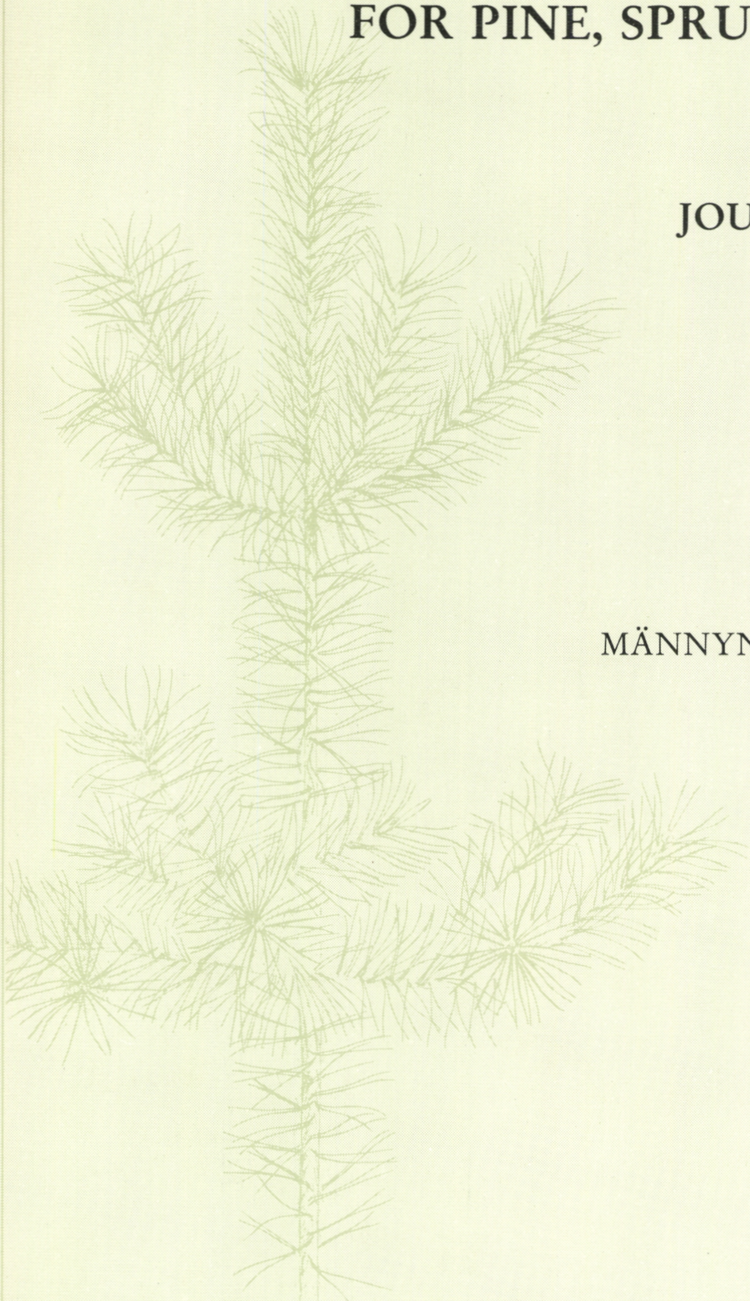
COMMUNICATIONES INSTITUTI FORESTALIS FENNIAE 108

TAPER CURVE AND VOLUME FUNCTIONS FOR PINE, SPRUCE AND BIRCH



JOUKO LAASASENAHO

SELOSTE

MÄNNYN, KUUSEN JA KOIVUN
RUNKOKÄYRÄ- JA
TILAVUUSYHTÄLÖT

HELSINKI 1982

COMMUNICATIONES INSTITUTI FORESTALIS FENNIAE

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Cover (front & back): Scots pine *(Pinus sylvestris* L.) is the most important tree species in Finland. Pine dominated forest covers about 60 per cent of forest land and its total volume is nearly 700 mill. cu.m. The front cover shows a young Scots pine and the back cover a 30-metre-high, 140-year-old tree.

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LAASASENAHO, J. 1982. Taper curve and volume functions for pine, spruce and birch. Seloste: Männyn, kuusen ja koivun runkokäyrä- ja tilavuusyhtälöt. Commun.Inst.For.Fenn. 108:1—74.

Models for determining the taper curve and volume of trees are developed in the study for the most important tree species in Finland: Scots pine, Norway spruce and birch.

The basic population for the study included all the pines, spruces and birches growing in Finland. Sample tree material was collected from about 100 tracts used in the National Forest Inventory, the tracts being randomly selected from different areas. The material consisted of 2 326 pine, 1 864 spruce and 863 birch sample trees.

Two alternative methods of calculating taper curves are presented: A polynomial model, in which the dependent variable is the ratio of stem diameter to the diameter at 20 % of tree height and the independent variable the relative height, and a simultaneous model containing all the diameters measured at different relative heights. The taper curve models are not restricted to any particular diameter measurement but calculation methods suitable for each instance can be constructed from them.

Volume functions were derived and calculated for those tree measurement combinations which are most important from the point of view of practical forestry. Combinations of height and diameters at certain relative heights were also used.

Tutkimuksessa esitetään malleja puun runkokäyrän ja tilavuuden määrittämiseksi Suomen pääpuulajeille: männylle, kuuselle ja koivulle.

Tutkimuksen perusjoukkona olivat kaikki Suomen männyt, kuuset ja koivut. Koepuuaineisto kerättiin noin sadalta valtakunnan metsien inventoinnin lohkolta, jotka valittiin alueittaista satunnaisotantaa käyttäen. Aineisto käsitti 2 326 mäntyä, 1 864 kuusta ja 863 koivua.

Runkokäyrän laskennalle esitetään kaksi vaihtoehtoista menetelmää: Polynomimalli, jossa rungon läpimitan suhde kahdenkymmenen prosentin korkeudella olevaan läpimittaan on selitettävänä muuttujana ja selittäjänä suhteellinen korkeus sekä kaikki mitatut suhteellisen korkeuden läpimitat sisältävä simultaanimalli. Runkokäyrien laskentamenetelmät eivät ole sidottuja tiettyjen mittaustunnusten käyttöön, vaan niistä voidaan rakentaa kuhunkin tilanteeseen sopiva laskentamenetelmä.

Tärkeimmille käytännön mittaustunnusyhdistelmille sekä lisäksi joidenkin suhteellisen korkeuden läpimittojen ja pituuden yhdistelmille johdettiin ja laskettiin tilavuusyhtälöt.

ODC 524+174.7 *Pinus sylvestris*+174.7 *Picea abies* +176.1 *Betula*
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Helsinki 1982. Valtion painatuskeskus

PREFACE

In 1968, the Department of Forest Inventory and Yield and Department of Forest Technology, of the Finnish Forest Research Institute, initiated a research project into the determination of the volume and dry weight of standing trees. The data collection phase was completed by 1972. So far, the material has been utilized in a number of different studies. One of the main aims of the project, the preparation of taper curve and volume functions, is dealt with in the study in hand.

The preparatory work for the study, as well as the planning of the field work, was carried out under the leadership of Prof. Kullervo Kuusela. I have received considerable assistance in the planning phase and later on in discussions with Prof. Aarne Nyyssönen and Dr. Pekka Kilkki.

One field work group was lead for four years by Mr. Tapani Juhe. The role played by Mr. Matti Kujala in carrying out the field work has been almost as great. In addition to the above, Mr. Matti Ahola, Mr. Pentti Savilampi and Mr. Antero Koskinen have acted as field work group leader for either one summer or part of a summer.

The material was handled and the methods initially developed on the IBM-1620 computer and later on the Burroughs 6700 computer, both owned by the University of Helsinki. The final results were calculated on the VAX-11 computer of the Finnish Forest Research Institute.

Mr. Jaakko Heinonen modified the calculation methods for the PDP computer of the Finnish Forest Research Institute, and Mr. Carl-Gustaf Snellman calculated the final equations and programmed the methods on the VAX computer. Mr. Snellman also made a number of valuable suggestions during the development of the methods.

Professors Kuusela and Nyyssönen, Dr. Kilkki, Assistant Professor Simo Poso, Acting Professor Pertti Hari and Mr. Timo Pekkonen have read the manuscript. Dr. Kilkki and Dr. Poso, especially, have given much of their valuable time in refining the final manuscript.

The manuscript was translated into English by Mr. John Derome, typed by Mrs. Anja Leskinen and the figures drawn by Mrs. Kaarina Ridanpää. I would like to thank all those mentioned here, as well as many other people, for their invaluable assistance during the course of this work.

Helsinki, August 1982
Jouko Laasasenaho

CONTENTS

Symbols for tree characteristics

The symbols used to denote tree characteristics are in agreement with those recommended by IUFRO (1959) and are hence the same as those commonly in use in forestry literature.

The most important symbols used in this paper are as follows:

d = diameter over bark at a height of 1,3 m above ground level, i.e. diameter at breast height (DBH), cm

d_u = diameter under bark at breast height, cm.

d_l = diameter over bark at a height of l m, e.g. d_6 = diameter at a height of 6 m, cm.

$d_{,ih}$ = diameter over bark at the relative height i, e.g. $d_{,3h}$ = diameter at a height equivalent to 30 % of the height of the tree from the ground.

h = tree height, m.

$g = \pi(\frac{d}{2})^2$ = cross-sectional area of tree at a height of 1,3 m over bark, cm^2.

v = volume of stemwood with bark from uppermost root collar affecting cutting to the top of the tree, dm^3.

$f = \frac{v}{g \cdot h}$ = breast height form factor, i.e. relationship between volume and a cylinder determined by height and cross-sectional area at breast height.

$f_{,1h} = \frac{v}{h \cdot g_{,1h}}$ = form factor based on a cylinder determined by height and cross-sectional area at 10 %-height, i.e. the true stemwood form factor.

1. INTRODUCTION

Determination of the stem volume of standing tree is the central task of almost all forest mensuration. The development of methods for calculating stem volume has thus been an important sector in the development of mensuration methods. As wood consumption has continued to increase over the years, determination of the extent, structure and increment of forest timber resources, as well as following changes in them, has presupposed the development of more and more accurate methods for measuring and calculating stem volume.

Following the establishment of forestry as a subject in the curriculum of universities in Central Europe at the end of the 18th century, the development of forest mensuration methods also started. First publications dealing with tree measurements appeared during the last century, as can be seen from the extensive forestry textbook "Handbuch der Forstwissenschaft" (Guttenberg 1903), for instance.

When forestry education was introduced in Finland in 1862, information obtained from Germany was applied to forest mensuration teaching. The research and development of measuring techniques for trees in Finland has taken place almost exclusively during the present century (cf. Nyyssönen 1959).

Information about the form of the tree stem provides the basis for the determination of stem volume. The factors affecting the stem form of trees were first studied in Central Europe over a hundred years ago. Of the large number of stem form theories which were put forward at that time, the mechanical stem form theory (Metzger 1893) gave the most impetus to further studies and to the development of practical methods for determining stem volume.

Considerable interest arose in Finland during the first half of the present century into methods for studying stem form (Cajanus 1911). In 1927, Lönnroth published an extensive theoretical exposition on formulae for calculating stem volume, in which functions for taper curves, as well as formulae for stem volume, were derived. Finnish studies on tree form and taper curve have mainly concentrated on Scots pine and birch, and less on Norway spruce (Lakari 1920, Hildén 1926, Lindholm 1934, Lappi-Seppälä 1937 and 1952, Tiihonen 1961 a and b, Kuusela 1965, Kilkki et al. 1978, Kilkki and Varmola 1979 and 1981). The study carried out by Ylinen (1952) on mechanical stem form theory, in which he used a wind tunnel, is perhaps the most comprehensive one so far on this theory.

Owing to the lack of domestic volume tables, tree volume was determined using formulae developed for solids of certain geometrical form as well as the Swedish (Jonson 1918) volume tables for forest mensuration and the timber trade at the beginning of the century. These were used, for instance, in calculating the results of the First National Forest Inventory (1921—24). In the second inventory (1936—38), the volume of the sample trees was calculated from tables based on material collected in Finland. The volume tables published by Ilvessalo in 1947 are still today the officially approved standards for determining the volume of standing trees in commercial and harvesting activities.

The application of computers and statistical methods has opened up a new era in the development and utilization of methods for measuring tree volume. The first volume functions calculated using linear regression analysis on a computer in Finland, were drawn up for larch by Vuokila in 1960. The predicting variables of volume for trees over 10 m tall were d, h and d_6, in other words the same as in Ilvessalo's tables for trees at least 7,5 m tall. The variables for trees under 10 m were d and h. Separate functions were drawn up for volumes with and without bark.

An extensive study on the volume and dry-matter weight of standing trees was started in 1968 at the Finnish Forest Research Institute as a part of studies being carried out in the field of forest inventory. The author of this paper has been responsible for the volume research. The sample tree material has already been utilized in a number of studies (Laasasenaho and Sevola 1971 and 1972, Hakkila et al. 1972, Laasasenaho 1975 b). These studies have dealt with the volume of stems, the value of stems, top form quotients in the logs, branch data for harvesting technology and the dependence of the amount of utilizable wood in the stem on stump height and top cutting diameter.

It was intended to develop a new, more accurate method for volume determination for use in the 6th National Forest Inventory (started in 1971). Since international measuring standards (the reference point for measurements is ground level) were used for the tree measurements carried out in this inventory, a new method was needed for volume calculation; in Ilvessalo's tables the starting point for determining all the measurement characteristics is the uppermost root collar which makes cutting difficult.

Theoretical models which could produce unbiased taper curves were not available when this study was started. Since some experience had already been gained in Finland about volume functions, the preparation of volume functions was taken as the first goal. The equations and the methods used to construct them were developed by the author and have been presented in a mimeographed paper (Laasasenaho 1976) and in Hakkila's publication (1979). The proportions of different timber assortments in accordance with the prevailing scaling system, were obtained using other equations (cf. Laasasenaho and Sevola 1971).

In order to determine a taper curve, the diameter of the stem at one height at least, as well as tree height, are required. If the method used for calculating the taper curve is a flexible one, then diameters measured at any height can be utilized, although the precision of the taper curve is dependent on the position of the diameters along the stem. The methods should cover all instances where taper curves are required. If there is a sufficient number of diameters measured along the stem, the taper curve can be accurately determined using spline functions (Lahtinen and Laasasenaho 1979).

Different methods for calculating taper curves are developed and tested in this study. In addition, the construction of volume function models based on common tree variables (d, d_6 and h), as well as on height and diameters at different relative heights, are presented. The aim of the study is to describe methods for calculating taper curve, as well as volume functions based on the most important tree measurements and to present taper curve and volume equations, which would be applicable in all parts of the country for the three most important tree species in Finland: Scots pine (*Pinus sylvestris*), Norway spruce (*Picea abies*) and birch (*Betula pendula* and *B. pubescens*).

2. STUDY MATERIAL

21. Sampling method and material

The applicability of the functions is greatly dependent on the material from which they have been constructed. Careful planning of the collection of the study material for volume and taper curve functions is especially important in Finland because methods for calculating the stem volume from standing tree measurements have been used during the last few years in determining the stumpage prices and wages for harvesting work in timber lots of over 10 mill. m^3 annually.

As well as being affected by genetical factors, the stem form of trees is also affected by a great number of environmental factors which influence tree growth. The topography and type of soil in Finnish forests are heterogeneous. For these reasons, and also as a result of the different regeneration and silvicultural methods used by large numbers of forest owners, stand compartments are usually small. The tree species composition, density, age and other stand factors in turn affect the variation in stem form within the stand.

The size and representability of the study material become the more important, the smaller is the number of predicting variables included in a function. The greater the number of characteristics on which the models are based, the smaller the likelihood of systematic errors arising as a result of poor representability of the material (see e.g. Kilkki and Siitonen 1975, p. 28). For instance, volume functions based only on tree species and diameter at breast height set high demands on the representability of the material. Functions based on tree species, diameter and height may also give biased results for certain groups of stand, unless the study material represents the trees in question sufficiently well. Neither is a correct picture of the reliability of the functions obtained if the material is not representative. If an upper diameter is included in the model as an additional predicting variable, poor representability of the material will not probably produce any serious errors.

Errors caused by poor representability of the material can thus be reduced by including numerous predicting variables in the functions. Another alternative is to give more weight to those observations which represent a larger portion of the basic population when the equations are being computed. Those tree or stand variables by which such weights can be calculated should therefore be measured when the material is being collected.

The relative precision of the volume estimation models tends to be indifferent of the stem size (see e.g. Cunia 1964). The models should, however, give the relatively most accurate estimates in the case of large trees because such trees constitute the major portion of the volume and value of the growing stock.

Accordingly, an important criterion in selecting the sample trees should be the size of the tree. Furthermore, costs and the practical arrangements involved place limits on research work directed at a large population. Thus, some sort of compromise has to be made as regards the representability of the material.

In a number of similar studies carried out during the last few years, the material has been collected using sampling by d and h-classes (Päivinen 1978, Kilkki and Varmola 1979). The material therefore includes trees of considerable variation in form, the total number of sample trees remaining within reasonable limits. However, the representability of the material suffers and it is not possible to determine the accuracy of the method for the basic population.

In the practical arrangement of the sampling it was decided to utilize the tracts of the National Forest Inventory (NFI). In order to ensure good geographical representability, the country was divided up into rectangular sub-areas, each representing an area which included 50 tracts of the NFI. One tract was selected randomly from each area, giving a total of 95 tracts. Sample plots were selected at 200 m intervals within each tract, one tract thus containing 26 sample plots (cf. Kuusela and Salminen 1969, p. 10).

The sample trees within each sample plot were selected using relascope (BAF 2). In each sample plot 5 sample trees at the most were picked according to a given rule. As selection was carried out with a relascope the DBH series of the sample trees closely follows the normal distribution and hence does not resemble the distribution prevailing in nature.

This selection method provided material which is sufficiently varied and by restricting the selection to the tracts and to sample plots the costs were kept at a reasonable level.

In addition to the material collected from

Table 1. Distribution of sample trees into diameter and height classes. Pine.
Taulukko 1. Koepuiden jakaantuminen läpimitta- ja pituusluokkiin. Mänty.

d, cm	Tree height, m – *Puun pituus, m* 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	Total *Yht.*
	Number of trees – *Puita, kpl*																											
1	3																											3
2	3	3																										6
3	3	7	1	1																								12
4		8	15	4	2																							29
5		4	15	12	7	2																						40
6		4	15	16	16	3	2																					56
7			6	16	15	7	4	2																				50
8			3	15	11	22	11	4	2	1			1															70
9			3	8	14	16	14	7	5	3		1																71
10				1	10	14	15	16	4	4	2		1															67
11			1	3	4	13	13	13	12	4	2			1														66
12				2	3	9	8	16	19	7	5	2	2															73
13					1	7	8	8	14	19	9	5	5	3														79
14						1	9	6	7	15	6	10	3	2														59
15					2	2	7	12	13	10	7	15	8	4	3		1											84
16					1	4	1	6	8	16	8	6	8	11	6	2	1											78
17								1	4	11	15	14	11	11	3	3	1	1										75
18					1	1	4	4	9	11	8	15	13	8	10	6	9	1	1	1								102
19								2	5	10	8	8	10	8	11	10	5	2		1								80
20							1	2	4	6	7	10	19	12	10	3	7	5	4	1	1							92
21									1	4	6	5	16	11	8.	11	8	9	1		1							81
22								2	4	3	8	5	8	17	11	8	14		7	2	1							90
23							1		3	6	4	12	11	14	9	20	12	4	1	1	1	1	1					101
24									3	2	3	5	15	14	10	11	9	8	11	1	3	2						97
25						2			2		4	4	6	16	12	7	5	9	5	6	3	1						82
26										1	1	3	6	3	4	8	10	7	10	10	5	1	2					71
27										2	2	4	5	13	8	7	10	11	8	3	6	4	2	2	2			89
28											1	5	3	5	6	5	14	5	8	9	4	3	2	1	1			72
29										2	2	2	3	5	4	1	8	7	6	10	4	2	1	1	1			59
30												4	2	3	7	9	6	3	7	5	3	4	3	2				58
31										1	1	1	3	6	7	4	6	5	8	4	4	5	1		1			57
32											3	2	4	1	2	4	3	5	1	3	5	4	2	1		1		41
33												3	3	3	2	2	2	4	8	7	2	6	1		2		1	46
34											1	3	1	2		4	6	3	5	5	1	6	2	2			1	42
35												1	4	2		5		3	3	2	1	2		2				25
36													3	1	1	5	4	2	2		3		1	2		1		25
37								1			1					4	2	1	1	2	1	2	3	1	1		1	21
38												1				2	2	1		5	1	1		2		1	1	17
39													1	1	2				3	5		3	1	1	1			18
40															1		1		1	3	1	2	1	1				11
41											1				2		1		2	1			2		1	1		11
42																	2			2	2							6
43																					1					1		2
44																		1										1
45																		1										1
46																	1	1										2
47															1		1											2
48																	1										1	2
49																				1								1
50																						1						1
51															1	1												2
Total *Yht.*	9	26	59	78	87	103	98	102	119	138	115	146	175	177	141	142	152	99	103	90	54	50	25	18	10	5	5	2 326

the tracts of the NFI, sample trees were also measured in the forests of the experimental areas of the Finnish Forest Research Institute. These trees, as well as those from tracts in the forests of the State Board of Forestry, were felled and additional measurements made.

The major portion of the study material was collected during three summer work periods (1968—70). Two three-man field teams performed the field measurements. Additional birch material was collected in 1971 and exceptionally large conifers and trees with only small tapering were collected in 1972 for control purposes. The distribution of the material into diameter and height classes is presented in Tables 1—3 and the geographical distribution of the material by tree species in Figs. 1—3. The control material measured in 1972 is not included in these tables and figures.

The proportion of birch out of the total number of sample trees was, despite the inclusion of supplementary material, only 17 %. Geographically the birch material is not as representative as the pine and spruce materials. The size of the birch material, however, can be considered to be sufficient.

Table 2. Distribution of sample trees into diameter and height classes. **Spruce.**
Taulukko 2. Koepuiden jakaantuminen läpimitta- ja pituusluokkiin. ***Kuusi.***

d, cm	Tree height, m - *Puun pituus, m* 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	33	Total *Yht.*
	Number of trees - *Puita, kpl*																														
2	8	1																													9
3	3	13																													16
4	1	13	12	3																											29
5		8	16	9	7																										40
6		1	5	20	22	3	2																								53
7			2	13	15	12	4																								46
8			1	12	18	18	6	2	1																						58
9				1	5	17	11	16	4	2																					56
10				1	2	10	23	18	12	8		1																			75
11					2	6	16	18	18	4	11	3	1																		79
12				1	4	1	13	24	20	17	10	6	2		1																99
13						1	4	5	18	14	17	11	2	3																	75
14						1	2	5	15	10	12	18	6	5	2																76
15						2	3	2	4	4	13	15	13	14	3	2															75
16								2	8	9	13	14	14	10	9	5	1	1													86
17								2	2	8	14	12	7	12	8	8	2														75
18								1	2	4	4	11	15	12	9	5	6														69
19							1	2	1	7	6	5	10	14	7	14	9	1	1	1											79
20								1	1		7	7	9	16	6	12	10	2	8	1											80
21									1		5	4	4	10	17	8	7	6	4		1										67
22											1	4	8	8	12	12	7	9	3	4	2										70
23													5	11	9	8	7	4	11	9	3	2									69
24													3	3	11	7	6	5	4	6	2	1									48
25												1	3	1	10	5	7	11	6	7	5	5									61
26												1	1	1	3	2	7	13	6	6	7	5									52
27											1	1	2	3	3	4	5	5	7	6	8	4	2	3							54
28														1		8	4	8	6	9	9	6	2								53
29															2	3	3	5	9	4	4	3	2	1	1						37
30													1		1	2	3		2	7	6	1	3	1							27
31													2			1	2		1	4	1	4	3	6	1	1					26
32																2	1	2	1	1	4		1	1							13
33																	2	2	3	3	2	5	2	1	1	1					22
34																	1	1	2		2	3	1	1	2		1				14
35																2	1		1		2	1	5		2						14
36												1						1	2	1		1	1	1	4	1					13
37																				2	1			3	1	4		1			12
38																					2		2		2	1	1				8
39																					1	1		1		2		1			6
40																				1		1	1		1						4
41																								2	1	1					4
42																						1	1	1		1					4
43																										1	1				2
44																									1			1			2
45																													1		1
46																											1				1
47																						1						1			2
.																															
54																						1							1		2
.																															
62																														1	1
Total *Yht.*	12	36	36	60	75	71	85	98	107	87	114	115	108	124	113	110	91	76	77	72	62	46	26	22	16	14	3	5	2	1	1 864

The diameter-height distribution of the material for each tree species is broad, as is to be expected, because the material is very variable as regards both the geographical and the site-type distribution.

The ranges and the means of the diameter at breast height, height and volume of the trees in the study material are presented in the following set-up:

Set-up 21.1

	Pine			Spruce			Birch		
	min.	max.	mean	min.	max.	mean	min.	max.	mean
d, cm	0,9	50,6	20,23	1,5	61,9	18,04	1,2	49,7	16,65
h, m	1,5	28,3	13,67	1,8	32,7	13,82	2,4	29,5	15,33
v, dm^3	0,4	1939	313	0,7	3790	265	0,4	2018	229

Table 3. Distribution of sample trees into diameter and height classes. Birch.
Taulukko 3. Koepuiden jakaantuminen läpimitta- ja pituusluokkiin. Koivu.

d,	Tree height, m - *Puun pituus, m*																													
cm	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	Total *Yht.*
	Number of trees - *Puita, kpl*																													
1	1	1																												2
2		2	3																											5
3			5	5	2																									12
4			2	4	4		1																							11
5				3	8	6	2	2																						21
6				2	4	11	12	1																						30
7				1	3	5	7	6	7	4	1																			34
8						3	7	6	6	6	4		1																	33
9					1	2	4	8	8	5	6	5		1																40
10							2	4	8	4	7	2	2	1	1															31
11						2	1	6	7	8	4	4	3	2	1	2														40
12								7	2	1	9	3	5	5	3	2	1	2												40
13								2	3	3	5	7	2	5	2		2													31
14									1	7	4	6	3	4	1	4	1	1	2											34
15									1	3	5	2	2	3	2	2	2	4	1	1	1									29
16										3	3	4	11	5	3	6	5	1	4											45
17											5	6	3	2	5	5	4	1	2	1	1									35
18									1	1	1			3	6	3	9	4	4	5	2	1								40
19										2	1	1	3	4	4	5	3	4	4	4	3									38
20										1	1		1	2	3	3	5	2	6	5	2		1							37
21												1		2	4	5	4	1	5	4	2	2								30
22											1	2		1		1	3	5	2	3	6	4								28
23												2			2	3	7	7	2	3	7	2	3	4	1					43
24														1	4		2	4	2	4	2	2	3	3	1					28
25													1		1	1	1	5	7	2	6	3	4	1	1					33
26													1		1	1	1	3	2	5	2	4	2		1					23
27																		1	2	4	1	2	4	1		1				16
28																	1	3	2		1	1		2		3				13
29																1	1		1	1	3	1	3	1						12
30																			2	2	1	1	1	2		1			1	11
31																		1			3		1	2	1					8
32																		1		1	1		2	1		2				8
33																					1		1	1						3
34																				2		1	1		2					6
35																				1	1		1							3
36																				1			1		1					3
37																									1					1
38																														
39																								1						1
40																														
41																					1		1							2
42																														
43																								1						1
44																														
45																							1							1
.																														
50																										1				1
Total *Yht.*	1	3	10	15	22	29	36	42	44	48	57	45	38	41	43	44	52	53	46	50	50	26	29	21	9	8			1	863

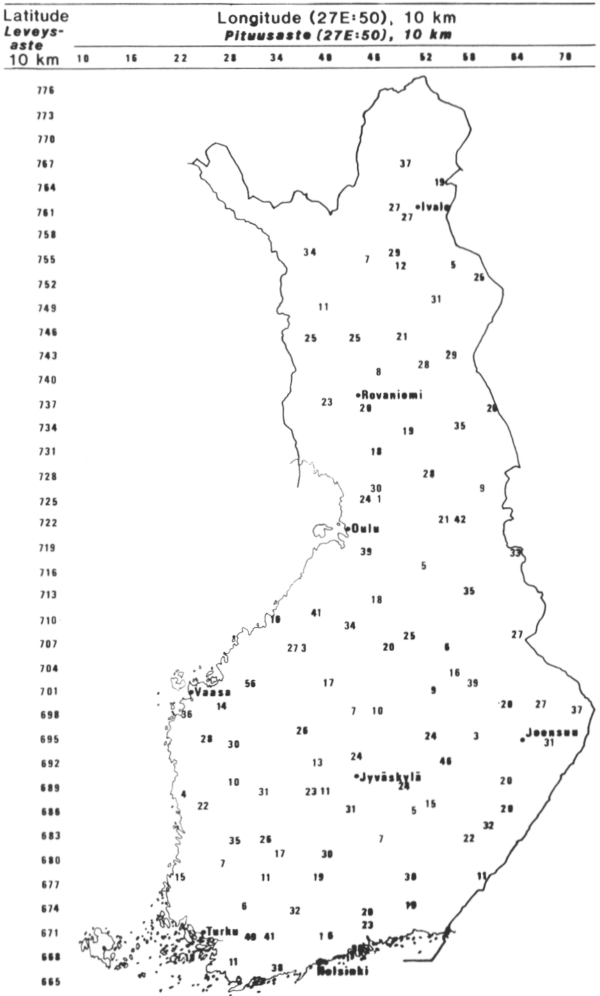

Fig. 1. Geographical distribution of the sample trees according to the Finnish Grid System. Pine.
Kuva 1. Koepuiden maantieteellinen jakauma yhtenäiskoordinaatiston mukaan. Mänty.

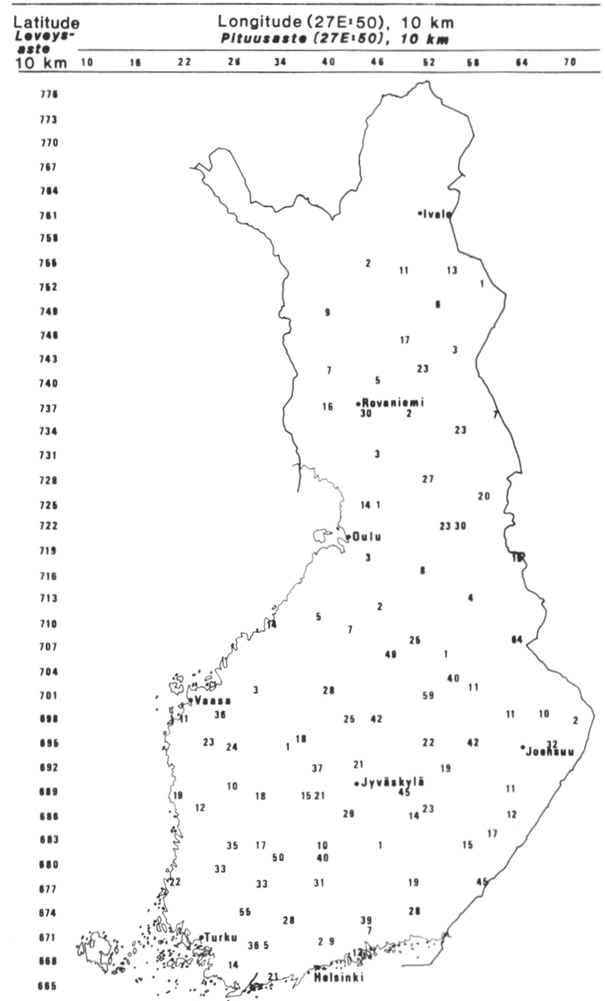

Fig. 2. Geographical distribution of the sample trees according to the Finnish Grid System. Spruce.
Kuva 2. Koepuiden maantieteellinen jakauma yhtenäiskoordinaatiston mukaan. Kuusi.

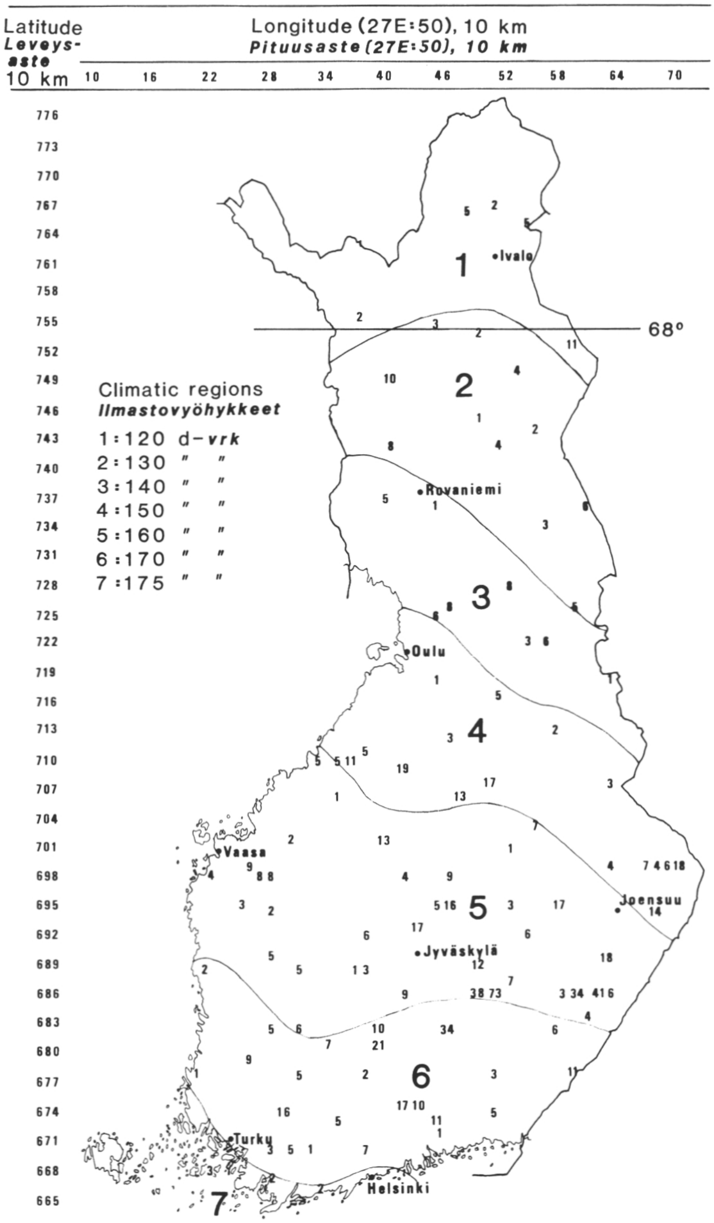

Fig. 3. Geographical distribution of the birch sample trees according to the Finnish Grid System and climatic regions based on the length of thermal vegetation season.

Kuva 3. Koivun koepuiden maantieteellinen jakauma yhtenäiskoordinaatiston mukaan ja termisen kasvukauden pituuteen perustuvat ilmastovyöhykkeet.

22. Measurement of the sample trees and sample plots

Since almost all the trees were measured as standing trees, tree height was determined to an accuracy of one decimeter only. Diameter and bark thickness were measured at the following relative heights, which are given as percentages of total tree height: 1, 2,5, 5, 7,5, 10, 15, 20, 30, 40, 50, 60, 70, 80, and 90. The measurement points along the stem of a sample tree are shown in Fig. 4. Measurements at 85 and 95 % of total height were only carried out on felled sample trees.

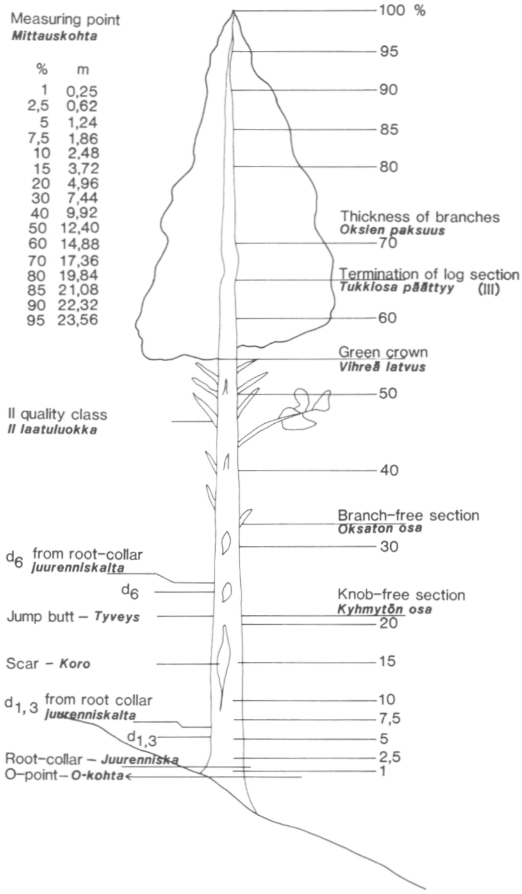

Fig. 4. Points where diameter was measured on the sample trees and the stem branchiness and quality measurement data for sample tree 24,8 m high.

Kuva 4. Koepuiden läpimittojen mittauskohdat sekä rungon oksaisuuden ja laadun mittaustiedot 24,8 m pitkällä esimerkkipuulla.

Diameter and bark thickness were both measured in two directions at right angles to each other. Diameter at breast height and associated bark thickness were measured at a height of 1,3 m above the ground (see e.g. Instruction ... 1971) and 1,3 m above the uppermost root collar (Ilvessalo 1947). The measuring point for measurements made at the height of six meters was also determined from both the ground surface and the root collar. The diameter at a height of 3,5 m was not measured on trees 5,5—7,5 m high (cf. Ilvessalo 1947).

Since the height of stems of all different sizes was measured to the nearest decimeter, the diameter and the portion of bark in the diameter to the nearest millimeter, the relative accuracy of the stem measurements for stems of different size was not quite the same. However, the accuracy can be considered as quite sufficient for the purposes of this study, even in the case of small trees.

In addition to the information required for calculating volume, many other variables were measured on the trees; tree characteristics connected with the quality classification of timber assortments, especially, were examined. As well as tree measurement variables, a number of variables pertaining to the sample plot and growth environment of the trees were measured, and the map coordinates of the sample plots registered.

23. Examination and revision of the material

After the field measurements had been carried out the data records were checked and whenever necessary and possible, slight amendments were made. After the material had been transferred to the computer the data was checked using a computer programme. These checkings were made as comprehensive as possible. Most of the programmes tested the logicality of the data. Most of the errors which had arisen during the measurement or card-punching stages were eliminated by employing upper and lower limits and by comparing different data with each other. A few sample trees had to be rejected during the data-checking because there was no information available for correcting the evident errors.

After checking, the material was condensed. At this stage mean values of diameters measured at right-angles to each other were calculated and the new values thereafter used as the diameter values. The final records made for each tree contained both stand and tree data.

24. Calculation of sample tree volume

Determination of stump height is important from the point of view of stem volume because the volume is concentrated in the butt part of the stem. Even in large trees the proportion of stemwood in the 10 cm-high section above the root collar is of the order of one per cent of the total amount of stemwood (Laasasenaho 1975 b).

When trees are felled in harvesting during the snow-free period the height of the stump is at least 5—10 cm, even in the case of small trees which have no root collars which would affect cutting. In practical harvesting the trees are in most cases cut off above the highest root collar which affects cutting, especially when the snow cover is thick (Laasasenaho 1975 a). Only on large spruces and birches is the uppermost root collar so high that cutting takes place either at that height or slightly below it.

Either a certain fixed height or a height determined by the height of the tree might be used as the stump height. In Sweden the stump height is taken as one percent of the height of the tree (Näslund 1947). Percentage stump height does not give as good an estimate of the amount of wood which will become available as the height of the root collar, because the root collar is a better indicator of the cutting point on larger trees. For this reason, the volume of each sample tree was calculated as the volume of the stemwood extending from the highest root collar affecting cutting, up to the top of the stem. If there were no such root collars or if they were below a height of 10 cm, the butt part of the stem up to a height of 10 cm above the ground was not included in the volume calculations. Despite the subjective problems associated with defining the root collar and the additional variation caused by variation in the height of the root collar, the actual amount of stem wood obtained in harvesting can best be obtained with this method.

The stem volume can be calculated in a number of alternative ways using the measurements made in this study. When the preliminary equations were being constructed, sample tree volume was calculated using interpolation parabola at the butt and Simpson's formula from a height of 5 % upwards (cf. Lahtinen and Laasasenaho 1979, p. 33). As spline-functions proved to be a suitable technique for producing accurate taper curves, the sample tree volumes were recalculated using spline-functions. The starting values for the spline-functions (cf. Lahtinen and Laasasenaho 1979, p. 30) were determined using additional diameters calculated at heights of 3,5 and 95 %.

The differences of the volumes when either the spline technique or the parabola—Simpson combination have been used are statistically highly significant (ibid. pp. 50—51).

The spline technique gives results which are different from those obtained when calculating the volume of each stem section as a cylinder according to the mid-point diameter. Ilvessalo (1947), for instance, used one-meter-long stem sections when the tree was less than 12 m high, and two meters when taller than 12 m. The butt section, however, was always one meter long.

The error associated with the use of the mid-point measurement has been studied in a number of investigations (e.g. Petrini 1928, Laasasenaho and Sevola 1972, Kärkkäinen 1974). The percentage underestimates and their standard deviations when the stem volumes of the present material have been calculated as cylinders based on mid-point measurement of sections either one or two meters long are given in Table 4.

The figures in the table have been calculated using the formula

$$100 \cdot \frac{V_s - V_c}{V_s}, \text{ where}$$

V_s = volume obtained using the integral of the cubic spline taper curve
V_c = volume obtained by cylindrical sections.

When 2 m sections have been used, the first section at the butt end is 1 m and the

Table 4. Percentage underestimates ($\bar{x}$) from the actual volumes and their standard deviations (s) of stem volumes obtained by summing cylindrical volumes of 1 or 2 m-long stem sections and the number of observations by tree species and height classes.

Taulukko 4. Runkojen tilavuuksien prosentuaaliset aliarviot ($\bar{x}$) oikeasta tilavuudesta, kun tilavuudet on saatu yhden tai kahden metrin pituisten pätkien keskuskiintomittoina, sekä %-erojen hajonnat (s) ja havaintojen lukumäärät puulajeittain ja pituusluokittain.

h	Pine - *Mänty*					Spruce - *Kuusi*					Birch - *Koivu*				
m	N	1 m		2 m		N	1 m		2 m		N	1 m		2 m	
		$\bar{x}$	s	$\bar{x}$	s		$\bar{x}$	s	$\bar{x}$	s		$\bar{x}$	s	$\bar{x}$	s
3	26	3,1	6,6	5,4	5,5	36	4,2	4,4	5,9	5,7	3	-3,5	14,2	3,4	16,1
4	59	3,5	3,9	4,3	5,5	36	2,6	3,2	4,0	4,6	10	4,5	2,7	8,3	3,6
5	78	3,2	3,4	4,2	3,5	60	2,1	2,6	3,6	2,5	15	4,0	3,4	6,8	4,5
6	87	2,3	2,3	3,4	2,6	75	2,0	2,0	2,6	2,5	22	3,5	2,3	4,7	3,3
7	103	2,3	1,6	3,0	2,6	71	1,9	1,6	2,2	2,4	29	3,6	2,9	4,5	4,1
8	98	1,9	1,4	2,5	2,1	85	1,9	1,5	2,4	2,0	36	2,1	2,4	3,1	3,4
9	102	1,8	1,1	2,6	1,9	98	1,6	1,1	2,1	1,6	42	2,4	2,3	2,8	3,2
10	119	1,7	1,0	2,1	1,5	107	1,5	1,1	1,8	1,6	44	2,0	1,6	2,0	2,7
11	138	1,5	0,9	1,9	1,1	87	1,4	1,0	1,7	1,2	48	1,5	1,2	2,4	1,4
12	115	1,3	0,8	1,6	0,9	114	1,2	0,7	1,5	1,0	57	1,3	1,4	1,9	1,9
13	146	1,1	0,7	1,4	0,9	115	1,1	0,7	1,1	0,9	45	1,4	0,9	1,6	1,4
14	175	0,9	0,6	1,2	0,9	108	1,1	0,6	1,1	0,9	38	1,5	1,3	1,9	1,7
15	177	0,8	0,5	1,2	0,7	124	1,0	0,6	1,2	0,9	41	1,2	0,8	1,5	0,9
16	141	0,8	0,4	1,0	0,5	113	0,8	0,5	0,9	0,6	43	1,0	0,6	1,4	0,9
17	142	0,7	0,3	1,0	0,5	110	0,7	0,3	0,9	0,4	44	0,8	0,4	1,0	0,6
18	152	0,6	0,2	0,9	0,5	91	0,7	0,3	0,9	0,6	52	0,9	0,4	1,1	0,9
19	99	0,6	0,2	0,8	0,5	76	0,6	0,2	0,7	0,5	53	0,7	0,3	0,9	1,0
20	103	0,6	0,2	0,8	0,5	77	0,6	0,2	0,7	0,5	46	0,6	0,3	0,9	0,8
21	90	0,5	0,2	0,7	0,3	72	0,5	0,2	0,6	0,4	50	0,6	0,3	0,8	0,6
22	54	0,6	0,3	0,8	0,4	62	0,4	0,2	0,6	0,4	50	0,6	0,3	0,9	0,5
23	50	0,6	0,2	0,8	0,3	46	0,4	0,2	0,6	0,2	26	0,7	0,3	0,9	0,5
24	25	0,4	0,2	0,6	0,3	26	0,4	0,2	0,5	0,3	29	0,8	0,5	1,1	0,8
25	18	0,5	0,2	0,6	0,3	22	0,4	0,2	0,5	0,2	21	0,5	0,3	0,6	0,5
26	10	0,5	0,2	0,6	0,3	16	0,4	0,2	0,4	0,3	9	0,6	0,3	0,8	0,8
27	5	0,3	0,1	0,7	0,2	14	0,4	0,2	0,5	0,4	8	0,4	0,3	0,7	0,4
28	5	0,5	0,2	0,6	0,4	3	0,3	0,1	0,3	0,1					
29						5	0,3	0,2	0,6	0,1					
30						2	0,9	0,2	0,9	0,1	1	0,1	0,0	0,9	0,0

other sections 2 m except for the top section. The volume of the top section has also been calculated using the middle diameter.

The figures in Table 4 show that the shorter the tree is, the more negatively biased is the volume estimate obtained by cylindrical sections. The underestimation is clearly greater when 2 m sections are used than with 1 m sections. The underestimation is greatest in the case of birch.

It was apparent that a major part of the error due to cylindrical sections is incurred in calculating the volume of the lower part of the stem (cf. Kärkkäinen 1974, p. 57). The smaller the tree, the greater is the relative effect of the butt section on the total error.

If the section is conical-shaped, the volume obtained using Simpson's rule is the actual one. Denote the central diameter by d_c, height by l and the difference between the butt and top diameters by 2 k. According to Simpson's formula the volume of the section is thus:

$$\frac{\pi}{4 \cdot 6}\left\{(d_c + k)^2 + 4 \cdot d_c^2 + (d_c - k)^2\right\} \cdot l =$$
$$\frac{\pi}{4 \cdot 6}(6 \cdot d_c^2 + 2k^2) \cdot l$$

The volume obtained using this method is either as great or greater than the volume given by mid-point measurement. The difference between the volumes is obtained as a percentage from the following formula: $p\,\% = \frac{100}{3} \cdot (\frac{k}{d_c})^2$. In other words the magnitude of the relative error is dependent on the diameter as well as on the taper.

The magnitude of the error percentage with some different values of d_c and k can be seen from the following set-up:

Set-up 24.1

		k			
		1	2	4	8
d_c	10	0,333	1,333	5,333	21,333
	15	0,148	0,593	2,370	9,481
	20	0,083	0,333	1,333	5,333
	25	0,053	0,213	0,853	3,413
	30	0,037	0,148	0,593	2,370
	50	0,013	0,053	0,213	0,853

If the section is neiloid-shaped, as the butt part of the stem in most cases is, the error is even greater.

3. CONSTRUCTION OF THE TAPER CURVE MODELS

31. Development of stem form

The factors which most affect stem appearance are stand density, site type and climate, as well as genetical factors. The requirements of different tree species as regards the site differ from each other. The location of the growing site also has an effect on the stem form in some special cases.

The space available for growth affects the development of the crown already at the seedling stage. The branches of trees growing on the same site become thicker, the greater the amount of space available for growth (cf. e.g. Varmola 1980, Kellomäki and Tuimala 1981). The degree of branchiness is also a genetically-determined property (e.g. Ehrenberg 1970).

The correlation between the cross-sectional area of the stem and the quantity of branches in three different-sized and different-aged Scots pines is presented in Fig. 5. The quantity of branches is indicated by the cumulative square sum of the diameters at the butt end of the branches. The sum is calculated starting from the top of the stem.

The greater the square sum of the butt diameters of the branches in a whorl, the more the stem diameter increases below the branch whorl. The more branches there are in a branch whorl, the greater the mass of photo-synthesising needles. This results in a higher rate of radial growth in the stem below the branch whorl. The diameter of the stem within the crown changes stepwise according to the branch whorls. The diameter in the branch-free portion of the stem

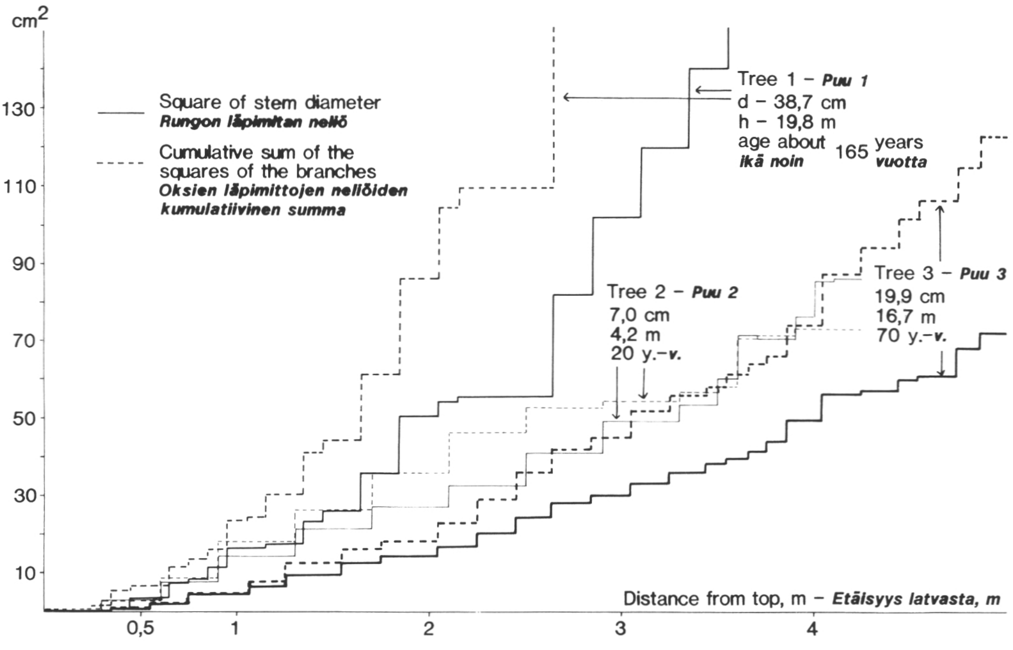

Fig. 5. Square of the stem diameter over bark and cumulative sum of the squares of the diameters of all (living and dead) branches over bark at each branch whorl for three different-aged pines.
Kuva 5. Rungon kuorellisen läpimitan neliön ja elävien sekä kuolleiden oksien oksakynnysten yläpuolelta mitattujen läpimittojen neliöiden kumulatiivisen summan kuvaajat kolmella eri-ikäisellä männyllä.

increases uniformly towards the butt.

The square sum of the diameters of the branches lying above each branch whorl was greater within the crowns of the pines studied than the square of the stem diameter at the corresponding point. The square of the stem diameter was, however, greater at the base of the tree than the sum of the squares of all the branch diameters.

As the stand starts to close, the crowns are no longer able to grow in length at the same rate as earlier and the stem form therefore starts to change. At this stage the stem form of planted stands, for instance, improves as the lowest branches die off and are pruned naturally. It is thus evident that the way in which stands are established, the planting or seeding density used, and subsequent tending measures, all have an effect on the development of stem form and the quality of the wood produced.

As the height growth of the tree gradually slows down, radial growth within the crown continues and the stem form changes (cf. Fig. 5, tree 1). Since trees usually have considerably more space for growth at this stage, radial growth at the base of the stem is, in accordance with the mechanical stem form theory strong.

Factors which affect the stem form of different sized trees are so numerous that all of them cannot be included in the taper curve models.

32. Mathematical modelling of the taper curves

Many attempts have been made to explain the form of the taper curve by several biological and physical factors. In practice it is not possible to include all of these factors in the taper curve models because they are either difficult or impossible to measure. For this reason relatively simple mathematical models have been presented for taper curves. The classic model for taper curves is Höjer's formula (Höjer 1903):

$$\text{(32.1)} \quad \frac{d_l}{D} = C \cdot \log \frac{c + l}{c}, \text{ where}$$

D = the reference diameter at the base of the tree,
l = the distance from the top (% of tree height),
d_l = the diameter at a distance of l from the top, and C and c are constants.

This formula and different modifications of it have been the starting point in many taper curve studies. The use of relative variables effectively eliminates the large absolute variation caused by differences in tree size. The following type of parabola has been found to give rather good results in models where diameter at breast height is used as the reference diameter (Kozak et al. 1969):

$$\text{(32.3)} \quad \frac{d_l^2}{d^2} = \beta_0 + \beta_1 \frac{l}{h} + \beta_2 \frac{l^2}{h^2}, \text{ where}$$

d_l is the stem diameter at height l.

Peters (1971) used the diameter at 10 % height as the basic diameter and a fifth-degree polynomial. Fries and Matern (1965) have tested polynomial functions of a very high power.

Quite different types of mathematical solutions have to be used in cases where volume equations have already been constructed, and taper curve functions which are compatible with them are required. In this connection, compatible means that the integral of the taper curve function gives the same volume as the function for trees of all different sizes. Demaerschalk (1972) has studied taper curves obtained with different volume function models and the accuracy of these models. The method is based on the use of non-linear regression analysis.

Pöytäniemi (1981) has also used the volume given by a volume function as the restriction in a taper curve function. After trying polynomials of different powers, the following one was found to be the best (cf. Pöytäniemi 1981, p. 66).

$$\text{(32.3)} \quad \left(\frac{d_l}{d}\right)^2 = \sum_{k=1}^{4} \beta_k \left(\frac{h - l}{h - 1{,}3}\right)^k + \beta_5 \left(\frac{h - l}{h - 1{,}3}\right)^7$$

Describing the taper curve by means of one equation has not given satisfactory results in all practical applications. The fit of the taper curve models has been improved by calculating individual functions for different part-intervals along the stem (cf. e.g. Roiko-Jokela 1974, Max and Burkhart 1976, Demaerschalk and Kozak 1977). These subfunctions have to fulfil certain continuity conditions at common knot points, if the taper curve given by the functions is to correspond to the actual taper curve. The most natural condition in this method is that the adjacent subfunctions take the same values up to the first derivatives at the knot points.

Kuusela (1965) presented a method, based on the use of form coefficients calculated for 10 % height and diameters at relative heights, for estimating stem volume and taper curve and for constructing volume functions. Kilkki et al. (1978), Kilkki and Varmola (1979 and 1981) and Kilkki (1979) have studied the prediction of diameters at relative heights using simultaneous equation models. The continuous taper curve is calculated in this method using some interpolation formulae.

As can be seen from the above, a taper curve model can be constructed in many ways. When the square of the diameter is used as the dependent variable, the error with respect to the cross-sectional areas is minimised and it is possible to obtain the unbiased estimate of the stem volume from the integral of the taper curve function. Another way is employ the error variance of the estimated taper curve via Taylor's expansion (see Kilkki and Varmola 1981).

Using the relative diameter as the dependent variable is advantageous since every tree then has an equal weight in the analysis. The use of the squared diameters

Table 5. Relative diameters of the sample tree stems at different measuring points at some diameter classes.
Taulukko 5. Suhteellisia läpimitta-arvoja rungon eräillä osakorkeuksilla joissakin läpimittaluokissa.

d	N	Relative height, % - *Suhteellinen korkeus, %*											
		1	2,5	5	10	20	30	40	50	60	70	80	90
							Pine - *Mänty*						
3	12	142,7	134,9	126,4	110,6	100	91,0	81,9	72,4	61,1	49,2	34,7	21,1
7	50	147,6	134,9	122,1	111,3	100	90,7	81,9	72,2	62,2	49,9	36,4	21,9
11	66	149,2	135,5	122,3	111,6	100	91,6	84,0	75,7	65,6	53,2	36,9	20,3
15	84	148,4	134,2	121,8	111,3	100	92,4	85,4	77,4	67,0	54,1	38,9	21,3
19	80	147,9	131,3	120,2	109,7	100	93,5	86,3	78,2	68,3	55,5	40,6	21,8
23	101	147,4	131,1	119,9	109,7	100	93,0	85,4	77,3	66,8	53,9	38,4	19,7
27	89	148,8	132,5	120,8	109,0	100	92,9	86,2	77,7	67,5	54,6	39,4	20,0
31	57	151,8	133,1	120,7	109,4	100	93,9	86,9	78,8	68,7	56,3	40,6	20,9
35	25	148,4	133,1	120,5	109,2	100	94,0	86,4	78,5	66,5	54,7	38,9	20,5
39	18	148,4	133,2	120,3	108,8	100	93,0	85,5	78,0	67,5	54,7	38,0	18,8
43	2	147,6	133,8	120,1	111,1	100	92,4	84,3	80,7	68,4	56,8	40,0	23,1
							Spruce - *Kuusi*						
3	16	147,2	133,5	116,9	109,7	100	94,6	86,1	74,5	63,3	50,4	37,3	25,5
7	46	153,7	132,2	117,1	108,7	100	91,2	82,2	72,5	61,2	49,0	35,3	20,1
11	79	149,2	126,8	115,6	108,1	100	92,4	83,6	73,0	61,5	49,0	34,8	19,9
15	75	146,5	125,8	114,6	107,4	100	93,1	84,4	74,3	62,6	49,7	35,9	20,7
19	79	152,4	128,3	116,2	108,0	100	92,7	84,0	74,2	63,4	50,3	36,4	21,0
23	69	150,4	128,2	115,7	107,5	100	92,8	84,3	74,8	63,7	51,1	36,9	21,1
27	54	154,1	131,6	116,5	107,9	100	92,4	83,9	73,9	62,9	50,0	36,7	20,8
31	26	150,1	127,3	114,9	107,1	100	93,0	85,0	74,8	63,9	50,9	37,3	20,4
35	14	158,9	133,9	119,2	109,0	100	91,3	81,6	71,6	60,8	48,0	34,6	20,1
39	6	153,3	128,1	115,2	107,1	100	93,2	83,9	74,4	62,8	50,4	37,8	20,3
43	2	163,5	135,2	115,4	109,2	100	93,0	85,7	75,2	64,1	51,8	39,0	24,0
							Birch - *Koivu*						
3	12	162,4	137,6	122,7	111,2	100	88,8	77,3	64,8	52,1	40,0	27,9	14,5
7	34	155,5	131,3	120,0	109,9	100	88,0	78,8	67,4	55,9	42,3	27,8	13,8
11	40	157,1	128,5	116,8	108,0	100	91,0	81,9	71,6	59,3	45,5	28,2	12,9
15	29	153,6	130,1	116,1	109,3	100	92,2	84,4	72,4	59,9	45,9	29,4	14,1
19	38	152,6	126,5	114,7	106,6	100	92,2	84,9	74,5	61,6	47,0	29,1	12,8
23	43	149,7	125,4	114,4	106,6	100	93,0	84,6	74,5	62,6	47,0	29,5	12,8
27	16	152,0	126,0	114,5	107,8	100	94,3	84,8	75,4	62,2	46,0	28,4	12,2
31	8	157,2	132,7	123,4	109,6	100	94,4	87,5	78,5	63,7	48,0	28,2	10,5
35	3	158,9	141,5	134,3	118,7	100	93,2	85,4	74,5	62,3	44,4	24,8	11,3
39	1	164,7	135,1	121,8	106,3	100	95,1	80,8	72,4	56,3	37,4	22,7	9,2
43	1	174,3	161,4	143,6	112,9	100	93,7	84,5	81,5	65,7	58,1	34,0	21,8

makes calculation of the stem volume easy. Use of the relative height as the independent variable has been justified owing to the similarity in the form of stems of different size (cf. e.g. Demaerschalk and Kozak 1977).

33. Single equation models

331. Basic models

The relative diameters at relative heights along the stem do not change very much according to tree size. This can be seen in Table 5 and Figs. 6—8. The mean diameters of the sample trees within 1 cm diameter classes have been calculated at each relative measuring height for the different tree species. In individual cases the differences in the tree form can be rather large, mainly due to the tree environment.

Graphs of the mean taper curves are presented by tree species in Fig. 9. The graphs have been obtained by calculating the ratios of the mean diameters at different measuring heights to the mean diameter at 20 % height.

The taper curves differ rather clearly in different trees species. The whip-shaped top of birch is clearly evident, while its taper curve at the base of the stem lies somewhere between pine and spruce, except at 1 % height where it has the greatest relative value.

The fact that the taper curve in different-sized trees of the same species is of the same shape enables the model for taper curve to be formed with the help of relative diameters and heights.

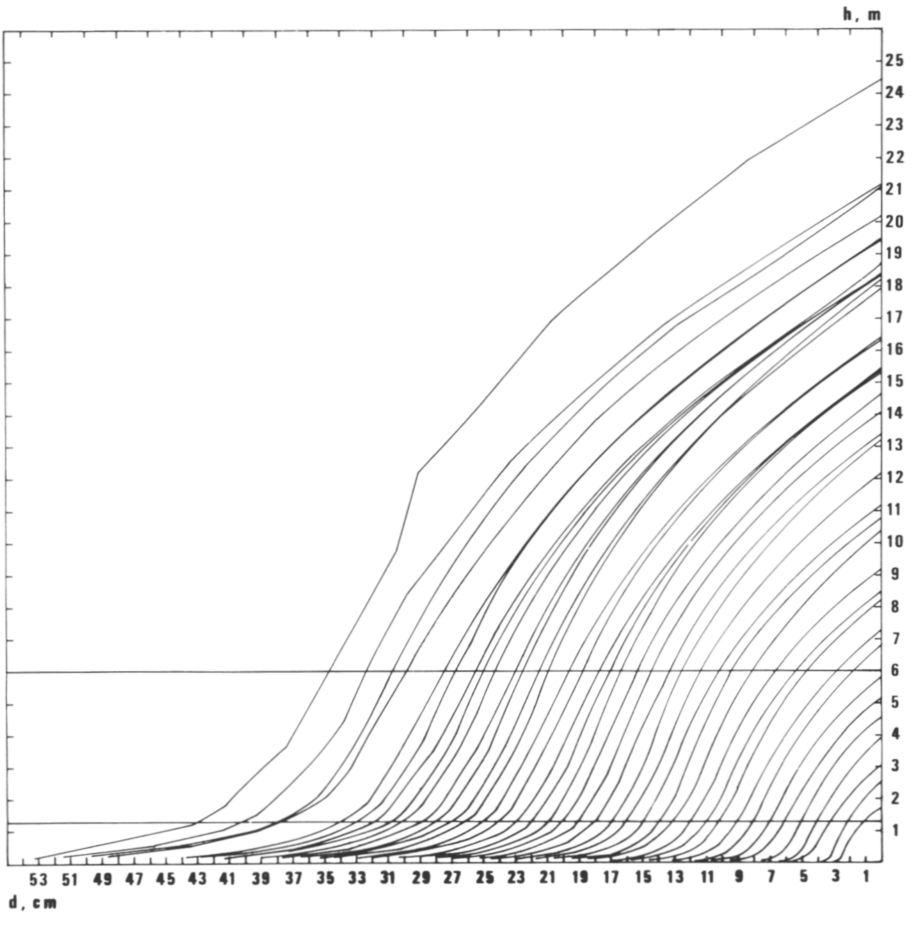

Fig. 6. Mean taper curves by diameter class. Pine.
Kuva 6. Keskimääräisiä runkokäyriä läpimittaluokittain. Mänty.

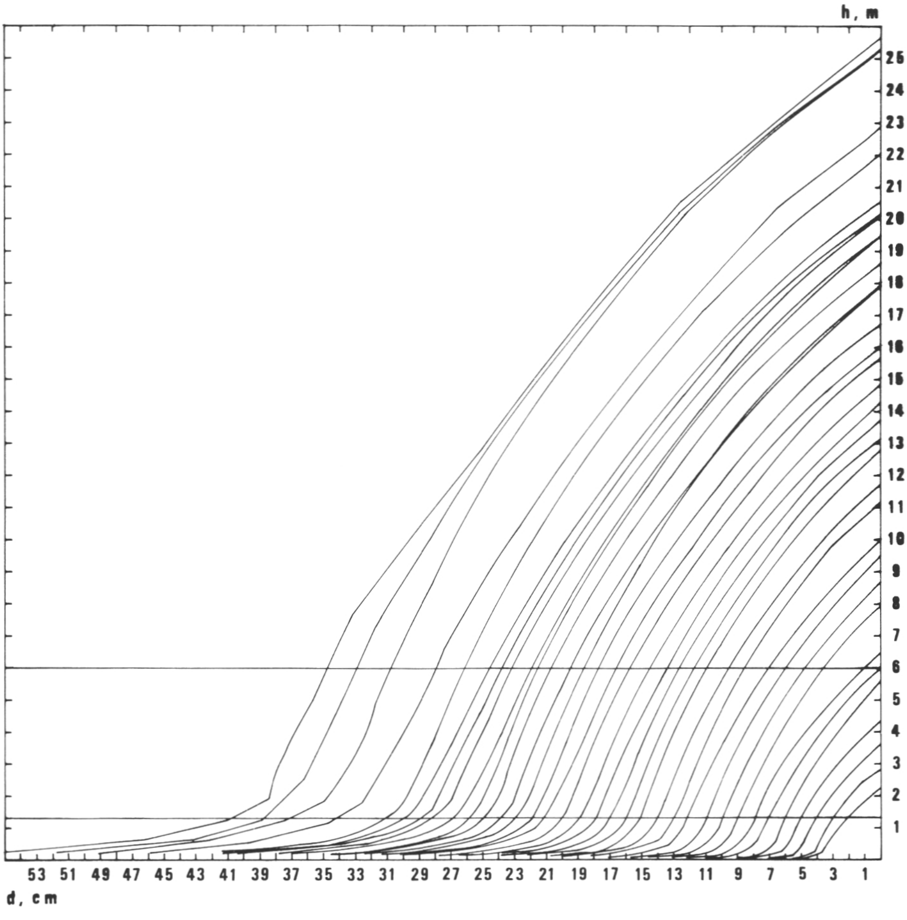

Fig. 7. Mean taper curves by diameter class. Spruce.
Kuva 7. Keskimääräisiä runkokäyriä läpimittaluokittain. Kuusi.

The following polynomial model was found to depict the curves in Fig. 9 rather well; the powers used in the model are in accordance with the so-called Fibonacci series:

$$(33.1)\quad \frac{d_l}{d_{,2h}} = b_1x + b_2x^2 + b_3x^3 + b_4x^5 + b_5x^8 + b_6x^{13} + b_7x^{21} + b_8x^{34}$$

where $d_{,2h}$ = the basic diameter at 20 % height,
d_l = the diameter at a height of l from the ground and $x = 1 - \frac{l}{h}$ or the relative distance from the top.

The model gives the value 0 at the origin, and by setting the requirement $1 = 0{,}8 \cdot b_1 + 0{,}64 \cdot b_2 + 0{,}512 \cdot b_3 + ... + 0{,}00050706 \cdot b_8$ for the coefficients, the coefficients can be solved so that the function passes through the point (0,8; 1).

The following combination of a lower-power polynomial and logarithmic terms gave the same degree of accuracy as the previous polynomial model:

$$(33.2)\quad \frac{d_l}{d_{,2h}} = b_0x + b_1x + b_2x^2 + b_3x^3 + b_4x^4 + b_5\ln(x) + b_6\{\ln(x)\}^2, \text{ where } x = \frac{l}{h}$$

Owing to the use of logarithmic terms, the model is able to take the butt swelling into account effectively, but with very small values of x (e.g. $x < 0{,}005$) the model is no longer usable. As the stump is not normally included in the tree volume, this does not reduce the applicability of the model. The model should give 1 when x = 0,2 and 0 when x = 1. Thus two requirements are needed

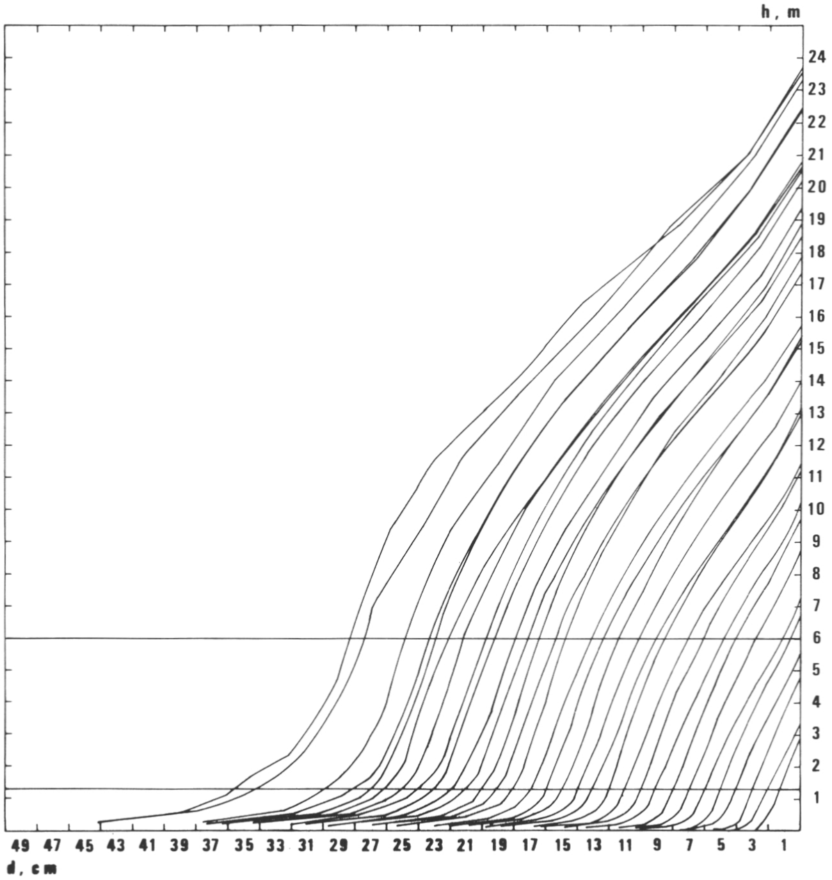

Fig. 8. Mean taper curves by diameter class. Birch.
Kuva 8. Keskimääräisiä runkokäyriä läpimittaluokittain. Koivu.

$0 = b_0 + b_1 + b_2 + b_3 + b_4$

$1 = b_0 + 0{,}2 \cdot b_1 + 0{,}04 \cdot b_2 + 0{,}008 \cdot b_3 + 0{,}0016 \cdot b_4 - 1{,}60944 \cdot b_5 + 2{,}59029 \cdot b_6$

Both models can be considered to be equally suitable as basic models for taper curves. The models are rather flexible and can be used to describe taper curves of different shape. If height and diameter at any height are known a taper curve function can be obtained using the following procedure.

Denote, for instance, equation (33.2) by $f_b(x)$ and let the diameter of the tree be measured at a particular point l. The value for $\frac{l}{h}$ is k_l and for $f_b(k_l)$ is t_l. The estimate, $\hat{d}_{,2h} = \frac{d_l}{t_l}$ is thus obtained for $d_{,2h}$. In most cases the diameter at breast height is measured and l = 1,3.

According to this common model the taper curve function for the tree is obtained as the function of diameter and height:

$$f = \hat{d}_{,2h} \cdot f_b(x) \tag{33.3}$$

332. Adjustment of the basic models using d and h

The relative height diameters along stems of different height but of the same diameter at breast height are not quite the same but there are certain regular differences. The basic equations of the taper curve can be adjusted by assuming that tree height and diameter at breast height are known.

Denote $z_i = \frac{d_i}{\hat{d}_{,2h}}$ and $t_i = f_b(i)$, where d_i is

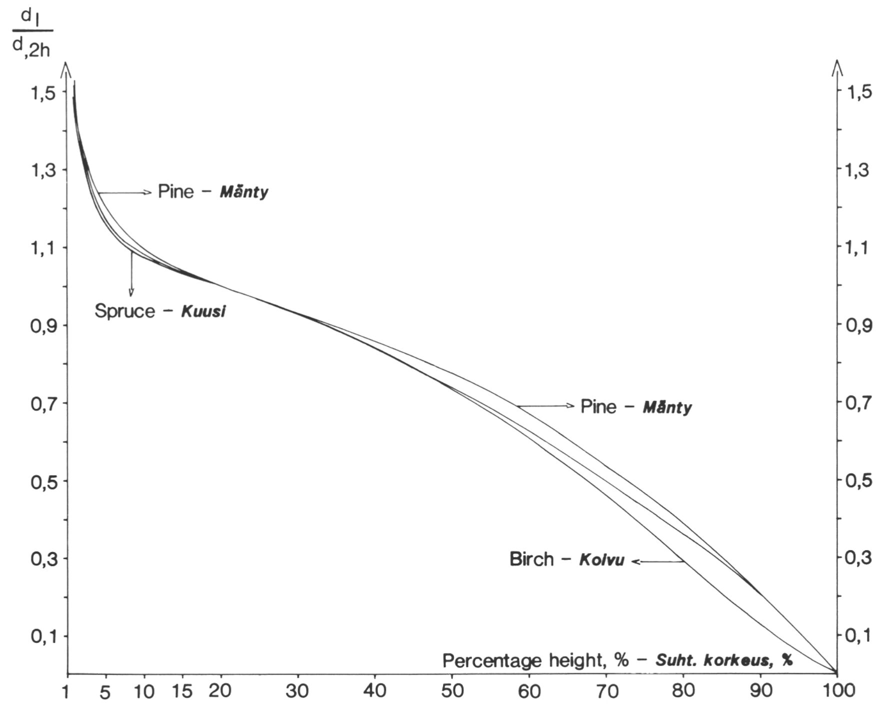

Fig. 9. The mean relative taper curves for pine, spruce and birch ($d_{,2h} = 1$) as a function of relative distance from ground.

Kuva 9. Männyn, kuusen ja koivun keskimääräiset suhteelliset runkokäyrät ($d_{,2h} = 1$) suhteellisen korkeuden funktiona.

the actual diameter at the relative height i and $\hat{d}_{,2h} = \dfrac{d}{f_b\left(\frac{1,3}{h}\right)}$. Based on measured data an equation can be determined for the differences $z_i - t_i$ at each relative height i as a function of diameter at breast height and height.

(33.4) $z_i - t_i = f_i\ (d, h) = f_i$

An adjusted estimate for diameter at the height i can be calculated using equation (33.5):

(33.5) $\hat{d}_i = \hat{d}_{,2h} \cdot (t_i + f_i)$

Three relative heights along the stem (e.g. ,1, ,4 and ,7) can be selected for which the functions (33.4) are to be computed.

A cubic interpolation polynomial can be calculated to pass through these three points determined by f_i and i and the top (correction = 0). The coefficients of the correction polynomial can be summed together with the corresponding coefficients for the basic curve f_b (x) to give the adjusted form curve function.

Denote the correction polynomial as $f_r(x)$. Define the corrected polynomial f_c as the sum of f_b and f_r, i.e.

(33.6) $f_c\ (x) = f_b\ (x) + f_r\ (x)$

Function f_c (x) gives the value 0 when x = 1, but no longer the value 1 at the point x = 0,2.

When the equation f_c (x) has been calculated for the tree, the adjusted taper curve for the tree is obtained by recalculating the value of $\hat{d}_{,2h}$ using f_c (x). The obtained value for $d_{,2h}$ is not the value of the taper curve at point x = 0,2, but instead at the point where function f_c (x) has the value 1.

The correction polynomial does not have to be a cubic one and the position of the checking points can be different from those suggested here. Neither does the power of the correction polynomial have to be one less than the number of checking points. For example, by adding a derivative condition to the correction polynomial a cubic polynomial is obtained with the help of two checking points and the top point. It is important that the correction polynomial is not too free but passes logically between the given points.

333. *Adjustment of the basic models using d, h and d_6*

An upper diameter, d_6, is also measured on sample trees in Finland. Then the taper curves of the sample trees have to pass through the measured diameter at a height of six meters. This condition is fulfilled when the following calculation procedure is used.

The estimate for $d_{,2h}$ is calculated using the diameter at breast height. Denote

$$k1 = \frac{1,3}{h},\ k6 = \frac{6}{h},\ y6 = f_b\ (k6) \text{ and } z6 = d_6/\hat{d}_{,2h}$$

The value of the correction polynomial at the height of 6 m is thus obtained as the difference

$$(33.7)\quad df = z6 - y6$$

By calculating the correction polynomial which passes through the points (k1, 0), (k6, df) and (1,0) and combining it into the basic model, a taper curve is obtained which also passes through the diameter point at a height of six meters.

In this case, too, there are many alternative ways of constructing the correction polynomial. As the correction polynomial should pass through these three points, a parabola will meet these conditions. The relative height of the 6 m point varies according to the height of the tree. A parabola would be far too inflexible to serve as the correction polynomial in most cases.

A unique third-degree polynomial is obtained by setting conditions on the first derivative of the correction polynomial (e.g. derivative = 0 at point (k1, 0)). The shape of the correction polynomial can be regulated by means of such derivative conditions and the taper curve is thus made to follow a certain degree of regularity (cf. Kuusela 1965, p. 11, Fig. 7). By setting two zero points (e.g. points (k1, 0) and (1,0)) on the first derivative of the correction polynomial, a certain portion of the curve of the quartic polynomial is obtained as the correction polynomial. The location of the maximum and minimum values of the correction polynomial can also be regulated by means of a derivative condition.

The correction polynomial can also be solved if the value of the correction at one additional point is estimated. This estimate can be dependent on the size of df (33.7).

Such additional points can be, for instance:

$$(33.8)\quad \begin{aligned} x1 &= 3,65/h \quad \text{(mid-way between 1,3 and 6 m)} \\ x2 &= 0,5 + 3/h \quad \text{(mid-way between 6 m and the top)} \end{aligned}$$

The values of the basic curve at these additional points are denoted as $f_b\ (x1) = y1$, $f_b\ (x2) = y2$ and $f_b\ (\frac{1,3}{h}) = y13$. To make the value of the correction polynomial f_r logical at points x1 and x2, the following conditions, for instance, may be set:

$$(33.9)\quad \begin{aligned} f_r\ (x1) &= df \cdot (y13 - y1)/(y13 - y6) \\ f_r\ (x2) &= df \cdot y2/y6 \end{aligned}$$

A unique fourth-degree polynomial is obtained by including these two points when solving the correction polynomial f_r. If only one of the additional points is selected, a cubic polynomial is obtained.

Functions for the corrections (cf. (33.4)) at the additional points can be computed with the help of the sample tree material. The estimates of the diameters at the additional points (33.8) can be calculated using the diameters at relative heights along the stem of each sample tree. The correction values are then calculated (cf. calculation of df (33.7)). Correction equations are computed for these values. The possible independent variables for these equations are d, h, d_6, df and modifications and combinations of these variables, as well as the type of variables in formulae (33.9).

34. Simultaneous equation models

Kilkki et al. (1978) presented a method for determining the taper curve using a number of diameters at different relative heights along the stem, predicted by means of a linear simultaneous equation model. In further studies carried out on this method, non-linear simultaneous equation models and linear models of the logarithms of the diameters have also been tested (Kilkki and Varmola 1979 and 1981, Kilkki 1979).

In this method, equations are calculated for every relative height diameter d_i, $i = 1, ..., n$ included in the model, by means of regression analysis. Other relative height diameters (endogenous variables) and tree height, as well as other possible variables (exogenous variables), are used as the independent variables. Kilkki et al. (1978) used height and its square as exogenous variables. The equations were of the following form:

$$(34.1) \quad d_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} \cdot d_j + \sum_{j=1}^{2} b_{ij} \cdot h^j$$

When tree height h is known, the effect (y_i) of the exogenous variables associated with each equation d_i, $i = 1, ..., n$ can be calculated. The set of equations to be solved is thus in matrix form (cf. also Kilkki 1979, p. 376):

$$(34.2) \quad \begin{bmatrix} 1 & -a_{12} \dots & -a_{1n} \\ -a_{21} & 1 \quad \dots & -a_{2n} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ -a_{n1} & -a_{n2} \dots & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \cdot \\ \cdot \\ \cdot \\ d_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix}$$

$$(34.3) \quad Ad = y$$

The vector d is solved by multiplying both sides of the equation by the inverse matrix A^{-1}. A basic solution for the set of equations is thus obtained which corresponds, to some extent, to the taper curve function (33.6). If some of the diameters d_i, $i = 1, ..., n$ are known, then the coefficients a_{ij} of the rows i in the set of equations (34.2) are set at zero and y_i replaced by the diameters d_i. The other diameters are obtained by solving the set of equations. When the diameter at some arbitrary point along the stem is known, corresponding changes are made in row i closest to the height in question. In this case the final set of the relative height diameter estimates is derived by iteration (Kilkki et al. 1978, p. 124).

A diameter series obtained by means of a simultaneous model is usually regularly monotonic, and hence the taper curve can be determined in a number of different ways. As the taper curve changes rapidly at the base of the stem, the diameters to be predicted at the bottom part of the stem should be at shorter intervals than higher up along the stem. However, a diameter series which is more comprehensive than the measurement series used in this study, i.e. 1, 2,5, 5, 7,5, 10, 15, 20, 30 ... 90 %, hardly improves the accuracy at all (cf. Lahtinen and Laasasenaho 1979, pp. 52—54).

Determination of the taper curve by means of simultaneous equations requires considerably more computer time than, for instance, the polynomial function of the taper curve does, even if the calculations would be done with the most effective algorithms. The amount of calculations are further increased by the correction procedure required when logarithms of the diameters are used as endogenous variables (cf. Kilkki and Varmola 1981).

The calculations in the method can be speeded up if the regression equations (34.1) are prepared in such a way that only the adjacent relative height diameters of the endogenous variables are used as the independent variables for each diameter. The coefficients of determination of the equations prepared in this way are almost as high as in the full model, because the dependencies on adjacent diameters are rather fixed. The coefficients matrix A (34.2) is thus a tridiagonal one, which can be solved using effective algorithms (cf. e.g. Lahtinen and Laasasenaho 1979, p. 12).

Owing to the simplification of the coefficient matrix and the improved efficiency of the calculation procedure the simultaneous equation method does not require essentially more computer time when the number of the equations for diameters d_i to be solved is increased. If a sufficient number of stem diameters are calculated, the intermediary diameters can be estimated reliably by means of parabola estimation, using simple algorithms (cf. Henrici 1964, p. 206). In the same way, stem volume can be calculated using, for instance, Simpson's formula, with the result that a continuous taper curve does not have to be constructed for calculating volume.

A polynomial taper curve calculated by the methods presented in Section 33 can be used to estimate a value for the d_i (34.1) closest to the height of the measured diameter. In this case, iteration does not have to be used nor the interpolation formula put into the set of equations (cf. Kilkki and Varmola 1979, p. 298), and the calculation can be carried out even more quickly.

If more than three diameters along the stem, located sufficiently far apart from each other, are known, they can all be easily taken into account in the simultaneous model when calculating the taper curve. The model thus enables a better taper curve to be constructed in these special cases, than can be obtained with other methods. The method provides an effective tool for studying factors which have an effect on the taper curve by including them as exogenous variables in simultaneous equations (cf. Kilkki and Varmola 1981). It would appear that as well as height, other variables such as diameter at breast height, have to be included in the equations as exogenous variables. This is due to the fact that when the diameter at one of the relative heights changes, the other diameters probably change in a more complicated fashion than when the model (34.2) is being solved.

4. TAPER CURVE EQUATIONS

41. Single equations models

411. Basic equations

The basic equations of the taper curve functions were calculated using models (33.1) and (33.2) with the necessary requirements. Parameters were estimated using linear regression analysis programme LM (cf. Pekkonen 1979). The mean diameters at each relative height were calculated by tree species. The diameter at 20 % height was chosen as the reference diameter. The ratios of the diameters at other relative heights to the reference diameter were calculated by the formula:

$$\frac{\sum_j^N d_{,ih}/N}{\sum_j^N d_{,2h}/N} = \frac{\sum_j^N d_{2h} \cdot \frac{d_{,ih}}{d_{,2h}}}{\sum_j^N d_{,2h}} = \frac{\bar{d}_{,ih}}{\bar{d}_{,2h}}$$

As can be seen from the second member of the formula the mean ratio has been weighted by the reference diameters.

As a result of selecting the sample trees with a relascope and using the abovementioned calculation method, large trees have a major weight on the shape of the basic curve. However, as the curves are mainly of the same form for all trees, this weighting has no strong effect on the taper curve equation.

Owing to the limited number of measuring points (14), some additional points were interpolated from the curves shown in Fig. 9. The relative heights and the respective diameters used in calculating the coefficients are presented by tree species in the following set-up:

Set-up 41.1

Relative height, i	Pine ($\bar{d}_{,ih}/\bar{d}_{,2h}$)	Spruce ($\bar{d}_{,ih}/\bar{d}_{,2h}$)	Birch ($\bar{d}_{,ih}/\bar{d}_{,2h}$)
0,01	1,48901	1,51478	1,52682
0,025	1,32747	1,28787	1,28766
0,05	1,20549	1,15993	1,16866
0,075	1,14011	1,10682	1,11573
0,10	1,09780	1,07664	1,08089
0,15	1,04286	1,03561	1,03534
0,20	1,00000	1,00000	1,00000
0,25	0,96480	0,96550	0,96447
0,30	0,92967	0,92577	0,92689
0,35	0,89390	0,88400	0,88640
0,40	0,85769	0,83826	0,84158
0,45	0,81900	0,79000	0,79117
0,50	0,77418	0,73748	0,73501
0,55	0,72720	0,68200	
0,60	0,67143	0,62523	0,60636
0,65	0,61050	0,56400	
0,70	0,54396	0,49849	0,45733
0,75	0,46900	0,43370	
0,80	0,38956	0,35969	0,28969
0,85	0,30000	0,29000	
0,90	0,20395	0,20579	0,12722
0,95	0,10440	0,11500	
1,00	0,00000	0,00000	0,00000

In preparing the basic function for birch, a spline function was calculated with the help of the values at the measuring points and additional values for regression analysis were taken from this curve at the following intervals:

Relative height	
0,02—0,15	0,5 per cent interval
0,16—0,45	1 per cent interval
0,46—1,00	2 per cent interval

The total number of the diameters employed for birch was thus 88. This was found to be a better way to describe the basic curve for birch than using only relative heights of set up 41.1 as was used in the calculations for conifers.

The coefficients for model (33.1) for each tree species were as follows:

		b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
	Pine	2,1288	—0,63157	—1,6082	2,4886	—2,4147	2,3619	—1,7539	1,0817
(41.1)	Spruce	2,3366	—3,2684	3,6513	—2,2608	0,0	2,1501	—2,7412	1,8876
	Birch	0,93838	4,1060	—7,8517	7,8993	—7,5018	6,3863	—4,3918	2,1604

The t-value of the coefficient b_5 in the equation for spruce was not statistically significant and hence this variable was omitted.

412. *Adjustment of the basic equations*

The basic taper curve equations give the same shape of taper curve for trees of all different sizes. The corrections f_i (33.4) for the taper curve, when the tree variables — d and h — are known, are calculated for the relative heights 0,1, 0,4 and 0,7.

First $\hat{d}_{,2h}$ was estimated for each tree from the basic curve with the aid of d and h. Then the difference

$$y_i = z_i - t_i = \frac{d_{,ih}}{\hat{d}_{,2h}} - f_b(i) \quad , i = ,1, ,4, \text{ and } ,7$$

(cf. 33.4) to be used as the dependent variable was calculated. The means ($\bar{y}_i$) and standard deviations (s_i) of the dependent variable can be seen by tree species in the following set-up.

Set-up 41.2

	$\bar{y}_{,1}$	$s_{,1}$	$\bar{y}_{,4}$	$s_{,4}$	$\bar{y}_{,7}$	$s_{,7}$
Pine	0,004317	0,0388	—0,002369	0,0468	—0,002805	0,0669
Spruce	—0,000001	0,0411	—0,000029	0,0536	—0,000424	0,0667
Birch	0,001325	0,0449	—0,009654	0,0637	—0,006199	0,0707

The means of the differences are almost zero. The standard deviations at the relative height 0,7 are the largest.

The height-diameter ratio ((h—1,3) /d) and its modifications appeared to be the most appropriate independent variables in the correction equations. Independent variables which proved to be good for one tree species were often inferior to other variables in the case of other tree species. For this reason different variable combinations have been employed for different tree species.

The equations which gave the best fit, and their coefficients of determination ($100 \cdot R^2$), were as follows:

(41.2)

Pine: $100 \cdot R^2$

$y_{,1} = 0{,}26222 - 0{,}0016245 \cdot d + 0{,}010074 \cdot h + 0{,}06273 \cdot d/(h-1{,}3) - 0{,}011071 \cdot d^2/(h-1{,}3)^2 - 0{,}15496 \cdot \ln(h) - 0{,}45333/h$ — 11,5

$y_{,4} = -0{,}38383 - 0{,}0055445 \cdot h - 0{,}014121 \cdot \ln(d) + 0{,}17496 \cdot \ln(h) + 0{,}62221/h$ — 10,7

$y_{,7} = -0{,}179 + 0{,}037116 \cdot d/(h-1{,}3) - 0{,}12667 \cdot \ln(d) + 0{,}18974 \cdot \ln(h)$ — 16,7

Spruce:

$y_{,1} = -0{,}003133 \cdot d + 0{,}01172 \cdot h + 0{,}48952 \cdot d/(h-1{,}3) - 0{,}078688 \cdot d^2/(h-1{,}3)^2 - 0{,}31296 \cdot \ln(d) + 0{,}13242 \cdot \ln(h) - 1{,}2967/h$ — 5,8

$y_{,4} = -0{,}0065534 \cdot d + 0{,}011587 \cdot h - 0{,}054213 \cdot d/(h-1{,}3) + 0{,}011557 \cdot d^2(h-1{,}3)^2 + 0{,}12598/h$ — 29,1

$y_{,7} = 0{,}084893 - 0{,}0064871 \cdot d + 0{,}012711 \cdot h - 0{,}10287 \cdot d/(h-1{,}3) + 0{,}026841 \cdot d^2/(h-1{,}3)^2 - 0{,}01932 \cdot \ln(d)$ — 24,6

Birch:

$y_{,1} = 0{,}59848 + 0{,}011356 \cdot d - 0{,}49612 \cdot \ln(d) + 0{,}46137 \cdot \ln(h) - 0{,}92116 \cdot (h-1{,}3)/d + 0{,}25182 \cdot (h-1{,}3)^2/d^2 - 0{,}00019947 \cdot d^2$ — 33,2

$y_{,4} = -0{,}96443 + 0{,}011401 \cdot d + 0{,}1387 \cdot \ln(d) + 1{,}5003/h + 0{,}57278 \cdot (h-1{,}3)/d - 0{,}18735 \cdot (h-1{,}3)^2/d^2 - 0{,}00026 \cdot d^2$ — 29,0

$y_{,7} = -2{,}1147 + 0{,}79368 \cdot \ln(d) - 0{,}5181 \cdot \ln(h) + 2{,}9061/h + 1{,}6811 \cdot (h-1{,}3)/d - 0{,}40778 \cdot (h-1{,}3)^2/d^2 - 0{,}00011148 \cdot d^2$ — 25,7

Equations (41.2) give rather large absolute values for some rare d—h — combinations. As the means of the deviations actually measured from the material did not indicate corresponding features, the maximum absolute value 0,1 were set on the taper curve programme for the equations $y_{,1}$, $y_{,4}$ and $y_{,7}$ of each tree species.

When these restrictions were applied, the cubic correction polynomial became logical in the case of d—h — combinations lying outside the limits of the material.

In order to form correction polynomials in cases where d_6 is also known, correction equations were calculated for the relative heights presented in formula (33.8). In this case, only one additional point is required for the cubic polynomial. The point can be either between a height of 1,3 and 6 meters or between a height of 6 meters and the top, depending on the height of the tree. In order to prevent the additional point chosen from producing abrupt changes in the taper curve equation, both of these additional points, suitably weighted, must be taken into account in certain height classes.

The height limits of the sample trees used in constructing the correction equations and the number of trees in the regression analysis were as follows:

			pine	spruce	birch
Additional point:	0,5 + 3/h	$h \geq 7{,}1$ m	2 013	1 604	795
	3,65/h	$6 \text{ m} \leq h \leq 16$ m	1 304	1 010	422

The means ($\bar{y}$) and standard deviations (s) of the differences y_i (cf. df (33.7)), at these additional points, as well as at a height of six meters in the above-mentioned sub-materials are presented in the following set-up:

Set-up 41.3

	Pine		Spruce		Birch	
	$\bar{y}$	s	$\bar{y}$	s	$\bar{y}$	s
Height: 0,5 + 3/h	0,00501	0,0587	0,00273	0,0618	0,00480	0,0628
6 m	0,00327	0,0496	0,00376	0,0579	—0,00738	0,0599
3,65/h	—0,00827	0,0441	—0,00257	0,0517	—0,02913	0,0598
6 m	—0,00182	0,0537	—0,00084	0,0642	—0,01853	0,0644

By comparing the value df (33.7) with the standard deviation s one can make conclusions about normality of the stem form of the tree.

The following equations were derived to describe the deviation between the actual taper curve and the basic equation at the heights (3,65/h) and (0,5 + 3/h):

(41.3)

	$100 \cdot R^2$
Pine:	
$Y36 = -0{,}039003 + 0{,}39739 \cdot df + 0{,}062884 \cdot x2 - 0{,}07917 \cdot x3 + 0{,}04201 \cdot x4$	46,6
$Y05 = -0{,}31017 + 1{,}2036 \cdot df - 0{,}16066 \cdot x1 + 0{,}21072 \cdot x2 + 0{,}096447 \cdot x3 + 0{,}32458 \cdot x5$	47,9
Spruce:	
$Y36 = -0{,}037635 + 0{,}53502 \cdot df + 0{,}032871 \cdot x2 - 0{,}065727 \cdot x3 + 0{,}072073 \cdot x4$	68,4
$Y05 = -0{,}51243 + 1{,}0204 \cdot df + 0{,}40615 \cdot x2 + 0{,}19315 \cdot x3 - 0{,}10077 \cdot x4 + 0{,}52327 \cdot x5$	67,0
Birch:	
$Y36 = -0{,}083783 + 0{,}37229 \cdot df + 0{,}097625 \cdot x2$	39,4
$Y05 = -0{,}86846 + 0{,}61482 \cdot df + 0{,}71465 \cdot x2 + 0{,}079054 \cdot x4 + 0{,}84701 \cdot x5$	43,5

where

Y36 = the correction at the height 3,65/h

Y05 = the correction at the height 0,5 + 3/h

$x1 = df \cdot (1 + |h - 10{,}7|/9{,}4) \cdot (h + 4) / (h - 0{,}7)$

$x2 = d_6/d_{,2h}$

$x3 = (h - 6)/d_6$

$x4 = (h - 1{,}3)/d$

$x5 = (d - d_6)/d$

The variable x1 was derived on the basis of assumption that the deviation of the taper curve from the basic equation (33.1) is at its greatest in the mid point between 1,3 m and the top.

In order to make the taper curve which changes uniformly according to height independent of the choice of the additional point (3,65/h or 0,5 + 3/h) and in order to ensure the logicality of the taper curve, the cubic correction polynomials were calculated for trees 8,35—13,05 m high as follows:

1 °　　　　$h < 8{,}35$ m

The additional point 3,65/h and equation for Y36 are used, but a check is made to ensure that the stem at the height of 3,65 m is not thicker than at the height of 1,3 m.

2 ° $13{,}05\ m \geq h \geq 8{,}35\ m$
a) $h \leq 10{,}7\ m$

The correction value at the height 3,65/h is calculated using (0,5 + 3/h, Y05) as the additional point. The value obtained is denoted by Y1 and TT = (h—8,35) /4,7. The additional point 3,65/h is used and the correction value:

$$TT \cdot Y1 + (1-TT) \cdot Y36$$

The same check is carried out as in 1°.

b) $h > 10{,}7\ m$

The correction value at the height 0,5 + 3/h is calculated using (3,65/h, Y36) as the additional point. This is denoted by Y2. The additional point 0,5 + 3/h is used and the correction value:

$$TT \cdot Y05 + (1-TT) \cdot Y2$$

3 ° $h \geq 13{,}05\ m$

The additional point 0,5 + 3/h and equation for Y05 are used.

42. Simultaneous equations

When the simultaneous equation method was being tested an attempt was first made to elucidate:

1 ° What is the effect on the accuracy of the method if the coefficient matrix A (34.3) is reduced to a tridiagonal one, in which case, Thoma's calculation algorithm (cf. Young 1971, p. 441), for instance, can be used.
2 ° How is the stem volume affected by the fact that the unbiased prediction of d_i is not the unbiased expected value of d_i^2 (cf. Kilkki et al. 1978).

Equations were formed for the diameters at eleven relative heights (1, 5, 10, 20, 30, 40, 50, 60, 70, 80 and 90 percent in order to study point 1 ° (cf. Kilkki et al. 1978). The independent endogenous variables in the complete model were the other diameters at each diameter $d_{,ih}$, and only the adjacent diameters in the tridiagonal model. The exogenous variables were d, h and ln (h).

As an example, regression coefficients and their t-values for the $d_{,3h}$ equations in the spruce material are given:

	Complete model		Tridiagonal model	
variable	coefficient	t-value	coefficient	t-value
constant	0,01651		0,004983	
$d_{,01h}$	—0,005002	—1,581		
$d_{,05h}$	—0,000836	—0,068		
$d_{,1h}$	0,031209	1,728		
$d_{,2h}$	0,438888	24,878	0,466162	32,410
$d_{,4h}$	0,475414	21,752	0,513947	45,902
$d_{,5h}$	0,113709	4,484		
$d_{,6h}$	—0,050320	—2,011		
$d_{,7h}$	—0,001024	—0,043		
$d_{,8h}$	—0,014494	—0,682		
$d_{,9h}$	—0,029939	—1,746		
d	0,015488	1,032	0,021416	2,338
h	0,007434	1,197	0,003503	0,638
ln(h)	—0,009279	—0,195	0,013958	0,296

As can be seen from the t-values of the coefficients for the complete equation, most of the variation in the diameter is explained by the adjacent diameters. The significance of the other diameters, as well as of the exogenous variables, is much smaller. The relative standard error of the estimate (cf. formula (54.1)) had a value of 1,950 % for the complete equation and 1,966 % for the simpler equation.

There was hardly any difference between the complete and tridiagonal simultaneous equation models when the methods were tested on the sample tree material. When testing the solutions given by the set of equations for a number of exceptional trees, e.g. when $d_6 > d$, the taper curve solved using the simpler equations was more logical. The results of these tests indicate that the tridiagonal coefficient matrix is sufficient and justified for endogenous variables.

Since the coefficient matrices can be condensed and effective calculation algorithms, used, an equation for each of the 14 relative heights measured were included in the set of equations. Thus the intervals at the butt end of the stem are short and the error which may arise when the nearest relative height diameter is predicted from the measured diameter by means of the basic polynomial taper curve (33.3), for instance, is insignificant. If several measured diameters are available, then the information which they provide can be used effectively because the same number of equations can be replaced by measurement data.

A system of simultaneous equations including all 14 relative heights was constructed to study point 2 °. The error between the volume obtained using the simultaneous model and the actual volume calculated using measured values was determined for each tree and the error was compared to the actual volume. The volumes of the sample trees were calculated using interpolation parabola and Simpson's formula (see p. 15). In addition, the error between the predicted and actual total volumes was calculated in a similar way. The percentage errors (x_t) of the total volumes (= by volume weighted error percentage) and the means of the percentage errors ($\bar{x}$) calculated for each tree, when one or more of the equations are replaced by the measured diameter ($d_{,ih}$), can be seen in the following set-up. The test was carried out using the spruce material.

Set-up 42.1

$d_{,ih}$	$\bar{x}$, %	x_t, %	$d_{,ih}$	$\bar{x}$, %	x_t, %
$d_{,025}$	0,36	—0,20	$d_{,025}$ and $d_{,4}$	—0,04	—0,06
$d_{,05}$	0,33	—0,20	$d_{,05}$ and $d_{,5}$	—0,02	—0,03
$d_{,1}$	0,14	—0,20	$d_{,1}$ and $d_{,5}$	—0,06	—0,03
$d_{,2}$	0,08	—0,09	$d_{,2}$ and $d_{,7}$	0,03	—0,01
$d_{,3}$	0,06	—0,05	$d_{,3}$ and $d_{,7}$	0,06	0,00
$d_{,4}$	0,03	—0,03	$d_{,1}$, $d_{,4}$ and $d_{,7}$	0,00	0,00

The figures show that when only one diameter is used, the volume obtained for the total volume is slightly too small as the theory indicates (see Kilkki et al. 1978). On the other hand, the means of the percentage errors calculated for each tree are slightly positive on an average.

This indicates that the models give slightly too large volumes for small trees and slightly too small volumes for large trees. However, these errors are so small that they are of no practical importance if more than one diameter is measured on the tree. The effect of the error in question is of the order of a few tenths of one percent if only one diameter is measured. The error is greater if the diameter is measured near the butt because the residual variances of the equations are then greater.

Some idea of the effect of point 2 ° can also be obtained by means of the residual variances of the diameters determined by the simultaneous equations (see Kilkki and Varmola 1981, p. 24).

The mean diameters ($\bar{d}$) of the spruce material and standard errors of the diameters (s) at different relative heights obtained with the tridiagonal model, in which d, h and h^2 were the exogenous variables and $d_{,2h}$ the measured value, were as follows:

Set-up 42,2.

	Relative-height, %													
	1	2,5	5	7,5	10	15	20	30	40	50	60	70	80	90
$\bar{d}$, cm	25,10	21,34	19,22	18,34	17,84	17,16	16,57	15,34	13,89	12,22	10,36	8,26	5,96	3,41
s, cm	2,40	1,34	0,67	0,48	0,42	0,34	0,00	0,44	0,63	0,77	0,87	0,90	0,84	0,66

If there are N trees, which have the same height and the relative height mean diameters and their standard errors as in the set-up 42,2, we can calculate the approximate total volume by formula:

$$v_1 \cong N \sum_i g_{,ih} \cdot l_i$$

where i = ,025, ,075, ,15, ,3, ,5, ,7 and ,9 both

$l_{,025} = 0{,}05 \cdot h = l_{,075}$, $l_{,15} = 0{,}1 \cdot h$ and others

$l_i = 0{,}2 \cdot h$ and $g_{,ih} = \frac{\pi}{4} (\bar{d}^2_{,ih} + s^2_{\bar{d}_{,ih}})$

(see Ilvessalo 1965, p. 199 and Kilkki and Varmola 1981, p. 24).

If we denote

$$v^2 = N \sum_i \frac{\pi}{4} \bar{d}^2_{,ih} \cdot l_i,$$

then we get

$$100 \cdot \frac{v_1 - v_2}{v_1} \cong 0{,}32 \ \%$$

It is quite clear that the result obtained in this way is dependent on the material. In addition, the effect is different in different-sized and different-shaped trees because the accuracy of the equations is poorer at the extreme ends of the material. It should also be pointed out that random direction in measurement of tree diameter tends to lead to overestimation for the volume of the tree. Thus these errors have opposite signs.

Large trees have a greater effect on the coefficients of the linear simultaneous equations (34.1) than small trees because the residual variance of the predicted diameters is clearly greater in the case of large trees, especially at the base of the stem. This is partly due to the fact that the measuring points are situated at relative heights, i.e. the taller the tree, the further they are from each other. As the relative standard error of the diameter estimate is almost constant, the residual variance of the equations is made more homoscedastic by taking logarithms of the variables (cf. Kilkki and Varmola 1981).

Linear regression equations of the simultaneous model for the spruce material were also determined using logarithms of diameters. The exogenous variables for the logarithmic equations were ln(d), ln(h) and $(\ln(h))^2$. By solving the logarithmic equations without correcting the constant (the correction procedure is handled later in section 51) and by replacing the equation of $d_{,2h}$ with

Table 6. Regression coefficients of the simultaneous equations.
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d_i	$b(d_{,i-1})$	$b(d_{,i+1})$	Constant *Vakio*	b(h)	b(d)	$b(h^2)$
			Pine - *Mänty*			
$d_{,01}$	-	1,087248	-,932290	,177595	,009741	-,003978
$d_{,025}$	,331914	,821352	,130481	-,071619	-,118685	,001834
$d_{,05}$	,298023	,639190	,155452	-,022086	,078169	,000175
$d_{,075}$	,354476	,503194	,267336	-,003202	,143073	-,000912
$d_{,1}$	,447359	,407061	,287498	-,001647	,141069	-,000676
$d_{,15}$	,430407	,520176	,143415	-,004027	,039389	,000097
$d_{,2}$	,588495	,383089	,089270	-,014792	,030428	,000246
$d_{,3}$	,491812	,505737	,002080	-,006921	,007116	,000077
$d_{,4}$	,544469	,422078	-,134078	,015574	,026821	-,000777
$d_{,5}$	,471346	,478448	,040910	-,002099	,044098	-,000084
$d_{,6}$	,512931	,480152	-,134950	,026939	,010120	-,000928
$d_{,7}$	,577500	,460664	,047589	-,009805	-,027588	,001031
$d_{,8}$	,560614	,516534	-,097420	-,000457	-,018588	,000416
$d_{,9}$	,545237	-	,174813	,014711	-,027591	,000233
			Spruce - *Kuusi*			
$d_{,01}$	-	1,172479	-,181940	,029400	-,025380	,001355
$d_{,025}$	,256819	,917383	,190010	-,092684	-,123787	,002639
$d_{,05}$	,254357	,587579	,051778	,002367	,186710	-,001945
$d_{,075}$	,309251	,533838	,223150	-,001887	,158407	-,000840
$d_{,1}$	,494979	,395151	,216289	-,018443	,111525	,000052
$d_{,15}$	,338965	,590714	,114421	,000031	,068836	-,000136
$d_{,2}$	,563782	,393229	,038497	,004245	,043910	-,000117
$d_{,3}$	,462868	,516135	-,030958	,013687	,023158	-,000343
$d_{,4}$	,465117	,534914	,050635	-,007752	,011826	,000285
$d_{,5}$	,506565	,522457	-,030397	,015636	-,022485	-,000038
$d_{,6}$	,520184	,503527	-,011262	-,003295	-,007018	,000122
$d_{,7}$	,549628	,486852	-,019198	,010540	-,025709	-,000007
$d_{,8}$	,608233	,395324	-,079438	-,003837	-,019291	,000320
$d_{,9}$	,607975	-	,131230	-,009266	-,016972	,000383
			Birch - *Koivu*			
$d_{,01}$	-	1,309034	-1,176491	,245192	-,226346	-,004463
$d_{,025}$	,301491	,761728	,286117	-,066183	-,016603	,000233
$d_{,05}$	,284686	,233923	,776312	-,064944	,498961	,000083
$d_{,075}$	,253650	,444908	,289145	-,001353	,302879	-,000893
$d_{,1}$	,300422	,509400	,234459	,006522	,187849	-,000710
$d_{,15}$	,426966	,568468	-,058911	,030696	-,008893	-,000685
$d_{,2}$	,552091	,380857	-,010639	,022691	,057950	-,000643
$d_{,3}$	,470993	,464906	,084271	-,015049	,057963	,000562
$d_{,4}$	,530625	,474988	-,187788	,022290	,000498	-,000571
$d_{,5}$	,478935	,456033	,008408	-,008263	,048163	,000551
$d_{,6}$	,541045	,476824	-,008287	-,004305	-,006870	,000180
$d_{,7}$	,461856	,565082	-,013186	,008374	-,000779	,000379
$d_{,8}$	,463331	,757485	-,145929	,015946	-,018028	-,000287
$d_{,9}$	,387040	-	,055823	,023764	-,007265	-,000272

the logarithm of the measured diameter, the mean deviations ($\bar{x}$) of the diameters and their standard errors at different relative heights in the whole spruce material were:

Set-up 42.3

%-height	$\bar{x}$, cm	s, cm	%-height	$\bar{x}$, cm	s, cm
1	—0,09	2,47	30	0,00	0,44
2,5	—0,03	1,45	40	—0,01	0,63
5	—0,01	0,77	50	—0,01	0,78
7,5	0,00	0,52	60	—0,02	0,88
10	0,00	0,43	70	—0,03	0,91
15	0,00	0,34	80	—0,04	0,85
20	0,00	0,00	90	—0,05	0,67

The bias at the 1 % height is greatest because the residual variance of the equation for this height is the largest. The mean of the percentage errors of the volumes calculated for each tree was —0,24 and the standard deviation 6,38. A difference of only —0,08 % occurred in the total volumes.

The bias in the total volume was —0,09 % when the simultaneous model for diameters were used and the mean of the percentage errors calculated for each tree was then 0,08 % (cf. set-up 42.1 on p. 32) and the standard deviation 6,18 %. As none of the other tests indicated any apparent advantage in using logarithmic models, the simultaneous linear equations were computed for diameters, and d, h and h^2 were taken as the exogenous variables. The coefficients of the equations are shown in Table 6.

5. CONSTRUCTION OF VOLUME FUNCTIONS

51. Prerequisites

A logical structure of volume functions is essential because the functions should give accurate volume estimates for trees of all sizes and shapes. If the model is logical, the functions give sensible results for extreme values in the material and even outside these limits.

Volume function models have been derived either by integrating the taper curve models (e.g. Lönnroth 1927) or by applying the volume formulae of geometric rotational sections. The volume of the tree (v) is usually presented using the formula:

(51.1) $v = g \cdot h \cdot f$, where
g = the cross-sectional area of the tree at breast height,
h = tree height, and
f = the form factor of the tree at breast height.

If the dependent variable has been presented as the product of the independent variables, as in Formula (51.1), the model is of the multiplicative type. When solving the parameters of a multiplicative model by means of linear regression analysis, the error term is considered to be a member in the product. Thus, for instance, by denoting the form factor f by the parameter β and the error factor by ϵ, a model for the volume is obtained from Formula (51.1):

(51.2) $v = \beta \cdot g \cdot h \cdot \epsilon$

The function can be linearised by taking the logarithm of both sides of the function

$$\log v = \beta + \log (g \cdot h) + \log \epsilon$$

The requirements on the tests and confidence limits is that $\log \epsilon$ is $N(0, I\sigma^2)$, i.e. that the logarithm of the residual term is normally distributed with a mean of 0 and a standard error of σ.

When the transformation: $f(x) = \log x$ is used, the values close to zero have a large standard deviation after the transformation. If, for instance, when using model (51.2) and the trees in the material are only slightly over 1,3 m in height, the standard deviation of the coefficient determined by regression analysis will be large and the coefficient can easily vary according to the material. Thus the basic prerequisite of regression analysis, that the standard error of the residue term should be homoscedastic, i.e. the standard error be of the same order of magnitude with all combinations of independent variables, has not been fulfilled.

A correction is needed when the logarithmic result is transformed. Let the result given by a linear equation be z_i, then the volume v_i is thus e^{z_i}. Assume that the random variable $\underline{z}_i$ is normally distributed with a mean μ_i and variance σ^2 or $\underline{z}_i \sim N(\mu_i, \sigma^2)$. Thus $\underline{z}_i$ can be presented in the form $\underline{z}_i = \mu_i + \sigma\underline{\xi}$, where $\underline{\xi}$ is N(0,1) and

$$E\underline{v}_i = E(e^{\mu_i + \sigma\underline{\xi}}) = e^{\mu_i}E(e^{\sigma\underline{\xi}}) = e^{\mu_i}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{\sigma\xi}e^{-\frac{1}{2}\xi^2}\,d\xi$$

(51.3)

$$= e^{\mu_i + \frac{1}{2}\sigma^2}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{1}{2}(\xi^2 - 2\sigma\xi + \sigma^2)}\,d\xi = e^{\mu_i + \frac{1}{2}\sigma^2}$$

The correction coefficient $\frac{\sigma^2}{2}$ should be added to the constant coefficient of the linear model (cf. Meyer 1941). If a multiplicativetype model is used and the error term is in a form permitting it to be summed, the parameters should be solved using methods for non-linear parameter estimation (Draper-Smith 1967, p. 132).

If the tree volume is used as the dependent variable and the regression coefficients computed by the method of least squares, the square sum to be minimised is:

$$\sum_{i=1}^{N} (v_i - \hat{v}_i)^2, \text{ where}$$

$\hat{v}_i$ refers to the estimated volume of the ith tree. Thus large deviations, which occur with the largest sample trees, have excessively large weight. Measurement error and protuberances along the stem have the most effect in large trees. This is especially the case when the same number of diameter measurements is made on each stem, because a single measurement thus represents a larger volume in the larger trees. The variances of the regression coefficients are large and in this sense the coefficients are ineffective (Silvey 1970, p. 55). Thus the confidence intervals of the linear combinations of the coefficients, or the estimates obtainable with the equation, are large.

Tree volume can be used as the dependent variable in regression analysis, although in this case regression analysis of the weighted least squares must be carried out. The choice of a suitable weighting variable can be statistically argued using, for instance, model (51.2).

When the variance $D^2(v)$ of v is calculated from the model (51.2) we get:

$$D^2(v) = D^2(\beta \cdot g \cdot h \cdot \epsilon) = \beta^2 \cdot g^2 \cdot h^2 \cdot D^2(\epsilon)$$

It can be seen from this that the variance is proportional to the square of $g \cdot h$. The variance has also been experimentally found to be approximately proportional to the square of the volume (e.g. Cunia 1964). The weight $1/(g_i \cdot h_i)^2$ can thus be used for each tree in the regression analysis. The square sum to be minimised is thus:

$$\sum_{i=1}^{N} \left(\frac{v_i - \hat{v}_i}{g_i \cdot h_i}\right)^2$$

The dependent variable is thus the same as when explaining the breast height form factor ($f = \frac{v}{g \cdot h}$), the independent variables being divided by $g \cdot h$. When the form factor f is used as the dependent variable in the equations, the normality hypothesis, $f = N(0, \sigma^2)$, leads to the distribution hypothesis of the volume: $v = N(0, g^2 \cdot h^2 \cdot \sigma^2)$.

When the breast height form factor is used as the dependent variable, each tree receives almost the same weight in the regression analysis. The variation of the dependent variable is small and the standard error is of the same order of magnitude if there are no really small trees included.

The homoscedasticy of the residual variance means that normal significance tests for regression coefficients can be used and that the confidence limits can be calculated for the estimates.

The derivation of multiplicative volume models and different form factor models are presented in the following. Predictors d, h, and d_6 are included in this order in the models.

52. Multiplicative models

As the stem of a tree can be considered as a three-dimensional rotating object, the natural starting point for constructing a volume function is a multiplicative model. The three factors affecting stem volume can be seen in Formula (51.1)

An equation based only on diameter at breast height is usually computed using the following simple model:

(52.1) $$v = \beta_0 \cdot d^{\beta_1}$$

Model (52.1) does not work satisfactorily with all sizes of tree because diameter d is measured at absolute height, i.e. at a height of 1,3 m above the ground. The diameter at breast height, d, can be used to estimate the stump diameter d_k, e.g. from the equation (cf. Laasasenaho 1975b, p. 7):

(52.2) $$\hat{d}_k = 2{,}0 + 1{,}25 \cdot d$$

Thus the equations obtained using the model

(52.3) $$v = \beta_0 \cdot (2{,}0 + 1{,}25 \cdot d)^{\beta_1}$$

should give better results for small trees than the equations constructed according to model (52.1).

The basic model based on diameter at breast height and height:

(52.4) $$v = \beta_0 \cdot d^{\beta_1} \cdot h^{\beta_2}$$

can be improved if d is replaced by x according to the ratio:

$$\frac{x}{h} = \frac{d}{h - 1{,}3} \text{ or } x = h \cdot \frac{d}{h - 1{,}3}$$

We thus get the following model (cf. Brandel 1974), when each factor is written separately and parametered again:

(52.5) $v = \beta_0 \cdot d^{\beta_1} \cdot h^{\beta_2} \cdot (h - 1{,}3)^{\beta_3}$

A model suitable even for rather small trees is obtained by adding the height h to model (52.3):

(52.6) $v = \beta_0 \cdot (2{,}0 + 1{,}25 \cdot d)^{\beta_1} \cdot h^{\beta_2}$

A third way of improving the model based on diameter and height is to estimate the relative height diameter $\hat{d}_{,2h}$ with the aid of the polynomial basic taper curve model (33.1) and d is replaced by $\hat{d}_{,2h} = \dfrac{d}{f_b(\frac{1{,}3}{h})}$.

More accurate models can be constructed for large trees if the upper diameter, d_6, is assumed to be known. Constructing a multiplicative model with the aid of d, h and d_6 is not as straightforward a task as in the previous case. In this case a better model is arrived at through taper curve derivation.

A curve which approximates the taper curve can be drawn using height, two diameters and their measuring heights. The volume represented by the taper curve is obtained by calculating the degree of parabolity of the curve and then by integrating the curve (Petrini 1948, pp. 11—14).

The degree of parabolity can be obtained on the basis of the diameter at breast height d, the diameter at a height of six meters d_6 and tree height as follows:

$$\frac{h-6}{h-1{,}3} = \left(\frac{d_6}{d}\right)^n \Rightarrow \frac{d_6}{d} = \left(\frac{h-6}{h-1{,}3}\right)^{\frac{1}{n}} \text{ and}$$

$$n = \frac{\log\left(\dfrac{h-6}{h-1{,}3}\right)}{\log\left(\dfrac{d_6}{d}\right)}$$

Thus the diameter d_l at the height h_l from the top is obtained using the formula:

(52.7) $d_l = d \cdot (h - 1{,}3)^{-\frac{1}{n}} \cdot h_l^{\frac{1}{n}}$

Since $\dfrac{d_6^2}{d^2} = \dfrac{g_6}{g}$ we get

$$g_6 = g \cdot \left(\frac{h-6}{h-1{,}3}\right)^{\frac{2}{n}} \quad \text{or generally}$$

(52.8) $g_l = a \cdot h_l^{\upsilon}$ where

$$a = \frac{g}{(h-1{,}3)^{\upsilon}} \quad \text{and } \upsilon = \frac{2}{n}$$

The volume v is obtained by integrating Formula (52.8):

$$v = \int_0^h g_l dl = \int_0^h a h_l^{\upsilon} dl = a \int_0^h h_l^{\upsilon} dl =$$

$$a \Big/_0^h \frac{h_l^{\upsilon+1}}{\upsilon+1} = a \cdot h^{\upsilon} \cdot \frac{h}{\upsilon+1}$$

The regression analysis model is thus obtained

(52.9) $v = \beta_0 \cdot \dfrac{G \cdot h}{\upsilon + 1}$, where $G = g\left(\dfrac{h}{h-1{,}3}\right)^{\upsilon}$

This result is written in the following form for regression analysis:

(52.10) $v = \beta_0 \cdot d^{\beta_1} \cdot h^{\beta_2} \cdot (h - 1{,}3)^{\beta_3} \cdot \left(\dfrac{1}{\upsilon + 1}\right)^{\beta_4}$

The additional variable $\dfrac{1}{\upsilon+1}$ has been included in the model in comparison to model (52.5).

If the degree of parabolity of the stem would be constant throughout the length of the stem, the volume of the stemwood lying above different heights along the stem could be calculated using this model. However, the model cannot be applied for this purpose because the parabolity of the stem is not constant even during the early growing stage.

53. Form models

531. Form height models

The form height ($fh = \frac{v}{g}$) is dependent on the size of the stem because tree height is included in this factor. In the case of quite small trees, when tree height is only slightly over 1,3 m and d therefore very small, the form height value is very large and its variation is also large. Form height based on diameter at breast height is thus not serviceable in the case of small trees. For small trees v/d_k^2 (d_k is the stump diameter estimated using d), for instance, would be a better form height variable.

Tree height and diameter at breast height are in rather fixed correlation with each other and, because the form value f varies only slightly, the construction of the form height model may be based on the model, which depicts height as a function of the diameter.

A relatively simple form height model can be used if its use is restricted to trees with d greater than 5 cm. The parabola, for instance, is an applicable model:

(53.1) $$fh = \beta_0 + \beta_1 \cdot d + \beta_2 \cdot d^2$$

The model for the volume thus becomes

$$v = b_0 \cdot d^2 + b_1 \cdot d^3 + b_2 \cdot d^4$$

A form height model can be made more flexible by using additional variables. Thus a complete quartic polynomial is obtained for the volume by adding the variables 1/d and $1/d^2$ to the model. This polynomial gives almost unbiased volumes, even for small trees, because a constant is also included in the volume equation.

532. Breast height form factor models

If the tree height, as well as diameter at breast height, is known, then the volume of the tree can be considered to consist of two parts: the butt part which extends to a height of 1,3 m, and the rest of the stem above this point. When Equation (52.2), for instance, is used for the stump diameter, then the volume of the butt section can be approximated as a truncated cone and thus the model obtained for the volume is:

(53.2) $$v = \beta_0 + \beta_1 \cdot (d_k^2 + d_k \cdot d + d^2) + \beta_2 \cdot d^2 \cdot (h - 1.3)$$

The breast height form factor is obtained as the dependent variable by dividing both sides of the model by g · h, resulting in the following model:

(53.3) $$f = \frac{\beta_0}{g \cdot h} + \beta_1 \cdot \frac{d_k^2 + d_k \cdot d + d^2}{g \cdot h} + \beta_2 \cdot (1 - \frac{1,3}{h})$$

The coefficient β_2 in model (53.2) is dependent on the size of the tree and hence additional variables are needed for the model. The model is not very good for small trees, and the residual variation is more heteroscedastic than with the corresponding multiplicative model.

The procedure can also be applied to a situation where trees are over 6 m high and for which d_6, as well as d and h, are known. In this case the volume is considered to consist of three parts: the butt section, the section between the heights 1,3 and 6 m, and the section of the stem above the height of 6 m. If the butt section is considered as a cylinder, the section between the heights 1,3 and 6 m as a truncated cone, and the uppermost section as a cone, then the following model is obtained:

(53.4) $$v = \beta_0 + \beta_1 \cdot d^2 + \beta_2 \cdot (d^2 + d \cdot d_6 + d_6^2) + \beta_3 \cdot d_6^2 \cdot (h - 6,0)$$

The breast height form factor model thus becomes:

(53.5) $$f = \frac{\beta_0}{g \cdot h} + \frac{\beta_1}{h} + \beta_2 \frac{d^2 + d \cdot d_6 + d_6^2}{g \cdot h} + \beta_3 \frac{d_6^2 \cdot (h - 6,0)}{g \cdot h}$$

Coefficient β_3 in Model (53.4) is dependent to some extent on the size of the tree, as β_2 in the model (53.2), and so the explanatory power of the model can be improved by using additional variables.

533. *Normal form factor models*

Tree volume and stem form often have to be determined more accurately for research purposes than is possible with usual sample tree measurements. Diameters measured at different relative heights have been used to describe the form of stems (see e.g. Prodan 1961, pp. 30—50). These diameters can also be used to construct models for computing volume functions which can be applied to all sizes of tree and which are usually more accurate than models encorporating the same number of diameters measured at absolute heights.

The model can be constructed in a corresponding way as for the breast height form factor model. Let the diameters $d_{,1h}$, $d_{,3h}$ and $d_{,5h}$, as well as tree height, be known. The tree volume v can be presented, for instance, by the model:

$$v = \beta_0 + \beta_1 \cdot d^2_{,1h} \cdot h + \beta_2 \cdot d^2_{,3h} \cdot h + \beta_3 \cdot d^2_{,5h} \cdot h \tag{53.6}$$

The following model is thus obtained for the normal form factor

$$f_{,1h} = \frac{v}{g_{,1h} \cdot h} :$$

$$f_{,1h} = \frac{\beta_0}{d^2_{,1h} \cdot h} + \beta_1 + \beta_2 \cdot \frac{d^2_{,3h}}{d^2_{,1h}} + \beta_3 \cdot \frac{d^2_{,5h}}{d^2_{,1h}} \tag{53.7}$$

As model (53.7) does not take the stem form completely into account, additional variables, such as $d_{,5h}/d_{,1h}$, $1/d_{,1h}$ or $1/h$ can make the model more precise.

When diameters at relative heights are used, different variations of the model can be constructed according to the above-mentioned principle. As far as regression analysis is concerned, the same model as (53.6) is arrived at if the following model is used for tree volume:

$$v = \beta_0 + \beta_1 \cdot d^2_{,1h} \cdot h + \beta_2 \cdot (d^2_{,1h} + 4 \cdot d^2_{,3h} + d^2_{,5h}) \cdot h + \beta_3 \cdot d^2_{,5h} \cdot h \tag{53.8}$$

If additional relative height diameters are included in the model, it can be done in a corresponding fashion. However, if diameter at breast height is included in the model, construction of the model is no longer as clear-cut. Pollanschütz (1965) has presented a way to predict the breast height form factor f by the model:

$$f = \beta_0 + \beta_1 \frac{d^2_{,3h}}{d^2} + \beta_2 \frac{d_{,1h} \cdot d_{,5h}}{d^2} + \beta_3 \frac{h}{d^2} \tag{53.9}$$

If d, h and $d_{,3h}$ as a relative height diameter, for instance, are known, the following model can be used for the tree volume:

$$v = \beta_0 + \beta_1 \cdot (d_k^2 + d \cdot d_k + d^2) + \beta_2 \cdot (0{,}3 - 1{,}3) \cdot (d^2 + d \cdot d_{,3h} + d^2_{,3h}) + \beta_3 \cdot d^2_{,3h} \cdot h \tag{53.10}$$

where d_k (52.2) is estimated using diameter at breast height.

In regression analysis f or also $f_{,3h} = \frac{v}{g_{,3h} \cdot h}$ can then be used as the dependent form factor.

54. Comparison of the models and reliability

A commonly used yardstick in estimating the reliability of functions is their coefficient of determination. However, this parameter cannot be used for comparisons between form factor equations and logarithmic equations obtained from multiplicative models. This is due to the fact that the dependent variables are not the same. Therefore, if the comparison is to be performed using a single parameter, then some other parameter has to be used.

The error in a multiplicative model is the product factor, as can be seen from model (51.2), and the error is thus of the same relative order of magnitude in trees of different size. The standard deviation of the observed values about a regression line, i.e. the standard error of estimate s, as a percentage of the mean response (e.g. Draper-Smith 1967, p. 119), thus gives quite a good picture of the reliability of the equation. In the case of a multiplicative model this parameter is easy to calculate as it can be seen from the following derivation procedure (cf. Meyer 1938).

The expected value of the squared volume (cf. (51.3)) is

$$Ev_i^2 = E\,(e^{2\mu_i + 2\sigma\xi}) = e^{\mu_i + \frac{(2\sigma)^2}{2}}$$

The variance Dv_i^2 is thus

$$D\,v_i^2 = Ev_i^2 - (Ev_i)^2 = e^{2(\mu_i + \sigma^2)} - e^{2(\mu_i + \frac{\sigma^2}{2})}$$
$$= e^{2\mu_i + 2\sigma^2} \cdot (1 - e^{-\sigma^2})$$

The relative standard error of the estimate (s_r) is obtained using the formula:

$$(54.1) \qquad s_r = \frac{Dv_i}{Ev_i} = \frac{e^{\mu_i + \sigma^2}\sqrt{1 - e^{-\sigma^2}}}{e^{\mu_i + \frac{\sigma^2}{2}}}$$
$$= e^{\frac{\sigma^2}{2}}\sqrt{1 - e^{-\sigma^2}} = \sqrt{e^{\sigma^2} - 1}$$

The Taylor series shows that the result is practically of the same size as σ, if σ is very small as is the case in models (52.5) and (52.10). The result given by Formula (54.1) can be approximated by calculating the difference between the estimated and actual value of each observation, divided by the estimated value, and then calculating the standard deviation of these relative errors (cf. e.g. Vuokila 1960, p. 13).

The relative standard error of the equation in the form factor model is obtained directly by comparing the residual standard error of the form factor equation with the mean value of the form factor. This parameter expressed as percentage has been used as the criterion in determining the fitness of the equations in the following section. A more precise comparison of the equations requires examination of the residual variance.

6. VOLUME EQUATIONS

61. Equations based on fixed height diameters and height

The equations are divided into three groups according to the predicting variables used in deriving the volume function models:

Predicting variables	Models
1° d	52.1, 52.3 and 53.1
2° d and h	52.4, 52.5, 52.6 and 53.3
3° d, h and d_6	52.10 and 53.5

The equations were computed for each basic model by tree species. Only sample trees measured on forest land were utilized in computing the equations of group 1°. With the type of sampling method used, the equations give too small volumes even for trees growing on forest land. The only real use for these equations is hence as some sort of quideline curves, their calibration always has to be done separately. Only those sample trees with d > 1 cm were included when computing the equations of group 1° and 2°. When this condition was set only three sample trees in the pine material had to be omitted. This restriction was set because large residual deviations were found with these trees. Sample trees over 7,0 m in height were included in computing the equations of group 3°.

The equations were first computed using the basic variables of the models in section 5. The equations can be made more precise with additional variables. Therefore, the tree measurements variables and different modifications of them were tested as additional variables. However, the relative standard errors s_r (cf. 54.1) of the equations obtained with the basic models were only little decreased by including these additional variables. Since no significant degree of bias was found when the residual variations of the equations were examined, only a few additional variables were included in the equations.

The same variables were included in each model for all three tree species when selecting the final equations. Although an additional variable, other than the one selected, gave in some cases a slightly smaller s_r-value for a certain tree species, the differences were statistically insignificant. As the variables are the same for all the tree species, the differences in the volumes given by the equations for different tree species are dependent on the tree species and also to some extent on the material, but not on the model. The differences between tree species can then be easily compared.

Equations were determined for volumes with bark. In addition equations based on d_u and h were computed for volume without bark. These equations are regarded as useable in growth estimation. The multiplicative models were linearised by taking the logarithm and the correction coefficients $\frac{\sigma^2}{2}$ (see (51.3)) have been added to the constant terms of the given logarithmic equations.

611. Equations based on d

The relative standard errors of the estimates obtained with multiplicative models were clearly smaller than those for equations obtained using form height models. For this reason, only the equations derived using multiplicative models are presented here. All the sample trees growing on forest land, 2 050 pine, 1 841 spruce and 834 birch, were included in the analyses.

The equations obtained using model (52.1) and their relative standard errors (s_r, %) were:

			s_r, %
	Pine	$\ln(v) = -2{,}29450 + 2{,}57025 \cdot \ln(d)$	17,9
(61.1)	Spruce	$\ln(v) = -2{,}41218 + 2{,}62463 \cdot \ln(d)$	19,7
	Birch	$\ln(v) = -2{,}09787 + 2{,}55058 \cdot \ln(d)$	19,9

The difference between the powers of the diameters for pine and birch is only about 0,02, but the constant is larger in the case of birch. The values of the coefficients are partly material-specific, because the value of the power is also to some extent dependent on the size of the tree.

The standard errors of the equations given by model (52.3) were greater than those of equation (61.1). However, when d was included in logarithmic version of model (52.3) as the additional variable, then more precise equations were obtained than if d had been included in logarithmic version of model (52.1). None of the other additional variables were able to reduce the residual variances to any noticeable degree.

The accuracy of equations based on diameter at breast height alone is in any case poor, and the equations are materialspecific. In some inventory tasks guideline volume curves, based on diameter only, are required. The following equations, which have been obtained by adding d as the variable to the logarithmic version of model (52.3), can be used for this type of task.

			s_r, %
	Pine	$\ln(v) = -5{,}39417 + 3{,}48060 \cdot \ln(2 + 1{,}25 \cdot d) - 0{,}039884 \cdot d$	17,2
(61.2)	Spruce	$\ln(v) = -5{,}39934 + 3{,}46468 \cdot \ln(2 + 1{,}25 \cdot d) - 0{,}0273199 \cdot d$	18,7
	Birch	$\ln(v) = -5{,}41948 + 3{,}57630 \cdot \ln(2 + 1{,}25 \cdot d) - 0{,}0395855 \cdot d$	18,8

612. *Equations based on d and h*

Multiplicative models also proved to be the best in constructing equations based on diameter and height. As the residual variance of the logarithmic equations was rather homoscedastic, the relative precision of the equations is thus rather constant with respect to the tree size.

The equations were computed according to models (52.4), (52.5) and (52.6). The residual variances of model (52.5) were clearly the smallest for all three tree species. The precision of the models was improved to some extent by additional variables. When diameter d alone was used as the additional variable in the logarithmic models, the decrease of the residual variances of all the models was highly significant. The "basal diameter", ln(20 + d) of the tree, presented by Brandel (1974), also significantly decreased the residual variance of model (52.5).

Adding the variable ln(h—1,3) to the logarithmic model (52.6) decreased its residual variance the most. When the variables ln(d), d and ln(h—1,3) were added to model (52.6), the regression coefficient of variable ln(2+1,25·d) was no longer significant in spruce and its additional explanatory value with the other tree species was very small. However, the diameter at breast height proved to be the best additional independent variable, and so the final equations were computed using model (52.5) with the variable d added to the logarithmic equation.

The relative standard errors of the different equations for the different tree species are presented in the following set-up.

Set-up 61.1

	Model			
	(52.4)	(52.5)	(52.6)	final-eq.
	Relative standard errors of the equations, %			
Pine	8,04	7,14	7,86	7,10
Spruce	10,24	7,66	8,61	7,47
Birch	9,96	8,65	9,74	8,23

The differences between the models are quite clear. Adding diameter to the logarithmic model (52.5) had different effects on the precision with different tree species. Birch has an irregular stem form and the precision of its equations are the poorest but the improvement in the precision of the model when d is added to the model is greatest.

The coefficients of the final logarithmic equations and the t-values of the coefficients were:

		constant	ln(d)	ln(h)	ln(h—1,3)	d
	Pine	—3,32176	2,01395	2,07025	—1,07209	—0,0032473
(61.3)	Spruce	—3,77543	1,91505	2,82541	—1,53547	—0,0085726
	Birch	—4,49213	2,10253	3,98519	—2,65900	—0,0140970

The t-values of the coefficients

	constant	ln(d)	ln(h)	ln (h—1,3)	d
Pine	78,35	143,84	37,18	22,20	4,75
Spruce	84,49	107,11	54,10	34,39	9,97
Birch	49,93	82,40	25,62	19,22	9,54

The t-values of the coefficients are high, even though the variables are strongly intercorrelated. However, omitting variable d in equation for pine increased the standard error of the equation by only 0,04 %-units even though the t-value for the coefficient of this variable is highly significant.

The coefficients and precision of the equations for the volumes without bark (v_u) when using the under-bark diameter were as follows.

		constant	$\ln(d_u)$	ln(h)	ln(h—1.3)	d_u	s_r, %
	Pine	—3,61554	2,05534	2,30886	—1,21013	—0,0057527	7,5
(61.4)	Spruce	—3,88390	1,89496	2,98696	—1,64418	—0,0091650	8,1
	Birch	—4,45962	2,06695	3,95373	—2,61998	—0,0115929	8,4

The coefficients of the equations for the tree species differ quite considerably from each other. As the coefficients are dependent on each other, the magnitude of the individual coefficients cannot be used to draw conclusions from the differences between tree species. Such examinations have to be made using the results given by the equations or their derivatives.

The partial derivatives of the equations are negative with respect to height in the case of small trees. With equation (61.4) the null point for pine is at 2,70 m, for spruce at 2,85 m and for birch at 3,91 m. Thus, for shorter trees with the same diameter, the volume decreases as the height increases. This is theoretically possible if the stem form, i.e. the ratios between the diameters at different relative heights, are the same. The variable ln(h—1,3) has a decisive effect on the derivative with respect to height in the case of small trees. However, the use of these equations is not recommended for pine and spruce under three meters high and for birch under four meters high.

613. Equations based on d, h and d_6

The equations in this group were calculated according to both multiplicative model (52.10) and form factor model (53.5). The inclusion of additional variables only a little increased the precision of the equations in the case of the multiplicative model. Including d and h as additional variables significantly increased the precision of the model (53.5) for all three tree species. Variable $1/(d^2 \cdot h)$ was no longer statistically significant and was therefore omitted from the final form factor equations. The equations obtained using the form factor were slightly more precise than equations derived from the multiplicative model. As this form factor model has a rather simple structure and no biases were found in the residual variances, the form factor model was accepted as the basis for the final equations.

The relative standard errors s_r of the equations for models (52.10) and (53.5), and for the final form factor equation are presented in the following set-up:

Set-up 61.2)

	(52.10)	(53.5)	final-eq.
Pine	3,59	3,63	3,53
Spruce	3,58	3,52	3,38
Birch	5,16	5,13	4,96

The standard errors s_r for the models in this group are about half of those for the group 2° (Set-up 61.1) models. The equations for spruce are the most precise and for birch clearly the least.

The coefficients of the equations for the final model and the t-values of the coefficients were as follows:

		constant	d	h	1/h	$\frac{d^2+d\cdot d_6+d_6^2}{d^2\cdot h}$	$\frac{d_6^2\cdot(h-6{,}0)}{d^2\cdot h}$
	Pine	—0,185311	—0,000609408	0,00425391	3,42019	1,23905	0,561137
(61.5)	Spruce	—0,190435	—0,00145666	0,00556127	2,64888	1,70546	0,476954
	Birch	—0,133297	—0,00155664	0,00557721	2,88448	1,26257	0,426327

The t-values of the coefficients

Pine	13,68	8,27	8,86	44,57	25,51	42,84
Spruce	15,10	10,97	10,84	38,03	36,01	37,15
Birch	4,34	7,30	5,18	22,84	11,69	20,67

The volume is obtained in liters (i.e. dm^3) using the formula:

$$v = 0{,}07854 \cdot h \cdot f$$

When the variables were chosen according to the forward selection procedure for equations (61.5), i.e. on the basis of the increase in the explanatory value of each remaining variable, the variables for the pine equation came in the following order:

$(d^2+d\cdot d_6+d_6^2)/(d^2\cdot h)$, h, 1/h, $d_6^2\cdot(h-6{,}0)/(d^2\cdot h)$ and d

The corresponding values for the standard errors of the equations were as follows: 5,87, 5,18, 4,94, 3,59 and 3,53 %. The standard error of the estimate is already reduced to a greater extent by using one variable, i.e. the truncated cone between heights of 1,3 and 6 m, than with the two variable equation (61.3). The clear decrease in the residual variance obtained by adding fourth variable shows that the pieces of this model fit together. The inclusion of d increases the explanatory power only little.

The above order of the variables do not indicate that, for instance, the first two variables in the list would give the best two-variable equation. The best two variable equation was given by the variables 1/h and $d_6^2 \cdot (h-6{,}0)/(d^2 \cdot h)$. The precision of this equation is even better than that of the equations formed from the first three variables in the list.

When equations (61.3) are transformed into multiplicative form and the coefficients and variables of form factor equations (61.5) are multiplied by the factor $\frac{\pi}{40} \cdot d^2 \cdot h$, the equations for calculating the volume are as follows:

v = f(d, h)

(61.6)

Pine $v = 0{,}036089 \cdot d^{2{,}01395} \cdot (0{,}99676)^d \cdot h^{2{,}07025} \cdot (h-1{,}3)^{-1{,}07209}$

Spruce $v = 0{,}022927 \cdot d^{1{,}91505} \cdot (0{,}99146)^d \cdot h^{2{,}82541} \cdot (h-1{,}3)^{-1{,}53547}$

Birch $v = 0{,}011197 \cdot d^{2{,}10253} \cdot (0{,}98600)^d \cdot h^{3{,}98519} \cdot (h-1{,}3)^{-2{,}65900}$

v = f(d, d_6, h)

(61.7)

Pine $v = 0{,}268621 \cdot d^2 - 0{,}0145543 \cdot d^2 \cdot h - 0{,}0000478628 \cdot d^3 \cdot h + 0{,}000334101 \cdot d^2 \cdot h^2 + 0{,}0973148 \cdot (d^2 + d \cdot d_6 + d_6^2) + 0{,}0440716 \cdot d_6^2 \cdot (h-6)$

Spruce $v = 0{,}208043 \cdot d^2 - 0{,}0149567 \cdot d^2 \cdot h - 0{,}000114406 \cdot d^3 \cdot h + 0{,}000436781 \cdot d^2 \cdot h^2 + 0{,}133947 \cdot (d^2 + d \cdot d_6 + d_6) + 0{,}0374599 \cdot d_6^2 \cdot (h-6)$

Birch $v = 0{,}226547 \cdot d^2 - 0{,}0104691 \cdot d^2 \cdot h - 0{,}000122258 \cdot d^3 \cdot h + 0{,}000438033 \cdot d^2 \cdot h^2 + 0{,}0991620 \cdot (d^2 + d \cdot d_6 + d_6^2) + 0{,}0334836 \cdot d_6^2 \cdot (h-6)$

The diameters are expressed in centimeters, the height in meters and the resulting volumes in dm^3. Tree height and the measuring height of the diameters used in the equations are determined with respect to the ground surface.

62. Equations based on relative height diameters and height

The models (52.1), (52.4) and (52.10) can be employed when the diameter is measured at a relative height. If the normal form factor $f_{,1h}$ is used as the dependent variable, then model (53.6) can be employed as the basis for the equations. The precision of the equations is dependent both on the number of the known diameters and on their position along the stem.

The standard errors of the equations calculated for relative height diameter model according to models (52.1) and (52.4) are presented in the following set-up for different tree species. Diameter has been measured alternatively at heights corresponding to 10, 20, 30, 40 or 50 % of the tree height.

Set-up 62.1

	Model (52.1)					Model (52.4)				
	Measurement height of diameter, %									
	10 %	20 %	30 %	40 %	50 %	10 %	20 %	30 %	40 %	50 %
	Relative standard errors of the equations, %									
Pine	28,7	27,9	26,6	25,0	24,3	7,2	6,1	5,6	6,2	8,2
Spruce	26,1	21,0	18,8	18,2	19,8	8,0	6,0	5,1	6,7	10,4
Birch	24,0	22,1	21,1	20,4	20,9	7,3	6,6	6,3	7,9	10,6

It can be seen from the results for model (52.1), that the tree volume cannot be estimated as reliably using one relative height diameter alone as with diameter at breast height. The absolute measuring height also includes some information about the tree height.

The precision of the equations given by model (52.4) is significantly dependent on the height at which diameter is measured. Out of all the diameters tested, $d_{,3h}$ was found to be overwhelmingly the best diameter for this model. The amount of butt swelling at that height is no longer significant and the degree of knobbiness and branchiness of the stem is small and the cross-section of the stem almost circular. The good usability of this diameter has also been recognised by Pollanschütz (1965). The equations based on $d_{,3h}$ and height were:

(62.1)

$$\text{Pine} \quad \ln(v) = -2{,}95660 + 1{,}93299 \cdot \ln(d_{,3h}) + 1{,}09664 \cdot \ln(h)$$
$$\text{Spruce} \quad \ln(v) = -2{,}98123 + 1{,}96332 \cdot \ln(d_{,3h}) + 1{,}04942 \cdot \ln(h)$$
$$\text{Birch} \quad \ln(v) = -2{,}98462 + 1{,}91355 \cdot \ln(d_{,3h}) + 1{,}09111 \cdot \ln(h)$$

The coefficients for the equations are very similar in the case of all three tree species. The sum of the powers of diameter and height is almost exactly three for each tree species. Proposals have been put forward for constructing volume functions on the basis of this "3-rule" (Spurr 1952, p. 93). This rule does not seem to hold true for other relative heights.

When carrying out the calculations according to model (53.6), the diameters $d_{,025h}$ and $d_{,7h}$ were tested with $d_{,1h}$, $d_{,3h}$ and $d_{,5h}$. The following set-up demonstrates how the standard error of the equation varies when different diameter combination are used in the normal form factor model (53.6).

Set-up 62.2

	Measurement-heights of diameters in the model, %				
	10 and 30	10 and 50	10, 30 and 50	10, 30, 50 and 70	2,5 , 10, 30 50 and 70
	Relative standard errors of the equations, %				
Pine	5,24	3,58	3,24	2,53	2,35
Spruce	4,53	3,16	2,74	2,34	2,23
Birch	5,20	4,04	3,20	2,76	2,57

Combination of $d_{,1h}$ and $d_{,5h}$ clearly gave a better result than combination of $d_{,1h}$ and $d_{,3h}$. The precision of the equations obtained using diameters $d_{,1h}$, $d_{,5h}$ and tree height were better in the case of spruce and birch than with equations (61.5), but about equal for pine. However, results cannot be compared directly to the precision of equations (61.5), because all the sample trees were not included when calculating these equations.

When additional variables 1/h and $1/d_{,1h}$ were included in the form factor equation with diameters $d_{,1h}$ and $d_{,5h}$, the standard error for the pine equation was 3,23 %.

When these additional variables 1/h and $1/d_{,1h}$ were added to the model (53.7), the equations and standard errors were:

			s_r, %
	Pine	$v = 0{,}0012899 \cdot d^2_{,1h} \cdot h + 0{,}00107444 \cdot d^2_{,3h} \cdot h + 0{,}00224611 \cdot d^2_{,5h} \cdot h + 0{,}000534442 \cdot d_{,1h} \cdot h - 0{,}00123071 \cdot d^2_{,1h}$	2,79
(62.2)	Spruce	$v = 0{,}00122392 \cdot d^2_{,1h} \cdot h + 0{,}00105269 \cdot d^2_{,1h} \cdot h + 0{,}00227640 \cdot d^2_{,5h} \cdot h + 0{,}000479235 \cdot d_{,1h} \cdot h - 0{,}00092256 \cdot d^2_{,1h}$	2,40
	Birch	$v = 0{,}00128054 \cdot d^2_{,1h} \cdot h + 0{,}00119148 \cdot d^2_{,3h} \cdot h + 0{,}00186415 \cdot d^2_{,5h} \cdot h + 0{,}000748341 \cdot d_{,1h} \cdot h - 0{,}00141218 \cdot d^2_{,1h}$	3,07

The standard errors of the equations based on the model (53.9) were clearly greater than in equations based on model (53.7). Using $f_{,1h}$ as the dependent variable gave a better result than f in the materials, which included trees 2—3 m tall.

63. Other equations tested

In addition to the equations presented above, equations based on other models were also computed. For instance, the volume equations models currently in use in Sweden were tested. Trees with a diameter $d_u \geq 3$ cm were included in the computation. The equations were calculated separately using the variables of large and small equations with diameters over bark (Näslund 1947, p. 13). The breast height form factor f was used as the dependent variable. The coefficients of the equations obtained from the material for this study (F) and those applicable for the whole of Sweden (S) are for pine:

(63.1) Small equations	s^2	$d^2 \cdot h$	$d \cdot h^2$
F:	0,0906	0,03040	0,002537
S:	0,1028	0,02705	0,005215

(63.2) Large equations	d^2	$d^2 \cdot h$	$d^2 \cdot k$	$d \cdot h^2$	$d \cdot h \cdot b$
F:	0,1055	0,03084	0,006489	0,001025	—0,002931
S:	0,1121	0,02919	0,006285	0,002460	—0,003574

The symbol k denotes the height (m) of the low limit of the living crown from the ground and b double the bark thickness (mm) at breast height.

The coefficients obtained for all the other variables, apart from $d \cdot h^2$, are of the same order of magnitude. The derivatives of the equations with respect to height differ considerably from each other owing to the differences between the coefficients of the variable $d \cdot h^2$. The t-values of the coefficients of all the variables were statistically significant.

The standard errors of the small equations (F) and of the equations (denote T) computed from the same material using variables of the multiplicative equations (61.6) are presented by tree species in the following set-up:

Set-up 63.1

	Pine	Spruce Standard errors, %	Birch
F	7,37	7,79	8,31
T	7,03	7,25	8,16

The standard errors given by the multiplicative model were clearly smaller than these for the model of the form factor equation (63.1). When the additional variables according to model (53.3) were added to the form factor equations, the t-values of the coefficients of these variables were significant and the standard errors then became considerably closer to the standard errors of the multiplicative model. When the standard errors of the multiplicative models presented in section 61 are compared to the above values, it can be seen that omitting the smaller sample trees has clearly reduced the residual variance. The diameter restriction resulted in the exclusion of 18 of the smallest pines, 24 of the smallest spruce and 17 of the smallest birch from the analyses.

Adding the low limit of the crown and the bark thickness to the equations, according to the equations used in Sweden, only slightly decreased the residual variance, as can be seen from the following standard errors of the large equations:

Pine	Spruce	Birch
6,93	7,35	8,29

The superiority of the second diameter in comparison to these factors becomes indisputable when the precision of the estimate for the volume of individual trees is examined with the models used here. Presumably crown length and even bark thickness enable tree volume to be described considerably more effectively using simultaneous taper curve models (cf. Kilkki and Varmola 1981).

A considerable amount of the residual variance in the volume equations is caused by the variation in stump height. A stump height of at least 10 cm, which was used when calculating the volume, causes a small amount of bias in the result obtained for very small trees. For instance model (53.7) would give a more accurate result if the stump height were in the model as an additional variable. Adding the variable h_k/h (h_k = stump height) to the equations computed with the principle of model (53.7), reduced the standard errors statistically significantly. When the diameters $d_{,025h}$, $d_{,1h}$, $d_{,3h}$, $d_{,5h}$ and $d_{,7h}$ were included in the model, as well as variable h_k/h, the standard errors were: pine 1,57, spruce 1,37 and birch 2,09 per cent. Stump height is a better regressor of volume in these equations than, for instance, the diameter $d_{,025h}$, as can be seen when these standard errors are compared with the values presented in the set-up 62.2. The standard error of the equation for spruce, 1,37, was even smaller than the standard deviation between the exact volume calculated using the Simpson rule and that obtained using corresponding diameters in the spline functions, 1,39 % (cf. Lahtinen and Laasasenaho 1979, p. 33 and 45).

7. RELIABILITY OF THE MODELS

71. Methods for estimating reliability

The methods for assessing the reliability of the models can be divided into two groups:

— theoretical examinations
— empirical tests

When theoretical examinations are carried out, the maximum error of the mathematical probability of different-sized errors, for instance, are studied. Empirical tests are done by either examining the distributions of the errors with respect to different variables in the original material, or by testing the models in a separately measured control material.

Reliability is often examined with respect to variables other than those included in the equations, so as to get some idea about any possible omissions of important variables. The use of a separate test material is an effective technique because this material can be selected in such a way which is interesting from the point of view of the testing. The results obtained with the models can of course also be compared with earlier results.

The relative standard errors of regression equations (see (54.1)) have already been used to determine the reliability of volume equations. If diameters are measured at absolute heights, the values of the relative standard error are not the same for trees of all different sizes (see p. 41 and cf. Kilkki and Varmola 1981, p. 36).

Volume equations are most reliable in the middle range of the sample trees. The variance of the estimate given by a linear regression equation can be calculated as follows (cf. e.g. Huang 1970, p. 79):

(71.1) $D^2(\hat{v}_e) = x_p(X'X)^{-1}\, x_p' \cdot s^2$, where

$X'X$ = the moment matrix of the independent variables,

s^2 = the residual mean square of the regression equation, and

x_p = the vector containing the independent variables used.

The variance for a predicted value of an individual observation, square of the prediction error, is obtained by adding the residual mean square of the regression equation to formula (71.1), i.e. through the formula:

(71.2) $D^2(\hat{v}_p) = (x_p(X'X)^{-1}\, x_p' + 1)\, s^2$

The variance obtained with formula (71.1) is very small in the volume equations compared to the residual variance (s^2).

The field of applications of the taper curve models is wide and the reliability of the results obtained from each taper curve model should be studied separately. Owing to the way in which the basic and correction equations of the polynomial taper curve models presented in this study were formed, the theoretical reliability values for the diameters obtained with the taper curve equations can be presented only for those points along the stem for which correction equations have been prepared. It is not possible to derive theoretical formulae which would give reliability values for other diameters and for volume. The logical requirements of the taper curve, as for instance the monotonity, can be tested and also regulated by means of a correction equation. By comparing the diameters given by the taper curve model with the measured values, an empirical estimate of the reliability of the taper curve model over the area covered by the material is obtained. The reliability of the volume can be examined in the corresponding manner.

Theoretical precisions for the points along the stem included in the model can be calculated if the simultaneous taper curve model is applied (cf. Kilkki and Varmola 1981). As well as depending on the accuracy of the equations in the simultaneous model, the accuracy of other points of the taper curve also depends on the method used for interpolating the curve. The spline function can be used to calculate, for instance, the theoretical maximum error for the intermediate points (cf. Lahtinen and Laasasenaho 1979, p. 22). Formulae can be derived for the reliability of stem volume using simultaneous equations (cf. Kilkki and Varmola 1981). These results can, of course, be compared to reliability values obtained empirically.

Quite a good picture of the reliability of the taper curve is obtained by examining the taper curve visually. The curve passes through the given points (d, d_6, h) and if the curve is also logical elsewhere, then the volume obtained as an integral is close to the correct one. Computer programs were developed for plotting taper curves. In addition to the spline function based on actual measurement data, three tapes curves estimated could be displayed simultaneously on the same picture. As an example the polynomial taper curves and the measured taper curve for a spruce sample tree and the volumes obtained by these curves are presented in Fig. 10.

72. Reliability of the taper curves in predicting the diameters

The biases and standard errors of the diameter estimates using taper curve models at each relative height in the whole material, d and h being known, are presented in Table 7. The polynomial model (33.1) is slightly biased. The terms in the polynomial used as the basic taper curve should be of a still higher power if there are to be no systematic errors for the butt. The basic model (33.2) gave better estimates than the model (33.1) for the diameters at the relative heights 1, 2,5 and 5 % in the case of birch, although it is not applicable right at the

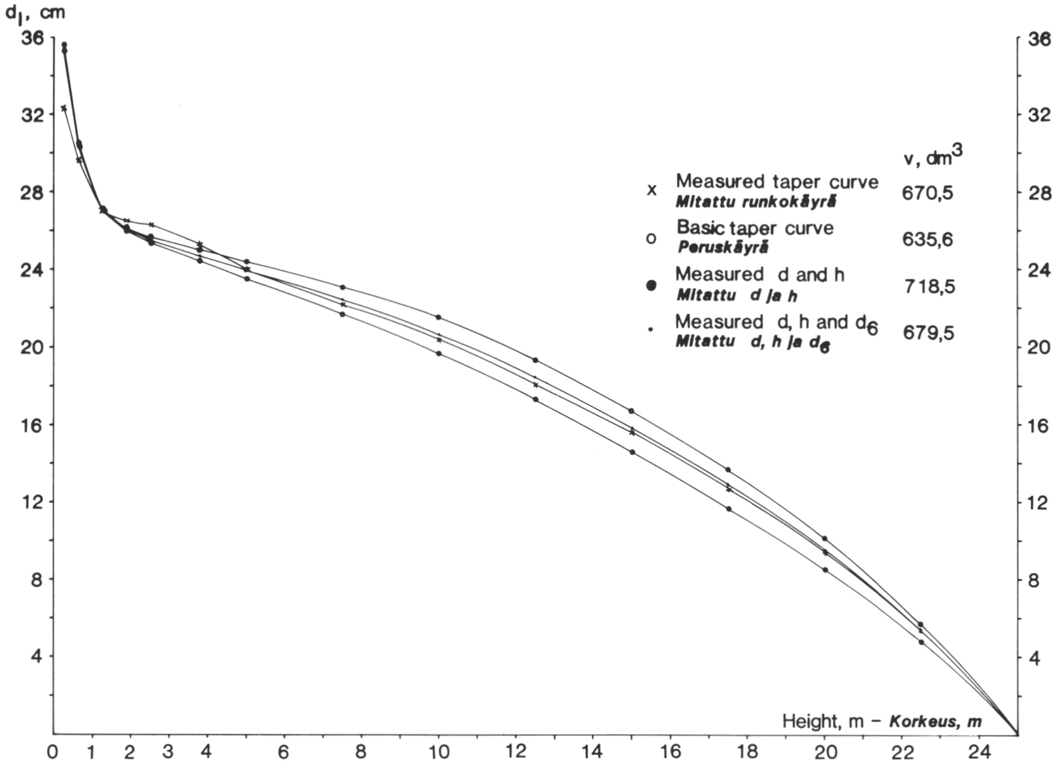

Fig. 10. Polynomial taper curves and measured taper curve and volumes from these curves for a sample spruce.
Kuva 10. Polynomirunkokäyrät ja mittaustietojen avulla saatu runkokäyrä sekä vastaavat tilavuudet eräälle kuusikoepuulle.

base of the stem.

When predicting diameters by means of simultaneous equations, the relative-height diameter closest to 1,3 m was first calculated using a corrected polynomial model (33.1) defined with the aid of d and h. Unbiased diameter estimates in the whole material can be obtained by combining the simultaneous

Table 7. Means ($\bar{x}$, cm) and standard deviations (s, cm) of the differences between the predicted and measured diameters when the predicted diameters are obtained by the polynomial taper curve (t_f) and simultaneous equations (s_f) and when diameter at breast height and height are known.
Taulukko 7. Ennustettujen ja mitattujen läpimittojen erojen keskiarvot ($\bar{x}$, cm) ja hajonnat (s, cm) polynomikäyrillä (t_f) ja simultaaniyhtälöillä (s_f), kun puusta tunnetaan rinnankorkeusläpimitta ja pituus.

Measuring height, %	Pine - *Mänty*				Spruce - *Kuusi*				Birch - *Koivu*			
	t_f		s_f		t_f		s_f		t_f		s_f	
Mittaus-korkeus, %	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
1	-0,04	2,00	-0,01	1,96	-0,05	2,40	0,02	2,38	-0,05	2,26	0,01	2,23
2,5	0,06	1,31	0,00	1,28	0,17	1,30	0,01	1,30	0,42	1,25	0,01	1,22
5	-0,08	0,73	0,00	0,73	-0,13	0,63	0,01	0,63	0,05	0,63	0,00	0,62
7,5	-0,02	0,50	0,00	0,52	-0,03	0,46	0,01	0,47	0,08	0,59	0,02	0,58
10	0,05	0,48	0,00	0,49	0,09	0,47	0,01	0,47	0,17	0,69	0,01	0,63
15	0,03	0,62	0,00	0,61	0,11	0,56	0,00	0,55	0,13	0,89	0,01	0,80
20	0,01	0,73	0,00	0,72	0,03	0,62	0,00	0,61	0,05	0,91	0,01	0,85
30	0,03	0,84	0,00	0,84	0,03	0,70	0,01	0,70	0,05	0,92	0,01	0,92
40	0,03	0,92	0,00	0,92	0,07	0,81	0,01	0,80	-0,03	0,96	0,01	1,00
50	0,02	1,01	0,00	1,01	0,02	0,89	0,01	0,89	-0,08	0,97	0,01	1,00
60	0,02	1,14	0,00	1,13	-0,02	0,93	0,01	0,93	-0,05	1,04	0,01	1,06
70	0,03	1,22	0,00	1,21	0,05	0,91	0,01	0,91	-0,03	0,98	0,01	0,99
80	0,00	1,14	0,00	1,14	0,10	0,82	0,01	0,83	0,00	0,82	0,00	0,82
90	0,01	0,85	0,00	0,84	-0,01	0,65	0,00	0,65	-0,02	0,45	0,00	0,44

Table 8. Means ($\bar{x}$, cm) and standard deviations (s, cm) of the differences between the predicted and measured diameters when the predicted diameters are obtained by the polynomial taper curve (t_f) and simultaneous equations (s_f) and when d, d_6 and h are known. All sample trees with $h \geq 7{,}1$ m are included.
Taulukko 8. Ennustettujen ja mitattujen läpimittojen erojen keskiarvot ($\bar{x}$, cm) ja hajonnat (s, cm) polynomikäyrillä (t_f) ja simultaaniyhtälöillä (s_f), kun puusta tunnetaan d, d_6 ja h. Mukana koepuut, joilla $h \geq 7{,}1$ m.

Measuring height, %	Pine[1)] - *Mänty*[1)]				Spruce[2)] - *Kuusi*[2)]				Birch[3)] - *Koivu*[3)]			
	t_f		s_f		t_f		s_f		t_f		s_f	
Mittaus-korkeus, %	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
1	-0,18	2,19	-0,02	2,05	-0,14	2,54	0,01	2,50	-0,06	2,47	0,02	2,29
2,5	0,02	1,41	-0,02	1,31	0,16	1,35	0,02	1,34	0,46	1,44	0,02	1,23
5	-0,11	0,76	0,00	0,69	-0,17	0,65	0,02	0,62	0,05	0,67	0,01	0,62
7,5	-0,03	0,47	-0,01	0,46	-0,05	0,43	0,02	0,43	0,05	0,58	0,02	0,56
10	0,05	0,41	0,00	0,41	0,09	0,43	0,01	0,42	0,14	0,57	0,02	0,52
15	0,06	0,47	0,01	0,47	0,11	0,45	0,02	0,43	0,09	0,57	0,01	0,54
20	0,05	0,53	0,03	0,52	0,03	0,42	0,03	0,42	-0,00	0,56	0,02	0,54
30	0,07	0,49	0,03	0,48	0,04	0,39	0,04	0,38	0,01	0,59	0,01	0,56
40	0,05	0,54	0,04	0,53	0,09	0,48	0,04	0,44	-0,05	0,61	0,02	0,58
50	0,02	0,62	0,04	0,62	0,03	0,57	0,05	0,54	-0,07	0,70	0,01	0,67
60	0,00	0,81	0,04	0,82	-0,03	0,68	0,04	0,66	-0,01	0,86	0,00	0,86
70	-0,01	0,99	0,03	0,98	0,04	0,74	0,04	0,72	0,04	0,89	0,00	0,88
80	-0,04	1,03	0,03	1,02	0,09	0,75	0,03	0,73	0,09	0,81	0,00	0,78
90	0,00	0,82	0,02	0,81	-0,02	0,64	0,01	0,63	0,06	0,49	0,00	0,44

1) 2 013 sample trees - *koepuuta*
2) 1 604 -"- - -"-
3) 795 -"- - -"-

model and the polynomial model, as is shown in Table 7. The standard errors of the diameter estimates at different relative heights are almost exactly the same with the polynomial and simultaneous models, except at the two lowest measurement points, where the simultaneous model gave better results.

The biases and standard errors of the diameter estimates of the sample trees over 7,0 m in height, when the taper curve is passed through the value for d_6 by means of the correction polynomial, can be seen in Table 8. The biases are slightly larger than those in Table 7 because only a part of the material was in the calculation. The taper curve is clearly more accurate in the middle and upper parts of the stem, as can be seen when the standard errors are compared with the corresponding values in Table 7.

A polynomial taper curve equation was first calculated on the basis of d, h and d_6 when a simultaneous model was applied to sample trees over 7 m in height. The diameters at relative heights closest to the heights 1,3 and 6 m were obtained from this taper curve and then inserted in the simultaneous equations when solving the set of equations. The method was tested in this case and also with a taper curve based on d and h by calculating the spline function which passes through the diameter points given by the simultaneous model and then comparing the diameter at breast height given by this curve with the measured diameter at breast height. The standard deviations of the differences varied from 0,3—0,5 mm and the means were close to zero.

The biases of the diameter estimates are close to zero when using a simultaneous model. The standard errors of the estimates are of the same order of magnitude, although usually slightly smaller than in the case of a polynomial model.

When d and h were predicting variables in the polynomial taper curve model, no systematic errors by d—h classes at the relative heights 0,4 and 0,7 were found. The magnitude of the standard error is greatly dependent on the size of the tree, especially at the bottom and top of the tree. The empirical standard errors of the estimates calculated with a simultaneous model when d and h were assumed to be known were in some 2 cm diameter-classes at certain relative heights in the pine material as follows:

Set-up 72.1

		Relative height				
d, lk	N	1 %	10 %	40 %	60 %	80 %
		Standard error of the diameter estimates, cm				
1	6	0,74	0,53	0,41	0,24	0,29
11	143	1,35	0,35	0,42	0,59	0,63
21	173	2,11	0,32	0,71	0,99	1,13
31	116	2,45	0,57	1,10	1,40	1,52
41	20	2,93	1,04	1,97	1,64	2,03

In the case of large pines, errors of over two centimeters in the diameter estimate are not rare in the crown part of the stem. The standard errors in spruce and birch were not as strongly correlated with tree size.

73. Reliability of the volume estimates

The relative precision of the volume estimates obtained by volume equations and by taper curve models are the best for the medium-sized trees in the study material. The standard errors of the volume estimates obtained using formulae (71.1) and (71.2) for pine equation (61.3) are presented for some diameter-height classes in Table 9. The confidence region resulting from the variance of the coefficients of the volume equation is very narrow in the center of the table. The variance of the estimate resulting from the variances of the coefficients in comparison to the residual mean square of the equation is small also at the edges of the table. Thus the reliability of the estimates is not much lower at the edges of the table.

The percentage biases and standard errors of different stem volume estimates in different height classes, when d and h or d, h and d_6 are used as measured variables, are shown for different tree species in Tables 10, 11 and 12. The same stump height is used for both actual measured volumes and for volumes estimated by the taper curve models and hence the results obtained with the taper curve models give a slightly too good picture of the reliability. In the simultaneous equation method, the volume was calculated using Simpson's formula from 5 % height upwards to 0,4 cm diameter at 100 % height. A parabola was calculated through the 1, 2,5 and 5 % diameters at the bottom of the stem. The integral of the parabola was used to calculate the volume of the section between stump height and 5 % height. The volume based on the measured diameters was used as the reference volume.

If the mean of the error percentages is positive, then too large volumes have been obtained with the prediction method.

It can be seen from the tables that the volume of the stem can be obtained to the same decree of accuracy with all three methods. As the methods differ from each other quite considerably, slightly different results will be obtained with these methods for individual trees. The biases between adjacent height classes may differ from each other by some percentage units when diameter at breast height and height are used as the measured variables.

The biases in the whole material are clearly positive with each method and each tree species, when either two or three measured variables are used. These positive total differences are caused by the systematic errors in the height classes from 5 to 12 meters. For instance, this is very clear in the 9 m height class in spruce. The methods used were not able to remove this systematic error completely. When the variable d_6, as well as d and h, are known, this bias is very small. The standard errors of the estimates are at their smallest in the case of trees about 15 m high, but the differences are relatively small between the standard errors in the different height classes.

The relative standard error of the volume estimates presented in Tables 10, 11 and 12 differ only slightly from those calculated using the residual mean square of the

Table 9. The standard error of the estimate of the equation (61.3) for pine (upper figure) and the prediction error of the equation (lower figure) (cf. formulae (71.1), (71.2) and (54.1)) in some of diameter-height classes.

Taulukko 9. Männyn tilavuusestimaatin (yhtälö (61.3)) keskivirhe (ylempi luku) ja tilavuuden ennustamisvirhe (alempi luku) (ks. kaavat (71.1), (71.2) ja (54.1)) eräissä läpimittapituusluokissa.

d, cm	h, m							
	3	7	11	15	19	23	27	30
1	1,81 7,34							
5	0,86 7,17	0,72 7,15	0,94 7,18					
10	1,29 7,23	0,26 7,12	0,34 7,12	0,50 7,13				
15		0,40 7,12	0,20 7,12	0,28 7,12	0,43 7,13			
20		0,53 7,13	0,25 7,12	0,20 7,12	0,31 7,12	0,44 7,13	0,56 7,14	
25			0,33 7,12	0,19 7,12	0,22 7,12	0,34 7,12	0,46 7,13	0,54 7,13
30			0,43 7,13	0,26 7,12	0,22 7,12	0,29 7,12	0,39 7,12	0,47 7,13
35			0,55 7,13	0,39 7,12	0,32 7,12	0,33 7,12	0,39 7,12	0,45 7,13
40			0,69 7,15	0,55 7,13	0,48 7,13	0,46 7,13	0,48 7,13	0,51 7,13

volume equations. When the restrictions of the equations are considered {f(d, h): pine and spruce h > 3 m and birch h > 4 m, f(d, d_6, h): h ≥ 7,5 m} it can be seen that the theoretical standard errors of the tree volume estimates are of the same order of magnitude as the empirical standard errors for almost all sizes of tree. The biases of the three-variable volume equation estimates for the total volume are clearly smaller than with the taper curve models. When errors in the stump height estimates required in the taper curve models are taken account the volume equations are at least as reliable for obtaining volume estimates as the taper curve models. Furthermore the volume equations are more simple computationally.

The systematic errors of the volume estimates expressed as a percentage of the total volume of the sample trees, are generally smaller than the mean of the percentage deviations calculated for each tree. The deviations arising in the estimation of the volumes of large trees affect, above all, the magnitude of this deviation. The percentage differences of the sums of the volumes calculated by different methods and the actual volumes are presented by tree species in the following set-up:

Table 10. The means ($\bar{x}$) and standard errors (s) of the percentage differences between the actual volumes and estimates obtained by volume equations (v_f), polynomial taper curve equations (t_f) and simultaneous equations of the diameters (s_f) by height class. Pine.

Taulukko 10. Tilavuusyhtälöitä (v_f), polynomirunkokäyriä (t_f) ja läpimittojen simultaaniyhtälöitä (s_f) käyttäen ennustettujen sekä oikeitten tilavuuksien prosentuaalisten erojen keskiarvot ($\bar{x}$) ja hajonnat (s) pituusluokittain. Mänty.

		Measured d and h *Mittaustiedot d ja h*						Measured d, h and d_6 *Mittaustiedot d, h ja d_6*					
h, m	N	v_f		t_f		s_f		v_f		t_f		s_f	
		$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
2	9	-10,04	19,63	4,88	28,14	-7,57	20,75						
3	26	1,73	9,77	2,47	9,75	5,58	15,88						
4	59	0,32	9,75	-0,14	9,78	0,97	10,93						
5	78	2,12	8,32	1,86	8,19	2,35	8,43						
6	87	1,28	7,24	1,17	7,13	1,24	7,20	1)		1)		1)	
7	103	1,10	6,81	0,36	6,89	0,72	6,68	-0,18	4,27	-0,37	4,07	0,14	3,98
8	98	0,35	6,27	-0,40	6,43	0,26	6,29	0,40	4,75	0,77	5,21	1,08	4,85
9	102	1,71	7,22	1,00	7,05	1,68	7,30	-0,22	4,15	-0,17	4,18	0,46	4,41
10	119	1,25	7,76	0,34	7,56	1,13	7,63	0,91	3,22	1,01	3,08	0,94	3,23
11	138	-0,25	6,62	-0,68	6,97	0,03	6,76	-0,05	3,30	0,20	3,33	0,04	3,33
12	115	-1,22	7,60	-1,26	7,65	-0,81	7,66	-0,40	3,50	-0,08	3,39	-0,28	3,47
13	146	-0,60	7,48	-0,40	7,30	-0,23	7,27	0,24	2,94	0,37	2,78	0,30	2,84
14	175	-0,73	6,73	-0,29	6,58	-0,32	6,57	0,33	3,47	0,47	3,38	0,47	3,34
15	177	0,24	6,52	0,79	6,47	0,25	6,47	0,16	3,31	0,31	3,16	0,28	3,20
16	141	0,31	7,05	1,07	6,96	0,11	6,98	-0,06	3,38	0,15	3,31	0,08	3,39
17	142	-0,24	6,21	0,44	6,43	-0,57	6,39	0,57	3,38	0,75	3,31	0,64	3,35
18	152	1,30	6,86	2,03	7,09	1,01	6,98	-0,15	3,27	0,30	3,19	0,50	3,25
19	99	1,12	6,95	1,41	7,08	0,53	7,00	-0,08	3,02	0,24	2,97	0,40	3,16
20	103	0,31	5,61	0,20	5,81	-0,48	5,74	-0,25	3,16	0,04	3,21	0,16	3,22
21	90	1,37	7,00	0,93	7,43	0,26	7,29	-0,07	3,67	0,01	3,64	0,09	3,69
22	54	2,59	6,75	1,36	6,69	0,92	6,71	0,49	3,24	0,80	3,30	0,81	3,27
23	50	0,57	7,53	-1,02	7,92	-1,07	7,88	-0,32	3,73	-0,31	3,80	-0,34	3,89
24	25	1,58	6,41	-1,33	6,22	-0,96	6,22	-0,25	3,52	-0,75	3,14	-0,95	3,03
25	18	1,09	5,86	-2,45	5,48	-1,53	5,58	0,20	3,08	-0,45	2,96	-1,17	2,80
26	10	0,96	5,14	-4,24	5,78	-2,83	5,92	1,68	4,25	0,42	4,13	-0,80	4,22
27	5	6,19	8,94	1,95	8,57	4,09	8,95	4,61	6,79	3,92	6,25	3,27	5,54
28	5	0,52	8,87	-5,47	9,23	-2,80	9,66	0,63	1,70	-1,34	1,83	-2,27	2,79
Total *Yht.*	2326	0,48	7,20	0,40	7,31	0,32	7,37	0,12	3,49	0,31	3,46	0,32	3,49

1) 49 observations - *havaintoja 49*

Set-up 73.1

	V = f(d, h)			V = f(f, h, d_6)		
	v_f	t_f	s_f	v_f	t_f	s_f
	total volume differences, %					
Pine	0,22	0,15	—0,23	0,05	0,09	0,04
Spruce	—0,05	0,61	—0,14	0,11	0,77	0,26
Birch	0,22	1,26	—0,20	0,05	0,44	—0,28

All the methods gave an almost correct estimate for the total volume in the case of pine. The polynomial taper curve model gave too large volumes for large trees in the case of birch and spruce.

When the polynomial taper curve model

Table 11. The means ($\bar{x}$) and standard errors (s) of the percentage differences between the actual volumes and estimates obtained by volume equations (v_f), polynomial taper curve equations (t_f) and simultaneous equations of the diameters (s_f) by height class. Spruce.
Taulukko 11. Tilavuusyhtälöitä (v_f), polynomirunkokäyriä (t_f) ja läpimittojen simultaaniyhtälöitä (s_f) käyttäen ennustettujen sekä oikeitten tilavuuksien prosentuaalisten erojen keskiarvot ($\bar{x}$) ja hajonnat (s) pituusluokittain. Kuusi.

		Measured d and h *Mittaustiedot d ja h*						Measured d, h and d_6 *Mittaustiedot d, h ja d_6*					
h, m	N	v_f		t_f		s_f		v_f		t_f		s_f	
		$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
2	12	3,67	19,71	-2,75	19,60	-15,30	15,92						
3	36	1,42	12,81	0,87	13,05	-0,99	11,81						
4	36	-1,33	6,13	-1,04	6,63	-0,13	6,25						
5	60	-0,03	7,97	1,17	7,12	1,80	7,76						
6	75	-0,31	7,20	0,19	6,79	0,93	6,95	1)		1)		1)	
7	71	-0,29	8,61	-0,31	8,35	0,12	8,42	-0,06	5,36	-0,73	4,50	0,51	4,57
8	85	0,42	8,06	-0,10	8,61	1,11	8,45	-0,14	4,45	-0,35	4,56	1,01	4,61
9	98	2,83	9,02	1,56	8,64	2,68	8,77	0,65	4,10	0,81	3,68	1,57	3,86
10	107	1,58	8,89	0,13	8,79	1,11	8,90	0,27	3,94	0,82	3,86	1,03	3,84
11	87	0,88	7,00	-0,39	7,12	0,71	7,06	0,35	3,28	1,04	3,45	1,05	3,44
12	114	0,58	7,60	-0,60	7,20	0,21	7,32	-0,03	2,90	0,34	2,73	0,43	2,75
13	115	-0,63	6,83	-1,39	6,51	-0,73	6,62	-0,45	3,27	-0,36	3,21	-0,20	3,20
14	108	0,23	7,09	-0,44	6,89	-0,06	7,03	0,29	3,12	0,27	2,93	0,91	2,96
15	124	0,97	7,72	0,61	7,54	0,12	7,67	0,24	2,73	-0,09	2,67	0,22	2,72
16	113	0,70	7,16	0,80	7,24	-0,39	7,25	0,20	2,66	0,14	2,66	0,19	2,65
17	110	0,39	6,65	1,13	6,57	0,00	6,70	-0,43	2,68	-0,10	2,52	-0,10	2,55
18	91	1,97	7,08	2,81	7,15	1,53	7,14	0,46	3,41	0,66	3,13	0,41	3,05
19	76	-0,38	5,87	0,30	5,48	-1,24	5,47	-0,22	3,25	-0,14	3,36	-0,44	3,41
20	77	1,38	5,58	2,68	5,46	1,29	5,45	0,34	3,72	0,79	3,59	0,53	3,61
21	72	-0,27	5,34	1,21	5,24	-0,61	5,10	0,35	3,69	1,05	3,78	0,79	3,73
22	62	1,30	6,69	2,94	6,56	0,81	6,78	0,34	3,19	1,17	3,30	0,67	3,18
23	46	-1,39	6,15	0,54	7,03	-1,14	6,72	-1,10	3,37	0,02	3,65	-0,37	3,41
24	26	1,28	5,63	2,78	5,89	1,31	5,95	0,61	3,58	1,69	3,88	0,75	3,83
25	22	-0,92	5,07	0,23	4,67	-0,88	4,60	0,12	3,41	0,94	4,58	-0,03	3,58
26	16	-0,46	7,33	1,02	7,56	0,43	7,58	1,07	4,41	2,52	4,18	1,18	3,88
27	14	-1,92	8,70	-0,75	9,14	-0,73	9,01	-0,37	4,75	0,89	4,98	-0,62	4,97
28	3	2,29	4,14	1,41	2,44	3,11	4,05	2,62	3,16	2,25	3,27	0,21	3,30
29	5	1,54	4,39	2,73	4,96	3,98	5,42	1,59	4,72	2,66	4,26	1,28	3,12
30	2	-5,76	6,68	-4,39	5,41	0,80	8,85	-0,50	0,62	3,15	1,35	0,09	2,30
33	1	-8,27	-	-11,31	-	0,06	-	2,40	-	4,09	-	-1,06	-
Total *Yht.*	1864	0,56	7,56	0,55	7,49	0,29	7,53	0,12	3,44	0,39	3,41	0,49	3,38

1) 30 observations - *havaintoja 30*

was tested with the control material for spruce too large estimates were obtained for slightly-tapering trees over 28 m high. The over-estimates for spruce shown in the above set-up were not apparent in another control material consisting of 1 170 sample trees collected from stands marked for cutting. However, this material did not contain any trees over 27 m high. No systematic errors were found in a birch material collected from marked stands but in a pine material the total volume estimate was a little (0,65 %) too small.

It is important, to know whether the taper curve models and volume functions are equally applicable in the different parts of Finland. In order to clarify this question the material was divided into climatic zones based on the length of the thermal growing season (see fig. 3, p. 13). The means and standard deviations of the percentage errors of the volume estimates and the number of sample trees in these seven climatic zones are presented by tree species in Table 13.

It is evident from the results of the two-variable equation (61.3) for pine, that there

Table 12. The means ($\bar{x}$) and standard errors (s) of the percentage differences between the actual volumes and estimates obtained by volume equations (v_f), polynomial taper curve equations (t_f) and simultaneous equations of the diameters (s_f) by height class. Birch.
Taulukko 12. Tilavuusyhtälöitä (v_f), polynomirunkokäyriä (t_f) ja läpimittojen simultaaniyhtälöitä (s_f) käyttäen ennustettujen sekä oikeitten tilavuuksien prosentuaalisten erojen keskiarvot ($\bar{x}$) ja hajonnat (s) pituusluokittain. Koivu.

h, m	N	Measured d and h *Mittaustiedot d ja h*						Measured d, h and d_6 *Mittaustiedot d, h ja d_6*					
		v_f		t_f		s_f		v_f		t_f		s_f	
		$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
2	1	6,88	-	-17,21	-	-21,46	-						
3	3	-9,15	14,96	-20,26	6,41	-22,62	12,11						
4	10	2,25	11,09	3,92	10,27	-0,34	10,74						
5	15	4,34	14,56	7,48	14,06	2,33	10,45						
6	22	-0,75	7,79	1,48	8,52	-1,01	9,01	1)		1)		1)	
7	29	-0,74	8,77	0,57	8,03	-1,40	8,11	-1,75	8,56	-2,62	8,29	-4,16	8,06
8	36	0,93	7,00	1,54	6,32	2,42	6,49	1,66	5,74	1,57	5,35	2,05	5,18
9	42	1,44	7,66	0,86	7,72	1,93	7,99	0,13	5,78	0,85	6,13	1,15	6,17
10	44	0,58	9,58	-0,49	9,66	1,16	10,11	-0,42	5,16	0,33	4,96	0,64	5,47
11	48	2,67	9,00	1,77	8,98	5,12	9,75	1,15	5,31	2,30	5,10	3,40	5,94
12	57	0,98	8,82	-0,76	8,64	2,53	9,96	0,15	5,14	0,77	4,98	1,47	5,56
13	45	-0,10	6,97	-1,15	6,94	1,32	7,51	-0,22	5,48	0,68	5,80	1,00	5,75
14	38	2,85	10,37	1,47	10,32	3,83	10,95	1,08	5,49	1,34	5,66	1,65	5,99
15	41	-0,54	7,26	-1,93	6,68	0,56	9,14	0,68	3,85	1,11	3,40	0,89	3,53
16	43	0,78	9,42	0,33	9,77	0,95	9,33	0,49	4,94	0,69	4,66	0,16	4,56
17	44	-2,71	5,87	-3,43	5,60	-2,43	6,56	-0,92	4,26	-0,69	4,05	-1,28	3,95
18	52	-1,81	6,18	-2,28	6,18	-2,24	6,68	-0,43	4,01	-0,48	4,10	-0,85	3,93
19	53	0,72	7,83	0,87	8,19	0,51	8,48	0,33	4,84	0,44	4,91	0,10	4,85
20	46	2,04	8,18	2,42	8,54	1,79	8,36	0,41	4,47	0,72	4,60	0,39	4,45
21	50	0,31	7,47	0,83	7,92	-0,67	7,93	0,58	4,25	0,66	4,50	-0,06	4,37
22	50	-0,30	6,46	0,12	7,06	-1,47	7,13	-0,56	4,45	-0,42	4,51	-1,16	4,37
23	26	1,85	8,16	2,62	8,16	0,30	7,95	0,30	3,87	0,67	3,98	-0,12	4,07
24	29	4,93	12,96	7,19	15,17	4,36	15,53	0,94	6,01	1,34	6,35	0,24	6,25
25	21	0,71	6,54	1,61	7,13	-0,63	7,76	-0,16	3,61	0,29	3,78	-0,24	4,44
26	9	2,66	8,14	3,54	9,00	1,24	9,42	1,51	5,24	1,78	5,81	1,35	6,73
27	8	-2,05	7,68	-0,13	7,52	-3,11	7,63	0,17	4,29	0,81	3,17	-0,61	3,80
28	-	-	-	-	-	-	-	-	-	-	-	-	-
29	-	-	-	-	-	-	-	-	-	-	-	-	-
30	1	8,87	-	8,39	-	4,56	-	3,35	-	2,04	-	2,36	-
Total *Yht.*	863	0,68	8,45	0,53	8,79	0,75	9,14	0,24	4,93	0,61	4,95	0,46	5,17

1) 12 observations – *havaintoja 12*

is a quite clear difference in the stem form of the trees in the different climatic zones. The results for the other tree species are not so clear, although too small volumes are generally obtained for pine and birch in the northern parts of the country when equations based on diameter at breast height and height are used. The standard deviations are of the same order of magnitude in the different climatic zones, except in the north where they are slightly larger.

The results obtained using three variable equations show similar trends as with the two variable ones, but the differences between climatic zones are smaller. The standard errors of the estimates are clearly largest in the north. In the fifth climatic zone, where the most observations fell, the mean deviated highly significantly from zero in the case of pine and spruce, and significantly in the case of birch.

The biases and standard errors of the estimates in different development classes (cf. Kuusela and Salminen 1969, pp. 27—28) are presented by tree species in Table 14. Examining the material by development classes (DC) shows whether the trees have a different stem form at different stages of stand development.

The biases of the equations based on diameter and height vary irregularly, partly as a result of the fact that the numbers of observations were small. The equations (61.7) gave a little too large volumes in development classes 3, 4 and 5. In mature spruce forests (class 6), the equations yield underestimates. Owing to the large number of observations, these errors are

Table 13. Means ($\bar{x}$) and standard errors (s) of the percentage differences between volumes estimated with the volume equation and the actual volumes by climatic zones (see Fig. 3) and tree species. N is the number of observations.

Taulukko 13. Tilavuusyhtälöillä ennustettujen ja oikeitten tilavuuksien prosentuaalisten erojen keskiarvot ($\bar{x}$) ja hajonnat (s) ilmastovyöhykkeittäin (ks. kuva 3) ja puulajeittain. N on havaintojen lukumäärä.

Climatic zone	v = f(d,h) = (61.6)			v = f(d,h,d_6) = (61.7)		
Ilmastovyöhyke	N	$\bar{x}$	s	N	$\bar{x}$	s
			Pine - *Mänty*			
1	202	-3,09	7,38	183	-0,53	4,26
2	259	-2,54	6,69	220	0,10	3,70
3	243	-1,78	6,50	200	-0,29	3,20
4	478	0,62	6,84	380	0,10	3,24
5	656	1,74	6,50	579	0,40	3,48
6	459	2,63	7,06	427	0,15	3,29
7	29	6,42	12,58	24	1,68	4,20
			Spruce - *Kuusi*			
1	1	6,64	-	1	-0,09	-
2	126	0,37	7,99	107	-1,12	4,53
3	170	-1,64	7,47	129	-0,56	3,11
4	331	-1,15	6,53	274	0,01	3,20
5	700	0,85	7,55	616	0,40	3,33
6	522	1,95	7,75	463	0,30	3,45
7	14	3,31	8,12	14	-0,83	2,28
			Birch - *Koivu*			
1	25	-0,19	8,70	17	-0,47	5,76
2	43	-1,52	8,44	36	-1,65	4,70
3	37	-0,25	12,25	32	0,27	5,37
4	168	-0,57	7,71	149	-0,11	5,03
5	398	1,27	8,29	377	0,57	4,87
6	187	1,40	8,45	180	0,37	4,74
7	5	-0,58	4,21	4	-3,21	7,25

statistically significant and can be explained by the development of stem form with regard to the age. The corresponding phenomenon can be seen in Tables 10—12 for height classes.

As has earlier been mentioned (section 31), many other factors have an effect on stem form, such as stand density and tree position in a stand. It was apparent when each crown layer was examined separately, that the volume obtained for dominating trees was slightly too large, and for intermediate pine and birch trees too small. The fertility of the site also has an effect. On poorly productive and waste land, the stem form of pine varies the most, as can be

Table 14. Means ($\bar{x}$) and standard deviations (s) of the percentage differences between volumes estimated with the volume equations and the actual volumes by development classes and tree species. N is the number of observations.

Taulukko 14. Tilavuusyhtälöillä ennustettujen ja todellisten tilavuuksien prosentuaalisten erojen keskiarvot ($\bar{x}$) ja hajonnat (s) eri kehitysluokissa puulajeittain. N on havaintojen lukumäärä.

Development class 1) *Kehitys-luokka 1)*	v = f(d,h) = (61.6)			v = f(d,h,d_6) = (61.7)		
	N	$\bar{x}$	s	N	$\bar{x}$	s
			Pine - *Mänty*			
0	287	-1,13	8,53	184	-0,61	4,41
1	14	2,80	7,09	11	0,30	3,65
2	113	1,89	7,02	79	0,15	3,72
3	149	2,65	7,93	43	0,55	3,41
4	531	1,51	6,63	491	0,56	3,10
5	532	0,49	6,36	523	0,21	3,20
6	360	0,59	6,93	359	-0,05	3,54
7	120	-0,52	6,72	117	0,00	3,14
8	220	-1,92	7,71	206	-0,21	4,03
			Spruce - *Kuusi*			
0	26	-3,19	7,56	15	-1,79	3,07
1	4	-14,80	17,20	1	-0,96	-
2	103	6,12	8,82	58	0,39	4,14
3	47	0,09	8,34	21	0,70	5,58
4	551	1,51	7,70	481	0,38	3,24
5	460	0,04	6,83	433	0,38	3,23
6	342	-1,06	6,73	324	-0,48	3,52
7	77	1,89	6,05	68	0,35	3,28
8	254	-0,32	7,56	203	-0,19	3,63
			Birch - *Koivu*			
0	30	1,20	12,62	14	-2,75	6,35
1	7	-9,83	9,29	3	-1,08	2,67
2	57	-1,06	8,81	48	-0,13	4,89
3	29	3,06	7,48	20	2,93	5,24
4	153	1,05	8,03	146	0,43	5,22
5	272	0,69	7,42	269	0,24	4,56
6	153	1,03	9,40	148	-0,07	4,80
7	36	1,94	9,35	35	0,65	5,16
8	126	0,15	8,02	112	0,41	5,19

0 = forestry land other than forest land - *kitumaa*
1 = open regeneration area and seed tree stand - *aukea ja siemenpuumetsikkö*
2 = seedling and sapling stands with standards - *pieni taimisto*
3 = seedling and sapling stands - *taimisto- ja riukuvaiheen metsikkö*
4 = thinning stands - *nuori kasvatusmetsikkö*
5 = preparatory stands - *varttunut kasvatusmetsikkö*
6 = mature stands - *uudistuskypsä metsikkö*
7 = shelterwood stands - *suojuspuumetsikkö*
8 = low-yielding stands - *vajaatuottoinen metsikkö*

seen from the standard errors of estimates for non forest land (DC = 0) in Table 14.

As these examinations show, methods based on diameter and height clearly give biased results in some cases. When the diameter at a height of 6 m is also known on trees over 7 m high, an estimate of the volume which is sufficiently precise for most purposes is obtained. In some special cases, such as in primeval pine stands, the volume may even then be underestimated and in some pine stands in Lapland perhaps by as much as 5—6 %.

8. DISCUSSION

81. Application of results

Methods to estimate the taper curve and volume of tree stems are presented in this study. The methods for calculating taper curves are not restricted to the use of ordinary tree measurements, such as d, h and d_6, only. The form and volume of a tree can be determined to the desired precision by planning the diameter measurements so as to conform to the precision requirements. In different situations, measurement and calculation methods can thus be chosen for each task. For instance, tree volume can be calculated in the simplest way using volume functions based on diameter at breast height, or in the most accurate way using a spline functions which incorporates any number of diameter values.

The models presented here for calculating pine, spruce and birch taper curves are based on over-bark diameter values. However, when the measured under-bark diameters are employed in these over-bark taper curve models the calculated under-bark diameter estimates are only slightly biased. Another possibility is to calculate first the over-bark taper curve and then subtract the bark by means of a separate model. A simple bark model could be: $b = a_0 + a_1 \cdot d_l$.

If, for instance, the double bark thickness in the upper terminal leader of spruce is assumed to be 3 mm and the bark thickness at diameter at breast height of a tree where d = 20 cm is found to be 10 mm, then the equation for the double bark thickness is, according to the above model, $b\ (mm) = 3 + \frac{7}{20} \cdot d_l$. Such models hold true rather well in the case of spruce, but for the rough barked part of pine stems, for instance, a separate model has to be used.

A function for volume without bark based on diameter and height has been prepared for each tree species (61.4). All the other volume functions give the stem volume with bark.

It is easy to determine volume growth using volume functions and taper curves if measurements for the beginning and end of the period in question are available. The height growth of conifers can be determined fairly reliably on the basis of the terminal leader, but this is not possible with birch. The diameter growth without bark can be determined from increment cores, for instance. The above mentioned approximation method, which is most inaccurate in the rough-barked part of the stem, has to be used for determining changes in the bark. Determining growth by means of height growth and diameter growth measured at breast height takes the form growth of the stem into account as an average value. By taking increment cores of diameter growth at other heights, as well as at breast height, changes in the stem form can be studied.

When volume is calculated by means of a taper curve, the stump height is needed for defining the integration limit. The height of the uppermost root collar (h_r) can be estimated using the following equations based on height and diameter at breast height.

			s_r, %
	Pine	$h_r(cm) = 0{,}4456 \cdot d + 0{,}0952 \cdot h$	68
(81.1)	Spruce	$h_r(cm) = 0{,}5089 \cdot d + 0{,}5600 \cdot h$	38
	Birch	$h_r(cm) = 0{,}4862 \cdot d + 0{,}4979 \cdot h$	44

As the relative standard errors of the estimate (s_r) show the precision of the equations is relatively poor. The height of the root collar in relation to tree height h_r/h, which was the dependent variable in the regression analysis, and the standard deviation (s) of this relationship, were as follows for the three tree species:

	Pine	Spruce	Birch
h_r/h	0,0075	0,0122	0,0101
s	0,0054	0,0048	0,0046

The uppermost root collar of pines is situated on the average, at a height equivalent to 0,75 % of tree height. However, its standard deviation was the largest, which explains why the pine function (81.1) had a poor precision. The height of the root collar in spruce is at a height of over 1 % of tree height and in birch at almost exactly 1 %.

The mean stump height left in harvesting is, in the case of small trees, higher than the root collar height. For this reason, a height of at least 10 cm was used as stump height when calculating the volume of the sample trees. When calculating the errors of the taper curve models (cf. Tables 10—12), the stump height was the same as when calculating the volume of the sample trees. When setting stump height as the maximum of the estimate from function (81.1) and 10 cm, the standard deviations of the error percentages of the volume estimates obtained with the polynomial taper curve model increased by less than 0,1 %. However, the systematic errors in certain height classes increased even by 0,5 %. When using taper curve models for the estimation of volume, estimation of the stump height would thus not appear to weaken appreciable the reliability of the method.

In estimating timber assortments, the height at which the stem fulfills the required diameter has to be calculated in order to determine the saw timber proportion. Owing to branch formation, the stem within the crown tapers stepwise and irregularly as shown in Figure 5. Predicting the height of the saw timber limit is thus not as reliable as would be decuded from the graph of the taper curve (cf. Fig. 9).

Owing to the tapering and branchiness of the stem, the saw timber portion of the stem is not completely used as a saw timber in practical scaling of felled trees. On the other hand, the stepwise tapering of the stem is utilized in scaling, because the diameter given by the taper curve at the top point of the saw logs is, on the average, slightly smaller than the actual top diameters of the saw logs. One method to use in scaling carried out by means of taper curve models, is to determine from practical data how much of the theoretically calculated saw timber height is used as saw logs (cf. Laasasenaho and Pekkonen 1982).

When calculating volume, the means of the error percentages of individual trees using two measurement variables (d and h) are between +0,29 — +0,75 % (cf. Tables 10—12) by different methods. The total sums of the volumes of the sample trees have usually been obtained more exactly (set-up 73.1, p. 54).

The means of the error percentages for the logarithmic volume functions are greatest. Addition of the correction factor (51.3) to the coefficient of the functions has, in these cases, led the overestimation. In the case of pine the correction factor increased the volume by 0,26 %, for instance. Without this correction the total volume of the sample trees would have been almost the correct one. This indicates that the normality assumptions used as the basis in the calculation of the correction factor are not valid in this material.

In the case of the method for calculating volume based on three measured variables (d, h and d_6), more precise estimates are obtained with the volume functions than with the taper curve models, as can be seen from Tables 10—12 and set-up 73.1. In the case of the simultaneous models, the estimates of the volume can be made more accurate by adding the unbiased correction formulae of the square of the diameter to the calculation programme (cf. Kilkki and Varmola 1982, pp. 40—45). The use of these correction formulae does not, on the basis of the values presented in set-up 42.1, have a significant effect.

As there is bias in all the methods, there are solid grounds for removing it by means of a separate correction factor. In principle, it would be possible to prepare the correction factor as a function of tree size, e.g. height. Defining the correction factor as the mean of the error percentage of the total volumes and the error percentages of individual trees would emphasise the importance of medium-sized trees in the correction factor. The correction factors calculated in this way from Tables 10—12 and set-up 73.1 would be between 0,9919—0,9991. The correction factors could be combined directly into the functions.

The unbiased result is, above all, important in conjunction with calculation of monetary results in measurement of standing trees for commercial purposes. As the stem form of trees in mature stands is usually more cylindrical, i.e. the methods for calculating volume give results which are too small (cf. Table 14), these correction coefficients have not been employed in present-day programmes.

82. Use of the taper curve models

A taper curve offer a starting point for almost all calculations concerning the tree stem. Taper curves are used when the diameter at a specific height is needed or when looking for the height along the stem where the diameter has a particular value. The volume of a certain portion of the stem, or of the whole stem, is obtained either through analytical of numerical integration of the taper curve function.

By the taper curve function f(l) which gives the diameter at height l, the volume of the interval l_1 — l_2 is obtained from the well-known integral:

$$v = \frac{\pi}{4} \int_{l_1}^{l_2} \{f(l)\}^2 \, dl$$

In the taper curve function (33.2) variable $x = \frac{l}{h} \Rightarrow l = h \cdot x$. By carrying out this variable exchange (cf. e.g. Myrberg 1968, p. 133) in integration we obtain

$$v = \frac{\pi}{4} \int_{l_1}^{l_2} \{f(l)\}^2 \, dl = \frac{\pi}{4} \int_{\frac{l_1}{h}}^{\frac{l_2}{h}} \{f(x)\}^2 \cdot h \cdot dx$$

Thus volume of the stem from the stump (l_s) to the top is obtained using the taper curve function according, for instance, to formula (33.3) with the integral:

$$v = \frac{\pi}{4} \cdot \hat{d}^2_{,2h} \cdot h \cdot \int_{\frac{l_s}{h}}^{1} \{f_b(x)\}^2 \, dx \quad (82.1)$$

The taper curve function can be used, for instance, in predicting the amount of various timber assortments and the value of the stem.

In studies requiring accurate measurements of the stem, such as in fertilization research, in which increment cores or measurements are used to follow tree growth, the magnitude of growth and its distribution between different parts of the stem can be effectively determined using taper curve functions. Similarly, plotting taper curves for different periods of time on the same graph clearly reveal any possible measurement errors (cf. Lahtinen and Laasasenaho 1979, p. 57). Taper curve functions also allow the bole area of the stem to be calculated. This can be used in, for instance, growth studies (Laasasenaho 1978).

Taper curve functions can also be used as an aid in drawing up weight tables for stems. The specific gravity of the wood changes on moving from the pith to the bark as a function of the radius r (cf. Hakkila 1966, p. 39).

Let the equation describing the wood density as a function of the radius r be:

$$\xi = f(r) \quad (82.2)$$

Denote the taper curve function $d_l = f(l)$.

The weight of the stem is then obtained with the methods frequently applied in physics to determine the mass of a non-homogeneous particle from the integral

$$M = \int_{l_s}^{h} \left(\int_{0}^{\frac{d_l}{2}} \xi \cdot 2 \cdot \pi \cdot r dr \right) dl = 2 \cdot \pi \int_{l_s}^{h} \left(\int_{0}^{\frac{d_l}{2}} \xi \cdot r dr \right) dl \quad (82.3)$$

Whether or not the integral can be analytically solved is dependent on the density function and on the taper curve function. In any case the integrals in question can be numerically calculated.

The taper curve function provides an effective means of studying changes in stem form as a function of age or, for instance, of studying the stem forms of different varieties of the same tree species. Form factors (e.g. f and $f_{.1h}$) describing stem form are easy to calculate with a taper curve function. The position of the turning point of the taper curve can also be studied with such a function.

Stand density, site type or other environmental factors can of course be included in the taper curve correction functions in the same way as Kilkki and Varmola (1981) tested the inclusion of crown length as an exogenous variable in a simultaneous model. Taper curves can thus be used to determine the factors affecting the development of stands.

83. Comparison of the different methods for estimating volume

Although the volume estimation methods presented in this study differ considerably from each other, the estimates are very similar over the typical areas of the study material. On the other hand, the differences due to the different methods at the extreme limits and outside the limits of the material may in some cases be considerable. If the three variable equations would be constructed using the basic variables according to model (53.5) only, the equations would probably be more accurate than equations (61.7) in extreme cases.

A good approximate value is always obtained for the volume when the basic coefficients (41.1) of the polynomial taper curve equation are used. If, for instance, the tree is short and almost uniformly-thick between the heights 1,3 and 6 m, the correction polynomial may be such that the taper curve does not decrease monotonously throughout the length of the stem. A corresponding phenomenon can also occur when using correction equations (41.2), based on diameter and height, without any restrictions. The estimate of the volume, in such cases, is of course erroneous for some stem sections at least.

A logical stem form is not always obtained with simultaneous equations in special cases. This is mainly due to the effect of exogenous variables. A more accurate diameter series is obtained for column-shaped trees in the case of a simultaneous model by including an additional diameter from the upper part of the stem. The accuracy of the taper curve can be improved by means of a number of additional diameters, because it is easy to arrange the inclusion of a number of additional measurements in the case of simultaneous equations.

The stem volume calculated using the polynomial taper curve model for pine and the percentage differences of the volumes calculated using the volume equations and simultaneous model for trees with a breast height diameter of 23 cm as a function of h and d_6, are presented in Table 15. The differences are the smallest in the case of average shaped trees and they change regularly. The difference percentages in the case of the simultaneous model do not change regularly with respect to height, because the measured absolute height diameters correspond to various relative height diameters in the set of equations (cf. Kilkki and Varmola 1981). The differences with the volume equation are usually smaller than with the simultaneous model and are less than one percent in the case of average shaped trees. The differences in other breast height diameter classes were similar. In extreme cases, the differences were over 10 percent. As the methods give the total stem volume estimates that are statistically of the same degree of precision, and there were no clear systematic errors in these, it is difficult to rank the models.

Comparisons were also made for the basic pine material using the simultaneous model of Kilkki and Varmola (1981). The the biases ($\bar{x}$) and standard errors (s) of the estimates as percentages for individual trees when two or three variables were used were as follows:

	$\bar{x}$, %	s, %
2 variables (d and h)	1,12	7,85
3 variables (d, d_6 and h)	1,03	3,52

The bias was almost same when the calculations were made using diameter at breast height and height, or when d_6 was used as well. The magnitude of the bias was clearly dependent on tree height, especially when the upper diameter was used. This bias was caused by the collected material. Owing to the bias, the standard errors of the estimates were also slightly larger than the values shown at the bottom of Table 10. The use of the model was found to involve a considerable amount of computer time.

The accuracy of the volume estimates given by Ilvessalo's (1947) volume tables was also studied. When the height of the root collar (the starting point for measurements in Ilvessalo's tables) is known, the polynomial taper curve function can be calculated using the measurements for these tables. As the variables required for the tables had been measured on the sample trees and the volumes determined from the tables, the table volumes were compared with the actual volumes. The means of the error percentages are presented by height class in Table 16.

The underestimation given by the tables can mainly be attributed to the different method of defining stump height and to the method used to calculate the volume of the sample trees when the tables were drawn up (cf. Table 4, p. 16).

The systematic errors were greatest in the extreme areas of the tables. In rare taper classes ($d—d_6$), the differences could even be as large as 30 % in comparison to the values obtained with the polynomial taper curve function. For instance, in the case of pines with diameter at breast height 36 cm, taper class 16 cm and height 8 m, the stem volumes obtained with the polynomial taper curve equation are 36 % greater than the values obtained from the tables. The polynomial taper curve function was logical in this case, and hence the respective volume estimate can be considered to be more

Table 15. Volume obtained by the polynomial taper curve equation (v_t) for pine and the percentage differences between this volume and the volumes calculated using the simultaneous equations (v_s) and the volume equation (v_f), when d = 23 cm.

Taulukko 15. Männyn polynomirunkokäyrällä saatu tilavuus (v_t) sekä siitä simultaanimallilla (v_s) ja tilavuusyhtälöllä (v_f) laskettujen tilavuuksien prosentuaaliset poikkeamat, kun d = 23 cm.

d_6, cm	Tree height, m – *Puun pituus, m*																						
	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
v_t, dm^3																							
6	156																						
7	162	159																					
8	168	166	164	163																			
9	175	173	171	170	170																		
10	182	180	179	178	178	180	183																
11	190	188	187	187	188	190	194	198	203														
12	198	196	196	197	198	201	205	210	216	222													
13	206	205	206	207	210	213	218	224	231	238	245	254											
14	215	215	216	219	222	227	232	239	246	254	263	272	282	292									
15	225	225	228	231	235	241	248	255	264	273	282	292	303	314	325								
16	234	235	239	244	250	256	264	273	282	292	303	314	325	337	350	362	375						
17	245	246	252	258	265	272	281	291	302	313	325	337	349	362	376	390	404	418					
18	255	258	265	273	281	290	300	311	323	335	348	361	375	389	404	419	434	449	465	480			
19	266	270	279	289	299	308	320	332	345	358	373	387	403	418	434	450	466	482	499	516	533	550	
20				305	317	328	340	354	368	383	399	415	432	448	465	483	500	518	536	554	572	590	
21					336	348	362	377	393	410	427	444	462	480	499	517	536	555	574	594	613	632	652
22						370	385	402	419	437	456	475	494	514	534	554	574	595	615	636	656	677	698
$100 \cdot (v_s - v_t)/v_t$																							
6	4,7																						
7	3,5	4,8																					
8	2,3	4,0	3,2	2,8																			
9	1,2	3,1	2,9	2,7	3,6																		
10	0,1	2,3	2,5	2,5	3,3	3,9	1,7																
11	-1,0	1,4	2,0	2,2	2,9	3,5	2,0	3,5	3,0														
12	-2,1	0,6	1,4	1,8	2,5	3,0	2,1	3,2	2,9	2,4													
13	-3,2	-0,2	0,8	1,3	2,0	2,4	2,1	2,8	2,6	2,2	3,4	2,2											
14	-4,2	-1,1	0,1	0,7	1,4	1,8	1,8	2,3	2,1	1,9	3,1	2,2	1,2	2,0									
15	-5,3	-1,9	-0,5	0,1	0,8	1,2	1,5	1,7	1,7	1,6	2,6	1,9	1,2	1,8	1,1								
16	-6,3	-2,7	-1,2	-0,6	0,1	0,6	1,0	1,1	1,1	1,1	1,9	1,5	1,0	1,4	0,9	0,4	-0,2						
17	-7,2	-3,4	-1,9	-1,3	-0,6	-0,1	0,5	0,4	0,6	0,7	1,2	1,0	0,8	1,0	0,7	0,3	0,0	1,2					
18	-8,1	-4,2	-2,7	-2,0	-1,3	-0,7	-0,1	-0,3	0,0	0,2	0,4	0,5	0,4	0,5	0,3	0,2	0,0	-0,1	-0,5	-0,9			
19	-9,0	-4,9	-3,4	-2,8	-2,0	-1,4	-0,8	-1,1	-0,7	-0,3	-0,4	-0,2	0,0	-0,1	0,0	0,0	0,1	-1,4	-1,6	-1,8	-2,0	-2,2	
20				-3,5	-2,7	-2,0	-1,5	-1,8	-1,3	-0,9	-1,2	-0,8	-0,5	-0,6	-0,4	-0,2	0,0	-2,8	-2,7	-2,7	-2,7	-2,7	
21					-3,4	-2,6	-2,2	-2,5	-1,9	-1,4	-2,1	-1,5	-1,0	-1,2	-0,8	-0,4	-0,1	-4,1	-3,9	-3,7	-3,5	-3,3	-3,1
22						-3,2	-2,9	-3,2	-2,5	-1,9	-2,9	-2,2	-1,5	-1,8	-1,2	-0,7	-0,2	-5,4	-5,0	-4,7	-4,3	-3,9	-3,6
$100 \cdot (v_f - v_t)/v_t$																							
6	1,7																						
7	0,8	0,5																					
8	0,0	0,1	-0,1	-0,3																			
9	-0,8	-0,3	-0,2	-0,1	-0,3																		
10	-1,5	-0,7	-0,3	0,0	0,0	-0,7	-1,8																
11	-2,2	-1,1	-0,5	0,0	0,2	-0,4	-1,2	-2,2	-3,3														
12	-2,9	-1,5	-0,8	-0,1	0,2	-0,1	-0,8	-1,6	-2,5	-3,5													
13	-3,6	-1,9	-1,1	-0,3	0,2	0,0	-0,5	-1,1	-1,9	-2,7	-3,6	-4,4											
14	-4,2	-2,3	-1,4	-0,6	0,0	0,0	-0,3	-0,8	-1,3	-2,0	-2,7	-3,5	-4,2	-4,9									
15	-4,8	-2,7	-1,8	-0,9	-0,3	-0,1	-0,2	-0,5	-1,0	-1,5	-2,1	-2,6	-3,2	-3,8	-4,3								
16	-5,4	-3,0	-2,1	-1,3	-0,6	-0,3	-0,2	-0,4	-0,7	-1,1	-1,5	-2,0	-2,4	-2,9	-3,3	-3,6	-4,0						
17	-5,9	-3,4	-2,5	-1,7	-1,0	-0,5	-0,3	-0,4	-0,6	-0,8	-1,1	-1,5	-1,8	-2,1	-2,4	-2,6	-2,9	-3,0					
18	-6,4	-3,8	-2,9	-2,2	-1,5	-0,8	-0,5	-0,4	-0,5	-0,7	-0,8	-1,1	-1,3	-1,5	-1,7	-1,8	-1,9	-2,0	-2,1	-2,1			
19	-6,9	-4,2	-3,3	-2,7	-1,9	-1,1	-0,7	-0,5	-0,5	-0,5	-0,6	-0,8	-0,9	-1,0	-1,1	-1,1	-1,2	-1,1	-1,1	-1,0	-0,9	-0,8	
20				-3,2	-2,4	-1,5	-1,0	-0,7	-0,6	-0,5	-0,5	-0,5	-0,5	-0,6	-0,6	-0,6	-0,5	-0,4	-0,3	-0,2	0,0	0,2	
21					-2,9	-1,8	-1,3	-0,9	-0,7	-0,5	-0,4	-0,4	-0,3	-0,3	-0,2	-0,1	0,0	0,2	0,3	0,6	0,8	1,0	1,3
22						-2,2	-1,6	-1,1	-0,8	-0,6	-0,4	-0,3	-0,1	0,0	0,1	0,3	0,5	0,7	0,9	1,2	1,4	1,7	2,1

reliable than that of Ilvessalo's table. The systematic errors in the tables can thus be considered to be due to exceptional tapering; the errors in the case with normal stem forms were small.

84. Differences between the tree species

It can be seen from the mean stem forms shown in Fig. 9, that there are systematic differences between the stem forms of these three tree species. The graphs of the integrals of the squares of the polynomial basic taper curve equations.

$$(84.1)\quad I = \frac{\pi}{4} \int_{\frac{t}{100}}^{1} \{f_b(x)\}^2\, dx$$

are presented in Fig. 11 as a function of the relative distance (t, %) from the top. When d_{2h} and h are known, the volume (V_l) of the part of the stem above a certain height l is obtained from the curves using the formula:

$$V_l = d^2_{,2h} \cdot h \cdot I,$$

whose I is obtained from the curve for the tree species in question, the value of t being $100 \cdot (1 - \frac{l}{h})$.

Table 16. Means ($\bar{x}$) and standard deviations of the percentage differences between measured volumes and volumes obtained using Ilvessalo's volume tables, by height classes.
Taulukkoi 16. Ilvessalon kuutioimistaulukoilla saatujen ja mitattujen tilavuuksien prosentuaalisten erojen keskiarvot ($\bar{x}$) ja hajonnat (s) pituusluokittain.

h, m	Pine - *Mänty*			Spruce - *Kuusi*			Birch - *Koivu*		
	N	$\bar{x}$	s	N	$\bar{x}$	s	N	$\bar{x}$	s
2	9	5,69	30,1	12	-3,78	33,6	1	-47,94	-
3	26	1,62	20,1	36	3,37	22,4	3	-23,27	3,2
4	59	3,80	15,1	36	4,09	17,0	10	18,63	30,0
5	78	1,67	13,4	60	2,38	12,2	15	5,25	17,2
6	87	-2,31	11,3	75	-0,96	8,7	22	3,98	15,7
7	103	-1,63	8,6	71	-3,31	9,9	29	0,36	10,7
8	98	-6,41	7,6	85	-7,24	7,8	36	-3,60	18,7
9	102	-4,79	7,6	98	-4,02	6,5	42	-4,48	9,8
10	119	-2,44	6,0	107	-4,11	6,6	44	-1,87	10,4
11	138	-4,33	5,1	87	-2,47	6,6	48	-1,65	9,1
12	115	-2,97	8,4	114	-2,90	5,6	57	-4,30	8,4
13	146	-2,93	5,2	115	-2,49	5,6	45	-4,39	7,3
14	175	-2,21	4,1	108	-2,97	4,7	38	-3,17	6,2
15	177	-2,26	5,6	124	-2,17	4,2	41	-0,94	6,7
16	141	-2,05	4,9	113	-2,04	4,1	43	-2,14	5,5
17	142	-1,12	4,7	110	-2,93	4,2	44	-2,71	6,1
18	152	-2,02	4,8	91	-0,98	4,7	52	-0,52	5,8
19	99	-2,08	4,7	76	-2,21	3,6	53	0,73	6,1
20	103	-2,42	4,4	77	-1,12	5,8	46	-0,58	9,0
21	90	-2,61	4,4	72	-2,03	3,8	50	-0,58	5,4
22	54	-1,82	5,1	62	-0,45	4,5	50	-1,23	5,2
23	50	-2,87	4,4	46	-3,46	5,9	26	1,50	5,6
24	25	-2,22	4,1	26	-1,45	5,7	29	1,36	6,0
25	18	-1,68	4,2	22	-2,83	5,1	21	0,37	4,8
26	10	-0,55	5,0	16	-1,46	3,7	9	0,59	5,7
27	5	0,67	7,7	14	-1,37	7,0	8	3,85	6,7
28	5	-0,85	3,3	3	-1,69	2,9			
29				5	-4,26	4,0			
30				3	-3,48	7,7	1	9,30	-
Total *Yht.*	2326	-1,12		1864	-2,27		863	-1,18	

The curves clearly demonstrate how great a share of the stem volume is in the butt part. For instance, when the stump height is reduced from 1 % of the tree height to 0,5 %, the stem volume in the case of spruce is increased by 2,1 %. The %-distribution of the stem volume, when the stump height for spruce is 1 % of tree height, is shown on the left-hand side of the figure. It can be seen from this scale that the volume of the stem section from the top down to a height of 20 % from the top comprises only 1,7 % of the total volume of the stem. Half of the total volume of the stem is in the bottom part of the stem upto a height of 25 %.

The I-values at the 99 % point in the curves of Fig. 11 are: pine 0,04766, spruce 0,04527 and birch 0,04433. Thus if trees of different species assume the same height and the same diameter at a height of 20 % and the stump height is 1 % of tree height, the volume of pine is on the average 5,3 % greater than that of spruce and 7,5 % greater than that of birch.

The differences between the tree species can also be examined by means of the volume equations (61.6) and (61.7). It is apparent, on the basis of the comparison between the coefficients of the equations, that the tree species differ rather much from each other. However, it is difficult to estimate the simultaneous effect of all the coefficients without applying the equations.

The means of the percentage differences of the volumes when the volumes of the pine and birch sample trees are calculated with equations (61.6) and (61.7) for spruce, are presented by height classes in Table 17. Equation (61.6) for spruce usually gives too small a volume for pine. The only clear exceptions are short trees and those over 20 m high. Equation (61.7) for spruce was less applicable for large pines than equation (61.6). The difference in the total volumes obtained with equation (61.6) was of the same order of magnitude as would have been expected on the basis of the taper curve.

When the spruce equation was used for birch, the two-predicting variable equation (61.6) gave a considerable overestimate. The bias is over 10 % in a number of height

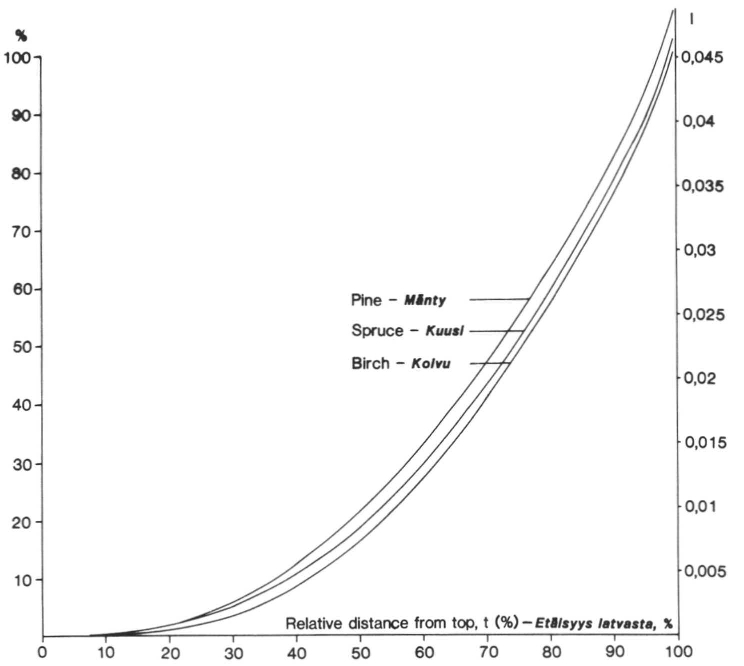

Fig. 11. The integrals (cf. I, (84.1)) of the basic taper curves $f_b(x)$ (cf. model (33.1)) as a function of the relative distance (t) from the top.
Kuva 11. Runkokäyrien perusyhtälöiden $f_b(x)$ (katso malli (33.1)) integraalit (ks. I, (84.1)) rungon latvasta olevan suhteellisen etäisyyden (t) funktiona.

classes, and only in the case of trees 2—3 m high did the spruce equation give too small volumes. The difference obtained with the three predicting variable equation (61.7) was less than that for pine.

Thus the differences between species as regards volume are generally clear, even though the measured variables are the same. As it is not always a question of level differences, the volume equations cannot be calculated from a combined material using dummy variables but instead separate equations for each tree species are required.

The differences between volumes estimates of *Betula pubescens* and *B. pendula* were found to be very small. Equation (61.6) for birch gave a slight overestimate (1,1 %) for *B. pendula,* while there were no clear differences when equation (61.7) was used. Combining the *B. pubescens* and *B. pendula* material is thus justified. However, it should be pointed out that the tallest birches were all *B. pendula,* while most of the medium-sized birches were *B. pubescens.*

Table 17. Percentage differences between the estimated and actual volumes of the pine and birch sample trees when the volumes are calculated by equations (61.6) and (61.7) for spruce, by height classes.
Taulukko 17. Männyn ja koivun koepuiden kuusen yhtälöillä (61.6) ja (61.7) ennustettujen ja todellisten tilavuuksien prosentuaaliset erot pituusluokittain.

h, m	Pine - *Mänty*			Birch - *Koivu*		
	N	(61.6)	(61.7)	N	(61.6)	(61.7)
2	9	13,0		1	-14,2	
3	26	-0,4		3	-18,3	
4	59	-5,5		10	7,5	
5	78	-4,7		15	12,2	
6	87	-5,1		22	10,5	
7	103	-5,8	-3,9 1)	29	9,6	-4,1 2)
8	98	-5,5	-1,3	36	13,5	0,6
9	102	-4,0	-1,1	42	12,0	0,4
10	119	-4,6	0,6	44	12,0	1,0
11	138	-5,6	-0,5	48	13,4	3,2
12	115	-7,3	-1,4	57	11,5	2,8
13	146	-6,3	-1,3	45	9,6	2,8
14	175	-6,1	-1,7	38	12,8	4,1
15	177	-4,6	-2,4	41	9,3	4,3
16	141	-4,3	-3,2	43	9,4	3,7
17	142	-4,7	-3,2	44	6,1	2,6
18	152	-2,4	-4,3	52	6,6	3,0
19	99	-2,2	-4,6	53	9,0	3,3
20	103	-2,6	-5,2	46	10,5	3,2
21	90	-2,4	-5,6	50	8,3	3,5
22	54	0,9	-5,0	50	7,6	2,1
23	50	-1,9	-6,3	26	9,8	2,7
24	25	-0,1	-6,3	29	12,6	2,6
25	18	-0,5	-6,2	21	8,4	1,9
26	10	1,4	-4,6	9	10,3	3,3
27	5	2,8	-2,9	8	5,3	2,2
28	5	-1,9	-6,6			
29						
30				1	16,9	4,2
Total *Yht.*	2326			863		
	Differences between total volumes *Kokonaistilavuuksien erot*					
		-5,6	-4,3		7,97	2,62

1) 49 observations - *havaintoja 49*
2) 12 -"- - -"- *12*

9. SUMMARY

Two alternative methods for calculating the taper curve of a tree are presented in this study. In addition, models are derived for volume equations. The equations required for calculating taper curves, as well as the volume equations, are presented for the three most important tree species in Finland: Scots pine, Norway spruce and birch. The reliability of the methods is examined using residual variances and control materials, as well as theoretical deductions.

The study material was collected during the period 1968—72 from 100 tracts utilized in the National Forest Inventory. The tracts were chosen so as to be representative of all forests growing in Finland. Representivity was improved by selecting the tracts randomly from the area strata. The sample trees were selected by means of a relascope. The material consisted of 2 326 pines, 1 864 spruces and 863 birches.

The diameters of the sample trees were measured at 14 relative heights. The diameters in the butt end of the stem were measured at shorter intervals because the volume of the stem is concentrated in this part of the stem and the taper curve there is irregular, too. The volumes of the sample trees were calculated using cubic spline functions (cf. Lahtinen and Laasasenaho 1979). These stem volumes were compared with the cylindric volumes calculated by mid-point measurement of sections one- or two-meters long. The cylindric volume estimations were found to give clear underestimates, which in the case of small, cordwood-sized stems (d ~ 7 cm) were 2—3 % and on larger trees of the order of 0,5 % when one-meter long sections were used. The errors were clearly greater when two-meter long sections were used. Most of the error was incurred in determining the volume of the first section at the butt end of the stem.

Graphical and numerical examination of the taper curve showed that the ratios of diameters measured at relative heights are of the same order of magnitude in the case of stems of different sizes within each tree species. However, some clear changes occur in the stem form at different stages of development. The mean diameters of all the sample trees were calculated at relative heights to depict the average taper curve for each tree species. The ratios between these mean diameters and the diameter measured at 20 %-height constituted the series of values describing the taper for each tree species. These relative diameter series were described by means of a polynomial, the powers of which were 1, 2, 3, 5, 8, 13, 21 and 34 and by a combination of logarithmic variables and a polynomial of lower power using relative height as the independent variable (cf. models (33.1) and (33.2)).

A polynomial model was chosen as the basic model of the taper curve, the coefficients for it being computed by tree species using the mean diameter series. A taper curve can be calculated for each tree using this basic equation if tree height and diameter at some height are known. This basic model can be made more accurate by means of a correction polynomial. The auxillary equations needed for determining the correction polynomial were computed when either d and h or d, d_6 and h are known. This correction polynomial can be calculated without any auxillary equations if the diameters at three heights along the stem are known. A number of logical conditions, which regulate the taper curve in some special cases, were set on the correction equation of the polynomial taper curve model.

A combination of simultaneous equations and a polynomial taper curve equation was also used in the study. Using the polynomial equation, the nearest relative height diameter included in the simultaneous model could be estimated with the help of the measured diameter. A simultaneous model based on a tridiagonal matrix,

which is much faster to use in calculations than a complete simultaneous model, was developed in the study.

After theoretical scrutiny of the models of the volume equation an attempt was made to find the best models for the most common combination of variables measured in practice and the models based on height and diameters measured at different relative heights along the stem. The models were derived by starting either from the multiplicative formula for volume or by presenting the volume as the sum of the different geometrical sections of the stem. The multiplicative models were linearized by taking the logarithm and the sum models were transformed into form factor models. The residual variances of these models were considered to be sufficiently homoscedastic for regression analysis. The equations were computed separately for each tree species using the same variables.

The relative standard error of the estimate was considered to be the best criterion in the comparison of the equations and in determining their reliabilities. The standard errors of the multiplicative models were smaller in the case of equations based on diameter at breast height and height than the corresponding form factor models. On the other hand, in the case of large trees where the upper diameter, d_6, had also been measured, the reliability obtained with the form factor equations was slightly better than that obtained with the multiplicative models.

Comparisons were made between the models developed in this study and a number of models presented by other authors. Multiplicative models based on height and diameter at breast height proved to be more precise than the equations obtained using the variables of equations currently in use in Sweden. Inclusion of the crown limit and bark thickness suggested by the Swedish model did not increase the precision of the equations by nearly as much as the inclusion of the upper diameter, d_6. Calculating volume by means of the new models proved to be more accurate than those by Ilvessalo's tables.

The reliability of the taper curve models was examined by comparing the diameter estimates with the values measured at different relative heights along the stem. There was a slight bias in the butt of the spruce and birch stems in the estimates given by the polynomial taper curve model (cf. Tables 7 and 8). The difference was statistically insignificant in the simultaneous model. The standard errors of the estimates were of the same order of magnitude with both methods. The graphic examination of the taper curves did not reveal any illogical features even in the cases of rare combinations of the variables.

The volumes obtained with the taper curve models and volume equations were compared to the volume obtained with the spline function. It can be seen from Tables 10—12, that the stem volume estimates are on the average equally reliable with all three methods. As the methods differ considerably from each other, they do give different results for individual trees. All the methods give small systematic errors in certain height classes.

The standard errors of the volume estimates as percentages were of the same order of magnitude in the case of trees of all different sizes, apart from quite small trees. When the estimation of stump height, needed in the taper curve method, is taken into account the volume equations can be seen to be at least as reliable a method for obtaining volume estimates as the taper curve models.

Reliability of the volume equations based on diameter at breast height and height varied between climatic zones. The differences between zones were the clearest for pine. The differences between the climatic zones are smaller when equations based on d, d_6 and h are used. The standard errors of the estimates are greatest in the northern parts of the country. The differences in the stem form at different stages of stand development were also visible when the errors were examined by development classes. However, the systematic errors with all the factors tested were so small, that they are of little practical significance.

Clear differences were found between pine, spruce and birch as regards stem form. On the other hand, stem forms of *Betula pendula* and *B. pubescens* were quite similar.

The taper curve and volume of stem can be calculated for different practical and research purposes by the methods developed in this study. The taper curves represent an adaptable calculation technique for a wide variety of uses and, for instance, a large number of diameter measurements can be employed. Using volume equations a large number of routine volume calculations can be carried out on a small programmable pocket calculator and the calculation can thus be even done in conjunction with the field measurements.

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SELOSTE

Männyn, kuusen ja koivun runkokäyrä- ja tilavuusyhtälöt

Tutkimustehtävä ja -aineisto

Tutkimuksessa esitetään malleja puun runkokäyrän ja tilavuuden määrittämiseksi. Runkokäyrän laskentamenetelmissä tarvittavat yhtälöt sekä tilavuusyhtälöt esitetään Suomen pääpuulajeille: männylle, kuuselle ja koivulle.

Tutkimusaineisto kerättiin vuosina 1968—72 noin sadalta valtakunnan metsien inventoinnin lohkolta. Perusjoukkona olivat kaikki Suomen männyt, kuuset ja koivut. Edustavuuden parantamiseksi lohkot valittiin alueittaista satunnaisotantaa käyttäen. Otannan tehokkuutta lisäsi koepuiden valinta relaskoopilla. Aineisto käsitti 2 326 mäntyä, 1 864 kuusta ja 863 koivua.

Koepuista mitattiin läpimitat 14:ltä suhteelliselta korkeudelta. Koska rungon tilavuus keskittyy tyviosaan ja runkokäyrä on siellä epäsäännöllisin, läpimittoja mitattiin tyviosassa lyhyimmin välein. Koepuiden tilavuudet saatiin kuutiollisen splinikäyrän (ks. Lahtinen ja Laasasenaho 1979) avulla. Näitä runkojen tarkkoja tilavuuksia verrattiin yhden ja kahden metrin pituisten mittauspätkien keskuskiintomittoina saataviin tilavuuksiin. Tällöin todettiin pätkittäisen tilavuuden laskentatavan antavan selvän aliarvion, jonka suuruus oli pienillä käyttöpuun kokoisilla rungoilla (d $\sim$ 7 cm) 2—3 prosenttia käytettäessä yhden metrin pätkiä (katso taulukko 4) ja suurilla puilla puolen prosentin luokkaa. Kahden metrin pätkiä käytettäessä virhe oli selvästi suurempi. Valtaosa erosta syntyi tyvellä ensimmäisessä pätkässä.

Runkokäyrän laadinta

Runkomuodon graafinen ja numeerinen tarkastelu osoitti, että suhteellisilta korkeuksilta mitattujen läpimittojen suhteet ovat erikokoisilla rungoilla puulajeittain samaa suuruusluokkaa, vaikka eri kehitysvaiheissa ilmenee joitakin selviä muutoksia runkomuodossa. Keskimääräistä runkokäyrää kuvaamaan laskettiin puulajeittain kaikkien koepuiden keskiläpimitat eri suhteellisilla mittauskorkeuksilla. Näiden keskiläpimittojen suhteet 20 prosentin korkeudelta laskettuun keskiläpimittaan muodostivat keskimääräistä runkokäyrää kuvaavan lukusarjan puulajeittain. Seuraava polynomimalli valittiin runkokäyräyhtälöiden perusmalliksi, jonka avulla kuvattiin tätä suhteellista läpimittasarjaa:

$$\frac{d_l}{d_{,2h}} = b_1x + b_2x^2 + b_3x^3 + b_4x^5 + b_5x^8 + b_6x^{13} + b_7x^{21} + b_8x^{34}$$

missä $d_{,2h}$ = 20 prosentin korkeudella oleva perusläpimitta

d_l = läpimitta korkeudella l maasta

$$x = 1 - \frac{l}{h} \text{ eli suhteellinen etäisyys maasta.}$$

Mallille laskettiin kertoimet puulajeittain (41.1).

Tämän perusyhtälön avulla voidaan jokaiselle puulle laskea runkokäyrä, kun tunnetaan puun pituus ja läpimitta joltakin korkeudelta. Tätä perusmallia voidaan tarkentaa kolmatta astetta olevan korjauspolynomin avulla. Korjauspolynomin määritystä varten laskettiin tarpeelliset apuyhtälöt, kun puusta tiedetään d ja h tai d, d_6 ja h. Tämä korjauspolynomi voidaan laskea ilman apuyhtälöitä, jos rungosta tiedetään kolmelta eri korkeudelta läpimitat.

Polynomimallilla ei voida joustavasti samassa yhtälössä hyödyntää useampaa kuin kolmea eri korkeudelta mitattua läpimittaa. Simultaaniyhtälöiden (ks. esim. Kilkki 1979) avulla voidaan määrittää rungon läpimitat yhtälöihin valituilla suhteellisilla korkeuksilla ja näiden yhtälöiden käytössä ottaa huomioon useampia läpimitan mittaustietoja. Tutkimuksessa kehitettiin kahteen selittävään läpimittaan perustuva simultaanimalli. Polynomikäyrää käyttäen estimoidaan kunkin mitatun läpimitan avulla lähin simultaanimallissa mukana olevista suhteellisen korkeuden läpimitoista. Menetelmä on laskennallisesti huomattavasti kevyempi kuin täydellinen simultaanimalli.

Tilavuusyhtälöiden laadinta

Tilavuusyhtälöiden mallien johtamisessa pyrittiin löytämään parhaat mallit tärkeimmille käytännön mittaustunnusyhdistelmille sekä puun suhteellisilta korkeuksilta mitattuihin läpimittoihin ja pituuteen perustuvat mallit. Ne johdettiin lähtemällä joko tilavuuden tulomuotoisesta kaavasta tai esittämällä tilavuus erilaisten geometristen kappaleiden summana. Tulomuotoiset mallit linearisoitiin logaritmin otolla ja summamallit muunnettiin muotolukumalleiksi jakamalla g · h:lla. Saatujen mallien jäännöshajonnat katsottiin riittävän homogeenisiksi regressioanalyysia varten. Yhtälöt laskettiin samoilla muuttujilla kullekin puulajille erikseen.

Runkokäyrien ja tilavuusyhtälöiden luotettavuus

Runkokäyrien luotettavuutta tarkasteltiin vertaamalla läpimittaennusteita rungon suhteellisilta korkeuksilta mitattuihin arvoihin ja näiden poikkeamien keskiarvoja ja hajontoja eri korkeuksilla tarkasteltiin jakamalla koepuut pituuden ja läpimitan mukaan luokkiin. Polynomirunkokäyrän ennusteissa oli lievää harhaisuutta rungon tyviosassa kuusella ja koivulla (ks. taulukot 7 ja 8). Simultaanimallissa harha oli merkityksetön. Poikkeamien hajonnat olivat samaa suuruusluokkaa molemmissa menetelmissä. Runkokäyrien graafinen tarkastelu ei osoittanut harvinaisillakaan mittaustunnusyhdistelmillä sellaista epäloogisuutta, joka tekisi ne käyttökelvottomiksi.

Tilavuusyhtälöiden vertailussa ja luotettavuuden määrittelyssä katsottiin ennusteen suhteellinen jäännöshajonta parhaaksi kriteeriksi (ks. (54.1), s. 40). Tulomuotoisten yhtälöiden jäännöshajonnat olivat rinnankorkeusläpimittaan ja pituuteen perustuvilla yhtälöillä pienemmät kuin vastaavilla muotolukumalleilla. Sen sijaan suurilla puilla, joista on mitattu myöskin ylempi läpimitta d_6, muotolukuyhtälöillä saatiin vähän parempi luotettavuus kuin tulomuotoisilla malleilla.

Kolmenkymmenen prosentin korkeudelta mitattuun läpimittaan ($d_{.3h}$) ja pituuteen perustuva tulomuotoinen yhtälö antoi pienemmän jäännöshajonnan kuin rinnankorkeusläpimittaan ja pituuteen perustuva vastaava malli. Kolmeen tai useampaan suhteelliselta korkeudelta mitattuun läpimittaan ja pituuteen perustuva normaalimuotoluvun malli osoittautui tarkaksi puun tilavuuden laskentamalliksi.

Vertailuja tehtiin myöskin eräiden muissa tutkimuksissa esitettyjen tilavuuden laskentamallien ja tässä työssä kehitettyjen mallien välillä. Rinnankorkeusläpimittaan ja pituuteen perustuvat tulomuotoiset mallit osoittautuivat tarkemmiksi kuin Ruotsissa käytössä olevien yhtälöiden muuttujilla saatavat yhtälöt. Latvusraja ja kuoren paksuus antoivat huomattavasti vähäisemmän lisätarkkuuden yhtälöille kuin ylempi läpimitta d_6.

Runkokäyrillä ja tilavuusyhtälöillä saatuja tilavuuksia verrattiin splini-käyrällä saatuun tilavuuteen ja näitä poikkeamia tarkasteltiin erikseen kunkin mittaustunnuksen suhteen sekä myöskin d—h -luokittain ja kapenemis ($d—d_6$) luokittain. Taulukoista 10—12 voidaan havaita, että rungon tilavuus saadaan keskimäärin yhtä luotettavasti kaikilla kolmella menetelmällä. Koska menetelmät poikkeavat huomattavasti toisistaan, saadaan niillä yksittäisille puille jonkin verran erilaisia tuloksia. Kaikki menetelmät antavat pieniä systemaattisia virheitä eräissä pituusluokissa, johtuen runkomuodon muuttumisesta metsiköiden eri kehitysvaiheissa. Tilavuuden poikkeamaprosenttien hajonnat olivat aivan pieniä puita lukuunottamatta samaa suuruusluokkaa erikokoisilla puilla. Kun otetaan huomioon runkokäyrissä tarvittava kannon korkeuden estimointi, saadaan tilavuusyhtälöillä rungon tilavuus ainakin yhtä luotettavasti kuin runkokäyrillä ja lisäksi ne ovat laskennallisesti erittäin yksinkertaiset.

Tilavuusyhtälöiden luotettavuudessa oli havaittavissa ilmastovyöhykkeittäin jonkinasteisia eroja, kun ennustetietoina oli rinnankorkeusläpimitta ja pituus. Selvimpiä erot olivat männyllä. Nämä erot johtuvat puuston runkomuodon eroista maan eri osissa. Kolmen tunnuksen (d, d_6, h) yhtälöillä poikkeamat ilmastovyöhykkeiden välillä ovat pienet. Poikkeamien hajonnat ovat pohjoisosissa kuitenkin suurimmat. Puuston runkomuodon erot eri kehitysvaiheissa olivat myös nähtävissä, kun virheitä tarkasteltiin kehitysluokittain. Systemaattiset virheet olivat kuitenkin kaikkien tarkasteltujen tekijöiden suhteen niin vähäisiä, ettei niillä ole käytännön toiminnalle merkitystä. Tilavuuden laskenta osoittautui näillä uusilla menetelmillä tarkemmaksi kuin käytännössä olevilla taulukoilla.

Puulajien välillä todettiin runkomuodossa selviä eroja. Siten esim. tilavuudet saattavat poiketa useita prosenttiyksiköitä, kun mittaustunnukset ovat samat. Koska kysymyksessä eivät ole tasoerot tai suhteelliset erot, puulajeittaisten tilavuusyhtälöiden yhdistäminen dummy-muuttujilla heikentäisi luotettavuutta. Raudus- ja hieskoivun välillä ei sen sijaan ollut havaittavissa selviä eroja.

Päätelmiä

Tässä tutkimuksessa kehitetyillä menetelmillä voidaan laskea puiden runkokäyrä ja tilavuus erilaisiin käytännön ja tutkimuksen tarkoituksiin. Runkokäyrät tarjoavat joustavan laskentamahdollisuuden mitä erilaisimpiin laskentatarpeisiin eivätkä ne vaadi määrättyjä mittaustunnuksia, vaan niitä käytettäessä voidaan hyödyntää monia läpimitan mittauksia. Niitä voidaan käyttää myös, kun mittausten lähtöpiste on ylin kaatoa haittaava juurenniska, ilmoittamalla mittaustapa. Tilavuusyhtälöitä käyttämällä voidaan monet tilavuuden laskentarutiinit suorittaa jopa pienillä ohjelmoitavilla taskulaskimilla ja täten siirtää laskentavaiheet tietojen tarvitsijoille tai jopa mittausten yhteyteen.

LAASASENAHO, J. 1982. Taper curve and volume functions for pine, spruce and birch. Seloste: Männyn, kuusen ja koivun runkokäyrä- ja tilavuusyhtälöt. Commun. Inst. For. Fenn. 108:1—74

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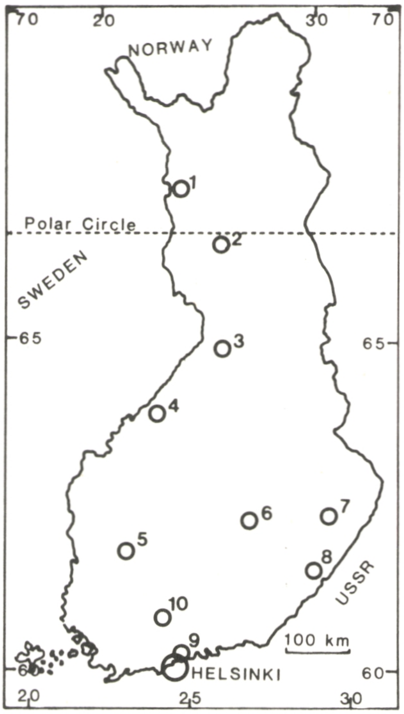

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FACTS ABOUT FINLAND

Total land area: 304 642 km^2 of which 60—70 per cent is forest land.

Mean temperature, °C:	Helsinki	Joensuu	Rovaniemi
January	-6,8	-10,2	-11,0
July	17,1	17,1	15,3
annual	4,4	2,9	0,8
Thermal winter (mean temp. < 0°C):	20.11.—4.4.	5.11.—10.4.	18.10.—21.4.

Most common tree species: Pinus sylvestris, Picea abies, Betula pendula, Betula pubescens

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