. https://doi.org/10.14214/sf.1057.
Siipilehto, J., Valkonen, S., Ojansuu, R., Hynynen, J., Miina, J., Saksa, T., 2014. Metsikön varhaiskehityksen kuvaus MOTTI-ohjelmistossa. Finnish Forest Research Institute. Res. Pap. 286, 43.
TAPIO, 2023. Best Practices for Sustainable Forest Management in Finland. https://metsanhoidonsuositukset.fi/en (accessed 24 November 2023).
Tonteri, T., Hotanen, J.-P., Kuusipalo, J., 1990. The Finnish forest site type approach: ordination and classification studies of mesic forest sites in southern Finland. Vegetatio 87, 85–98. https://doi.org/10.1007/BF00045658.

Tullus, A., Lukason, O., Vares, A., Padari, A., Lutter, R., Tullus, T., Karoles, K., Tullus, H., 2012. Economics of hybrid aspen (*Populus tremula* L. × *P. tremuloides* Michx.) and silver birch (*Betula pendula* Roth.) plantations on abandoned agricultural lands in Estonia. Balt. For. 18, 288–298.

Venäläinen, A., Tuomenvirta, H., Pirinen, P., Drebs, A., 2005. A basic Finnish climate data set 1961–2000 — description and illustrations. Finn. Meteorol. Inst., Rep. 5, 1–27.

Wykoff, W., 1986. Supplement to the User's Guide for the Stand Prognosis Model — Version 5.0. General Technical Report INT-208, USDA Forest Service, Intermountain Research Station 208, Ogden, UT, USA. doi:10.2737/INT-GTR-208.

Zellner, A., 1962. An efficient method of estimating seemingly unrelated regressions and tests for aggregation bias. J. Am. Stat. Assoc. 57, 348–368. https://doi.org/10.1080/01621459.1962.10480664.

Zeltiņš, P., Jansons, Ā., Baliuckas, V., Kangur, A., 2024. Height growth patterns of genetically improved Scots pine and silver birch. Forestry 97, 458–468. doi:10.1093/forestry/cpad057.