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Abstract

Decompositions of productivity indices contribute to our understanding of what drives the observed productivity changes by providing a detailed picture of their constituents. This paper presents the most comprehensive decomposition of total factor productivity (TFP) to date. Starting from the Fisher ideal TFP index, we systematically isolate the productivity effects of changes in production technology, technical efficiency, scale efficiency, allocative efficiency, and the market strength. The three efficiency components further decompose into input- and output-side effects. The proposed decomposition is illustrated with an empirical application to a sample of 459 Finnish farms over period 1992-2000.

Key Words: *index numbers and aggregation, Total Factor Productivity (TFP) measurement, Fisher ideal index, Malmquist index, decompositions, agriculture*

JEL classification: C43, D24, O30

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1. Introduction

Traditionally, productivity has been viewed as a synonym to technical progress, following the influential work of Solow (1957). Today, development of more sophisticated methods for measuring total factor productivity (TFP) enables us to distinguish technological development from other sources of productivity changes, most notably, changes in technical and scale efficiency (see e.g. Färe et al., 1994a). As a consequence, many recent studies interpret productivity in broader terms as a welfare measure, which includes the effect of technical change among other components (e.g. Basu and Fernald, 1997; Kumar and Russell, 2002). From the welfare perspective, it is interesting to investigate the relative importance of the different effects that influence productivity. Decomposing the overall productivity index into sub-components can provide more detailed information about the underlying sources of productivity changes.

Decompositions of productivity date back at least to the seminal work of Farrell (1957), who expressed the overall economic efficiency as a product of the technical efficiency and allocative (price) efficiency components. Farrell restricted to a static framework, which left no room for technical progress. Nishimizu and Page (1982) proposed a first dynamic extension of Farrell's decomposition, which included technical change and technical efficiency components. Färe et al. (1994a,c) generalized and developed the decomposition further, introducing the scale efficiency component. These decompositions are based on the Malmquist productivity index by Caves et al. (1982), which has gained momentum after Färe et al. presented their decomposition, especially in the firm-level applications. Indeed, the decomposability of the Malmquist index is generally seen as its main advantage to other productivity indices, together with its weaker data requirements. Similar decompositions have been extended to variants of the Malmquist index such as the Hicks-Moorsten type productivity index of Bjurek (1994) (see Lovell, 2003) and to the Malmquist-Luenberger type index (see Färe and Primont, 2003). However, the classic productivity indices such as the Fisher ideal TFP index and the Törnqvist TFP index currently lack such decompositions.

This paper intends to fill this gap at least partly by proposing an intuitive decomposition for the Fisher ideal TFP index (henceforth 'the Fisher index'). Following the approach of Färe et al. (1994a,c), we express the Fisher index as a product of five components that represent changes in 1) production technology, 2) technical efficiency, and 3) scale efficiency. In addition to these standard components, our decomposition includes 4) allocative efficiency, and 5) market strength components. Components 2) – 4) can be further decomposed into separate sub-components for inputs and outputs, offering a detailed picture of the constituents of productivity change as measured by the Fisher index.

Our decomposition contributes to the productivity literature in at least four important ways. Firstly, the decomposition further enhances our general understanding about how the Fisher index works (i.e., which effects it captures, and which ones not). While a number of decompositions of the Fisher ideal price and quantity indices have been proposed (see Balk, 2003, for a recent review), we are unaware of other decompositions in the present context of productivity indices. Secondly, the decomposition provides a more detailed picture about the driving forces behind productivity growth (or decline) in empirical applications (i.e., which effects have improved productivity, and which ones have deteriorated it). For example, in the application of Section 9 we will find that the decline in the market strength explains the slow productivity growth in the Finnish agriculture in the aftermath of Finland's EU accession in 1995. Thirdly, our decomposition recognizes price effects as important elements of productivity change. From the welfare perspective, not only the quantity of output is important – also quality matters. By attributing changes in economic value of inputs and outputs to the productivity index, we hope to capture at least some quality aspects of productivity. Fourthly, better understanding of the anatomy of the Fisher index also enables one to tailor the productivity index for the purposes of the study. If some components capture effects that do not fit in a given definition of productivity, the decomposition enables one to correct for (or eliminate) these undesirable components from the overall productivity index.

Since our decomposition is closely related to (and inspired by) the decomposition of the Malmquist index (in particular the version by Färe et al., 1994c; see Appendix 1 for details), it is useful to compare the two in more detail. We consider the classic Fisher index to be a useful alternative for the nowadays widely used Malmquist approach: the Fisher index is simple to calculate from the data; it does not require empirical estimation of production technology; it is not influenced by arbitrary assumptions about the technology or its functional form; it has a well-established axiomatic foundation (see Diewert, 1992); and it is consistent with the index number formulae widely used for price and quantity indices by statistical agencies around the world. By contrast, the Malmquist index relies on the technology distance functions, which must be estimated empirically. This estimation requires a number of assumptions to be made. To begin with, the direction or orientation of measurement must be chosen; the choice of input distance function can lead to very different results than the choice of output distance function. Next, the choice of the estimation method and the assumptions involved may influence the results considerably.

Of course, these disadvantages should be balanced against the two major advantages of the Malmquist index. The first concerns the data requirements: the Malmquist index only requires data on input and output quantities, whereas the Fisher index requires the complete price data in addition to the quantities. While the data requirements do favour the Malmquist index, it is worth to ask if it is easier to estimate the prices or the production technology when both are unknown. Moreover, if the assumption of allocative efficiency can be maintained, then the quantity data suffices for estimating bounds for the Fisher index, as shown by Kuosmanen et al. (2004) (who build on the work of Färe and Grosskopf, 1992; and Balk, 1993). The second benefit has been the decomposability of the Malmquist index. A number of alternative decompositions have been proposed (e.g., Färe et al. 1994a,c; Ray and Desli 1997). It should be noted, however, that the decompositions of the Malmquist index have also attracted critical comments. We refer to Lovell (2003) and Grosskopf (2003) for recent surveys of this debate. By starting from the Fisher ideal index, we hope to overcome some of the difficulties experienced with the Malmquist decompositions.

The remainder of this paper is organized as follows. Section 2 introduces the standard notation and terminology used in production theory, and presents a formal duality theorem regarding profitability function. Sections 3 to 6 introduce the components of productivity change in a step-by-step manner, comparing with the analogous components available for the Malmquist index. Section 3 starts with the technical efficiency component followed by technical change in section 4. Section 5 discusses the definition of scale efficiency component. Section 6 introduces the allocative efficiency component of the Fisher index and section 7 presents the market strength component. Having introduced all components, in Section 8 we are finally equipped to fit all the components together in our main theorem, which proves that the Fisher ideal TFP index is obtained as a product of these intuitive components. Section 9 provides an empirical application on Finnish farm data. The paper finishes with some concluding remarks.

2. Preliminaries

Productivity growth is the change in output not explained by change in input use. To measure productivity changes in the general multiple input multiple output setting, we must aggregate the inputs and outputs in one way or another. Inputs (outputs) measured in different units do not add up as such, but must first be converted to some common scale of measurement, a natural choice being money. Indeed, the classic index theory uses unit prices (or cost shares) as weights of quantity index. Using prices as weights has the added advantage that the more precious or important commodities are given a greater weight than the inexpensive ones. In this vein, the conventional index theory typically uses the prices (or cost/revenue) shares as the weights of quantity indexes. Of course, the prices often change from one period to another, so we inevitably face the question of which set of prices provide the most appropriate weights (consider e.g. difference between the classic Paashe and Laspeyres indexes). Fisher (1922) solved this question in an ingenious way evaluating productivity at the weights of the base period and at the weights of the target period, and taking the geometric average of the two.

We limit attention to two-period comparisons, and denote the base period as period 0 and the target period as period 1. Adopting the standard notation, $\mathbf{y}^t \in \mathbb{R}^s_+$ represents the output quantity vector of period $t \in \{0,1\}$ and $\mathbf{p}^t \in \mathbb{R}^s_{++}$ the associated price vector. Similarly, $\mathbf{x}^t \in \mathbb{R}^r_+$ denotes the input quantity vector of period *t* and $\mathbf{w}^t \in \mathbb{R}^r_{++}$ the associated price vector. The Fisher ideal output and input quantity indices are defined as

(1) $$F_y(\mathbf{p}^{0,1},\mathbf{y}^{0,1}) \equiv \left(\frac{\mathbf{p}^0 \cdot \mathbf{y}^1}{\mathbf{p}^0 \cdot \mathbf{y}^0} \cdot \frac{\mathbf{p}^1 \cdot \mathbf{y}^1}{\mathbf{p}^1 \cdot \mathbf{y}^0} \right)^{1/2} \text{ and } F_x(\mathbf{w}^{0,1},\mathbf{x}^{0,1}) \equiv \left(\frac{\mathbf{w}^0 \cdot \mathbf{x}^1}{\mathbf{w}^0 \cdot \mathbf{x}^0} \cdot \frac{\mathbf{w}^1 \cdot \mathbf{x}^1}{\mathbf{w}^1 \cdot \mathbf{x}^0} \right)^{1/2},$$

respectively.

Total factor productivity is usually defined as the ratio of the output quantity index to the input quantity index (e.g. Diewert, 1992; Bjurek, 1996). Using the Fisher ideal indexes to aggregate both inputs and outputs, the Fisher ideal TFP index is obtained simply as the ratio of the aggregate output to the aggregate input

(2) $$F_{TFP}(\mathbf{p}^{0,1},\mathbf{w}^{0,1},\mathbf{y}^{0,1},\mathbf{x}^{0,1}) \equiv \frac{F_y(\mathbf{p}^{0,1},\mathbf{y}^{0,1})}{F_x(\mathbf{w}^{0,1},\mathbf{x}^{0,1})}.$$

This productivity measure is easily calculated given the necessary price and quantity data. It does not require estimation of any sort. Unlike the Malmquist index, the Fisher TFP index for a firm is independent on performance of other firms in the sample.

To get a more detailed picture of the underlying sources of productivity changes, we next proceed to decompose the Fisher index (2) into subcomponents along the lines of Färe et al. (1994a,c). Our decomposition uses a number of alternative equivalent representations of technology, which will be introduced next. The production possibility set of period *t* is defined as

(3) $$T^t = \left\{ (\mathbf{x},\mathbf{y}) \in \mathbb{R}^{r+s}_+ \,\middle|\, \mathbf{x} \text{ can produce } \mathbf{y} \text{ in period } t \right\}.$$

These sets are assumed to be non-empty, closed and satisfy the scarcity and no-free-lunch assumptions (see Färe and Primont, 1995). Alternative set representations of technology are the input set $L^t(\mathbf{y}) = \left\{ \mathbf{x} \in \mathbb{R}^r_+ \,\middle|\, (\mathbf{x},\mathbf{y}) \in T^t \right\}$

and the output set $P^t(\mathbf{x}) = \left\{\mathbf{y} \in \mathbb{R}^s_+ \,\middle|\, (\mathbf{x},\mathbf{y}) \in T^t\right\}$.

Input-output vector (***x***,***y***) is considered technically efficient if it lies on the boundary of set *T*. The degree of inefficiency is traditionally measured using Shephard's (1953, 1970) distance functions. Slightly deviating from the original definition,[1] we define the input distance function as

$$(4) \qquad D^t_x(\mathbf{x},\mathbf{y}) \equiv \min\left\{\theta \,\middle|\, (\theta\,\mathbf{x},\mathbf{y}) \in T^t\right\},$$

and the output distance function as

$$(5) \qquad D^t_y(\mathbf{x},\mathbf{y}) \equiv \min\left\{\theta \,\middle|\, (\mathbf{x},\mathbf{y}/\theta) \in T^t\right\}.$$

Distance functions can also be seen as representations of technology: we can recover T^t from input (output) distance function when inputs (outputs) are weakly disposable (Färe and Primont, 1995).

The minimum cost of producing a given target output ***y*** at given input prices ***w*** is given by the cost function

$$(6) \qquad C^t(\mathbf{w},\mathbf{y}) \equiv \min_x\left\{\mathbf{w}\cdot\mathbf{x} \,\middle|\, (\mathbf{x},\mathbf{y}) \in T^t\right\}.$$

Analogously, the maximum revenue that can be obtained given inputs ***x*** and output prices ***p*** is given by the revenue function

$$(7) \qquad R^t(\mathbf{x},\mathbf{p}) \equiv \max_y\left\{\mathbf{p}\cdot\mathbf{y} \,\middle|\, (\mathbf{x},\mathbf{y}) \in T^t\right\}.$$

By duality theory, these monetary expressions can also be viewed as technology representations. Specifically, cost function C^t enables us to recover the convex monotonic hull of the input set L^t, and analogously, we can recover the convex monotonic hull of the output set P^t from the revenue function R^t.

As the only non-standard concept we introduce the profitability function

$$(8) \qquad \rho^t(\mathbf{w},\mathbf{p}) \equiv \max_{x,y}\left\{\frac{\mathbf{p}\cdot\mathbf{y}}{\mathbf{w}\cdot\mathbf{x}} \,\middle|\, (\mathbf{x},\mathbf{y}) \in T^t\right\},$$

which indicates the maximum return to the dollar achievable with the given non-negative input-output prices. The following duality theorem shows that (like profit function) profitability function can be taken as a representation of technology:

[1] In our definition, the values of both the input distance function (4) and the output distance function (5) are conveniently restricted to the half-open interval (0,1]. Since in this paper we restrict to measuring efficiency of observed firm behaviour (which must be technically feasible to be observable to begin with), and since the production possibility sets T^t are assumed to be closed, the minimum of (4) and (5) will always exists.

Theorem 1: If production possibility set T^t satisfies the assumptions of free disposability, convexity, and constant returns to scale, then profitability function ρ^t is a complete characterisation of the technology. In particular,

$$T^t = \left\{ (\mathbf{x},\mathbf{y}) \in \mathbb{R}_+^{r+s} \left| \frac{\mathbf{p}\cdot\mathbf{y}}{\mathbf{w}\cdot\mathbf{x}} \le \rho^t(\mathbf{w},\mathbf{p}) \ \forall (\mathbf{w},\mathbf{p}) \in \mathbb{R}_+^{r+s} \right. \right\}.$$

Proof. See Appendix 2.

Under the assumptions of Theorem 1, the input distance function can be expressed in terms of profitability as

(9) $$D_x^t(\mathbf{x},\mathbf{y}) = \max_{(\mathbf{w},\mathbf{p})\in\mathbb{R}_+^{r+s}} \left\{ \frac{\mathbf{p}\cdot\mathbf{y}}{\mathbf{w}\cdot\mathbf{x}} \middle| \rho^t(\mathbf{w},\mathbf{p}) \le 1 \right\}.$$

This result will be used in the developments of the subsequent sections.

As a final remark, it should be noted that production possibility sets, distance functions, cost and revenue functions, and profitability functions are usually not known and must be estimated from empirical data. A number of techniques for such estimation are available, including Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA) (e.g. Färe et al., 1994b, Kumbhakar and Lovell, 2000). This paper abstracts from the estimation issues and focuses solely on the decomposition. Recall that no estimations are needed for calculation of the Fisher input, output or TFP indices. Estimation is only needed for the decomposition.

3. Technical efficiency

Following Farrell (1957), we measure technical efficiency as a distance from the production frontier, using distance functions (4) and (5). In theory, operational inefficiency signals under-utilization of inputs, which in turn may depend on a number of factors such as managerial competence (Leibenstein, 1966) or incentive structure (Bogetoft, 1994, 1995). Changes in input-utilization over time may also be attributed to the gradual diffusion of innovations (Färe et al., 1994c) or to the business cycle. Though we usually cannot isolate the different effects that influence the input utilization, it is still interesting to measure how much its changes contribute to productivity changes.

Technical efficiency component is an integral part of the Malmquist decomposition. Decompositions following Färe et al. have traditionally oriented either towards inputs or towards outputs. The input oriented measure of technical efficiency change $D_x^1(\mathbf{x}^1,\mathbf{y}^1)/D_x^0(\mathbf{x}^0,\mathbf{y}^0)$ gauges the movements towards or away from the frontier in the direction of inputs, holding the output levels as constant. By contrast, the output oriented measure of technical efficiency change $D_y^1(\mathbf{x}^1,\mathbf{y}^1)/D_y^0(\mathbf{x}^0,\mathbf{y}^0)$ gauges the movements towards or away from the frontier in the direction of outputs, holding the input levels as constant. The well-known result of Färe and Lovell (1978) implies that these two approaches are equivalent if and only if the production sets T^0 and T^1 exhibit constant returns to scale.

If the technology exhibits variable returns to scale, the choice of orientation does matter. Since we have no particular reason to prefer input or output orientation over another, we propose to resolve the problem in the traditional fashion (of Irving Fisher) by measuring efficiency change by the geometric mean of the ratios of input distance functions and output distance functions. This gives the following overall measure of technical efficiency change

$$(10) \qquad \Delta TEff = \left(\frac{D_x^1(\mathbf{x}^1,\mathbf{y}^1)}{D_x^0(\mathbf{x}^0,\mathbf{y}^0)} \cdot \frac{D_y^1(\mathbf{x}^1,\mathbf{y}^1)}{D_y^0(\mathbf{x}^0,\mathbf{y}^0)} \right)^{1/2}$$

Under constant returns to scale, this measure is equivalent to the usual technical efficiency component of the Malmquist index. Attractively, under variable returns to scale this measure is invariant to the choice of orientation. If the orientation makes a difference, then we can distinguish between the sub-components of input efficiency $\left(D_x^1(\mathbf{x}^1,\mathbf{y}^1)/D_x^0(\mathbf{x}^0,\mathbf{y}^0)\right)^{1/2}$ and the output efficiency $\left(D_y^1(\mathbf{x}^1,\mathbf{y}^1)/D_y^0(\mathbf{x}^0,\mathbf{y}^0)\right)^{1/2}$. In our decomposition, both will be accounted for with the equal weight.

Measures of technical efficiency change are graphically illustrated in Figure 1. Figure 1a presents input oriented technical efficiency component in input-output space and Figure 1b in input-input space. In the figures, the input oriented technical efficiency change is $D_x^1(x^{1,},y^1)/D_x^0(x^{0,},y^0) = \frac{|OB| / |Ox^1|}{|OA| / |Ox^0|}$. Similar relationship can be found in the output orientation, which is presented in Figures 1c and 1d as $D_y^1(x^{1,},y^1)/D_y^0(x^{0,},y^0) = \frac{|Oy^1| / |OD|}{|Oy^0| / |OC|}$. Taking a geometric mean of these two provides our weighted average for technical efficiency.

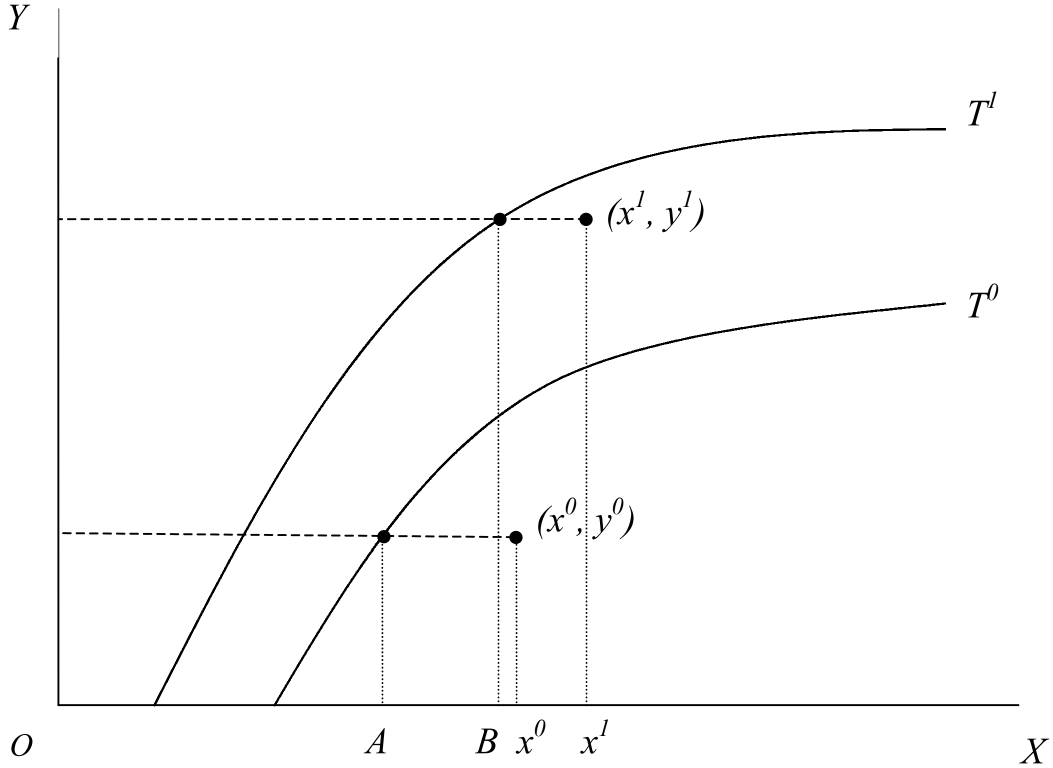

Figure 1a. Input oriented technical efficiency change in input-output space.

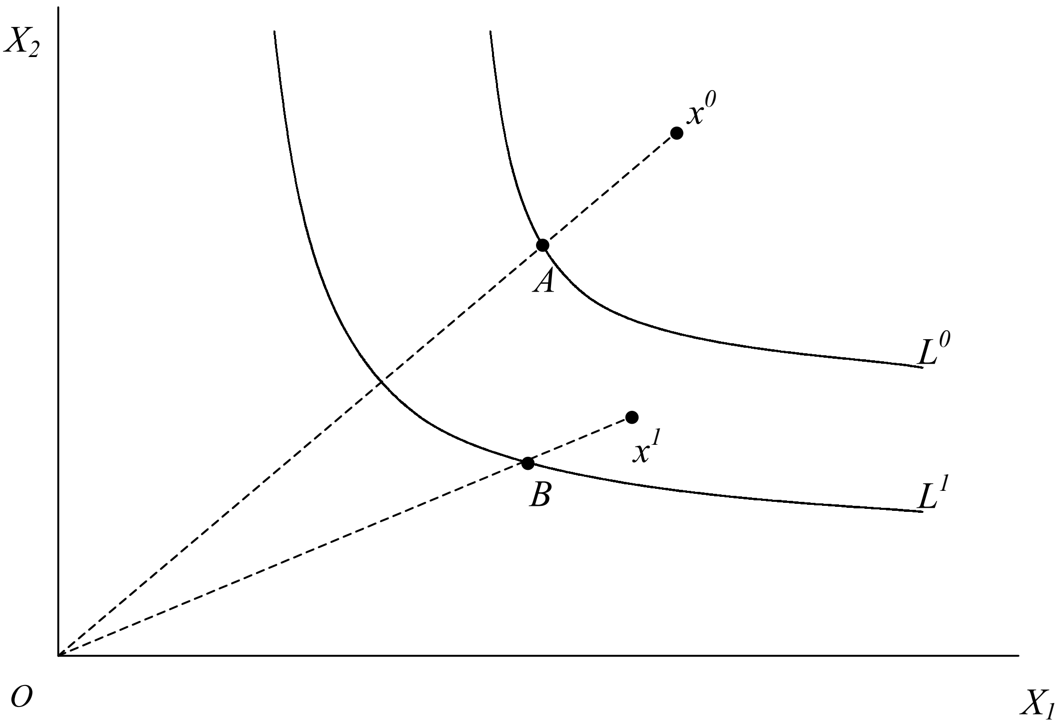

Figure 1b. Input oriented technical efficiency change in input-input space.

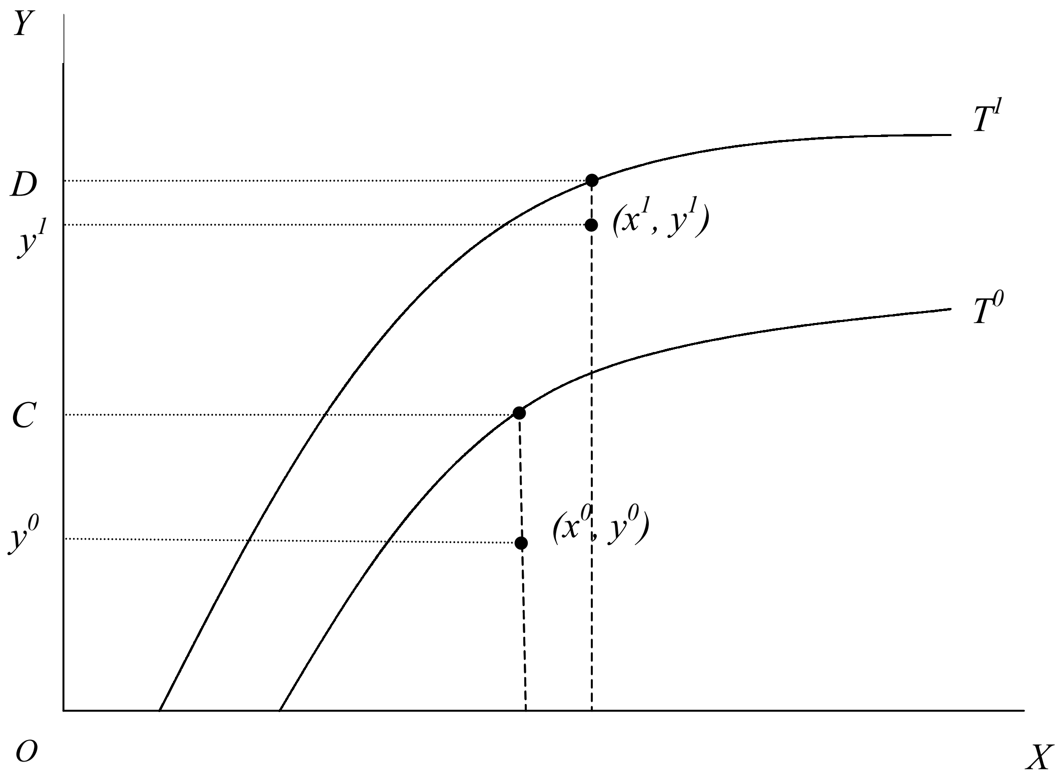

Figure 1c. Output oriented technical efficiency change in input-output space.

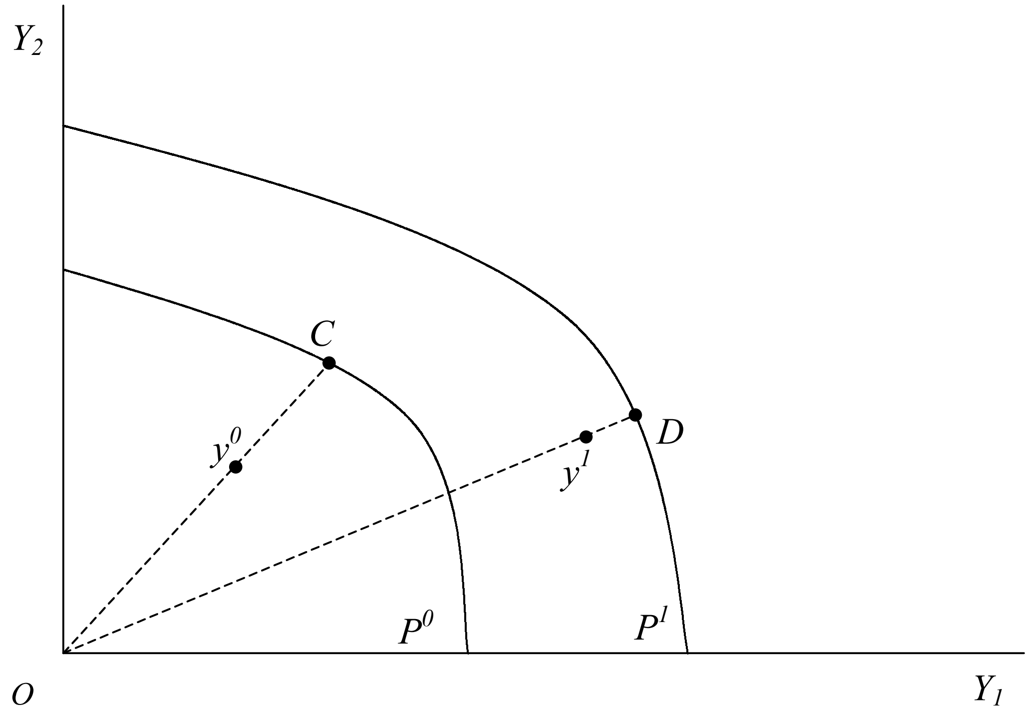

Figure 1d. Output oriented technical efficiency change in output-output space.

4. Technical change

The traditional perspective on technical progress is that of Hicks neutral frontier shift considered by Solow (1957). Recent economic literature has recognised the importance of biases in technical progress, which means that productivity of different inputs increases at different rates (e.g., Acemoglu, 2002). Both neutral and biased technical changes are accounted for by the technical change component of the Malmquist index, which measures the shifts in the production frontier by distance functions. By duality theory, input and output distance functions can be seen as equivalent representations of technology as production sets T^t.

In this paper we resort to another representation of technology, the profitability function ρ^t. By Theorem 1, ρ^t is an equally valid representation of technology as the distance function when T^t satisfies free disposability, convexity, and constant returns to scale. It is common to measure technical change in terms of the constant-returns-to-scale benchmark technology (e.g., Färe et al., 1994c). Thus, measuring technical change in terms of profitability functions ρ^t is equally legitimate as it is with distance functions.

Consider ratio $\frac{\rho^1(\mathbf{w}^0,\mathbf{p}^0)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)}$, which represents the change of maximum profitability from the base period to the target period, at the prices of the base period. As the same price vectors appear both in the nominator and the denominator, any change in profitability is inevitably due to the change of technology. Technical progress would tend to increase profitability, and hence this ratio. Technical regress would decrease this ratio. We could similarly measure technical changes using the prices of the target period. Again, as we do not have any reason to prefer the prices of the base or target period, we express the technical change component as a geometric mean of these two:

$$(11) \qquad \Delta Tech = \left(\frac{\rho^1(\mathbf{w}^0,\mathbf{p}^0)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)} \cdot \frac{\rho^1(\mathbf{w}^1,\mathbf{p}^1)}{\rho^0(\mathbf{w}^1,\mathbf{p}^1)} \right)^{1/2} .$$

We should note that the technical change component of Färe et al. (1994a,c) also measures technical change in terms of profitability, as the dual formulation of the distance function relative to a constant returns to scale technology can be expressed as profitability measure (see equality (9) above). Our measure for technical change can differ from that of Färe et al. to some extent because we measure profitability in terms of observed input-output prices, whereas Färe et al. use shadow prices. Like the component of Färe et al., our technical change measure does not capture technical progress that occurs in the part of the frontier exhibiting increasing or decreasing returns to scale if the frontier does not shift at the most productive scale size. In particular, our component only captures frontier shifts that have an impact on profitability; thus, it could be more precisely qualified as "profitability enhancing technical progress". Other types of frontier shifts are attributed to the allocative or scale efficiency components (as in Färe et al. decomposition). For example, suppose a technical innovation helps to improve productivity of small firms, but has no effect on the profitability of the leading firms operating at the most productive scale size. Such a local frontier shift makes the small-scale production more attractive, and thus improves scale efficiency of small firms even if these firms do not expand their scale. Even though this productivity improvement was ultimately due to a technical innovation, it does not show up in the technical change component because the same productivity could have been achieved with the earlier technology if economies of scale were appropriately utilised. Thus, excluding these local frontier shifts from the technical change components seems justified.

This measure is illustrated in Figure 2 where ρ^0 and ρ^1 represent the maximum return per dollar (most profitable) lines in period 0 and 1. The change in maximum profitability is simply given by the following geometric expression: $\Delta Tech = \dfrac{\tan\beta}{\tan\alpha}$.

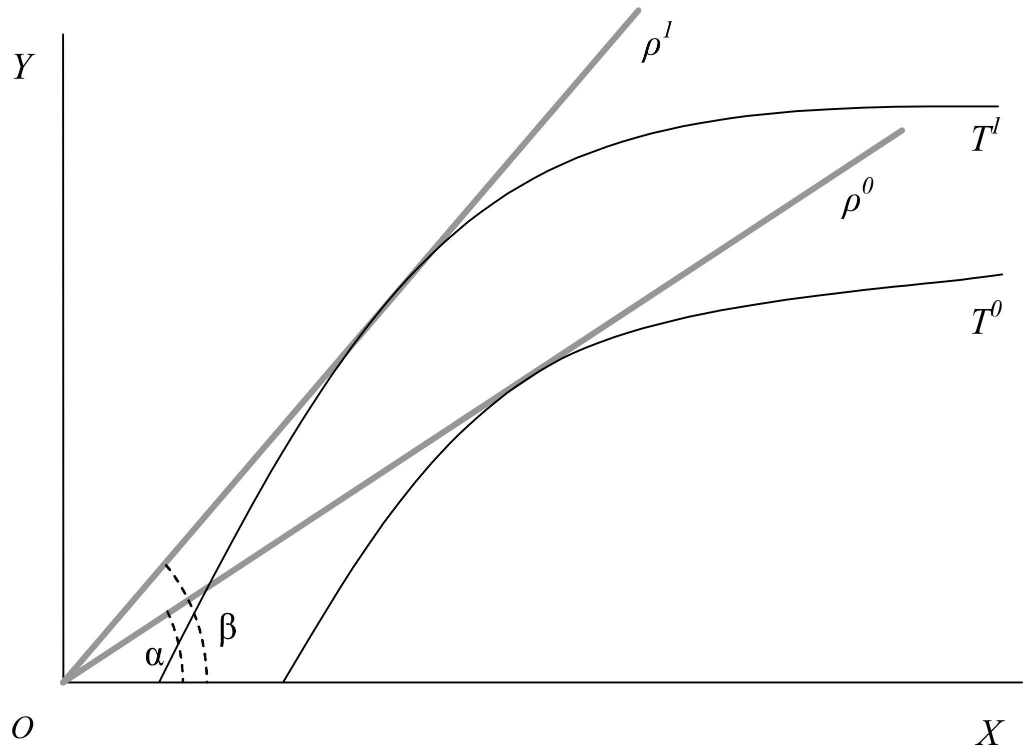

Figure 2. Technical change component, measured with profitability functions

5. Scale efficiency

Economies (and diseconomies) of scale can obviously contribute to productivity. Indeed, varying utilisation of scale economies in different stages of the business cycle is considered as one plausible explanation for pro-cyclical productivity (e.g. Basu and Fernald, 1997). Therefore, it is interesting to isolate the effect of scale economies component in the productivity change. While technical and allocative inefficiencies relate to managerial and organisational aspects within the firm, scale inefficiency typically signals structural inefficiency in the operating environment of the firm. Possible reasons for an ineconomically small scale include credit constraints, trade barriers, and shortage of essential resources, while imperfect competition and monopoly power may lead firms to produce on ineconomically large scale.

The decomposition of Färe et al. (1994c) utilises the intuitive scale efficiency measure by Färe et al. (1983), which compares the distance functions gauged relative to a variable returns to scale and constant returns to scale reference technologies. However, applying this conventional scale measure as such in the present framework is problematic, because the Färe et al. (1983) decomposition depends on the order in which its components are calculated, as pointed out by McDonald (1996) in the case of technical efficiency, congestion efficiency, and scale efficiency components (see also Färe and Grosskopf, 2000). The same problem extends to the present case with allocative and scale efficiency, where allocative efficiency may be different if we measure it relative to the variable returns to scale technology or the constant returns to scale benchmark that already accounts for scale inefficiency. Only when the input sets *L* and output sets *P* are homothetic the order of decomposition does not influence the result.

To avoid this arbitrariness, we propose to define scale efficiency in economic rather than technical terms. In line with our treatment of technical change, we adopt the dual perspective and characterise the optimal scale size in terms of the profitability function ρ^t. Thus, the most natural measure of scale efficiency is the ratio of the maximum profitability at the current scale size (scale constrained profitability function) to the overall (global) maximum profitability (unconstrained profitability function). In essence, this definition of scale efficiency assumes constraints as the reason for scale inefficiency.

The current scale size can be measured either in terms of inputs or outputs. If we fix the output level to $\mathbf{y}^t$, the maximum profitability (or return on the dollar) in period *t* is given by the ratio $\frac{\mathbf{p}^t \cdot \mathbf{y}^t}{C^t(\mathbf{w}^t, \mathbf{y}^t)}$. Thus, the input oriented profitability based scale efficiency measure is the ratio of output constrained maximum profitability to the unconstrained maximum profitability, that is,

$$(12)\qquad \left(\frac{\mathbf{p}^t\cdot\mathbf{y}^t}{C^t(\mathbf{w}^t,\mathbf{y}^t)}\right)\Big/\rho^t(\mathbf{w}^t,\mathbf{p}^t).$$

Similarly, if we fix the inputs to $\mathbf{x}^t$, the maximum profitability is $\frac{R^t(\mathbf{x}^t,\mathbf{p}^t)}{\mathbf{w}^t\cdot\mathbf{x}^t}$. Thus, the output oriented profitability based scale efficiency measure is

$$(13)\qquad \left(\frac{R^t(\mathbf{x}^t,\mathbf{p}^t)}{\mathbf{w}^t\cdot\mathbf{x}^t}\right)\Big/\rho^t(\mathbf{w}^t,\mathbf{p}^t)$$

In the case of the Malmquist index, the scale efficiency measure is especially dependent on the choice of orientation. To avoid this problem, we again follow the Fisher approach and take the geometric mean of the input oriented and output oriented scale efficiency measures. This results as the following expression for the scale efficiency component:

$$(14)\qquad \Delta SEff=\left(\frac{\left(\frac{\mathbf{p}^1\cdot\mathbf{y}^1}{C^1(\mathbf{w}^1,\mathbf{y}^1)}\right)}{\rho^1(\mathbf{w}^1,\mathbf{p}^1)}\cdot\frac{\left(\frac{R^1(\mathbf{x}^1,\mathbf{p}^1)}{\mathbf{w}^1\cdot\mathbf{x}^1}\right)}{\rho^1(\mathbf{w}^1,\mathbf{p}^1)}\right)^{\frac{1}{2}}\Bigg/\left(\frac{\left(\frac{\mathbf{p}^0\cdot\mathbf{y}^0}{C^0(\mathbf{w}^0,\mathbf{y}^0)}\right)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)}\cdot\frac{\left(\frac{R^0(\mathbf{x}^0,\mathbf{p}^0)}{\mathbf{w}^0\cdot\mathbf{x}^0}\right)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)}\right)^{\frac{1}{2}}.$$

Input oriented scale efficiency measure and its change is illustrated in the Figure 3a. Input scale efficiency $\Delta ISEff=\frac{|OK|/|OB|}{|OJ|/|OA|}$ is calculated as the change of input scale efficiency between periods 0 ($|OJ|/|OA|$) and 1 ($|OK|/|OB|$). Similarly, Figure 3b depicts the output oriented scale efficiency measure and its change. Output scale efficiency $\Delta OSEff=\frac{|OD|/|OM|}{|OC|/|OC|}$ is the change in output scale efficiency between periods 0 and 1, respectively. Note that in period 0 the output scale efficiency equals 1. The value is bigger (smaller) than one if the unit gets closer (further away) to (from) the most profitable scale of production.

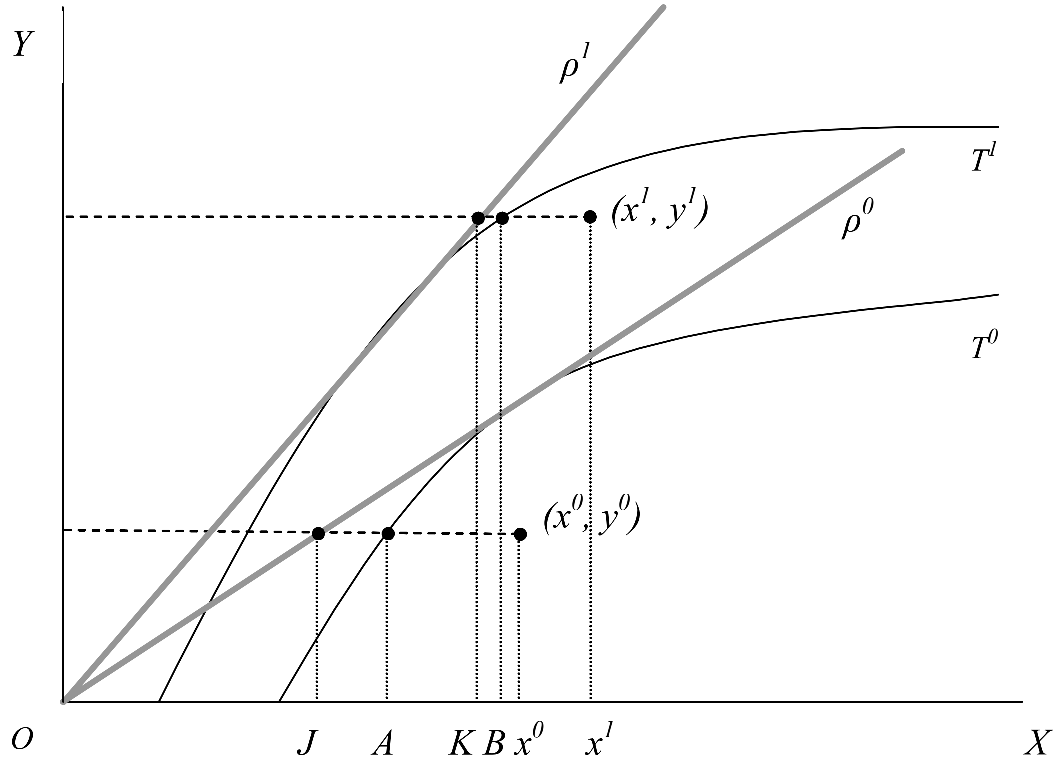

Figure 3a. Input oriented scale efficiency change.

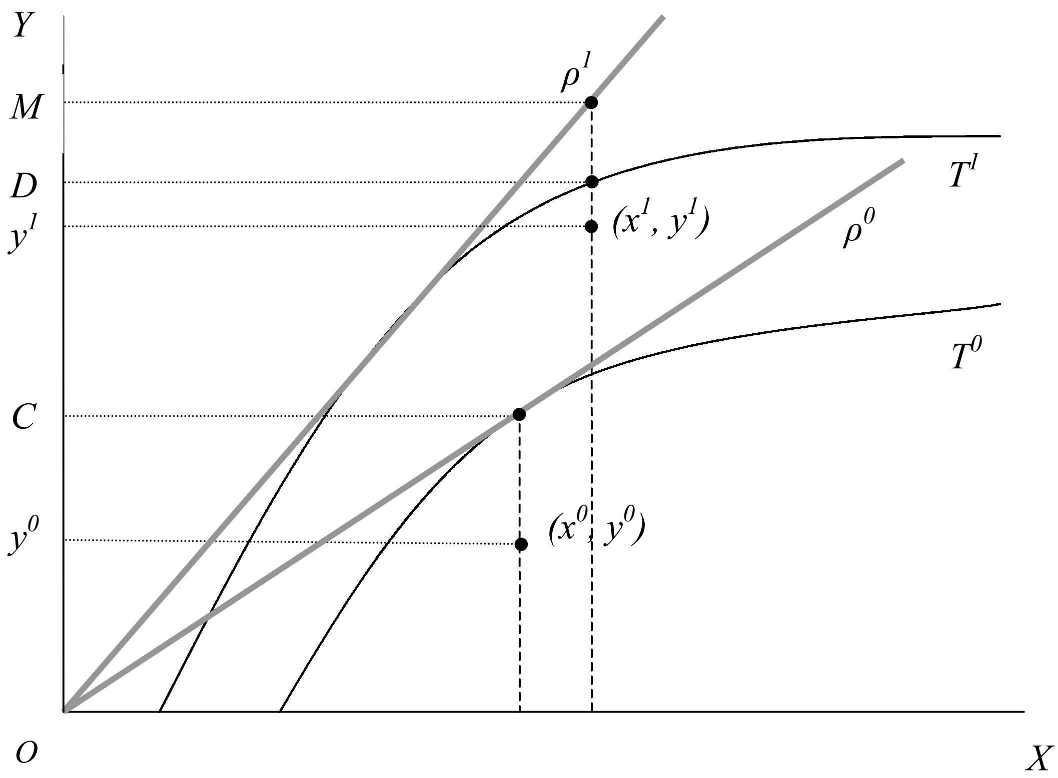

Figure 3b. Output oriented scale efficiency change.

6. Allocative efficiency

Thus far, allocative efficiency components have not appeared in the decompositions of productivity change, although it is a component of Farrell's (1957) static decomposition of productive efficiency.[2] Introducing allocative efficiency into the productivity index may appear unorthodox if we consider productivity and technical progress to be the same. Since the standard neoclassical assumption of rational profit maximizing firms leaves no room for allocative inefficiency, changes in allocative efficiency do not rank among the most standard explanations for productivity changes. Yet, in the real world, markets are far from perfectly competitive, incomplete information and other frictions are present, and adjustments to price changes can occur with considerable delay. Since changes of allocative efficiency do occur in the real life, especially in more turbulent times, it is well in line with the welfare interpretation of productivity to account for its changes. Ignoring such a potentially important component can only contribute to mis-measurement of welfare.

While technical efficiency captures the radial input reduction and output expansion potential, Farrell's (1957) allocative efficiency measure represents the non-radial productivity improvement potential, which is obtainable by restructuring production (i.e., changing the input-output mix). As Kuosmanen and Post (2001, Eq. 2.8) note, allocative efficiency can be expressed solely in terms of technology distance functions. Indeed, allocative efficiency measure can be viewed as a distance measure in the input-output quantity space, reflecting the technically feasible distance to the economic iso-cost (iso-revenue) surfaces representing the maximum aggregate, quality adjusted input (output) quantity. In this perspective, introducing allocative efficiency component to the TFP index, defined as a ratio of two quantity indices, seems fully legitimate.

Allocative efficiency can be measured in terms of inputs or outputs (costs or revenues). Input allocative efficiency is the ratio of the cost function and the cost of the technically efficient input vector that can be obtained by proportionately scaling the observed input vector downward to the efficient frontier, that is,

$$(10) \qquad IAEff^{t} \equiv \frac{C^{t}(\mathbf{w}^{t},\mathbf{y}^{t})}{\mathbf{w}^{t}\cdot(D_{x}^{t}(\mathbf{x}^{t},\mathbf{y}^{t})\mathbf{x}^{t})}.$$

Analogously, output allocative efficiency is the ratio of the hypothetical revenue of the technically efficient output vector, obtained by equiproportionate augmentation of the observed output vector, and the revenue function, that is,

[2] A notable exception is the decomposition of profitability by Grifell-Tatjé and Lovell (1999).

(11) $$OAEff^{t} = \frac{\mathbf{p}^{t} \cdot (\mathbf{y}^{t} / D_{y}^{t}(\mathbf{x}^{t}, \mathbf{y}^{t}))}{R^{t}(\mathbf{x}^{t}, \mathbf{p}^{t})}.$$

The overall allocative efficiency of the firm should take into account the changes of mix both in the input and the output side. As before, we measure the overall change of allocative efficiency as the geometric mean of the changes in allocative efficiencies in inputs and outputs, that is,

(12) $$\Delta AEff = \left(\frac{IAEff^{1}}{IAEff^{0}} \cdot \frac{OAEff^{1}}{OAEff^{0}} \right)^{1/2}.$$

This is in line with fact that changes in input and output allocative efficiency contribute to profitability with equal weight and in multiplicative form.

Input and output oriented allocative efficiency changes are illustrated graphically in Figure 4. Input oriented allocative efficiency change in Figure 4a is depicted as $\Delta IAEff = \frac{|OF| / |OB|}{|OE| / |OA|}$. The measure compares the deviation from the minimum cost due to wrong allocation of inputs between subsequent periods when given output is produced. Similarly, output oriented allocative efficiency change is illustrated in Figure 4b as $\Delta OAeff = \frac{|OD| / |OH|}{|OC| / |OG|}$. It compares the deviations from maximum revenue due to wrong allocation of outputs given inputs between two periods, respectively. Again, we can take a geometric mean of these two, which provides our weighted average for allocative efficiency.

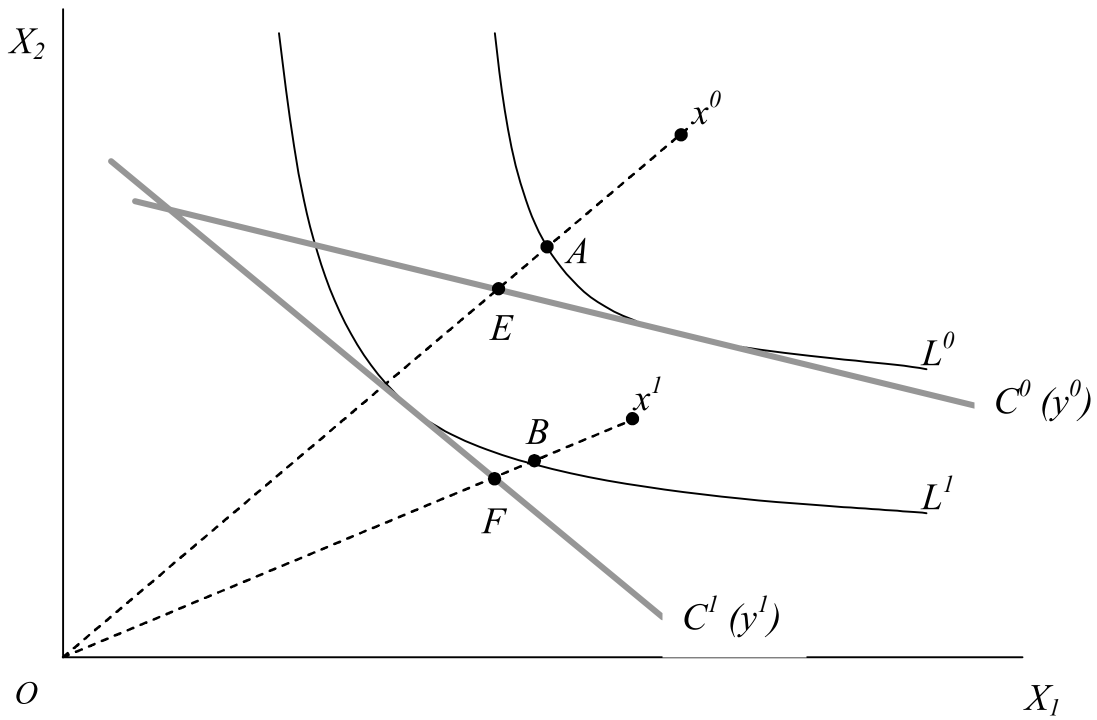

Figure 4a. Input allocative efficiency change.

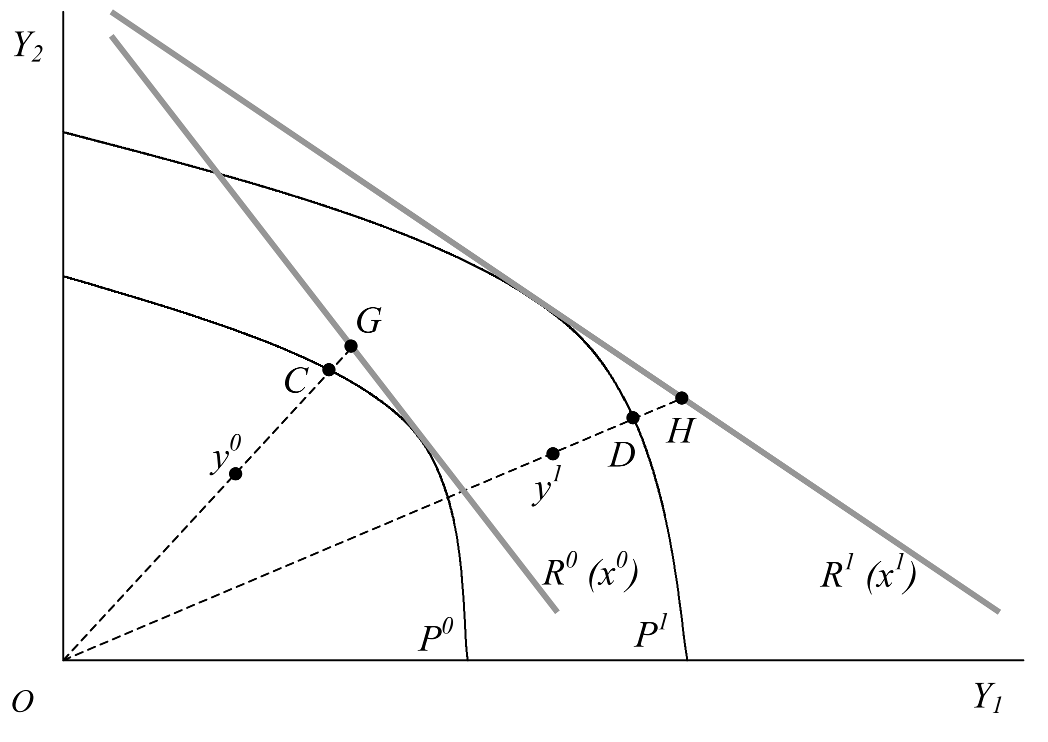

Figure 4b. Output allocative efficiency change.

7. Market strength

The market strength measure, to be introduced next, is the most atypical component of all. Therefore, we need to provide some motivating examples. Suppose a new technological innovation or learning by doing improves the quality of some outputs more than other outputs, such that the same inputs produce the same output quantities as before, but the high-quality outputs yield a higher price. Alternatively, quality of some inputs may improve such that a smaller amount of the same input is needed for producing the same output (consider e.g. the effect of education and training on labour input). Or an external change in the market environment (e.g. tariffs or trade barriers) or the government policies (e.g. taxes and subsidies) influences the competitiveness of the firms. As a result of these kinds of changes, firms may become more or less profitable, but the traditional technical change measures as well as technical, allocative, and scale efficiency components fail to account for this effect.

The increased capacity of firms to produce aggregate output with the same aggregate input could be measured by ratio $\frac{\rho^0(\mathbf{w}^1,\mathbf{p}^1)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)}$, representing the increase in profitability of the leading firms in the industry, given the base year technology. Alternatively, we could use the target year technology as the benchmark, and consider the ratio $\frac{\rho^1(\mathbf{w}^1,\mathbf{p}^1)}{\rho^1(\mathbf{w}^0,\mathbf{p}^0)}$. Since we have no reason to prefer either base or target year technology, we can take the geometric mean of these ratios:

$$(17) \qquad \left(\frac{\rho^0(\mathbf{w}^1,\mathbf{p}^1)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)} \cdot \frac{\rho^1(\mathbf{w}^1,\mathbf{p}^1)}{\rho^1(\mathbf{w}^0,\mathbf{p}^0)} \right)^{1/2}.$$

This geometric mean effectively captures the change in maximum profitability at given technology attributable to the price changes. However, this measure fails to account for possible profitability effects of changes in nominal prices due to inflation. To convert the profitability changes from nominal to real terms, we need an appropriate price deflator. The most natural deflator for profitability change is the ratio of Fisher output price index to the input price index, $F_p(\mathbf{x}^{0,1},\mathbf{y}^{0,1})/F_w(\mathbf{x}^{0,1},\mathbf{y}^{0,1})$, where the input and output vectors of the evaluated unit are used as the index weights. This deflator essentially measures the change of output prices relative to the change of input prices. If average output prices increase more (less) than the input prices, then the value of the deflator is greater (smaller) than one. If output and input prices change by the same factor, then this deflator is equal to one. To measure the change in market strength in terms of real prices, we propose the following measure:

(18) $$\Delta MS = \left(\frac{\rho^0(\mathbf{w}^1,\mathbf{p}^1)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)} \cdot \frac{\rho^1(\mathbf{w}^1,\mathbf{p}^1)}{\rho^1(\mathbf{w}^0,\mathbf{p}^0)} \right)^{\frac{1}{2}} \Bigg/ \frac{F_p(\mathbf{x}^{0,1},\mathbf{y}^{0,1})}{F_w(\mathbf{x}^{0,1},\mathbf{y}^{0,1})} .$$

The market strength measure does not change if all prices change by the same percentage. In fact, as the nominal profitability changes are adjusted by the price indices, the market strength remains constant even if output prices change by a different factor than input prices, if price changes are uniform across input factors and output goods. In essence, this component measures changes in the production possibilities of the quality-adjusted aggregate output due to changes in relative prices, measured in real terms. As these relative price changes are typically beyond the influence of an individual firm but rather depend on the external operating environment, we find market strength an appropriate name for this component. While non-uniform changes in quality of inputs and outputs may be one important reason for the change of market strength, as such, this component cannot be interpreted as a measure of quality change. For example, this component would likely fail to capture output quality improvements in the computer industry, where the quality improvement of outputs has coincided with decreasing output prices. If possible, the input and output quantity measures and indicators should be directly adjusted for their quality.

In the previous section we interpreted allocative efficiency change as a qualitative, non-radial, price-based component of efficiency change. In analogy with that interpretation, we may consider the market strength component as a qualitative, price-driven frontier shift. In other words, the market strength component can be viewed as the change in the aggregate production possibilities attributable to price changes. In this respect, it is instructive to multiply the market strength component by the associated technical change component to obtain

(19) $$\Delta Tech \times \Delta MS = \frac{\rho^1(\mathbf{w}^1,\mathbf{p}^1)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)} \Bigg/ \frac{F_p(\mathbf{x}^{0,1},\mathbf{y}^{0,1})}{F_w(\mathbf{x}^{0,1},\mathbf{y}^{0,1})} .$$

That is, the product of the technical change and market strength components is the change of maximum profitability measured in real terms (i.e., the nominal change of profitability function divided by the Fisher price index deflator).

The market strength component is difficult to illustrate graphically, but an attempt towards this is made in Figure 5. Figure 5a represents the input and Figure 5b the output orientation. In the form of cost and return function the market strength component can be presented as $\Delta MS = \frac{R^0(\mathbf{x}^{*0},\mathbf{p}^1)/C^o(\mathbf{y}^{*0},\mathbf{w}^1)}{R^0(\mathbf{x}^{*0},\mathbf{p}^0)/C^o(\mathbf{y}^{*0},\mathbf{w}^0)} \cdot \frac{R^1(\mathbf{x}^{*1},\mathbf{p}^1)/C^1(\mathbf{y}^{*1},\mathbf{w}^1)}{R^1(\mathbf{x}^{*1},\mathbf{p}^0)/C^1(\mathbf{y}^{*1},\mathbf{w}^0)}$

where x* and y * refer to optimal inputs and outputs given prices.

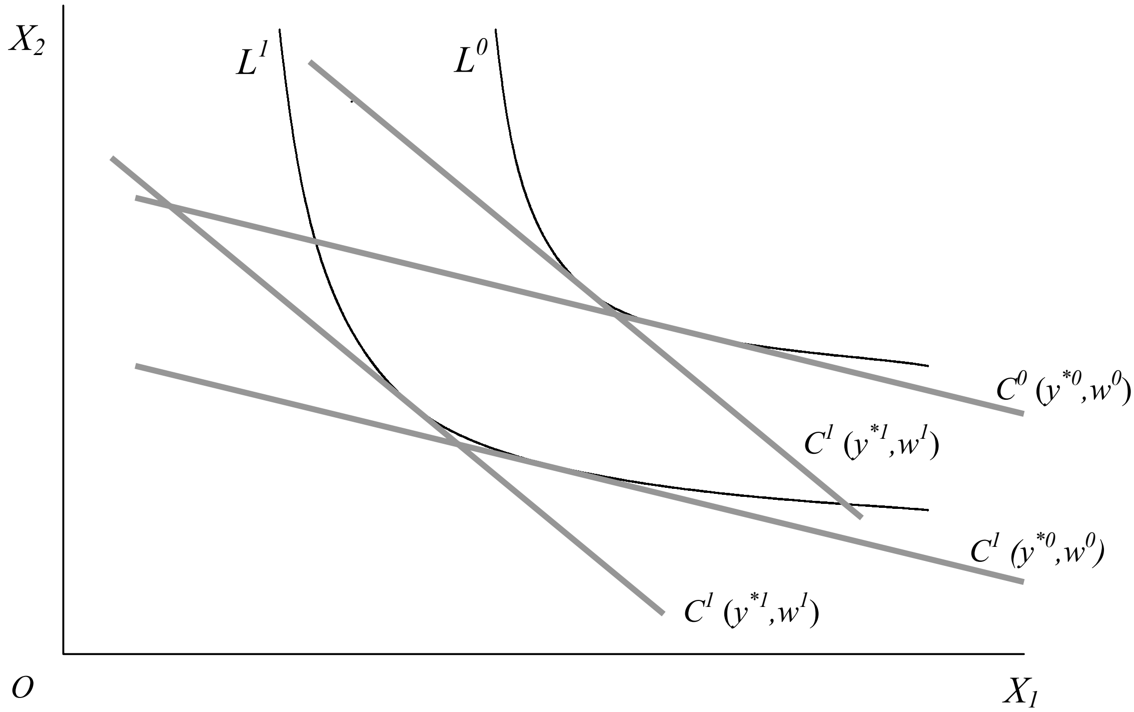

Figure 5a. Market strength (input orientation).

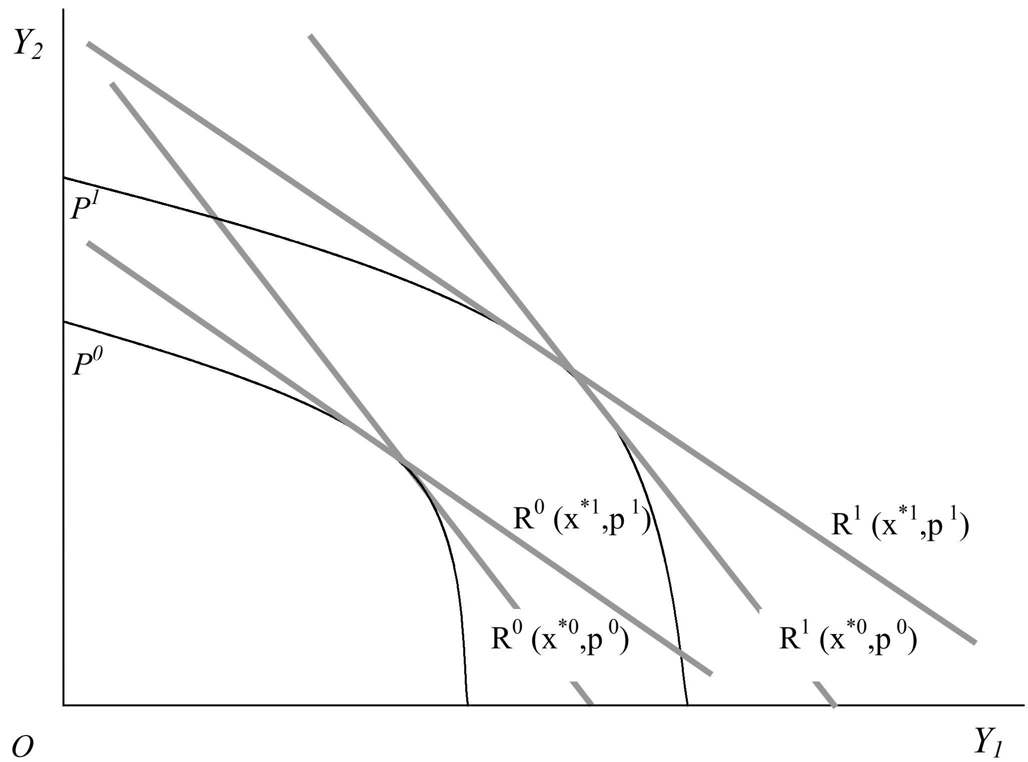

Figure 5b. Market strength (output orientation).

8. The main result

We are now ready to present the main result of the paper. It turns out that by multiplying the five components introduced above, we obtain the classic Fisher index.

Theorem 2: *The Fisher ideal TFP index can be expressed as the product of the technical efficiency, technical change, scale efficiency, allocative efficiency, and the market strength components:*

(20) $$F_{TFP} = \Delta TEff \times \Delta Tech \times \Delta SEff \times \Delta AEff \times \Delta MS$$

Proof. See Appendix 3

This result is interesting at least for the following reasons. Firstly, the decomposition further enhances our general understanding about how the Fisher index works. For example, we note that changes in allocative efficiency contribute to the Fisher index, but not to the Malmquist index (compare also with Färe and Grosskopf, 1992; Balk, 1993, 1998; and Kuosmanen et al., 2004). Secondly, the decomposition provides a more detailed, anatomical picture about the driving forces behind productivity changes. By distinguishing between input and output oriented sub-components, and by introducing allocative efficiency and a new market strength component, the present decomposition is by far the most detailed one ever presented. Thirdly, our decomposition recognises price-based allocative efficiency and market strength components as important elements of productivity change. By attributing changes in economic value of inputs and outputs to the productivity index, we hope to capture at least some quality aspects of productivity. Fourthly, better understanding of the anatomy of the Fisher index also enables one to tailor the productivity index for the purposes of the study. Even though the Fisher TFP index may not be the most appropriate index number formula for some applications, we believe our decomposition can provide insight for developing a tailored index that captures those effects that are important for the purposes of the study. If some components represent effects that do not fit in a given definition of productivity, the decomposition enables one to correct for (or eliminate) these undesirable components from the overall productivity index.

As a direct corollary of Theorem 2, we obtain an alternative decomposition of profitability (compare with Grifell-Tatjé and Lovell, 1999). In particular, the change of revenue can be expressed as the product of the Fisher output quantity and output price indices, and the change of costs can be expressed as the product of the Fisher input quantity and input price indices (Diewert, 1992). Note that the Fisher price indices were included as the deflator in the market strength component. Multiplying both sides of (18) with $F_p(\mathbf{x}^{0,1},\mathbf{y}^{0,1})/F_w(\mathbf{x}^{0,1},\mathbf{y}^{0,1})$ we obtain that the change in profitability is the product of our technical efficiency, technical change, scale efficiency, and allocative efficiency components and the nominal price version of the market strength component.

9. Application to Finnish farms

9.1 Motivation

We next apply the proposed decomposition to study productivity development in 459 Finnish bookkeeping farms through 1992-2000. The period under investigation is interesting because Finland joined the European Union (EU) in the middle of the study period in 1995. The EU accession meant increased international competition and a fall in price support, which together resulted as a drastic fall in output prices. Figure 6 illustrates the development of price indices for milk, meat and other animal products, and crops. The drop in prices of meat and other animal products and crop products was more than 50 percent, but the price of milk decreased only by 15 percent. Input prices have not fallen as dramatically as some output prices, although prices in input categories like land, animals, fertilizers, purchased feed, and materials have decreased. The fall has been more than 30 percent in purchased feed and fertilizers and even more than 40 percent in prices of purchased animals. These dramatic price changes have led farms to adjust their input-output mix over time, the effects of which should show up in the technical, allocative, and scale efficiency measures.

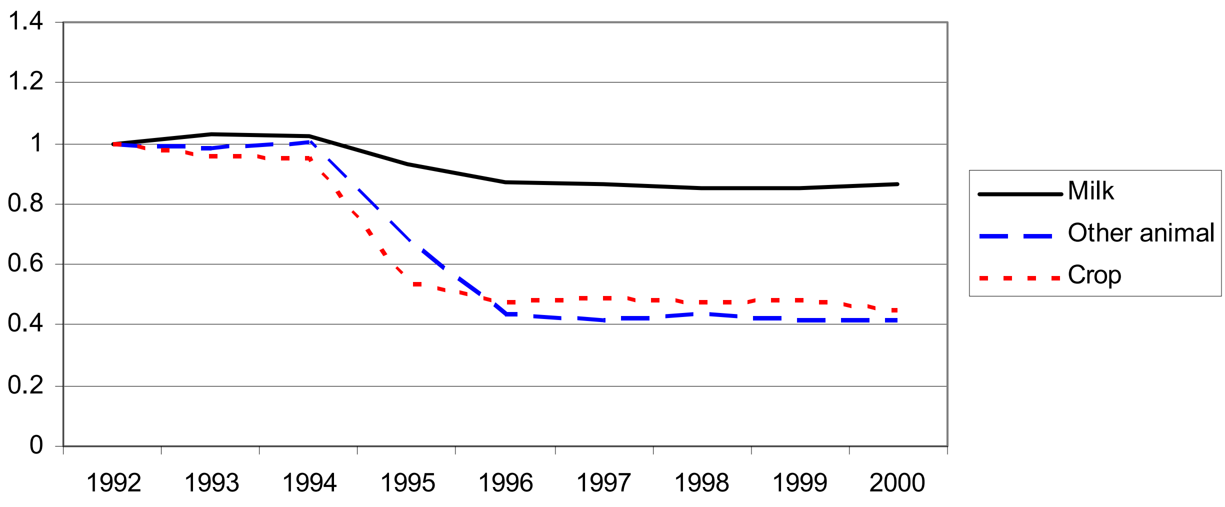

Figure 6. Development of average output prices (1992 = 1). (*Source: Statistics Finland*)

9.2 Data

Our balanced panel data includes all types of farms varying from specialized animal production and crop farms to more conventional farms engaged both in animal husbandry and crop farming. A typical farm produces several outputs including milk, meat and/or several types of plant products using a number of different inputs, but not all farms produce all products. For example, less than half of the sample farms produce milk. Thus, it was necessary to aggregate specific inputs and outputs into larger categories to avoid dimensionality problems in the empirical efficiency analysis.

Table 1 lists the input and output categories and presents the descriptive statistics of costs and revenues in 1992, 1996 and 2000. Four observations are worth noting. First, Finnish farms are typically small. Table 1 shows that the mean arable land area of sample farms in 1992 was only 37 hectares, the number of animal units being 31. Since 1992 output per farm has increased; the maxima and the standard deviations have also increased considerably. Second, the average crop output of farms did not change much, despite the drastic changes in price relations (see Table 1). Third, the capital stock per farm has increased more rapidly than output. Considerable public investment aids boosted investment activities. In year 2000, the capital stock of machinery was in real terms 85 percent and that of buildings 70 percent higher than in 1992. Capital intensity increased since the annual growth of labour input was only about 5 percent during the time period under study. Although the total labour input in agriculture has decreased because of the decreasing number of farms, simultaneously, the farms increasing in size have had to increase their total labour input, but relatively less than other inputs. Fourth, use of other inputs per farm has increased during the research period. For example, fertiliser use on sample farms has increased 25 percent together with the increase in arable land area. However, the use of fertilisers per hectare has decreased. Although the intensity of crop production seems to have decreased, relative price changes have contributed to the intensification of production with respect to purchased feed, for example, in milk production.

Table 1. Inputs and outputs. Descriptive statistics of revenues and costs (in €): 1992, 1996 and 2000.

	1992		1996		2000	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Outputs:						
Milk	23354	25628	26732	30291	32403	40987
Other animal products	18331	30707	19984	40541	24710	55487
Crops	11272	18076	11109	18623	14311	21146
Inputs:						
Labour	27057	11283	29420	13147	28634	14415
Animal units	1242	1334	1385	1653	1500	2062
Land	4820	2842	5496	3223	6401	3711
Machinery	28917	20017	33749	21620	54025	39749
Buildings	25843	29903	29451	31168	42117	52952
Fertiliser	3785	2409	4552	3114	4742	3924
Energy	5411	3146	5933	3615	4802	3443
Purchased feed	9879	13487	10118	14195	14455	22899
Materials	10370	6280	12964	8361	16854	12225

9.3 The Fisher indices

Calculation of the Fisher quantity indices requires full price and quantity data on all inputs and outputs. Unfortunately, both prices and quantities were documented only for labour and land inputs, and for milk output. For other input and output categories only cost and revenue data are available; a common feature in farm-level production data. Since all farms are relatively small and their market power is low, the farms are assumed to take the prevailing market prices for inputs and outputs as given. All farms are assumed to face the same market prices for specific inputs and outputs (i.e., the law of one price is assumed to hold).[3] Changes in market prices over time were measured by price indices documented by Statistics Finland. For aggregated input and output categories (such as crops), farm-specific Fisher price indices were constructed from more specific price indices (e.g., indices for wheat, barley, oats, and so forth) and the farm-level quantity data. For input and output variables with missing quantity data, implicit quantity indices were calculated by dividing the costs or revenues by the respective price indices. These implicit quantities capture possible quality differences associated with higher returns or costs due to higher prices in quantities.

Given thus constructed farm-specific quantity and price data, the Fisher ideal output and input quantity indices could now be calculated. The total factor productivity index was subsequently obtained as the ratio of the two. Figure 7 presents cumulative cost-share weighted averages of Fisher input (F_x) and output (F_y) quantity indices and the Fisher index (F).

[3] Note that the same assumption has to be made when cost and/or revenue data are used as proxies for respective input and output quantities, as is often done in the Malmquist index approach. For more detailed discussion on the one price assumption, see Cross and Färe (2003) and Kuosmanen et al. (2005).

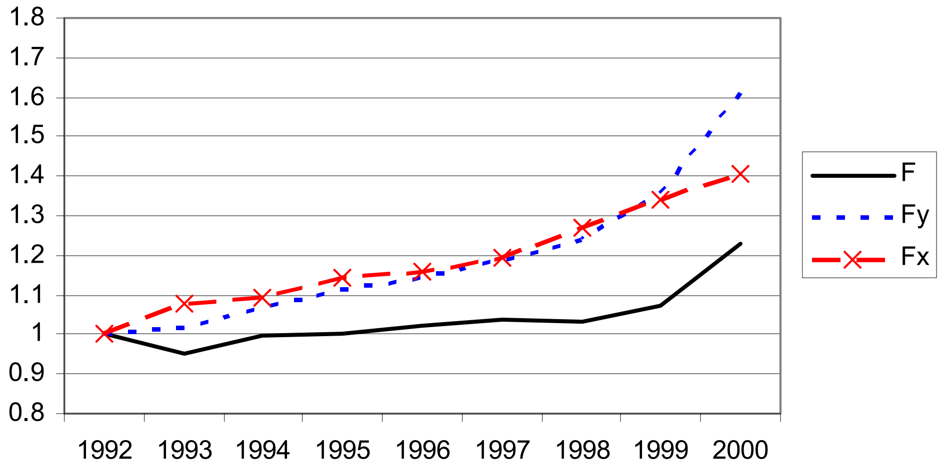

Figure 7. Cumulative Fisher index (F), Fisher input (F_x) and output (F_y) quantity indices.

The aggregate input has grown steadily over the whole research period. The output growth has followed the input growth but there have been two major deviations: in 1992/93 the output growth deviated downwards and in 1999/2000 upwards from the input growth. These deviations can also be observed in the Fisher index, which at first falls but then recovers, although very slowly until 1998. After 1998 the growth rate has increased being 14.6 percentage units from 1999 to 2000.

9.5 Decomposition of the Fisher TFP index

To implement the decomposition of Section 8, we utilise the nonparametric Data Envelopment Analysis (DEA) approach for estimating the production technology. We resort to the most standard convex, variable returns to scale benchmark technology. The values of distance functions as well as cost and revenue functions relative to DEA technology are obtained as optimal solution to specific Linear Programming (LP) problems. Detailed descriptions of these LP problems are presented e.g. in Färe et al. (1985, 1994b). The only non-standard measure, the profitability function, was calculated by simply enumerating through all observed combinations of price and quantity vectors to calculate the full profitability distribution at all observed prices. To make this critical profitability measure less sensitive to data errors and outliers, we used the 95 percentile of the profitability distribution as our empirical estimator for the profitability function.

9.5.1 Technical efficiency and technical change components

Figure 8 describes the development of technical efficiency and technical change components in cumulative fashion. The weighted annual averages of the input and output oriented sub-components of technical efficiency change are presented separately. Their geometric mean (the solid thick line) is the overall technical efficiency measure of our decomposition. Technical efficiency has grown slowly until 1998. Since then the growth of technical efficiency has accelerated considerably. Figure 8 shows that output oriented technical efficiency indices indicate approximately twice as high growth as input oriented ones. On the other hand, their geometric mean is close to the input oriented index. This is probably due to the fact that output orientation yields a larger variation in individual efficiencies.

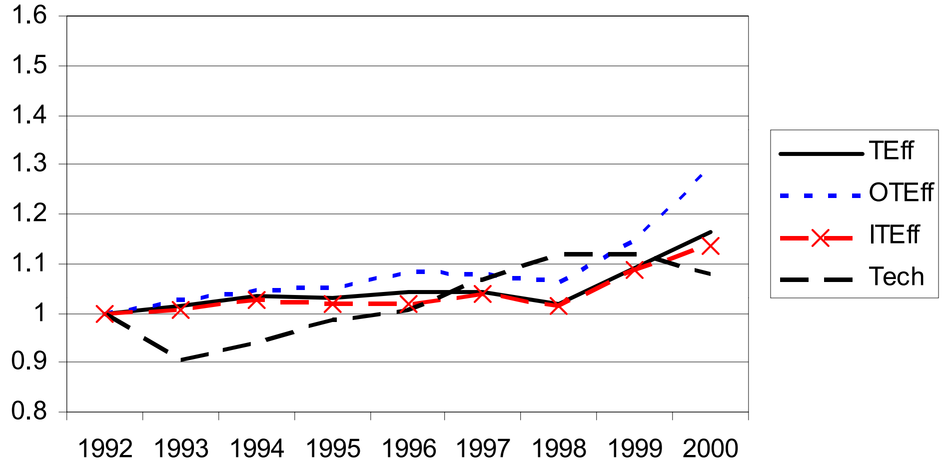

Figure 8. Cumulative input oriented (ITEff) and output oriented (OTEff) technical efficiency change, their geometric mean (TEff), and the technical change (Tech) components.

The technical change component shows a surprisingly steady growth path. The black broken line of Figure 8 represents the cumulative weighted average of farms' technical change components. This figure shows a considerable technical regress in the beginning of the sample period, which can be attributed to extensive set-aside programs and adverse weather conditions in 1993 and 1994. After these first adverse years, technological progress is rapid until 1998, showing impressive average cumulative growth of 25 percent in a five-year period. However, the progress slows down in 1999 and turns to regress in 2000, in spite of the fact that overall productivity growth is at its highest in that year. As we noted earlier, there was almost no change in technical efficiency but since 1998 technical efficiency improves 15 percentage units. Thus, average farms are getting closer to the frontier but partially due to the regress on the most profitable farms.

9.5.2 Scale efficiency component

Figure 9 presents the scale efficiency components analogous to the previous figure. The scale efficiency component shows large annual fluctuations, showing an increasing trend before the EU accession in 1995 and until 1996, then decline until 1999, and a sudden 20 percentage points jump in 2000. The fluctuations are especially large in the output-oriented measure of scale efficiency. This is surprising because farms' production scale exhibited relatively steady growth throughout the study period (compare with Figure 7), while the composition of input-output mix changed much more dramatically. The great fluctuations in scale efficiency must therefore arise from rapid changes in the most profitable scale size itself. It seems that the expansion of scale size was profitable in the market conditions prevailing before the EU accession, but the continuing expansion after 1995 was at odds with the deteriorated market conditions. Note that the input oriented component showed greater variation prior to 1995, while the output oriented sub-component fluctuates drastically after 1995.

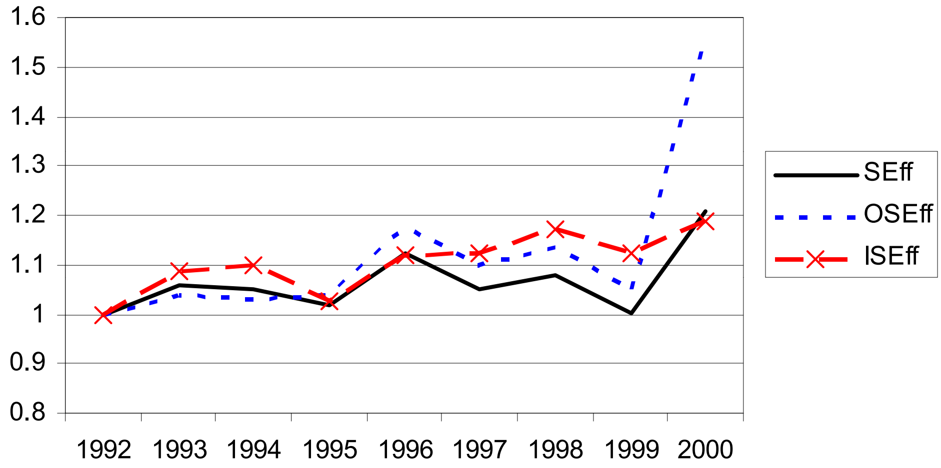

Figure 9. Cumulative input oriented (ISEff) and output oriented (OSEff) scale efficiency change and their geometric mean (SEff).

9.5.3 Allocative efficiency and market strength components

The decomposition of the Fisher index also includes allocative efficiency change. Analogous to Figure 8, Figure 9 presents the weighted annual averages of the input and output oriented sub-components and their geometric mean. This figure shows that allocative efficiency has on average improved less than 10 percent during the research period. At the end of the research period output oriented allocative efficiency has increased relatively more than input oriented allocative efficiency. A change in development can be observed at the time of the EU accession, and it is probably related to different incentives and constraints set by the changed agricultural policy.

Before that point the input oriented allocative efficiency improved faster than the efficiency in output orientation. Instead, after the EU accession output oriented allocative efficiency has shown faster growth than input oriented efficiency.

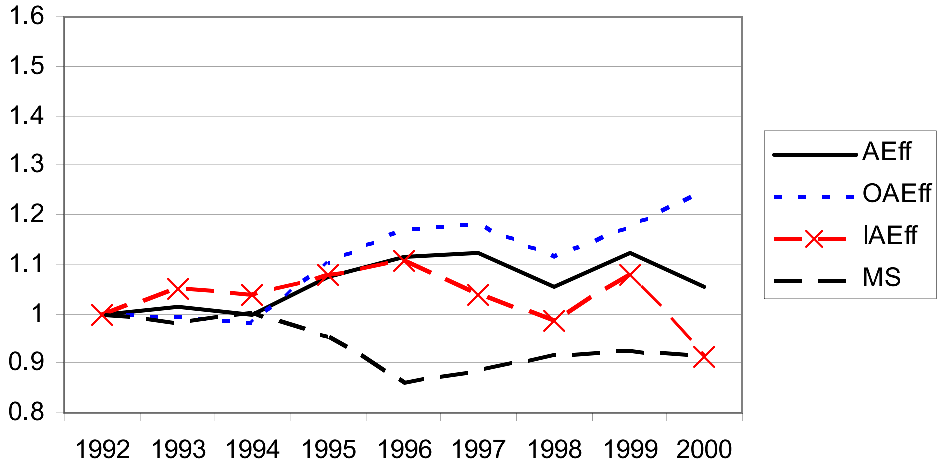

Figure 10. Cumulative input oriented (AECi) and output oriented (AECo) technical efficiency change and their geometric mean (AEC), and market strength change (MS).

The weighted average of the market strength components is represented by the broken line in Figure 10. There was little change in the market strength in the beginning of the study period. In 1995-1996, the first two years after the EU accession, the market strength dropped by almost 15 percentage points, reflecting the adverse effect of the price changes to farms' potential for producing aggregate output. After this sharp downfall, market strength recovered slowly, but never reached the original price conditions.

It is interesting to note that overall Fisher TFP index showed slow but steady growth throughout the study period, supported by both technical efficiency and technical change components. The effect of dramatic prices changes was revealed most clearly by the market strength component. The negative effect of the market strength component was countered at least to some extent by improved allocative efficiency.

9.4 Comparison of the Fisher and Malmquist TFP indices

For comparison, we also calculated the Malmquist productivity index of Färe et al. (1994c) using the same DEA reference technology and the same input and output quantity data as in the Fisher indices. While the Malmquist index does not require any price data, the assumptions of price taking behaviour and the law of one price are still required for recovering quantity data from the observed cost/revenue aggregates. Thus, the data requirements do not favour either approach in this application.

Figure 11 illustrates the average productivity growth measured by the Malmquist index (denoted by M100; the thin broken line) and the Fisher index (F100, solid black line). We observe that the weighted average Malmquist index shows considerably higher cumulative productivity growth than the Fisher ideal TFP index: the Malmquist index indicates an average productivity growth of 68.7 percent during the research period of eight years, which is three times higher than the average growth according to the Fisher index. Such a productivity growth seems incredibly high in the context of agricultural production.

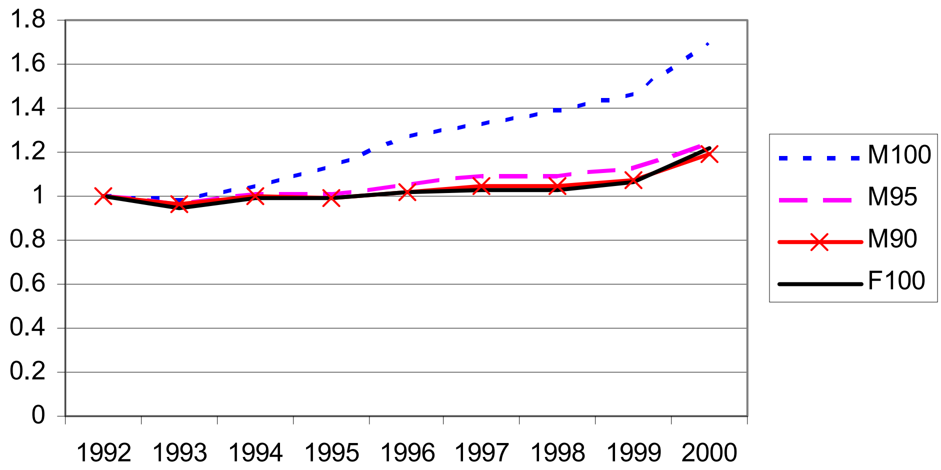

Figure 11. Cumulative Fisher index (F) versus Malmquist indices (M). The number after capital letter indicates the percentage of observations taken into account.

To investigate the distribution of productivity growth figures across farms, we clipped the extreme tails of the distribution, and plotted in Figure 12 the average values of the Malmquist productivity index for farms falling within the (2.5, 97.5) and (5, 95) percentile ranges (denoted by M95 and M90 respectively). After clipping the tails, the average Malmquist index follows closely the path of the average Fisher TFP index. This suggests that for the large majority of farms the two index formulae offer relatively similar results. For a small percentage of farms in our sample, the Malmquist index produces considerably higher index values than the Fisher TFP index. In our interpretation, the Fisher TFP index proves a more stable and robust index number formula of the two, providing more credible results in this application. Recall that the Fisher TFP index for an individual farm only depends on its own performance, while the Malmquist index compares the performance of each farm relative to the farms defining the best practice frontier.

We next compare the results component-wise to see where the differences between the Malmquist and Fisher TFP indices may occur. We focus on the most standard decomposition of the Malmquist index by Färe et al. (1994c), specified in detail in Appendix 1.

Firstly, technical efficiency change components of the Malmquist index are equal to their input- and output-oriented counterparts in the Fisher index. Applications of the Malmquist index typically assume either input or output orientation, while our Fisher decomposition incorporates both orientations applying the geometric mean of the two. Secondly, allocative efficiency change and the market strength components appear in our decomposition, but are not included in the Malmquist index decompositions. Thus, we can only compare the scale efficiency and technical change components empirically.

Figure 12 presents a comparison between the input oriented scale efficiency components of the Malmquist (MISEff) and the Fisher index (ISEff). As Figure 12 shows, the scale components of the Fisher index are typically larger than the corresponding Malmquist component. Moreover, the Fisher components exhibit more variation over time.

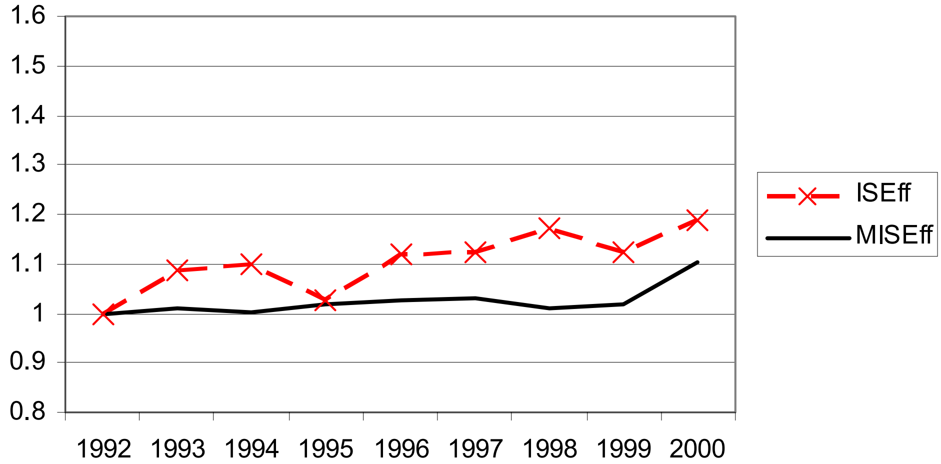

Figure 12. Input oriented scale efficiency change component of the Malmquist index (MISEff) and inputs oriented scale efficiency component (ISEff) and weighted scale efficiency component (SEff) of the Fisher index.

Figure 13 compares the technical change components of the Malmquist (MTech100: the thin broken line) and Fisher indices (Tech100: the solid black line). The Malmquist component suggests considerably faster technical development than the Fisher component, with the cumulative difference exceeding 20 percentage points at the end of the research period. Thus, the difference in the technical change components explains about half of the difference in the overall index. As in the case of the overall index (Figure 12), we also considered the technical change component of the mid-95 percentile of the farm distribution clipping away 5 percent of the extreme observations. The resulting average is illustrated by the broken line with diamonds (MTech95). Like in the case of the overall index, the majority of farms exhibited similar technical progress according to both Fisher and Malmquist components.

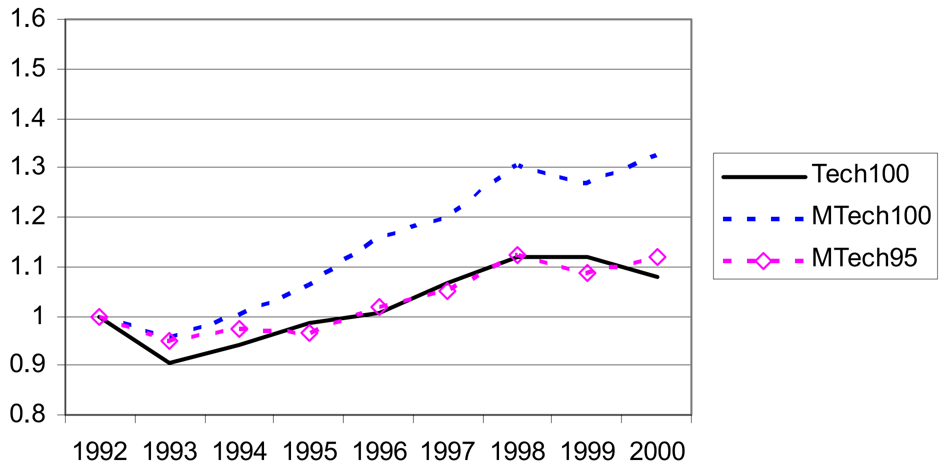

Figure 13. Technical change components of the Malmquist and the Fisher index.

10. Concluding remarks

We have shown that the Fisher ideal TFP index can be decomposed in the similar fashion to the numerous decompositions of the Malmquist index. We believe this decomposition enhances both our understanding about the different sources of productivity growth captured by the Fisher index and the position of the Fisher index as a useful index number formula for productivity analysis. The decomposition is readily implementable in empirical applications using the standard parametric or nonparametric frontier estimation techniques.

The proposed decomposition includes two new components – allocative efficiency and market strength – which have not been accounted for explicitly in the productivity decompositions before. The allocative efficiency component is based on the classic Farrell allocative efficiency measure, which indicates the nonradial efficiency improvement potential in production of quality-adjusted aggregate output. The market strength component captures changes in the market conditions, transmitted through changes in relative prices, which influence the firms' capability to produce the quality-adjusted aggregate output. While both these components represent economic rather than technical efficiency, we find it meaningful to account for these qualitative components in the productivity index (which is essentially a quantity index), because usually it is difficult to draw a sharp distinction between the technical input-output quantities and the associated economic costs and revenues. For example, the capital input is almost always measured by some kind of a cost aggregate.

The proposed decomposition may also provide useful insights for other decompositions. While the existing Malmquist decompositions usually assume either input or output orientation, our decomposition builds on geometric means of both input and output oriented sub-components. Of course, the same approach could be adapted to the Malmquist decompositions in a straightforward manner. On the other hand, we employed the dual representation of the technology, the profitability function, for our measure of technical change. By duality theory, it is equally legitimate to measure technical change by means of monetary profitability data as with more traditional input/output technology distance functions.

The usefulness of the new decomposition was illustrated by an empirical application where productivity developments in a large sample of Finnish farms were studied over the period 1992-2000. This period is interesting because of the drastic price changes due to Finland's EU accession in 1995, which increased international competition and led to major revisions in the agricultural policy. Despite major restructuring of production, the average productivity growth was found to be surprisingly stable, on the average about 2.5 percent per annum. Technical change and technical efficiency followed a similar stable growth path. The impacts of the EU accession presented themselves most clearly in the market strength component, which showed a sudden downfall in 1994-1996. This negative market strength effect was offset most importantly by improved allocative efficiency. The scale efficiency component showed relatively large annual fluctuations and proved more difficult to interpret.

We hope this decomposition might inspire debate about the relative merits of different index number formulae used in productivity measurement. We believe there is no single superior index number for all empirical studies, but different index formulae may be appropriate depending on the purposes of the analysis and the interpretation of productivity. Our decomposition suggests that it is possible to extend the approach of Färe et al. (1994a,c) from the domain of Malmquist type indices towards the more classic, price-weighted indices. Further research could consider other important indices such as the widely used Törnqvist productivity index.

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http://www.sls.wageningen-ur.nl/enr/staff/kuosmanen/program1/

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Appendix 1: The Malmquist index and its decomposition

For brevity, we focus on the input oriented Malmquist index of Färe et al. (1994b). Let $\bar{D}_x^t(\mathbf{x}^t, \mathbf{y}^t)$ denote the input distance function defined relative to the constant returns to scale reference technology (i.e., the conical hull $\lambda T^t, \lambda \geq 0$ rather than T^t). Given this CRS input distance function, the Malmquist index is defined as

$$M_x(\mathbf{x}^{0,1}, \mathbf{y}^{0,1}) \equiv \left(\frac{\bar{D}_x^0(\mathbf{x}^1, \mathbf{y}^1)}{\bar{D}_x^0(\mathbf{x}^0, \mathbf{y}^0)} \cdot \frac{\bar{D}_x^1(\mathbf{x}^1, \mathbf{y}^1)}{\bar{D}_x^1(\mathbf{x}^0, \mathbf{y}^0)} \right)^{1/2}.$$

The Malmquist index decomposes into components of efficiency change (Eff) and technical change (Tech) as

$$M_x(\mathbf{x}^{0,1}, \mathbf{y}^{0,1}) = Eff(\mathbf{x}^{0,1}, \mathbf{y}^{0,1}) \cdot Tech(\mathbf{x}^{0,1}, \mathbf{y}^{0,1}),$$

where

$$Eff(\mathbf{x}^{0,1}, \mathbf{y}^{0,1}) = \frac{\bar{D}_x^1(\mathbf{x}^1, \mathbf{y}^1)}{\bar{D}_x^0(\mathbf{x}^0, \mathbf{y}^0)}$$

and

$$Tech(\mathbf{x}^{0,1}, \mathbf{y}^{0,1}) \equiv \left(\frac{\bar{D}_x^0(\mathbf{x}^1, \mathbf{y}^1)}{\bar{D}_x^1(\mathbf{x}^1, \mathbf{y}^1)} \cdot \frac{\bar{D}_x^0(\mathbf{x}^0, \mathbf{y}^0)}{\bar{D}_x^1(\mathbf{x}^0, \mathbf{y}^0)} \right)^{1/2}.$$

Efficiency change can be further decomposed into 'pure' technical efficiency change (OEff) and scale efficiency change (SEff) as

$$Eff(\mathbf{x}^{0,1}, \mathbf{y}^{0,1}) = OEff(\mathbf{x}^{0,1}, \mathbf{y}^{0,1}) \cdot SEff(\mathbf{x}^{0,1}, \mathbf{y}^{0,1})$$

where

$$OEff(\mathbf{x}^{0,1}, \mathbf{y}^{0,1}) = \frac{D_x^1(\mathbf{x}^1, \mathbf{y}^1)}{D_x^0(\mathbf{x}^0, \mathbf{y}^0)}$$

and

$$SEff = \left(\frac{\bar{D}_x^1(\mathbf{x}^1, \mathbf{y}^1)}{D_x^1(\mathbf{x}^1, \mathbf{y}^1)} \cdot \frac{\bar{D}_x^0(\mathbf{x}^0, \mathbf{y}^0)}{D_x^0(\mathbf{x}^0, \mathbf{y}^0)} \right)^{1/2}.$$

Appendix 2: Proof of Theorem 1

This is a variant of the duality result on profit functions (e.g., Färe and Primont, 1995: Proposition 6.1.4). The result follows directly from Lemma 1 in Kuosmanen et al. (2004), which shows that if T^t satisfies free disposability, convexity, and constant returns to scale, then the input distance function can be expressed as

(i) $$D_x^t(\mathbf{x},\mathbf{y}) = \max_{(\mathbf{w},\mathbf{p})\in\mathbb{R}_+^{r+s}} \left\{ \frac{\mathbf{p}\cdot\mathbf{y}}{\mathbf{w}\cdot\mathbf{x}} \middle| \frac{\mathbf{p}\cdot\mathbf{y}'}{\mathbf{w}\cdot\mathbf{x}'} \le 1 \ \forall(\mathbf{x}',\mathbf{y}')\in\mathbb{R}_+^{r+s} \right\}.$$

By the definition of function ρ^t, this further implies that

(ii) $$D_x^t(\mathbf{x},\mathbf{y}) = \max_{(\mathbf{w},\mathbf{p})\in\mathbb{R}_+^{r+s}} \left\{ \frac{\mathbf{p}\cdot\mathbf{y}}{\mathbf{w}\cdot\mathbf{x}} \middle| \rho^t(\mathbf{w},\mathbf{p}) \le 1 \right\}.$$

Furthermore, we know that under weak disposability of inputs

(iii) $$T^t = \left\{ (\mathbf{x},\mathbf{y})\in\mathbb{R}_+^{r+s} \middle| D_{\mathbf{x}}^t(\mathbf{x},\mathbf{y}) \le 1 \right\}.$$

Substituting (ii) in (iii), we have that

(iv) $$T^t = \left\{ (\mathbf{x},\mathbf{y})\in\mathbb{R}_+^{r+s} \middle| \max_{(\mathbf{w},\mathbf{p})\in\mathbb{R}_+^{r+s}} \left\{ \frac{\mathbf{p}\cdot\mathbf{y}}{\mathbf{w}\cdot\mathbf{x}} \middle| \rho^t(\mathbf{w},\mathbf{p}) \le 1 \right\} \le 1 \right\}$$

(v) $$= \left\{ (\mathbf{x},\mathbf{y})\in\mathbb{R}_+^{r+s} \middle| \max_{(\mathbf{w},\mathbf{p})\in\mathbb{R}_+^{r+s}} \left[\left(\frac{\mathbf{p}\cdot\mathbf{y}}{\mathbf{w}\cdot\mathbf{x}} \right) \Big/ \rho^t(\mathbf{w},\mathbf{p}) \right] \le 1 \right\}$$

(vi) $$= \left\{ (\mathbf{x},\mathbf{y})\in\mathbb{R}_+^{r+s} \middle| \frac{\mathbf{p}\cdot\mathbf{y}}{\mathbf{w}\cdot\mathbf{x}} \le \rho^t(\mathbf{w},\mathbf{p}) \ \forall(\mathbf{w},\mathbf{p})\in\mathbb{R}_+^{r+s} \right\}. \square$$

Appendix 3: Proof of Theorem 2

We start from the definition of the Fisher TFP index

(i) $$F_{TFP}(\mathbf{p}^{0,1},\mathbf{w}^{0,1},\mathbf{y}^{0,1},\mathbf{x}^{0,1}) \equiv \frac{\left(\dfrac{\mathbf{p}^0\mathbf{y}^1}{\mathbf{p}^0\mathbf{y}^0}\cdot\dfrac{\mathbf{p}^1\mathbf{y}^1}{\mathbf{p}^1\mathbf{y}^0}\right)^{1/2}}{\left(\dfrac{\mathbf{w}^0\mathbf{x}^1}{\mathbf{w}^0\mathbf{x}^0}\cdot\dfrac{\mathbf{w}^1\mathbf{x}^1}{\mathbf{w}^1\mathbf{x}^0}\right)^{1/2}},$$

and modify it in order to isolate the specific components. (To simplify the expressions, the arguments of F_{TFP} will be suppressed.) As the first step, we divide the revenue terms by the corresponding revenue functions and the cost terms by the corresponding cost functions as

(ii) $$F_{TFP} = \frac{\left(\dfrac{\left(\dfrac{\mathbf{p}^0\cdot\mathbf{y}^1}{R^0(\mathbf{x}^0,\mathbf{p}^0)}\right)}{\left(\dfrac{\mathbf{p}^0\cdot\mathbf{y}^0}{R^0(\mathbf{x}^0,\mathbf{p}^0)}\right)}\cdot\dfrac{\left(\dfrac{\mathbf{p}^1\cdot\mathbf{y}^1}{R^1(\mathbf{x}^1,\mathbf{p}^1)}\right)}{\left(\dfrac{\mathbf{p}^1\cdot\mathbf{y}^0}{R^1(\mathbf{x}^1,\mathbf{p}^1)}\right)}\right)^{1/2}}{\left(\dfrac{\left(\dfrac{\mathbf{w}^0\cdot\mathbf{x}^1}{C^0(\mathbf{w}^0,\mathbf{y}^0)}\right)}{\left(\dfrac{\mathbf{w}^0\cdot\mathbf{x}^0}{C^0(\mathbf{w}^0,\mathbf{y}^0)}\right)}\cdot\dfrac{\left(\dfrac{\mathbf{w}^1\cdot\mathbf{x}^1}{C^1(\mathbf{w}^1,\mathbf{y}^1)}\right)}{\left(\dfrac{\mathbf{w}^1\cdot\mathbf{x}^0}{C^1(\mathbf{w}^1,\mathbf{y}^1)}\right)}\right)^{1/2}}.$$

We then substitute the numerators of the second-level ratios to obtain

(iii) $$F_{TFP} = \frac{\left(\dfrac{\left(\dfrac{\mathbf{p}^1\cdot\mathbf{y}^1}{R^1(\mathbf{x}^1,\mathbf{p}^1)}\right)}{\left(\dfrac{\mathbf{p}^0\cdot\mathbf{y}^0}{R^0(\mathbf{x}^0,\mathbf{p}^0)}\right)}\cdot\dfrac{\left(\dfrac{\mathbf{p}^0\cdot\mathbf{y}^1}{R^0(\mathbf{x}^0,\mathbf{p}^0)}\right)}{\left(\dfrac{\mathbf{p}^1\cdot\mathbf{y}^0}{R^1(\mathbf{x}^1,\mathbf{p}^1)}\right)}\right)^{1/2}}{\left(\dfrac{\left(\dfrac{\mathbf{w}^1\cdot\mathbf{x}^1}{C^1(\mathbf{w}^1,\mathbf{y}^1)}\right)}{\left(\dfrac{\mathbf{w}^0\cdot\mathbf{x}^0}{C^0(\mathbf{w}^0,\mathbf{y}^0)}\right)}\cdot\dfrac{\left(\dfrac{\mathbf{w}^0\cdot\mathbf{x}^1}{C^0(\mathbf{w}^0,\mathbf{y}^0)}\right)}{\left(\dfrac{\mathbf{w}^1\cdot\mathbf{x}^0}{C^1(\mathbf{w}^1,\mathbf{y}^1)}\right)}\right)^{1/2}}.$$

Next, we further decompose the left-hand side ratios by introducing revenues and costs of the technically efficient reference points as

(iv) $$F_{TFP} = \frac{\left(\frac{\left(\frac{\mathbf{p}^1\cdot\mathbf{y}^1}{\mathbf{p}^1\cdot(\mathbf{y}^1/D_o^1(\mathbf{x}^1,\mathbf{y}^1))}\cdot\frac{\mathbf{p}^1\cdot(\mathbf{y}^1/D_o^1(\mathbf{x}^1,\mathbf{y}^1))}{R^1(\mathbf{x}^1,\mathbf{p}^1)}\right)}{\left(\frac{\mathbf{p}^0\cdot\mathbf{y}^0}{\mathbf{p}^0\cdot(\mathbf{y}^0/D_o^0(\mathbf{x}^0,\mathbf{y}^0))}\cdot\frac{\mathbf{p}^0\cdot(\mathbf{y}^0/D_o^0(\mathbf{x}^0,\mathbf{y}^0))}{R^0(\mathbf{x}^0,\mathbf{p}^0)}\right)}\cdot\left(\frac{\mathbf{p}^0\cdot\mathbf{y}^1}{\mathbf{p}^1\cdot\mathbf{y}^0}\cdot\frac{R^1(\mathbf{x}^1,\mathbf{p}^1)}{R^0(\mathbf{x}^0,\mathbf{p}^0)}\right)\right)^{1/2}}{\left(\frac{\left(\frac{\mathbf{w}^1\cdot\mathbf{x}^1}{\mathbf{w}^1\cdot(D_i^1(\mathbf{x}^1,\mathbf{y}^1)\mathbf{x}^0)}\cdot\frac{\mathbf{w}^1\cdot(D_i^1(\mathbf{x}^1,\mathbf{y}^1)\mathbf{x}^0)}{C^1(\mathbf{w}^1,\mathbf{y}^1)}\right)}{\left(\frac{\mathbf{w}^0\cdot\mathbf{x}^0}{\mathbf{w}^0\cdot(D_i^0(\mathbf{x}^0,\mathbf{y}^1)\mathbf{x}^0)}\cdot\frac{\mathbf{w}^0\cdot(D_i^0(\mathbf{x}^0,\mathbf{y}^1)\mathbf{x}^0)}{C^0(\mathbf{w}^0,\mathbf{y}^0)}\right)}\cdot\left(\frac{\mathbf{w}^0\cdot\mathbf{x}^1}{\mathbf{w}^1\cdot\mathbf{x}^0}\cdot\frac{C^1(\mathbf{w}^1,\mathbf{y}^1)}{C^0(\mathbf{w}^0,\mathbf{y}^0)}\right)\right)^{1/2}}.$$

Reorganizing the left-hand components of (iv), we

(v) $F_{TFP} =$

$$\left(\frac{D_o^1(\mathbf{x}^1,\mathbf{y}^1)}{D_o^0(\mathbf{x}^0,\mathbf{y}^0)}\cdot\frac{D_i^1(\mathbf{x}^1,\mathbf{y}^1)}{D_i^0(\mathbf{x}^0,\mathbf{y}^0)}\right)^{1/2}\cdot\left(\frac{\left(\frac{\mathbf{p}^1(\mathbf{y}^1/D_o^1(\mathbf{x}^1,\mathbf{y}^1))}{R^1(\mathbf{x}^1,\mathbf{p}^1)}\right)}{\left(\frac{\mathbf{p}^0(\mathbf{y}^0/D_o^0(\mathbf{x}^0,\mathbf{y}^0))}{R^0(\mathbf{x}^0,\mathbf{p}^0)}\right)}\cdot\frac{\left(\frac{C^1(\mathbf{w}^1,\mathbf{y}^1)}{\mathbf{w}^1(D_i^1(\mathbf{x}^1,\mathbf{y}^1)\mathbf{x}^1)}\right)}{\left(\frac{C^0(\mathbf{w}^0,\mathbf{y}^0)}{\mathbf{w}^0(D_i^0(\mathbf{x}^0,\mathbf{y}^0)\mathbf{x}^0)}\right)}\right)^{1/2}\cdot\frac{\left(\frac{\mathbf{p}^0\mathbf{y}^1}{\mathbf{p}^1\mathbf{y}^0}\cdot\frac{R^1(\mathbf{x}^1,\mathbf{p}^1)}{R^0(\mathbf{x}^0,\mathbf{p}^0)}\right)^{1/2}}{\left(\frac{\mathbf{w}^0\mathbf{x}^1}{\mathbf{w}^1\mathbf{x}^0}\cdot\frac{C^1(\mathbf{w}^1,\mathbf{y}^1)}{C^0(\mathbf{w}^0,\mathbf{y}^0)}\right)^{1/2}}$$

The first component of (v) is the change in technical efficiency ($\Delta TEff$), the second component is the change in allocative efficiency ($\Delta AEff$), so we rewrite (v) as

(vi) $$F_{TFP} = \Delta TEff \times \Delta AEff \times \frac{\left(\frac{\mathbf{p}^0\mathbf{y}^1}{\mathbf{p}^1\mathbf{y}^0}\cdot\frac{R^1(\mathbf{x}^1,\mathbf{p}^1)}{R^0(\mathbf{x}^0,\mathbf{p}^0)}\right)^{1/2}}{\left(\frac{\mathbf{w}^0\mathbf{x}^1}{\mathbf{w}^1\mathbf{x}^0}\cdot\frac{C^1(\mathbf{w}^1,\mathbf{y}^1)}{C^0(\mathbf{w}^0,\mathbf{y}^0)}\right)^{1/2}}.$$

Next, we turn attention to the remaining undecomposed element. Firstly, we expand the revenue and cost ratios as

(vii) $$F_{TFP} = \Delta TEff \times \Delta AEff \times \frac{\left(\frac{\mathbf{p}^0\mathbf{y}^0}{\mathbf{p}^1\mathbf{y}^0}\cdot\frac{\mathbf{p}^0\mathbf{y}^1}{\mathbf{p}^1\mathbf{y}^1}\right)^{1/2}}{\left(\frac{\mathbf{w}^0\mathbf{x}^0}{\mathbf{w}^1\mathbf{x}^0}\cdot\frac{\mathbf{w}^0\mathbf{x}^1}{\mathbf{w}^1\mathbf{x}^1}\right)^{1/2}}\cdot\frac{\left(\frac{\mathbf{p}^1\mathbf{y}^1}{\mathbf{p}^0\mathbf{y}^0}\right)^{1/2}}{\left(\frac{\mathbf{w}^1\mathbf{x}^1}{\mathbf{w}^0\mathbf{x}^0}\right)^{1/2}}\cdot\frac{\left(\frac{R^1(\mathbf{x}^1,\mathbf{p}^1)}{R^0(\mathbf{x}^0,\mathbf{p}^0)}\right)^{1/2}}{\left(\frac{C^1(\mathbf{w}^1,\mathbf{y}^1)}{C^0(\mathbf{w}^0,\mathbf{y}^0)}\right)^{1/2}}.$$

The third component is the ratio of the inverse of Fisher output price index and the input price index:

(viii) $$F_{TFP} = \Delta TEff \times \Delta AEff \times \frac{1/F_p}{1/F_w}\cdot\frac{\left(\frac{\mathbf{p}^1\mathbf{y}^1}{\mathbf{p}^0\mathbf{y}^0}\right)^{1/2}}{\left(\frac{\mathbf{w}^1\mathbf{x}^1}{\mathbf{w}^0\mathbf{x}^0}\right)^{1/2}}\cdot\frac{\left(\frac{R^1(\mathbf{x}^1,\mathbf{p}^1)}{R^0(\mathbf{x}^0,\mathbf{p}^0)}\right)^{1/2}}{\left(\frac{C^1(\mathbf{w}^1,\mathbf{y}^1)}{C^0(\mathbf{w}^0,\mathbf{y}^0)}\right)^{1/2}}.$$

We next re-organize the remaining revenue and cost ratios as

(ix) $$F_{TFP} = \Delta TEff \times \Delta AEff \times \frac{\left(\dfrac{\mathbf{p}^1 \cdot \mathbf{y}^1}{C^1(\mathbf{w}^1,\mathbf{y}^1)} \cdot \dfrac{R^1(\mathbf{x}^1,\mathbf{p}^1)}{\mathbf{w}^1\mathbf{x}^1}\right)^{1/2}}{\left(\dfrac{\mathbf{p}^0 \cdot \mathbf{y}^0}{C^0(\mathbf{w}^0,\mathbf{y}^0)} \cdot \dfrac{R^0(\mathbf{x}^0,\mathbf{p}^0)}{\mathbf{w}^0 \cdot \mathbf{x}^0}\right)^{1/2}} \Bigg/ \frac{F_p}{F_w}$$

Further, we introduce the profitability functions as

(x) $$F_{TFP} = \Delta TEff \times \Delta AEff \times \frac{\left(\dfrac{\left(\dfrac{\mathbf{p}^1 \cdot \mathbf{y}^1}{C^1(\mathbf{w}^1,\mathbf{y}^1)}\right)}{\rho^1(\mathbf{w}^1,\mathbf{p}^1)} \cdot \dfrac{\left(\dfrac{R^1(\mathbf{x}^1,\mathbf{p}^1)}{\mathbf{w}^1\mathbf{x}^1}\right)}{\rho^1(\mathbf{w}^1,\mathbf{p}^1)}\right)^{1/2}}{\left(\dfrac{\left(\dfrac{\mathbf{p}^0 \cdot \mathbf{y}^0}{C^0(\mathbf{w}^0,\mathbf{y}^0)}\right)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)} \cdot \dfrac{\left(\dfrac{R^0(\mathbf{x}^0,\mathbf{p}^0)}{\mathbf{w}^0 \cdot \mathbf{x}^0}\right)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)}\right)^{1/2}} \cdot \left[\frac{\rho^1(\mathbf{w}^1,\mathbf{p}^1)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)} \Bigg/ \frac{F_p}{F_w}\right].$$

The third component is the change in scale efficiency ($\Delta SEff$), that is,

(xi) $$F_{TFP} = \Delta TEff \times \Delta AEff \times \Delta SEff \times \left[\frac{\rho^1(\mathbf{w}^1,\mathbf{p}^1)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)} \Bigg/ \frac{F_p}{F_w}\right].$$

Finally, we expand the ratio of profitability functions as

(xii) $$F_{TFP} = \Delta TEff \times \Delta AEff \times \Delta SEff$$

$$\times \left[\left(\frac{\rho^1(\mathbf{w}^0,\mathbf{p}^0)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)} \frac{\rho^1(\mathbf{w}^1,\mathbf{p}^1)}{\rho^0(\mathbf{w}^1,\mathbf{p}^1)}\right)^{1/2} \cdot \left(\frac{\rho^0(\mathbf{w}^1,\mathbf{p}^1)}{\rho^0(\mathbf{w}^0,\mathbf{p}^0)} \frac{\rho^1(\mathbf{w}^1,\mathbf{p}^1)}{\rho^1(\mathbf{w}^0,\mathbf{p}^0)}\right)^{1/2} \Bigg/ \frac{F_p}{F_w}\right]$$

The first component within the brackets is the technology change measure ($\Delta Tech$) and the remaining second component and the price deflator form the market strength component (ΔMS). Thus, reorganizing produces our decomposition of F_{TFP}

(xiii) $$F_{TFP} = \Delta TEff \times \Delta Tech \times \Delta SEff \times \Delta AEff \times \Delta MS. \qquad \square$$