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Integrating Economics and Ecology in Stand-Level Timber Production

Kari Hyytiäinen

VANTAAN TUTKIMUSKESKUS – VANTAA RESEARCH CENTRE

METLA

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Kari Hyytiäinen

Doctoral dissertation at the Department of Forest Economics, University of Helsinki

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Contents

Abstract

This dissertation examines the economics of stand-level timber production in even-aged conifer stands. Models containing a Faustmann-type description of timber production and forest ecological descriptions of tree and stand growth are developed and analyzed. Three well-established descriptions of growth with increasing degree of detail are used: a variable-density whole stand model, a distance-independent individual-tree model, and a process-based stand growth model. The analysis is extended to various Scots pine (*Pinus sylvestris* L.) and Norway spruce (*Picea abies* [L.] Karst.) site classes, initial stand states, and stand structures.

The thesis consists of a summary section and five separate studies, which present results, for example, on optimal rotation length, thinning patterns (type, intensity and frequency of thinnings), juvenile density, value of forests, cost structure, and the structure and level of long-run timber supply in Finland. It is found that optimal harvesting departs from silvicultural recommendations and in some cases violates the legal restrictions applied in Finland. Efficiency gains from following the economic principles are computed, the reasons for deviations are elaborated, and policy conclusions are drawn. It is further questioned whether and what type of state intervention is presently needed in timber production in Finland. The potential for economic optimization in forestry planning and extension is highlighted.

Inclusion of biological detail in economic optimization enables one to relax the assumptions concerning stand growth and harvest control, to better specify timber quality, and to deepen the economic analysis of various forestry activities. Numerical optimization turns out to be a useful research tool in combining economics and ecology. Sensitivity analysis of empirically validated numerical models provides information on the quantitative significance of the theoretical forest economics results. In addition, it highlights economically promising stand management strategies and ecological model components that may require further investigation and field experimentation.

Keywords: even-aged management, forest economics, forest policy, numerical optimization, *Picea abies*, *Pinus sylvestris*, timber quality.

Abstrakti

Metsänkasvatuksen taloutta voidaan tutkia matemaattisilla optimointimalleilla, joissa yhdistetään eri tieteenaloilla saatuja tutkimustuloksia metsikön kasvusta ja metsänkasvatukseen liittyvistä tuloista ja kustannuksista. Tässä väitöskirjatutkimuksessa yhdistetään erilaisia tasaikäisten havumetsiköiden metsikkö- ja puutason kasvumalleja taloudelliseen kuvaukseen raakapuun tuotannosta. Puiden elintoimintojen kuvaukseen perustuva prosessipohjainen kasvumalli liitetään ensimmäistä kertaa taloudelliseen optimointiin. Analyysit kattavat suuren määrän erilaisia lähtötiloja ja männyn ja kuusen kasvupaikkoja.

Väitöskirja koostuu viidestä artikkelista ja yhteenveto-osasta. Tutkimusartikkelit esittävät tuloksia muun muassa optimaalisista kiertoajoista ja harvennuksista (harvennusten lukumäärä, harvennuspoistuma ja harvennustapa), taimikkovaiheen kasvatustiheyksistä, metsiköiden ja metsämaan arvosta, metsänkasvatuksen kustannusrakenteesta ja pitkän aikavälin puuntarjonnasta Suomessa. Optimointitulosten mukaan metsänkasvatuksen kannattavuutta voidaan merkittävästi parantaa ottamalla taloudelliset ja metsikön rakenteeseen liittyvät tekijät eksplisiittisesti huomioon hakkuiden ja metsänhoidollisten toimenpiteiden suunnittelussa. Optimaaliset metsänkäsittelyt poikkeavat metsänhoidollisten suositusten mukaisista käsittelyistä ja ovat joissain tapauksissa ristiriidassa hakkuita koskevan lainsäädännön kanssa. Viimeaikaisten yhteiskunnallisten muutosten (esimerkiksi pääoma- ja metsämaamarkkinoiden vapautuminen) perusteella voidaan kysyä tarvitaanko Suomessa yhä hakkuiden lainsäädännöllistä rajoittamista vai voisivatko toimivat raakapuumarkkinat turvata puuntarjonnan jatkuvuuden.

Numeerinen optimointi on käyttökelpoinen menetelmä yhdistää metsäekologista, taloustieteellistä ja korjuuteknologiaan liittyvää tutkimustietoa. Optimointimallien analyysi antaa myös palautetta metsäekologisten mallien kehittämiseksi. Numeeristen mallien herkkyysanalyysi auttaa identifioimaan kasvumallien komponentteja ja taloudellisesti tehokkaita metsänkäsittelymenetelmiä, joiden lisätutkimukseen on tarpeen suunnata resursseja. Numeerisen optimoinnin avulla voidaan lisäksi saada käsitys teoreettisten metsäekonomisten tulosten kvantitatiivisesta merkityksestä.

Avainsanat: metsäekonomia, metsäpolitiikka, numeerinen optimointi, *Picea abies*, *Pinus sylvestris*, puun laatu, tasaikäinen metsikkö

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Helsinki, 12 December 2003

Kari Hyytiäinen

List of separate studies

The dissertation includes the following studies, which are referred to by roman numerals in the text:

I Hyytiäinen, K. and Tahvonen, O. 2001. The effects of legal limits and recommendations on timber production: the case of Finland. Forest Science 47: 443-454.

II Hyytiäinen, K. and Tahvonen, O. 2002. Economics of forest thinnings and rotation periods for Finnish conifer cultures. Scandinavian Journal of Forest Research 17: 274-288.

III Hyytiäinen, K. and Tahvonen, O. 2003. Maximum Sustained Yield, Forest Rent or Faustmann: Does it really matter? Scandinavian Journal of Forest Research 18: 457-469.

IV Hyytiäinen, K., Tahvonen, O., and Valsta, L. 2003. Optimum juvenile density, harvesting and stand structure in even-aged Scots pine stands. Accepted for publication in Forest Science.

V Hyytiäinen, K., Hari, P., Kokkila, T., Mäkelä, A., Tahvonen, O., and Taipale, J. 2003. Connecting a process-based forest growth model to stand-level economic optimization. Submitted manuscript.

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Author's contribution:

The research problems of all the separate studies were developed jointly with Olli Tahvonen. The optimization models and software were constructed jointly with Olli Tahvonen for studies **I–III** and with Pertti Hari, Tero Kokkila, Annikki Mäkelä, Olli Tahvonen, and Juhani Taipale for study **V**. The author modified the optimization program provided by Lauri Valsta for the purposes of study **IV**. The author acquired data, made the computations, interpreted the results, and assumed the main responsibility for writing the research articles.

1 Introduction

1.1 Background and Scope

Forest economics research can be considered a subfield of either resource and environmental economics or forest sciences. It employs a spectrum of theories and techniques developed under various disciplines (economics, mathematics, biology, forest ecology, computer sciences, forest engineering etc.) to investigate rational use of a renewable natural resource. A renewable resource can be viewed as a capital stock that is capable of yielding a sustainable consumption flow over time (Johansson and Löfgren 1985, p. 46). Managed forests meet this definition particularly in boreal and temperate zones, where they regenerate naturally or are regenerated artificially after clearcutting harvests.

A forest stand is a geographically contiguous parcel of land considered homogeneous in terms of tree vegetation (Davis et al. 2001, p. 65). It is typically the smallest operational unit, and thus offers an appropriate level for examining how the sequence of management activities should be designed in order to maximize the long-run economic surplus of timber production or some other objective. Stand-level results can be aggregated to forest or regional level if the management of individual stands does not affect optimal management of other stands (see Johansson and Löfgren 1985, p. 121).

This thesis aims to examine the economics of timber production in even-aged stands. Forestry and forest-based industries are important economic activities for many national economies. Thus, there is a practical need for economic research of timber production. There is a direct link between stand-level economic research results and practical forest management. Research results on stand establishment, harvesting, land value etc. consider daily decision making problems of non-industrial private forest owners as well as professional foresters working in forestry extension or managing industrially or publicly owned forest lands.

In addition to timber, forests provide other important products and services, such as habitats for plants and animals, amelioration of soil and atmosphere, outdoor recreation, berries, mushrooms, and grazing of livestock. Economic research on timber production provides a solid foundation to extend the analysis to the more complex problem of joint production of multiple forestry benefits. Moreover, research results on timber production can be used to compute pecuniary costs that may follow from policy prescriptions designed to promote environmental values of forests.

The theoretical basis of the economics of stand-level timber production is well established (Faustmann 1849, Pressler 1860, Ohlin 1921, Samuelson 1976, Johansson and Löfgren 1985). This thesis is confined to those extensions of the Faustmann model that include thinnings and other stand management activities as optimized variables, in addition to rotation length. These extensions require special emphasis on how stand growth, harvesting technology and harvest control are described. Such problem formulations are in most cases nonconvex, nondifferentiable, and too complex to be solved analytically.

Complex models can be assigned explicit parameter values using empirical data and the optimization problem can be solved numerically. Detailed and empirically validated numerical models combine microeconomic theory of investment, forest ecological theories on tree and stand growth, and forest engineering regarding logging and on-site transport of timber. Construction of such models requires collaboration across disciplines. Recent developments in computers and powerful solution algorithms have given us the requisites for quantitative analysis of forestry.

The production process behind stand-level timber production is extremely complex. It involves tree growth processes, interactions between individual trees, interactions between trees and other living beings (other plants, animals, bacteria and fungal organisms) and with abiotic factors (soil, precipitation, solar radiation, winds etc.). Capturing all these relationships in a single model may not be possible. However, stand growth models used in economic research of forestry must be detailed enough to reliably predict development of stand value over time and as a response to analyzed disturbances, harvests or other management activities. They must also include those attributes of tree populations that best reflect stand value.

Numerical analysis is a powerful tool in economics because it provides information on the quantitative significance of theoretical results (Judd 1999, p. 6-). It enables testing whether qualitative results obtained with stylized descriptions of the production process are valid with models that are more detailed. Sensitivity analysis of empirically validated models also highlights economic parameters that are most instrumental in determining the economic surplus of forestry and therefore need to be estimated or predicted carefully. Numerical optimization may provide feedback also for ecological research. Exploring a huge number of different stand management alternatives when searching for the optimum may reveal growth model components that need further research and field experimentation.

In forest economics research, there is a gap between theoretical and numerical studies that investigate the economics of stand-level forest management. The theoretical studies extend the rotation model with economic complexities and emphasize the search for intuitive interpretations of results obtained with stylized growth models. Numerical optimization studies, on the other hand, pay attention to specification of sophisticated growth models using empirical data. However, they typically focus less on interpreting the underlying reasons for numerical results. In many numerical studies, interpretation is hampered by constraints designed to ensure that solutions fall within reliable ranges of empirical growth models.

1.2 Aims and outline

The central theme of this thesis is to integrate economic and ecological knowledge so as to better understand the rational use of forest resources. This is done by developing and analyzing numerical optimization models that contain economic descriptions of timber production and increasingly detailed forest ecological descriptions of stand growth. Three well-established descriptions of growth are used: a density-dependent whole stand model specified for Finnish cultures of Scots pine (*Pinus sylvestris* L.) and Norway spruce (*Picea abies* [L.] Karst.), a distance-independent individual-tree model for Scots pine, and a process-based individual-tree model for Scots pine.

A particular aim is to extend the existing research by integrating a detailed process-based forest growth model with an economic optimization model. Process-based growth models are based on description of the fundamental ecophysiological processes such as photosynthesis. They explicitly include the causal relationships between stand management and development of tree characteristics. Therefore a process-based model, as compared to the empirical models used earlier in numerical optimization, provides a better foundation for extending the analysis to management strategies on which empirical data does not yet exist.

This study attempts to narrow the gap between theoretical and numerical research disciplines by providing solid interpretation for numerical optimization results obtained from increasingly detailed growth models. Attempts are made to offer intuitive interpretations for optimal solutions that include up to 25 optimized variables and may involve thousands of state variables. The underlying economic-ecological relationships of rational timber production are elaborated by conducting comprehensive sensitivity analyses (or comparative statics analyses) with respect to site fertility classes, initial stand states, and economic parameters (rates of interest, timber prices, cost parameters). Results concern, for example, optimal rotation length, thinning patterns (type, intensity, timing and frequency of thinnings), juvenile density, value of forests, cost structure, and long-run timber supply.

As a policy-relevant objective, this thesis aims at clarifying the roles of economics and ecology in the Finnish forestry extension and legislation concerning stand-level timber production. Results from earlier economic studies provide evidence that the economically optimal stand management may violate the Finnish legal restrictions and depart from the guidelines applied in forestry extension (e.g. Valsta 1992b, Vettenranta and Miina 1999). In this dissertation, efforts are made to determine the efficiency gains that follow from application of economic principles rather than some other decision criterion. The reasons for deviations are analyzed, practical applicability of the optimization results is considered, and policy conclusions are drawn.

The summary section of this thesis is organized as follows. The characteristics and limitations of different stand growth models used in optimization are explained in the second section. The classical Faustmann rotation model and an economic model for simultaneous optimization of thinnings and clearcutting are presented in section 3. Earlier studies are reviewed in section 4, and the data used in the separate studies are presented in section 5. The main results of the separate studies are presented in section 6 and discussed in section 7.

2 Stand growth models

2.1 Whole stand models

Stand growth can be described at various levels of aggregation. The most direct way is to express stand volume or value as a function of stand age. The curve is typically strictly concave, but may have a convex section at young ages (for illustration, see Chang 1984). These models are classified as density-free whole stand models (Davis et al. 2001, p. 188) or univariate models (Getz and Haight 1989, p. 228). They provide a tractable framework to extend the classical Faustmann rotation model with economic complexities, but do not enable inclusion of thinnings or other intermediate activities as endogenous variables in optimization.

The whole stand model can be extended to include stand density as an independent variable. This extension enables simultaneous determination of thinning patterns (timing, frequency and intensity of thinnings) and clearcutting. Such models are called variable-density whole stand models according to Davis et al. (2001, p. 194). Density is typically described as basal area (i.e. total cross-sectional area of trees measured at breast height per area unit). These models contain a few state variables (such as basal area, volume or number of trees) that evolve over stand age due to growth, mortality and harvesting.

A diameter distribution model is a refinement of the basic variable-density whole stand model. It assumes a certain diameter class structure that may depend on state variables. This refinement enables more accurate economic analysis that can consider harvesting costs and log values by size classes (Davis et al. 2001, p. 197-198).

The main limitation of the whole stand models is that description of stand structure is either neglected or is too aggregated to realistically describe stand development over time or in response to various thinning options. More detailed models are needed to predict growth and stand dynamics, for example, for stands with downward-sloping diameter distributions or when optimizing the type of thinning. The type of thinning determines the proportion of trees removed from different canopy layers or diameter classes.

2.2 Stage-structured models

In stage-structured models (Getz and Haight 1989, p. 230-239), stand state is described by tree size distribution. Trees are classified in discrete size classes, typically characterized by tree diameter measured at breast height. Each class is represented by average tree volume, tree height, and number of trees. Growth is described as the transition from one predefined class to another at discrete time intervals. Recruitment and survival functions define in-growth and mortality. Size class attributes are added up to obtain stand characteristics, such as stand basal area or volume.

The effects of in-growth and selective harvests on stand structure can be explicitly accounted for in stage-structured models. This is a favorable feature when describing the development of uneven-aged stands. Stage-structured models are also more compact and tractable for numerical analysis than individual-tree models that treat tree age and size attributes as separate state variables (Haight and Getz 1987). The main limitation of stage-structured models is that tree size is not always a valid representation of development stage.

2.3 Individual-tree models

Individual-tree (or single tree) models describe a forest stand using a list of tree records (Getz and Haight 1989, p. 239-242, Davis et al. 2001, p. 210-). Each tree is characterized by a number of state variables reflecting its current dimensions (diameter, height, crown ratio etc.). The tree vectors evolve over time due to growth, mortality, and harvesting. With distance-independent individual-tree models, growth and mortality are specified as functions of stand density variables. Each tree in the list is assigned an additional state variable representing the number of its kind in the stand. With distance-dependent (or spatial) individual-tree models, in contrast, growth depends explicitly on a tree's location and size relative to its neighbors.

Individual-tree models are the most detailed specifications of stand growth currently used in stand-level optimization. They enable one to explicitly account for the effects of with-in stand competition and harvesting on stand structure and growth of individual trees. Moreover, the number of trees, which describes stand structure, can be very high (Haight and Getz 1987).

2.4 Process-based stand growth models

The common feature of all earlier stand-level optimization studies (except for **V** in this thesis) is that the description of growth is based on "empirical" growth models. In these models, equations predicting tree or stand growth are based on statistically estimated relationships observed from the field measurement data. Process-based growth models, in contrast, emphasize description of the fundamental ecophysiological processes such as photosynthesis, carbon balance, water balance, and nutrient uptake (Bossel 1991, Korzukhin et al. 1996, Hari 1999, Mäkelä et al. 2000a).

A process-based forest growth model typically consists of a number of submodels or modules that together create a complex hierarchical structure. Hierarchical structure is also characteristic of a real system. The process-based models explicitly include the causal relationships and feedbacks that drive the development of trees. Model outcomes can be explained by analyzing the temporal sequence of the interconnected component processes (Mäkelä et al. 2000a). As an example, accumulation of biomass production in tree boles depends heavily on how tree crowns and foliage mass develop. Development of crown morphology, on the other hand, is affected by the tree's competitive status, development of stand density, and past stand management.

It is a common feature of process-based stand growth models that the balance between the carbon produced through photosynthesis and lost through respiration and senescence determines the accumulation of biomass in trees. Many process models also describe major structural relationships. For example, the pipe-model theory assumes that the cross-sectional area of sapwood in tree boles (functioning as a channel for water) is described by a constant ratio to foliage mass above that cross-section at any tree height (see e.g. Mäkelä 1997). Such relationships are used to determine how biomass production is allocated between and within different tree compartments: bole, branches, roots, and foliage (Hari et al. 1985).

The need to integrate complex layers of biological information and technical improvements in computers have given birth to a number of process-based growth models ranging from tree to landscape levels during the last 20 years (Battaglia and Sands 1998, Johnsen et al. 2001). The most useful process models from the point of view of economic research of forestry are obviously those that predict the growth of individual trees in a forest stand (i.e. follow the general structure of individual-tree models). These enable detailed descriptions of both stand structure and tree structure.

Process models are potentially very useful in economic analysis of timber production. Inclusion of feedback effects of stand management and detailed description of tree structure and crown morphology enable explicit inclusion of timber quality in the analysis. Owing to solid theoretical background, process models may provide good predictions of growth also in cases where empirical data do not yet exits. These may include, for example, unconventional management activities or changing environment with respect to nitrogen deposition or temperature. Limited application due to scarcity of data is a common problem in numerical optimization. It has forced researchers using empirical growth models to set constraints or penalty functions in their optimization codes, or to interpret cautiously any solutions falling outside of the boundaries of conventional stand management.

Ideally, all the submodels or modules of a process model can be tested independently against the empirical data. However, all the existing process-based models still share the problems of a lack of precise data and incomplete understanding of some important processes. A promising approach for solving this problem is to incorporate both empirical and process-based components at the same hierarchical level (Battaglia and Sands 1998, Mäkelä et al. 2000a). The parameter values of such "hybrid" models can be calibrated by using empirical data relating to the whole system. The process model applied in **V** (Mäkelä 1997, 2002, Mäkelä and Mäkinen 2003) falls into the category of hybrid models. Its outcomes have been tested extensively with empirical data (see e.g. Mäkelä et al. 2000b, c).

3 Stand-level economic models

The economic approach to stand-level timber production is straightforward: plan the sequence of stand establishment, silvicultural, and harvesting activities such that the net present value of forest stand is maximized. This approach was first applied by Faustmann (1849) to select optimal rotation length for even-aged stands. The classical forest rotation problem can be written as

$$\max_t V(t) = \left[pg(t)\,e^{-rt} - W\right]\left(1 - e^{-rt}\right)^{-1}, \qquad [1]$$

where V is the net present value of bare land, t is the stand age at clearcutting, p is the stumpage price of timber, r is the real market rate of interest, W is the exogenously given stand establishment cost, and $g(t)$ denotes the output from clearcutting measured in cubic meters of usable timber. It is normally assumed that the growth function $g(t)$ is twice continuously differentiable. With some additional assumptions, it is possible to determine a unique optimum[1]. Bare land value V can be maximized by differentiating [1] with respect to stand age t, and setting the resulting derivative equal to zero. For an interior solution (t*>0), the first order condition can be written as:

$$pg'(t^*) = rpg(t^*) + rV(t^*). \qquad [2]$$

This yields an intuitive decision rule: clearcut a forest stand at age t^* when its value growth rate, $pg'(t^*)$, equals the interest on the timber value, $rpg(t^*)$, and forestland, $rV(t^*)$ (see Johansson and Löfgren 1985, p. 73-80, for details).

The classical Faustmann rotation model is based on a set of "heroic" assumptions (Samuelson 1976, Johansson and Löfgren 1985). The markets for forestland and capital are assumed to be perfect, and all the future economic parameter values are assumed to be known with certainty. Timber is the only product considered; the existence of any other forest benefits or externalities is ignored. Scale returns from simultaneous management of neighboring stands are also neglected.

There is an expanding theoretical literature extending the classical Faustmann rotation model to imperfect capital markets (e.g. Tahvonen et al. 2001), non-timber benefits (e.g. Hartman 1976, Swallow and Wear 1993, Tahvonen and Salo 1999), endogenous timber prices (e.g. Mitra and Wan 1985, Salo and Tahvonen 2002, 2003), and nonlinear harvesting cost (e.g. Heaps 1984). The perfect foresight assumption has been relaxed in studies that focus on stochastic stand growth (e.g. Clarke and Reed 1989, Willasen 1998), timber prices (e.g. Brazee and Mendelsohn 1988, Plantinga 1998), rate of interest (Alvarez and Koskela 2003), and risk of natural hazards (Reed 1984).

There has been increasing interest in the last 35 years in extending the classical Faustmann model by including thinnings as optimized variables. The appropriate prob-

[1] The sufficient conditions for uniqueness of optimum are e.g. $g'(t)>0$ and $g''(t)<0$ (Johansson and Löfgren 1985, p.80), or $g'''(t) \le 0$ or $g''(t)<r$ for all t whenever $pg(t) \ge W$ (Tahvonen et al. 2001).

lem formulation depends on the number and character of optimized variables, applied growth model, and solution method. An exemplary problem formulation for simultaneous optimization of thinning schedule (number, timing, intensity and type of thinnings) and rotation length is presented next. This prototype model is suited for distance-independent individual-tree models that describe stand structure with a list of tree records, and can be solved using numerical optimization algorithms.

Let $\mathbf{Z}_t$ be a matrix describing stand state at stand age t:

$$\mathbf{Z}_t = \begin{bmatrix} X_{11t} & X_{12t} & \cdots & X_{1dt} \\ X_{21t} & X_{22t} & \cdots & X_{2dt} \\ \vdots & \vdots & & \vdots \\ X_{n1t} & X_{n2t} & \cdots & X_{ndt} \end{bmatrix}. \quad [3]$$

This matrix consists of row vectors for n mean trees representing the size classes. The number of state variables describing each class is denoted by d. The first element in each row vector X_{i1t}, $i=1,...,n$, represents the number of similar trees in the stand. The remaining elements in each row vector represent the characteristics of mean trees such as tree diameter at breast height, tree height etc. The number of tree state variables, d, typically increases with complexity of the model. The matrix evolves over time due to tree growth, mortality, and harvesting.

Let $\mathbf{h}_u$ denote an n-dimensional vector representing the proportion of trees removed from different classes in thinnings $u=1,...,k-1$; and q the number of optimized thinning rate variables for each thinning. The optimization problem is to select the timing of thinnings, t_u, $u=1,...,k-1$, and clearcutting t_k, and thinning rates γ_{us}, $u=1,...,k-1$, $s=1,...,q$, such that the value of bare land V is maximized:

$$\max_{\left\{\begin{smallmatrix} t_u, u=1,...,k, \\ \gamma_{us}, u=1,...,k-1; \\ s=1,...,q \end{smallmatrix}\right\}} V = \frac{\sum_{u=1}^{k} e^{-rt_u}\left[\sum_{i=1}^{n}\sum_{j=1}^{m} p_{uij}(\mathbf{Z}_{t_u}) g_{uij}(\mathbf{Z}_{t_u}, h_u(i)) - C_u(\mathbf{Z}_{t_u}, \mathbf{h}_u)\right] - W(\mathbf{Z}_{t_0})}{1-e^{-rt_k}}. \quad [4]$$

Gross harvest revenues are summed over n size classes and m roundwood categories as products of roadside prices, p_{uij}, and harvested volumes, g_{uij}. The net stumpage revenues are attained by subtracting the stump-to-truck harvesting cost, C_u. The stumpage revenues from all thinnings and clearcutting are discounted to the beginning of the rotation period. Stand establishment cost, W, is subtracted from discounted stumpage revenues. Finally, the discounted net revenues from one rotation are converted to discounted net revenues from infinite series of similar rotation periods (i.e. net present value of bare timberland) by using a multiplier.

The value of bare land in [4] is maximized subject to the stand dynamics equation that defines the stand state just before each harvest at age t_u, $u=1,...,k$. This is given as a function of stand state before the previous harvest ($\mathbf{Z}_{t_{u-1}}$), intensity of the previous harvest ($\mathbf{h}_{u-i}$), and time difference between the present (t_u) and previous harvest (t_{u-1}):

$$\mathbf{Z}_{t_u} = f(\mathbf{Z}_{t_{u-1}}, \mathbf{h}_{u-1}, t_u - t_{u-1}), \qquad u = 1,\dots,k. \qquad [5]$$

A large number of size classes are needed to describe the structure of stand in a detailed manner. However, optimizing the thinning rate separately for each class would lead to an excessively large number of decision variables. In order to keep the problem solvable, the number of optimized variables can be reduced e.g. by grouping into diameter classes (see e.g. Wikström 2001). Another option applied here is to determine thinning rates for the largest and the smallest tree, and some intermediate tree diameters, and to apply a set of piecewise linear restrictions to determine thinning rates for the other tree classes. The diameter difference between the largest, d_{max}, and smallest tree, d_{min}, can be divided into segments of equal length: $D_s=d_{min}+(d_{max}-d_{min})(s-1)/(q-1)$, $s=1,\dots,q$, where D_1 is the diameter of the smallest tree, D_q is the diameter of the largest tree, and D_s, $s=2,\dots,q-1$, denote the boundaries between segments. Flexibility in selecting trees for removal in thinnings increases with the exogenously given number of thinning rate variables, q. This can be set at a level, which reflects the loggers' capability to select trees in mechanized harvesting based on tree diameter distribution.

Let the optimized thinning rate variables, $\gamma_{us} \in [0, 1]$, $u=1,\dots,k-1$, $s=1,\dots,q$, satisfy the system

$$\begin{aligned}
a_{u1}\, D_{u1} + b_{u1} &= \gamma_{u1} \\
a_{u1}\, D_{u2} + b_{u1} &= \gamma_{u2} \\
a_{u2}\, D_{u2} + b_{u2} &= \gamma_{u2} \\
a_{u2}\, D_{u3} + b_{u2} &= \gamma_{u3} \\
&\vdots \\
a_{u,q-1}\, D_{u,q-1} + b_{u,q-1} &= \gamma_{u,q-1} \\
a_{u,q-1}\, D_{u,q} + b_{u,q-1} &= \gamma_{u,q},
\end{aligned} \qquad [6]$$

where a_{us} and b_{us}, $u=1,\dots,k-1$, $s=1,\dots,q-1$, are constants. Choosing γ_{us}, $s=1,\dots,q$, determines a_{us} and b_{us}, $s=1,\dots,q-1$. The thinning rate for each tree class $h_u(i)$ represented by tree diameter d_{ui}, $i=1,\dots,n$ is determined by

$$h_u(i) = a_{us} d_{ui} + b_{us}, \quad d_{ui} \in [D_{u,s}, D_{u,s+1}], i = 1,\dots,n, s = 1,\dots,g-1, u = 1,\dots,k-1. \quad [7]$$

Consequently, the selection of γ_{us}, $s=1,\dots,q$, determines $h_u(i)$, $i=1,\dots,n$, for each thinning $u=1,\dots,k-1$. An additional constraint
is needed to ensure that the harvests are in proper order.

$$t_{u-1} \leq t_u, \qquad u = 1,\dots,k, \qquad [8]$$

The harvesting problem [4]-[8] is solved for different numbers of thinnings in order to find the optimum number of thinnings and global maximum. Compared to the classical rotation model [1], this formulation enables richer and more detailed analysis of stand management. Timber prices, harvesting costs, and division of the growing stock into roundwood categories depend on stand state in [4]. This enables relaxation of the strict assumptions concerning stand development, prices, and costs. Note that [4] can also be extended to maximize the net present value of existing stands and to allow any initial state (see **III**).

4 Review of earlier literature

Simultaneous determination of thinnings and clearcutting was first specified as a continuous-time optimal control problem (Näslund 1969, Schreuder 1971). Clark (1976) gave an analytical solution to this problem. He used the volume growth equation by Kuusela and Kilkki (1963) to produce a solution where stand volume is kept at the optimal level by continuous thinning. This approach was later extended to study e.g. joint production of multiple forestry benefits (Steinkamp and Betters 1991, Strange et al. 1999) and the effects of natural hazards on optimal thinnings (Reed and Apoloo 1991). The results for the effects of economic parameters (Clark and De Pree 1979, Cawrse et al. 1984, Wernerheim 1989) and growth formulations (Betters et al. 1991) on optimal stocking are intuitively appealing. The drawback is that thinning forest stands continuously is unrealistic due to the positive fixed cost of harvesting.

Stand-level optimization studies that employ discrete thinnings are listed in Table 1. The list mainly consists of studies concerning even-aged management, and updates the earlier reviews in Brodie and Haight (1985) and Valsta (1993). The list is not intended to be exhaustive, but rather to illustrate trends and coverage of research results by geographical regions, tree species, and investigated topics.

The earliest studies that were able to determine optimum timing, number, and intensity of thinnings employed variable-density whole stand models (Amidon and Akin 1968, Kilkki and Väisänen 1969, Risvand 1969, Brodie et al. 1978). Since these pioneering studies, whole stand models have been extended to study, for example, the effects of stochastic prices (e.g. Haight and Smith 1991, Teeter and Caulfield 1991, Brazee and Bulte 2000), growth (Kao 1982, Gong 1998), natural hazards (Thorsen and Helles 1998), and climate change (Jacobsen and Thorsen 2003). The reviewed studies employing whole stand models, except for Torres-Rojo and Brodie (1990), neglect stand structure.

Stage-structured models have proved useful particularly in analyzing uneven-aged stand management (Haight et al. 1985b) and comparing different management systems (Haight 1987, 1991). However, during the last 10-15 years individual-tree models have gained increasing popularity. Vastly increased computational capacity, recent developments in growth and yield research, and availability of powerful solution algorithms have greatly contributed to this trend. Detailed discussion of benefits, disadvantages, and plausible applications of various optimization methods can be found in Yoshimoto et al. (1990), Valsta (1990, 1993), and Eriksson (1994).

Stand-level optimization literature concentrates on coniferous species: Douglas-fir (*Pseudotsuga menziesii*), pine (*Pinus sp.*), and spruce (*Picea sp.*) on boreal and temperate zones of the Northern hemisphere. The obvious reasons for this orientation are historical and economic. Investments in growth and yield research, which is prerequisite for quantitative economic analysis, have been directed to those species and regions that have been considered to possess the greatest economic potential. There have also been studies investigating uneven-aged management of mixed-species hardwood stands (e.g. Adams and Ek 1974, Buongiorno and Michie 1980). Much less emphasis have been devoted to economic analysis of pure even-aged hardwood stands (Rautiainen 2000, Mabvurira and Pukkala 2002), which may also have promising economic prospects.

Table 1. Stand-level optimization studies with discrete thinnings.

Studies	Location	Tree species	Topic
Whole stand models			
Amidon and Akin (1968)	USA	*Pinus taeda*	methods
Kilkki and Väisänen (1969)	Finland	*Pinus sylvestris*	methods & management
Risvand (1969)	Norway	*Picea abies*	methods & management
Brodie et al. (1978)	Northwest USA	*Pseudotsuga menziesii*	methods & management
Brodie and Kao (1979)	Northwest USA	*Pseudotsuga menziesii*	methods & management
Kao and Brodie (1979)	Northwest USA	*Pseudotsuga menziesii*	methods
Kao and Brodie (1980)	Northwest USA	*Pseudotsuga menziesii*	methods & management
Kao (1982)	Northwest USA	*Pseudotsuga menziesii*	stochastic
Riitters et al. (1982b)	Northwest USA	*Pseudotsuga menziesii*	management
Roise (1986a)	USA	*Pseudotsuga menziesii*	methods
Valsta (1986)	Finland	*Pinus sylvestris, Betula pendula*	management & mixed sp.
Paredes and Brodie (1987)	USA	*Pseudotsuga menziesii*	methods
Torres-Rojo and Brodie (1990)	Central Mexico	*Pinus hartwegii*	management
Valsta (1990)	Finland	*Betula pendula, Pinus sylvestris*	methods & mixed sp.
Haight and Smith (1991)	Southeast USA	*Pinus taeda*	mixed sp. & stochastic
Teeter and Caulfield (1991)	Southeast USA	*Pinus taeda*	management & stochastic
Filius and Dul (1992)	Netherlands	*Pseudotsuga menziesii*	management
Haight (1993a)	Southeast USA	*Pinus taeda*	management & stochastic
Salminen (1993a)	Finland	*Betula pend., Picea ab., Pinus sylv.*	management
Salminen (1993b)	Finland	*Picea abies, Pinus sylvestris*	management
Arthaud and Pelkki (1996)	Southeast USA	*Pinus taeda*	methods
Gong (1998)	Northern Sweden	*Pinus sylvestris*	management & stochastic
Thorsen and Helles (1998)	Denmark	*Picea abies*	management & stochastic
Zhou (1998)	Northern Sweden	*Pinus sylvestris*	management & mgmt.sys.
Brazee and Bulte (2000)	Netherlands	*Pinus sylvestris*	stochastic
Jacobsen and Thorsen (2003)	Denmark	*Picea abies, Picea sitchensis*	stochastic, mixed sp.
Stage-structured models			
Riitters et al. (1982a)	Arizona, USA	*Pinus ponderosa*	non-timber
Hann et al. (1983)	Arizona, USA	*Pinus ponderosa*	management
Bullard et al. (1985)	USA	*northern hardwoods*	methods & mixed sp.
Michie and McCandless (1986)	Northeast, USA	*Carya sp., Querqus sp.*	methods
Haight (1987)	Arizona, USA	*Pinus ponderosa*	management & mgmt.sys.
Haight (1991)	California, USA	*Abies concolor*	stochastic & mgmt.sys.
Solberg and Haight (1991)	Norway	*Picea abies*	management
Distance-independent individual-tree models			
Martin and Ek (1981)	Lake states, USA	*Pinus resinosa*	methods
Haight et al. (1985a)	Northwest USA	*Pinus contorta*	methods, quality

Table 1. Continues.

Studies	Location	Tree species	Topic
Roise (1986b)	Northwest USA	*Pseudotsuga menziesii*	methods
Arthaud and Klemperer (1988)	Southeast USA	*Pinus taeda*	management
Haight and Monserud (1990a)	Rocky mount., USA	*several species*	methods
Haight and Monserud (1990b)	Rocky mount., USA	*Pinus monticola*	management, mngt. syst.
Yoshimoto et al. (1990)	Rocky mount., USA	*several species*	methods
Haight et al. (1992)	Rocky mount., USA	*several species*	non-timber
Pesonen and Hirvelä (1992)	Finland	*Picea abies, Pinus sylvestris*	management
Valsta (1992a)	Finland	*Picea abies*	management, stochastic
Valsta (1992b)	Finland	*Picea abies*	management
Haight (1993b)	Northwest USA	*Alnus rubra, Pseudotsuga menziesii*	stochastic, mixed sp.
Eriksson (1994)	Sweden	*Picea abies*	methods
Eriksson (1997)	Sweden	*Pinus sylvestris*	management
Pukkala and Miina (1997)	Finland	*Picea abies, Pinus sylvestris*	stochastic, non-timber
Wikström and Eriksson (2000)	Sweden	*Picea abies, Betula pendula*	non-timber
Wikström (2000)	Sweden	*Picea abies*	methods & mgmt.sys.
Wikström (2001)	Southern Sweden	*Picea abies*	methods & mgmt.sys.
Mabvurira and Pukkala (2002)	Zimbabwe	*Eucalyptus grandis*	management
Huang and Kronrad (2002)	Texas, USA	*Pinus taeda*	management
Cao (2003)	Finland	*Picea abies*	management
Lu and Gong (2003)	Sweden	*Pinus sylvestris*	stochastic
Distance-dependent individual-tree models			
Miina (1996)	Finland	*Pinus sylvestris*	management, stochastic
Vettenranta (1996)	Finland	*Picea abies, Pinus sylvestris*	management, mixed sp.
Pukkala et al. (1998)	Finland	*Picea abies, Pinus sylvestris*	methods, mixed sp.
Pukkala and Miina (1998)	Finland	*Picea abies, Pinus sylvestris*	methods, mixed sp.
Vettenranta and Miina (1999)	Finland	*Picea abies, Pinus sylvestris*	management, mixed sp.
Rautiainen et al. (2000)	Nepal	*Shorea robusta*	management
Möykkynen and Miina (2002)	Finland	*Picea abies*	stochastic, quality

Specification of topics:

methods:	Developing optimization methods, specification of stand structure, or harvest control
management:	Presenting results on optimal harvests, planting density, efficiency gains etc.
stochastic:	Inclusion of risk or stochastic economic parameters, growth or natural hazards
quality:	Inclusion of timber quality or roundwood defects
non-timber:	Inclusion of non-timber benefits
mgmt.sys.:	Analysis of different management systems
mixed sp.:	Analysis of mixed-species stands

Some trends can be detected within the optimization results despite the species and region-specific nature of numerical computations. For example, most numerical studies report that it is optimal to establish forest stands at high initial density to enable returns from thinnings (e.g. Haight and Monserud 1990b, Teeter and Caulfield 1991, Solberg and Haight 1991, Valsta 1992b). However, thinnings may become suboptimal if the fixed cost of harvesting is sufficiently high (Eriksson 1997). Another common result for many conifer species is that thinnings removing trees from above or from the both ends of the diameter distribution are superior to conventional thinning from below (see e.g. Haight 1987, Haight and Monserud 1990b, Solberg and Haight 1991, Valsta 1992a,b, Eriksson 1994, Pukkala and Miina 1998, Vettenranta and Miina 1999).

In theoretical studies, much attention is paid to giving clear and intuitive interpretations of the results. To this end, theoretic studies often include comparative statics analysis with respect to economic parameters and other model characteristics. Most numerical studies, by contrast, place little stress on interpretation of optimization results. Many numerical studies have concentrated more on developing the optimization methods, rather than elaborating the results. Optimal solutions are typically presented for only one or a few economic parameter values. This is not enough to analyze the underlying relationships and determinants of the optimal solutions.

5 Materials and methods

Table 2 elaborates the characteristics of the optimization models used in the separate studies of this thesis. Study **I** is based on a discrete set of stand management alternatives obtained from the variable-density growth and yield tables specified for conifer cultures (Vuokila and Väliaho 1980). The tables are based on stand growth equations estimated with extensive sample plot data. These equations are combined with other Finnish growth and yield equations and converted to the continuous-time optimization model in **II**. This model contains three state variables: stand age, basal area, and number of stems. In **III**, this model is extended to allow any initial state. Non-linear optimization algorithms of Matlab-software are used to obtain optimal solutions in **II** and **III**.

Study **IV** employs the SMA optimization program by Valsta and Linkosalo (1996) and field measurement data for several initial densities of Scots pine. This program consists of the individual-tree growth equations designed initially for the Finnish forest planning system MELA. The equations (see Hynynen et al. 2002) are based on extensive

Table 2. Models and data used in separate studies of this thesis

	I	II	III	IV	V
Description of growth					
Growth tables	x				
Variable-density whole stand model		x	x		
Distance-independent individual-tree model				x	
Process-based individual-tree model					x
Timber species					
Scots pine	x	x	x	x	x
Norway spruce	x	x	x		
Timber pricing					
price premium	x	x	x	x	
several roundwood categories					x
Timber quality defects					
age-dependent		x	x		
dependent on growth rate at early ages				x	
age-, density- and treatment-dependent					x
Harvesting technology					
lumped model	x	x	x		
work productivity models				x	x
fixed cost		x	x	x	x

data sets from permanent sample plots of the Finnish National Forest Inventory. In addition to stand age, state variables include tree basal area, crown ratio and expansion factor representing the number of stems for each tree class.

The individual-tree process-based model PipeQual (Mäkelä 1997, 2002, Mäkelä and Mäkinen 2003) is combined with descriptions of roundwood categories, harvesting technology and economic data in order to analyze determinants of optimum stand management in **V**. This process-based (or hybrid) stand growth model consists of submodels at stand, tree, whorl, and branch levels, and is specified for pure Scots pine stands. Several state variables describe each hierarchical level. Trees interact through shading and availability of physical space. The direct search method by Hooke and Jeeves (1961) is used to obtain optimal solutions in **IV** and **V**.

Real timber prices are long-term averages obtained from the time series 1983-1997 (Finnish Statistical Yearbook of Forestry 1998, p. 159). In **I** - **III**, the unit prices for sawlogs and pulpwood vary with average tree diameter for both Scots pine and Norway spruce. In **IV**, there is a positive price premium for high-dimension pine sawlogs. In **V**, stem form and crown morphology information are used to buck tree stems into logs representing different roundwood categories.

The degree of detail in specifying timber quality defects gradually increases from **I** to **V**. Reduction of sawlog volume due to quality defects is assumed to be minor in regularly thinned stands and is thus ignored in **I**. Age-dependent cull reduction is used with more flexible thinnings in **II** and **III**. In **IV**, the average ring width around the mean trees' pith determines the proportion of trees with quality defects for each size class. Such specification reflects the negative effects of rapid early growth on timber quality. In **V**, the proportion of inferior-quality timber is based on vertical distribution of branches along the stem and on information about the crown rise and shedding of dry branches.

Harvesting cost is determined as a function of average volume of removed trees and total sawlog and pulpwood removals. Studies **I** - **III** employ a lumped equation, and **IV** and **V** a set of work productivity equations derived for different phases of felling and onsite transport (Kuitto et al. 1994). There are separate equations for thinnings and clearcutting. Positive fixed cost of harvesting is applied in models where time is continuous (studies **II** - **V**). Stand establishment costs are based on statistics (Finnish Statistical Yearbook of Forestry 1998).

6 Main results of the separate studies

6.1 The effects of legal limits and recommendations on timber production: the case of Finland (I)

One form of regulation of private forestry does not allow a stand to be clearcut until it exceeds a certain average diameter or age. Study **I** is the first attempt to systematically examine whether this type of regulation constrains optimal harvesting of Finnish conifer cultures. The analyzed regulations include clearcutting restrictions set out in forestry legislation (see Niskala 2000, p. 18-23) and official management prescriptions of the Forestry Development Centre Tapio (1994). It is further evaluated how regulation-constrained harvests deviate from unconstrained harvests and the seriousness of the consequences of regulations for long-run timber supply, working opportunities in forestry, and forest owner's income. The analysis is extended to six Scots pine and five Norway spruce sites covering virtually all relevant site classes for conifer cultures.

It is found that the minimum clearcutting restrictions typically constrain the optimal solution if the rate of interest exceeds 3% or 5% for Scots pine and Norway spruce stands, respectively, when applying a Faustmann type rotation model, a large number of alternatives for thinnings and clearcutting, and data reflecting past economic developments in Finland. The restrictions are the more binding, the lower the stand's growth capacity. Comparison with earlier Finnish stand-level optimization studies reveals that the results regarding optimum rotation lengths are quite consistent despite varying descriptions of growth.

It is further shown that following the clearcutting restrictions set out in legislation or recommended in forestry extension documents may cause substantial economic losses, especially for mature stands, when high rates of interest are applied. The restrictions increase long-run sawlog production but reduce pulpwood production, in comparison to the unconstrained optimum. Moreover, less frequently occurring final clearcutting harvests may reduce working opportunities in forestry.

6.2 Economics of forest thinnings and rotation periods for Finnish conifer cultures (II)

Study **II** investigates simultaneous optimization of thinnings and clearcutting. To this end, a variable-density whole stand model is specified for all relevant Finnish Norway spruce and Scots pine site classes. The main objective is to analyze the dependence of optimal thinnings and clearcutting on economic parameters and site quality. To meet this objective, sensitivity analyses with respect to rate of interest, fixed harvesting cost and site quality are conducted.

This study contributes to the literature by providing three extensions to the understanding on how the length of rotation period depends on economic factors and site quality. Firstly, the optimal rotation length does not decrease monotonously with the rate of interest. Secondly, an improvement in site fertility does not always reduce optimal rotation length. Furthermore, an increase in costs, if directed at fixed harvesting cost, is likely to shorten rather than lengthen the optimal rotation.

These extensions to classical forest rotation results follow from variations in the optimal number of thinnings and thinning intensity. Analysis of the growth model reveals that the relationship between thinning intensity and value growth of the residual stock is almost linear for average thinning intensities. Consequently, small changes in the economic parameters may lead to large changes in the optimal harvests. A small increase in the rate of interest may, for example, trigger a substantial increase in thinning intensity and length of rotation.

The number of thinnings is more sensitive to changes in the rate of interest, logging conditions, and site productivity for Scots pine versus Norway spruce. This follows from different growth responses to thinning and high price premia for large dimension Scots pine sawlogs. As a policy-relevant issue, economic optimization suggests that for both species the first thinning should be performed later than officially recommended. High growing density is rational at early age due to high growth rate and high thinning costs. Optimization results also suggest that the last thinning should be heavier than officially recommended, especially at high rates of interest.

6.3 Maximum Sustained Yield, Forest Rent or Faustmann: Does it really matter? (III)

Study **III** examines efficiency gains when harvests are determined using the Faustmann approach instead of the Maximum Sustained Yield, Forest Rent, or the silvicultural rules stated in the Finnish silvicultural recommendations (Tapio 2001). The variable-density stand growth model in **II** is extended to allow any initial state. A large number of solutions covering virtually all admissible combinations of initial stand age and basal area are computed for several Finnish Norway spruce and Scots pine sites.

This study contributes to the literature by giving a comprehensive description on the value of bare and forested conifer stands and the economic losses that follow from non-optimal criterion or silvicultural rules-of-thumb. The results show clearly that following the Maximum Sustained Yield or Forest Rent leads to a major reduction in the economic value of highly stocked stands, in particular. High relative losses are also realized for young sparsely stocked stands that are clearcut at economically premature age. Finnish silvicultural recommendations are found to encourage production of higher-than-optimal dimension timber. Applying Forest Rent, Maximum Sustained Yield, or silvicultural recommendations leads to respective reductions of 63%, 30% and 13% in the value of a typical conifer forest at a 4% rate of interest.

Initial states are split into two groups by clearcutting curves, according to whether or not immediate clearcutting is optimal. Comparison of the curves computed for different rates of interest reveal that the optimum timing of clearcutting is highly sensitive to both initial state and economic parameters. Clearcutting curves also define sets of initial states for which an increase in the rate of interest delays the clearcutting of the present stock. Such initial states are characterized by low initial basal area. They fall below the optimal path initiated at bare land.

Extended rotation as a consequence of an increase in the rate of interest follows from the high cost of artificial regeneration. At low rates of interest, juvenile stands with low enough basal area (so called "understocked" stands) are optimally replaced by new fully stocked seedling stands. At higher rates of interest, in contrast, retaining of these stands becomes optimal due to high (relative) value growth in existing stock and small or zero benefits (net of costly stand establishment) in subsequent periods.

6.4 Optimum juvenile density, harvesting and stand structure in even-aged Scots pine stands (IV)

Study **IV** investigates the economics of timber production using empirical data for various juvenile densities in sown Scots pine stands and a distance-independent individual-tree growth model. The study contributes to the literature by explicitly including the quality effects on timber prices. The negative effects of fast early growth are found to increase optimal juvenile density. In addition, quality effects render the economic surplus of forestry more sensitive to variations in juvenile density.

The results show that a Scots pine stand is optimally established at high juvenile density (2100-2500 trees ha^{-1}). High juvenile density enables thinnings that bring intermediate revenues and induce positive quality and quantity effects on the value of the final clearcut stock. An increase in the juvenile stand density lengthens rotation and reduces the final tree diameters. As a policy-relevant issue, the legal clearcutting restrictions are found to be the most constraining for those juvenile densities that yield the highest economic outcome.

Comparison of a sequence of diameter distributions shows that optimum thinnings remove the trees that have no prospects for high relative value growth. Optimum thinnings are selective thinnings that remove inferior-quality trees, some of the smallest trees and, contrary to convention, the largest trees that fulfill sawlog dimensions. Later thinnings homogenize stand structure by removing trees strictly from above and/or below. This results in a sharply peaked final diameter frequency distribution.

This study confirms the earlier result that thinning from above is superior to thinning from below. The new finding is the role of optimal thinnings as a means to homogenize stand structure. Sharply peaked diameter distribution is economically favorable because a stand can be clearcut promptly after the majority of the trees have reached minimum dimension requirements for sawlogs and have experienced the final high peak in value growth. Allowing trees to grow larger would lead to high capital costs, despite positive price premia.

6.5 Connecting a process-based forest growth model to stand-level economic optimization (V)

The main aim of study **V** is to integrate for the first time (to the authors' knowledge) a detailed process-based growth model into stand-level economic optimization. The economics of timber production is investigated using a distance-independent individual-tree process model specified for pure Scots pine stands. This model provides a great deal of information on allocation of biomass production between and within different tree compartments (bole, branches, roots, foliage). Due to the hierarchical structure, the model explicitly includes causal feedback effects of stand management on development of tree characteristics. Stem taper and crown morphology information are used for bucking the harvested trees into six commercial roundwood categories according to the quality and dimension requirements applied in the Finnish timber markets.

The underlying economic and ecological determinants of optimal harvests and long-run timber supply are analyzed by conducting sensitivity analyses for different rates of interest and roundwood prices. Sensitivity analysis is also used as a tool to evaluate whether the model outcomes are logical responses to any combination of analyzed forest management activities (i.e. over all possible thinning patterns). For example, economic activity is shown to reduce total biomass production, but to increase roundwood production.

In contrast to earlier studies, optimal rotation length is found to be insensitive to changes in the rate of interest above 4%. Insensitivity follows from nonmonotonic value growth caused by discontinuous shifts of timber volume between roundwood categories. Management strategies involving shorter than 73 yr rotations would severely reduce the proportion of sawlog yield from the final clearcutting or, alternatively, reduce roundwood quality and increase harvesting costs for the case study stand.

The first thinnings are light irrespective of the rate of interest due to their favorable feedback effects on the quality of the residual trees. Results concerning optimal type of thinning confirm to earlier results obtained with empirical models, and provide a novel basis for their interpretation. The most suppressed and largest dominant trees are removed in thinnings when maximizing long-run timber production. This follows from their low photosynthetic capacity and productivity in allocating biomass production to tree bole. Using the economic criterion, by contrast, optimal thinnings are selection thinnings that remove the dominant trees that have reached sawlog dimensions.

Study **V** also provides new results on the structure of long-run timber supply. An increase in the rate of interest is found to reduce the quality of roundwood supply. In particular, the long-run supply of upper sawlogs (middle logs and top logs) decreases with the rate of interest. Production of the highest-grade roundwood (superior quality butt logs) is found optimal only at 0-1% rates of interest, which are lower than the long-run average rates prevailing in the capital markets.

The objective function response surface is characterized by several local maxima represented by distinct timber production strategies. Optimality of different strategies is found to be highly sensitive to economic parameters: rate of interest and price ratios between the roundwood categories. A sensitivity analysis with respect to roundwood prices is conducted to illustrate the existence of multiple optima.

7 Discussion

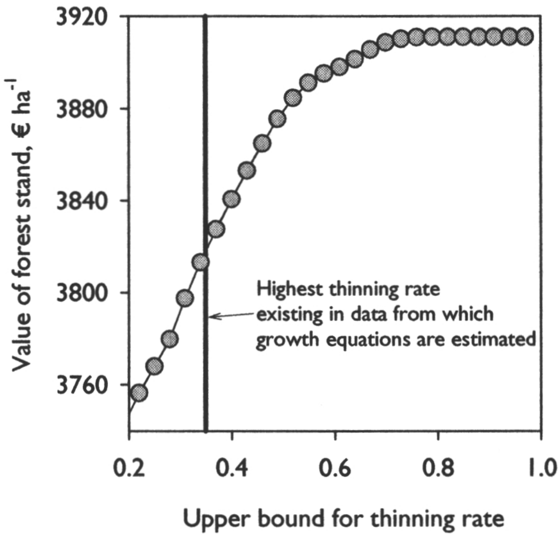
7.1 Advantages and difficulties of complex models

Increased biological detail provides opportunities to gradually relax the assumptions concerning stand growth and harvest control, and to deepen the economic analysis of different forestry activities. The separate studies of this thesis incorporate increasingly detailed descriptions of stand growth in economic optimization. Compared to growth tables (**I**), a continuous-time model enabled more flexible determination of timing, intensity and frequency of thinnings (**II**), and extension to multiple initial states (**III**). Next, an individual-tree model was used to explicitly account for the effects of with-in stand competition and harvesting on stand structure and tree growth (**IV**). This model enabled more itemized analysis of optimal stand management, including analysis of optimal type of thinning and juvenile density. Finally, hierarchical structure and a high degree of detail in the process-based model made it possible to explicitly account for feedback effects of thinnings on the quality characteristics and stem taper of the residual trees (**V**).

Numerical optimization can be used as a tool to evaluate whether qualitative results obtained with stylized theoretical models remain valid when more itemized and detailed descriptions of stand growth, harvest control and harvesting technology are applied. In general, the sensitivity analysis results in the separate studies of this thesis accord with earlier understanding. However, some alterations to established qualitative forest economics results were identified. It is shown in **II** that optimal rotation period length may occasionally increase with the rate of interest and site fertility, and decrease with harvesting cost due to indirect effects of optimized thinnings. In addition, an increase in the rate of interest may delay clearcutting of the present stock for some initial states with low basal area, as shown in **III**.

Sensitivity analysis of numerical optimization models may turn out to be a useful research tool in both forest economics and forest ecological research. It helps to highlight the economic parameters and growth model components that have the greatest impacts on the economic surplus of forestry. It also helps to identify economically interesting stand management strategies. Optimal solutions that are bound by the reliability constraints, enforced by penalty functions, or fall outside the most reliable range of application provide incentive to enlarge the scope of ecological research. This is illustrated in Figure 1, which shows the maximized value of a forested Norway spruce stand as a function of thinning intensity constraint.

In Figure 1, the maximized forest values are computed by optimizing thinnings and clearcutting with respect to the upper bound for the thinning rate, which is varied at 0.03 intervals. This computation is based on a whole stand model used in **III**. The curve implies that the maximized forest value increases with the thinning rate constraint up to 0.73. However, the growth predictions are reliable only over a narrow and suboptimal range: thinning rates vary between 0.15 and 0.35 in the data from which the growth equations were estimated (Vuokila and Väliaho 1980, p. 28).



Note: Initial stand age is 40 yrs, basal area 29.3 m^2 ha^{-1} and density 2005 trees ha^{-1}. Stand is growing on a *Myrtillus*-type site (dominant height is 27 m at age 100 yrs).

Figure 1. Maximized value of a densely stocked Norway spruce stand as a function of thinning intensity constraint.

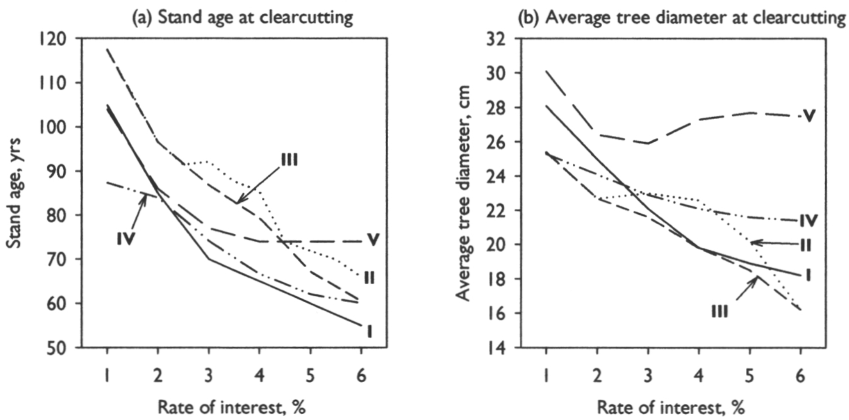
Figure 1 provides justification to extend the growth measurements to more heavily thinned stands, and to quantify other implications of abrupt reductions in stocking, such as increased probability of natural hazards. Several other important topics that may need further investigation were also identified in the separate studies of this thesis. These include description of tree mortality due to self-thinning (**II**, **IV**, **V**), timber quality (**IV**, **V**), and growth response of the most suppressed trees to increased growing space (**IV**, **V**).

Detailed description of stand growth and harvest control may also entail difficulties for economic optimization. For example, increasing the number of decision variables or including discontinuities in bucking equations typically increase the number of local maxima and thus hamper determination of the global maximum. Another problem following from the detailed description of initial stand states is that the optimization results apply only to the investigated stands. Information on more general applicability of the results can be obtained only by extending the computations to different initial stand structures.

7.2 Synopsis of optimal harvests

Rotation length

Figure 2 shows the relationship between rate of interest and optimal rotation length in terms of stand age and average tree diameter at clearcutting for a typical Scots pine site. Comparison of results from separate studies (denoted by roman numerals) reveals that optimal rotation is highly sensitive to the indirect effects of optimized thinnings and to how harvest control, roundwood categories and prices are described. For example, setting an upper bound for thinning intensity reduces stand age and average tree diameter at clearcutting (cf. curves for **II** and **III** at 3% and higher rates of interest in Figures 2a and b). When applying an individual-tree model with flexible harvest control, optimal thinnings tend to homogenize the stand structure towards the end of rotation. This leads to clearcutting promptly after all the residual trees have reached sawlog dimensions (see the curve for **IV** in Figure 2b). Optimal timing of clearcutting may become insensitive to the rate of interest over a certain range due to nonmonotonic value growth. In such case, the demands of a higher rate of interest are met by increasing thinning intensity. This leads to accelerated diameter growth and larger final diameters (cf. curves for **V** in Figures 2a and b).



Note: Initial stand density is 1800 trees ha^{-1} and age 20 yrs for **I, II** and **III**. Initial stand density is 2250 trees ha^{-1} and age 18 yrs for **IV** and **V**.

Figure 2. Optimal rotation length in terms of stand age and average tree diameter (weighed by basal area) in **I - V** for *Vaccinium*-type Scots pine stands.

Varying growth predictions from different models also affects optimal rotation results. For example, the process-based individual tree model used in **V** gives somewhat higher growth at low basal areas in comparison to the individual-tree model used in **IV**. This fact, along with different specifications of timber pricing and quality, leads to larger final diameters, particularly at higher rates of interest (cf. curves for **IV** and **V** in Figure 2b).

Initial stand structure is another important determinant of optimal rotation length. Homogeneous stand structure is economically advantageous: narrow and centered diameter distributions enable earlier clearcutting. Therefore, optimal rotations are typically shorter for intensively managed conifer cultures than for initial stand structures common for naturally grown undisturbed stands (see **IV**).

Thinnings

Figure 3 shows the development of stand basal area over rotation length for 1, 3 and 5% rates of interest for a *Vaccinium*-type Scots pine stand. The results are obtained from **V**.

The basal areas before and after thinnings are connected with smooth lines to indicate upper and lower bounds for stocking.

Optimal level of stocking is highly sensitive to the rate of interest. At low rates of interest (see Figure 3a), stand basal area is kept close to the level that maximizes long-run timber production. Thinnings are light and remove only the portion of timber stock that would otherwise be lost due to self-thinning. An increase in the rate of interest tends to increase the intensity of the last thinnings. Therefore, the level of basal area is gradually reduced towards the end of rotation period with 3% and higher rates of interest (Figures 3b and c).

Heavy thinnings are advantageous at high rates of interest because they increase thinning revenues and speed up diameter growth of the residual trees. However, heavy thinnings may become optimal only after they cannot reduce the quality of butt log, which is economically the most important part of a tree (see **V**). Therefore, stand density is kept high at young ages until the crown bases of the stock trees have lifted above the butt logs. It must also be noted that heavy thinnings may increase the probability of natural hazards (windfalls, snow damages) in some forest stands. Accounting for these effects would lead to lighter and more frequent thinnings.

The optimal solutions in the separate studies of this thesis can be summarized as a harvesting rule: remove trees soon after they have reached the sawlog dimensions. Optimal thinnings remove particularly those trees that have already reached sawlog dimensions or cannot qualify as sawlogs due to quality defects (**IV**, **V**). The residual stock is optimally clearcut after the majority of the remaining trees have reached the minimum dimension requirements for sawlog trees and no further upgrading to higher quality classes is expected. Regeneration of younger stands becomes rational only for very sparsely populated stands (basal area lower than 5 m^2 ha^{-1}) at 2% and lower rates of interest (see **III**).

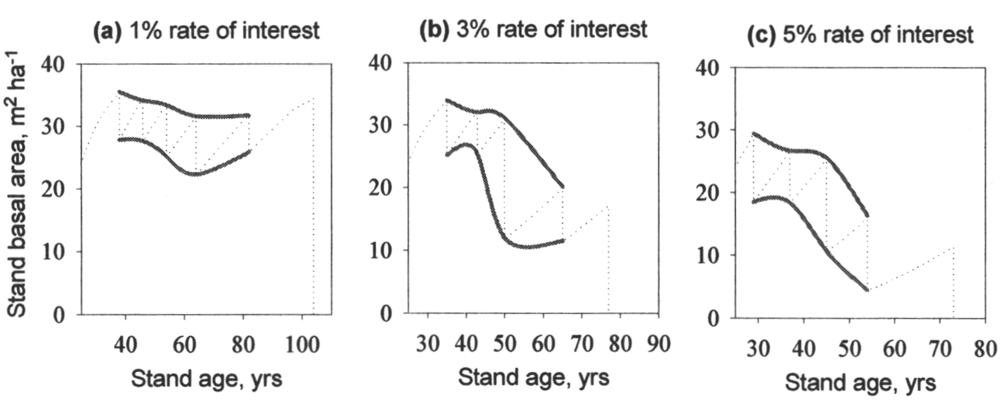


Figure 3. Optimal thinning prescriptions for a single Scots pine stand

7.3 Policy implications

Interpretation of optimization results

Accumulation of economic and ecological information in optimization models undoubtedly increases the applicability of economic research in policy analysis and forest management planning. However, the underlying assumptions of the optimization models as well as the descriptions of stand growth, harvesting technology, and initial data must be given due consideration when interpreting the quantitative research results.

A fruitful approach to interpret deterministic results is to consider how they are affected by the dynamics of stochastic prices, rates of interest, or other real-life phenomena that are still somewhat difficult to explicitly account for in detailed optimization models. There is a growing theoretical literature considering multiple forest benefits (e.g. Swallow and Wear 1993) and relaxing assumptions regarding e.g. perfect capital markets (Tahvonen et al. 2001) or perfect foresight (e.g. Willasen 1998). Results from these studies, along with empirical understanding of the importance of these assumptions, help to properly interpret the deterministic solutions.

Silvicultural recommendations

The silvicultural recommendations (Tapio 1994, 2001) include detailed prescriptions for stand establishment, thinnings, and clearcutting. They are specified separately for the major timber species, different stand fertility classes and regions in Finland. The guidelines for thinnings define the basal area level that triggers thinning and the lower and upper bounds for residual basal area (see Figure 4). It is recommended that the forest stand be clearcut once either the average diameter of the trees or stand age reaches the desired range (see Table 3). Tree diameter is considered the primary criterion for clearcutting. The silvicultural recommendations do not bind forest owners in legal terms, but are widely applied in forestry extension and training of forestry workers.

The recommended rotation lengths were initially based on computations by Nyyssönen (1958). He used the "marginal method" originated by Duerr and Bond (1952), stand growth tables for lightly thinned stands, inventory information on typical forest structures in the early 1960s, and a 3% rate of interest to compute optimal rotation lengths for different conifer sites (Nyyssönen 1958, 1999). In the present version of the silvicultural recommendations, these results are used as lower or upper bounds for stand age at clearcutting (see Table 3).

Recommended thinnings for conifers (an example is shown in Figure 4) are based on Finnish growth and yield research and, in particular, on studies by Vuokila and Väliaho (1980) and Vuokila (1983). The prime aim of these studies was to determine thinnings and stocking levels that lead to high or maximized sawlog yield in the long run. Vuokila (1983) designed guidelines for discrete numbers of thinnings in order to account for the fixed cost of harvesting. No other economic aspects were considered.

The silvicultural recommendations aim at encouraging rational timber production and promoting biological diversity of forests along with other uses (Tapio 2001, p. 7). However, it can be questioned whether the specific prescriptions for thinnings and clearcutting are valid from the point of view of rational timber production. Thinning

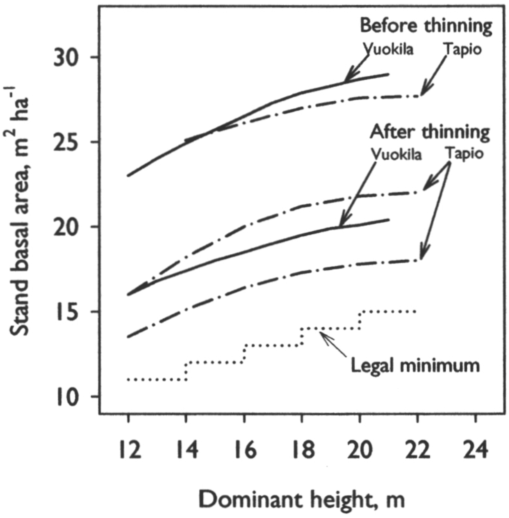


Figure 4. Comparison of recommendations (Tapio 2001), legal restrictions (see Niskala 2000, p. 18-23), and guidelines in Vuokila (1983) for thinning *Vaccinium*-type Scots pine stands in southern Finland

Table 3. Comparison of recommendations (Tapio 2001), legal restrictions (see Niskala 2000, p. 18-23), and results by Nyyssönen (1999) for clearcutting conifer stands in southern Finland

	Silvicultural recommendations		**Legal restrictions**		**Nyyssönen**
Site type	Diameter, cm	Age, yrs	Diameter, cm	Age, yrs	Age, yrs
Scots pine					
Myrtillus	29-31	80-90	27	70	80
Vaccinium	27-29	90-100	25	80	90
Calluna	25-27	110-120	23	100	120
Norway spruce					
Oxalis-Myrtillus	28-30	80-90	25	70	
Myrtillus	26-28	90-100	24	80	90

guidelines aim at maximizing sawlog yield rather than economic surplus of forestry. The marginal method that was used to determine recommended rotation length deviates from the true economic criterion and leads to shorter optimal rotation length (Chang 1990, Viitala 2002, 2003, Viitala and Tahvonen 2003)[2]. According to **III**, pursuing

[2] Note however that the results by Nyyssönen (1958, 1999) suggest somewhat longer rotations than the separate studies of this thesis (see Figure 2) or other Finnish studies applying the Faustmann approach and optimizing thinnings and clearcutting (see **I** for review). The obvious explanation for this is that Nyyssönen (1958, 1999) applied exogenously given light thinnings.

recommended (instead of optimal) harvesting leads to about a 13% reduction in the value of typical conifer forest at 4% rate of interest.

The recommended and economically optimal rotation lengths typically agree only over a narrow range of the rates of interest, varying around 1% or 2%. However, time series for real rates of interest (**I**) and forest estate prices (**III**) suggest that, at present, higher rates of interest varying between 2% and 6% may be more relevant for most forest owners. Therefore, the recommendations suggest longer-than-optimal rotation periods, particularly for intensively managed stands.

The recommended and optimal thinnings also deviate systematically. Recommended thinnings aim at gradual increases in basal area level, while optimal thinnings gradually decrease basal area towards the end of the rotation period when 3% or higher rates of interest are applied (compare Figures 3 and 4). Recommended thinnings typically remove trees from below. In the final thinnings, a proportion of the dominant trees may also be removed. Optimal thinnings, in contrast, are in most cases selective thinnings that systematically remove the largest sawlog trees.

Economically optimal stand management is highly sensitive to initial stand structure (**III**, **IV**, Cao 2003) as well as economic factors, some of which are owner-specific. Therefore, it may be impossible to design generalized guidelines for harvesting and other forestry activities that always, and under any conditions, lead to good economic outcomes. This calls for development of optimization tools that allow forestry planners, extension specialists, and forest owners to design tailored stand prescriptions. Modern remote sensing techniques may provide opportunities for inexpensive acquisition of high-resolution stand structure data for this purpose (see e.g. Smith et al. 2003).

Legal restrictions

The minimum legal requirements for harvesting are set out in the Decision by the Ministry of Agriculture and Forestry (see Niskala 2000, p. 18-23). They give minimum requirements for clearcutting and thinnings by tree species and site productivity (see Table 3). The minimum stand ages and average tree diameters before clearcutting, and residual basal areas after thinning, are somewhat lower than recommended (see Figure 4 and Table 3). Furthermore, it is required that the trees retained in thinnings be primarily high-quality and vigorous and belong to the upper canopy layers. Deviations from these restrictions may be allowed if they are well justified.

According to **IV**, the legal restrictions constrain optimal clearcutting decisions in particular for those initial Scots pine stand structures that yield the highest economic outcome when 3% or higher rate of interest is applied. The minimum residual basal area restriction may also become binding at high rates of interest (**II**, **V**). Optimal thinnings typically remove particularly the largest trees and retain the lower canopy trees with potential for large value increases (**IV**, **V**). Whether such selection thinning fulfills the legal requirements depends on thinning intensity and stand structure.

The harvesting restrictions have been applied in Finland at least since the early 1970s (Veijola 1971, Tapio 1974). It is possible that, at that time, economic incentives for long-term forestry decisions were distorted by heavily regulated markets for capital and forestland (Tahvonen et al. 2001). However, economic environment, forest ownership structure, and the forests have changed greatly since then. Deregulation of the markets for capital and forestland were completed by 1986 and 1997, respectively. A shift from uneven-aged management to intensive even-aged forestry in the 1950s and vastly increased artificial regeneration since the 1960s have led to steadily increasing growing stock and altered structure of forests.

It is somewhat unclear how and to what extent the recent changes in Finnish society and forestry are reflected in the harvesting restrictions. This raises questions such as whether state intervention is still needed to safeguard sustained yield of raw material for the forest industries, how effective and cost-efficient are the harvesting restrictions in comparison to alternative forest policy instruments, and what is their justification. Economic research may help policy makers to evaluate the implications of different policy instruments by providing information on gains and losses to different interest groups and by identifying possible market distortions.

7.4 Promising areas for future research

Developing and extending the use of detailed process-based growth models in economic analysis of forestry is one of the most promising future topics. Solid theoretical background and a high degree of detail provide good prerequisites for description of production processes for both timber and other forest benefits under varying conditions. Interactions between individual trees can best be captured by using distance-dependent growth models. One possible extension is to describe stand growth in detail in age-class models formulated at forest owner or regional level.

In forest economics research, there is a great need for empirical and process-based stand growth models that enable endogenous and continuous stand establishment investment. Such models would enhance comparison of different management systems and would yield increasingly reliable results on optimal planting density, optimal level of management intensity in young stands, and several other important topics. One interesting topic to explore is whether it is more advantageous to pay attention to successful stand establishment or careful planning of harvesting.

Another important area for future research is to develop various economic features of numerical optimization models. Relaxing assumptions regarding e.g. perfect foresight or capital markets enables more realistic description of forest owners' decision making. Theoretical studies clearly show that inclusion of stochasticity alters optimal rotation in comparison to the deterministic solutions (see e.g. Willasen 1998). However, it is still unclear how rational harvesting decisions are altered as a consequence of, for example, stochastic roundwood prices or rates of interest when both thinnings and clearcutting are considered endogenous and flexible.

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Paper I

I

Hyytiäinen, K. and Tahvonen, O. 2001. The effects of legal limits and recommendations on timber production: the case of Finland. Forest Science 47: 443-454.

The Effects of Legal Limits and Recommendations on Timber Production: The Case of Finland

Kari Hyytiäinen and Olli Tahvonen

ABSTRACT. One form of legislative regulation on private forestry does not allow a stand to be clearcut until it exceeds a certain average diameter or age. This study examines whether this type of regulation used in Finland, for example, constrains the optimal timber harvesting of various site types of Norway spruce and Scots pine cultures. Applying a Faustmann type of rotation model, well-established growth and yield data and economic time series, we find that the regulations constrain the optimal solution if the rate of interest exceeds 3% or 5% for Scots pine and Norway spruce stands, respectively. They turn out to be more binding, the lower the growth capacity of the stand. Especially for mature stands, the regulations may cause major economic losses. The regulations increase long-term sawlog production but decrease pulpwood production in comparison to the unconstrained optimum. Moreover, the regulations change the structure and reduce the level of costs, and may then reduce working opportunities in forestry. For. Sci. 47(4):443–454.

Key Words: Optimal rotation model, forest legislation, conifer cultures, long-term timber supply.

Depending on traditions and the state of economic and social development, forests or any natural resources may be utilized under different property right regimes such as private, government, common, or a free access regime. In the case of forests, one possible solution common to the Nordic countries and some parts of North America is a private property right regime dominated by nonindustrial private forest owners. This ownership form is often found attractive due to its favorable effects on rural development and because the small-scale nature may support forestry employment opportunities. However, in many countries, private ownership of forests is heavily regulated by the government. This leads to questions such as what is the driving force behind the regulation and how the regulation changes the distribution of rent or stumpage earnings that accrue to owners of forestland.

In Finland, nonindustrial private forest owners occupy about 62% of forestland and supply about 85% of the industrial timber (Finnish Statistical Yearbook of Forestry 1999, p. 45, 168). The history of regulating forest management decisions goes back to the Private Forest Acts of 1928 (Government of Finland 1928) or 1886 (Government of Finland 1886) where the "devastation of forests" was prohibited. Later the regulation has become more specific, and presently there are species- and site-specific legal restrictions requiring that either the age of the stand or the average tree diameter must exceed certain limits before a stand is allowed to be clearcut (Ministry of Agriculture and Forestry 1997).[1] Widely applied recommendations by the government-financed extension organization are of the same kind (Tapio 1994, p. 21).

The legal limits and recommendations are motivated by timber production and economic arguments, although the exact reasoning behind the specific figures has remained

[1] In addition, it is required that the regeneration of a new stand is assured within 5 yr after clearcutting.

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somewhat unclear (Viitala 2001).[2] From the points of view of different interest groups and whether the use of forest resources is economically and socially efficient, it is important to know the effects of regulation on income distribution, timber supply, and employment opportunities in forestry. The importance of the issue is increasing since regulations need to be revised in accordance with institutional changes in the economic environment of forestry such as liberalization of the markets for capital and forestland.

Existing Finnish studies on economically optimal forest management and rotation periods are not able to answer the above questions. There is no systematic research knowledge on what the optimal rotation period and the related tree diameter are needed for comparing regulations and economically optimal forest management. There are only a few studies that systematically cover all the relevant tree species and site types. No study reports on how regulation-constrained optimal clearcut/thinning management regimes deviate from the unconstrained optimum and how serious are the consequences of regulations and recommendations in terms of timber supply, working opportunities in forestry, and forest owners' income.

This study applies growth and yield data from extensive fieldwork, based on 354 sample plots that cover all the relevant site types for well managed Norway spruce (*Picea abies* [L.] Karst.) and Scots pine (*Pinus sylvestris* L.) cultures in Finland. The economic data on (real) timber prices, silvicultural costs, and the rate of interest are based on time series for the past 15 yr or more. The optimal forest management regime is defined by a Faustmann type model, which maximizes the forest owner's profit over an infinite time horizon. Our optimization method is straightforward: we compute the economic value of each of the 235 (on average) different forest management regimes covered in our growth and yield data for each 11 Norway spruce and Scots pine site types and then pick the most profitable one for each case.

According to Finnish time series, rates of interest between 2–6% should cover a large fraction of the forest owners who are borrowers or who are able to utilize better investment opportunities than an ordinary bank account. Nevertheless, our results indicate that the regulations restrict the forest owner's optimal forest management if the rate of interest exceeds 3% or 5% for Scots pine and Norway spruce cultures, respectively. The regulations turn out to be more binding in northern Finland, where the rural income level is also lower.

For a 4% rate of interest and a Scots pine stand with a typical fertility index, the per hectare losses caused by the regulations may exceed 10% if the stand is close to an optimal clearcut age. The regulations also affect the long-term timber supply and the proportions of timber assortments produced. The regulations may increase the supply and reduce the price of sawlogs, especially in the case of Scots pine.

This article is organized as follows: the second section, Criteria for Thinning and Clearcutting, presents the theoretical basis, and the third section the biological and economic data. The Results section gives the main results on optimal and constrained forest management, the economic losses from regulations, and the implications for long-term timber supply and structure of costs in forestry. The results are discussed further in the section entitled Discussion.

Criteria for Thinning and Clearcutting

We apply the Faustmann (1849) model to calculate the value of bare forestland. This is done by subtracting the silvicultural and stand establishment costs, c_{ik}, of activities $k = 1,\ldots,m$ from the harvest revenues R_i. The resulting annual net revenues are discounted to the beginning of the rotation using the term $(1 + r)^{-i}$, where r is the rate of interest and $i = 0,\ldots,s$ is the age of the stand in years. The value of the bare land, V_F, is calculated from an infinite series of similar rotation periods:

$$V_F = \frac{\sum_{i=0}^{s}\left(R_i - \sum_{k=1}^{m} c_{ik}\right)(1+r)^{-i}}{1-(1+r)^{-s}} \tag{1}$$

The value of forestland with standing timber, V_H, is obtained by discounting the net revenues from the remaining part of the ongoing rotation period, and the land value, V_F, from the end of the ongoing rotation:

$$V_H = \sum_{i=b}^{s}\left(R_i - \sum_{k=1}^{m} c_{ik}\right)(1+r)^{-(i-b)} + V_F(1+r)^{-(s-b)} \tag{2}$$

where b is the age of the existing stand.

The equations for the land value do not include taxes since prior to 1993 Finnish forest taxation was based on a lump sum tax. After the transition period 1993–2005, this taxation will be based solely on profits. Excluding transition effects, neither of these affects forest management decisions in the basic Faustmann framework (Johansson and Löfgren 1985, p. 96, 99).

When $r \to 0^+$, V_F approaches infinity. However, as shown by Clark (1976, p. 260), the case of zero discounting corresponds to maximization of annual average net income. The annual average net income or forest rent, V_{FR}, is calculated as the sum of net income from one rotation divided by the rotation age:

$$V_{FR} = \frac{\sum_{i=0}^{s}\left(R_i - \sum_{k=1}^{m} c_{ik}\right)}{s}. \tag{3}$$

We calculate mean annual increment, V_{MAI}, in order to analyze the effects of optimal and regulated forest management on timber supply. V_{MAI} is calculated as the sum of timber volumes, x_{ij}, of usable assortments $j = 1,\ldots,n$ removed during thinnings and clearcut in years $i = 0,\ldots,s$, divided by rotation age, s:

$$V_{MAI} = \frac{\sum_{i=0}^{s}\sum_{j=1}^{n} x_{ij}}{s}. \tag{4}$$

[2] This is in contrast to other types of forest legislation that attempt to protect various nontimber benefits like wildlife (see, e.g., Siegel 1997).

V_{MAI} can also be calculated separately for each timber assortment $j = 1,\dots,n$.

We seek the maximized value of each of the criteria (1), (3), and (4) by computing values for all the forest management alternatives that are described in our growth and yield data. Maximization of (4) is known as maximum sustainable yield.

Biological and Economic Data

Our biological data is adapted from the variable density whole-stand growth and yield models presented by Vuokila and Väliaho (1980) for conifer cultures in Finland. Their study is based on extensive field work on 354 temporary sample plots scattered over Finland, and presents 233 growth and yield tables and the underlying functions for well-managed Norway spruce and Scots pine stands. The models and the tables are based on thinning from below. The conventional intensities applied in the yield tables accord with the requirements on thinnings in the present legislation (Ministry of Agriculture and Forestry 1997).

The classification of the sites is based on the dominant height of standing trees at the age of 100 yr. Thus, for example, $H_{100} = 24$ means that the standing trees are 24 m high at the age of 100 yr. The growth tables are presented for three meter class intervals with $H_{100} = 21, 24,\dots, 33$ for Norway spruce and $H_{100} = 15, 18,\dots, 30$ for Scots pine, covering virtually all suitable sites for forestry in Finland.

Based on the yield tables, we obtain on average 235 alternative management regimes for each 11 Norway spruce and Scots pine site types. The variables are timing, removal rate and number of thinnings, and the rotation period. The harvest revenue equation is specified from the standpoint of a forest owner selling timber at stumpage. We denote the harvest revenue for year i as R_i and define it as:

$$R_i = \sum_{j=1}^{2} \alpha_{ij}(dbh_i)p_j x_{ij} + \beta_{ih}(v_i, x_i), \tag{5}$$

where $\alpha_{ij}(dbh)_i$ is the quality premium multiplier determined separately for the two species and two timber assortments[3] (j = 1 for sawlog and j = 2 for pulpwood) as a function of average tree diameter at breast height, dbh_i.[4] The quality premium is multiplied by the average stumpage price, p_j, and the harvested volume, x_{ij}. The average volume of the removed trunks and the total harvested volume affect revenue via the effect of logging costs. This is denoted by $\beta_{ih}(v_i, x_i)$, where v_i is the average size (l) of the removed trunks and $x_i = x_{i1} + x_{i2}$ the total removed usable volume (m^3), for thinnings ($h = 1$) and clearcutting (h = 2). The stand establishment and silvicultural activities are scaled so that they guarantee the development of fully stocked young stands, as required in the legislation. The parameters and functions for prices and costs are presented in Appendix 1.

[3] The assortment specification is based on timber dimensions only. The reduction of sawlog volume due to quality defects is assumed to be minor in well-maintained stands (which are frequently thinned from below), and is thus ignored in this study.

[4] The average dbh is weighted by the trees' basal area.

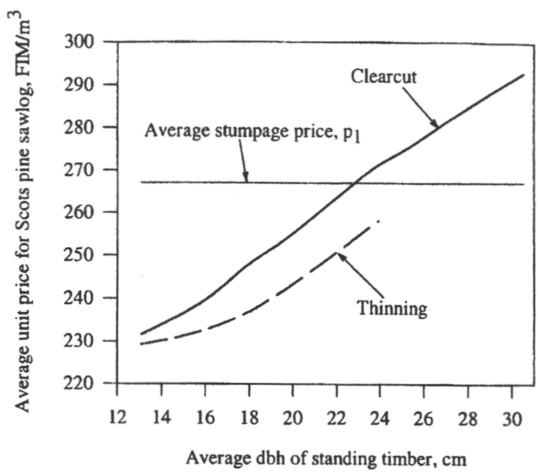


Figure 1. Average unit price for Scots pine sawlogs as a function of average dbh of standing timber for clearcut and thinning (30% removal).

Figure 1 shows the relationship between the average Scots pine sawlog price (including the effects of both quality premium and logging costs) and average dbh in cases of clearcut and thinning. The unit price increases with tree dimensions. The curves in our illustration are nonconvex because of the immediate effects of thinnings on average tree dimensions. At each respective dimension, the unit price is higher for clearcutting than for the thinning operation, mainly because of lower logging costs.

The rate of interest is one of the key factors in studying optimal forest management decisions. Figure 2 shows some of the most important time series for determining this parameter value.

The classical Faustmann model assumes that capital markets are perfect in the sense that the forest owner does not face any borrowing constraint and can borrow money at the same rate at which he can invest his savings. However, in Finland, capital markets were imperfect and

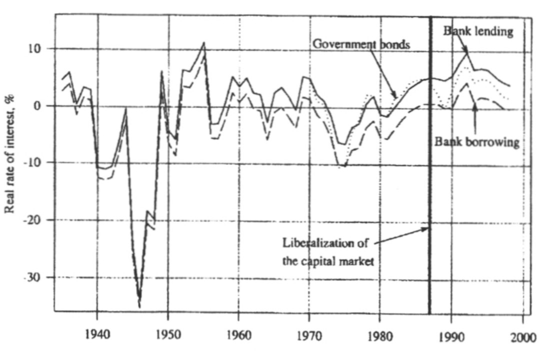


Figure 2. Average annual real rates of interest for bank lending and borrowing for 1935–1998 and government bonds for 1970–1998 derived from the nominal rates by using a cost-of-living index. (Sources: Statistics Finland, Statistical Yearbook of Finland 1970–1984, Finnish Bulletins of Statistics 1988–1998, Research Institute of the Finnish Economy)

Table 1. Legal and recommended rotation periods in southern Finland.

			Present law/recommendations						
			Legal minimum 1989		Recommendations 1989		Past recommendations		
							1975–1989		1971
Species	Site type	H_{100} class	Age	Dbh	Age	Dbh	Age	Dbh	Age
Scots pine	MT	27	70	27	80–90	29–31	80	29	80
	VT	24	80	25	90–100	27–29	90	27	90
	CT	18	100	23	110–120	25–27	120	25	120
Norway spruce	OMT	27	70	25	80–90	28–30	80	27	75
	MT	24	80	24	90–100	26–28	90	26	85

CLARIFICATIONS: Site type class (Cajander 1909) is based on classification of under-vegetation: OMT = Oxalis-Myrtillus, MT = Myrtillus, VT = Vaccinium, and CT = Calluna. Conversions between site type and dominant height site class, H_{100}, are according to Vuokila & Väliaho (1980, p. 26).

marked by credit rationing[5] until 1985 and thus clearly contradicted these assumptions, especially for those forest owners who faced a borrowing constraint. Tahvonen et al. (2001) have shown that under an effective borrowing constraint, optimal rotation periods are shorter than those suggested by a model based on perfect capital markets. This would lead to the interpretation that during the period before 1985 forest owners facing the borrowing constraint had incentives to make forestry decisions as if their rate of interest were higher than the bank lending rate of interest.

The average real bank lending rate of interest was negative (–1.3%) for the period 1935–1985. This period, however, includes World War II and the postwar years, which can be perceived as an exceptional time in the economy. The average bank lending rate for the regulated period after the war (1950–1985) is 1.2%. Since deregulation of the capital market in 1986, the average real lending rate of interest has been considerably higher (6.1%). This rate can now be considered as a realistic figure for those forest owners who are borrowers and who cannot deduct interest costs in taxation. The corresponding after-tax rate of interest[6] for those forest owners who are able to deduct interest costs in capital taxation is 3.5%.

The average real bank borrowing rates of interest have been –4.3% for the period of the regulated capital market (1935–1985), –2.1% for the postwar regulated period (1950–1985) and 1.3% in the conditions of liberalized capital markets (after 1985). However, a bank account hardly reflects the best low-risk investment opportunity for the average forest owner. The average after-tax rate of interest for government bonds has been 1.4% for the period 1970–1998, and 3.9% for the period of liberalized capital market (1986–1998). It is more difficult to obtain data on interest earned for other investment outlets, but it is expected that the average (pretax) return on market portfolio exceeds the return on government bonds.

Based on our data, it is difficult to exclude any rate of interest between 0 and 10% as a possible figure in Finnish forest owners' decision making. The taxation of capital earnings and the possibility to deduct interest costs in taxation imply that (after-tax) rates of interest around 2–6% may be relevant for a large fraction of forest owners.

Finally, we need data on clearcutting regulations and recommendations. These are summarized in Table 1 for southern Finland[7] from the beginning of the 1970s to the present.

According to requirements set out in statutory provisions (Ministry of Agriculture and Forestry 1997), a stand must reach either a certain minimum average dbh, expressed in cm, or a certain minimum age before it can be clearcut. The recommended conditions for maturity applied since 1989 (Tapio 1994, p. 21) are of the same kind. Before 1989, the relationship between legal limits and recommendations was somewhat unclear, but according to a typical interpretation a deviation from instructions could have been interpreted as forest devastation under the law originated from the Private Forest Act of 1928 by which devastation was prohibited. It was possible to cut a stand using dbh as a condition, but not earlier than 10 yr before the stated age (Tapio 1986, p. 212) (see Table 1). The earliest published recommendations from 1971 (Tapio 1971, p. 151) specified only the minimum ages to be applied.

The legal limits and recommendations were initially designed for natural forests, but they have been identically applied to artificially regenerated forests. The past instructions were more binding than the present recommendations. Gradual authorization of dbh as an alternative condition for maturity has increased the forest owners' flexibility to meet the requirements. During the 1970s, a logging ban was set at about 1.5% of the (private) forest area as a penalty for illegal forest management; in 1990 the rate was about 0.2% (Tapio 1992, p. 32).

Results

Optimal and Constrained Management Regimes

We can now apply the criteria for clearcutting and thinning together with the economic parameters and compute the outcomes of all the forest management alternatives described in our growth and yield data.

[5] Deregulation of bank lending was started in 1983, and the last regulations were removed by 1986.

[6] The after-tax rate of interest, r, can be derived as a function of real rate of interest, r', the inflation rate, ι, and the capital tax rate τ (0.29 in Finland) using the formula $r = r'(1-\tau) - \tau\iota$.

[7] We concentrate in our analysis on southern Finland, which accounts for about 75% of total annual volume increment in Finland, according to the present zone definition (Finnish Statistical Yearbook of Forestry 1998, p. 68).

In Figure 3, each dot represents one particular management alternative in terms of rotation age and tree diameter at breast height (dbh) for Norway spruce sites. The growth and yield data is based on thinning from below. This type of thinning accelerates the growth of dbh. Thus, lower points represent light, and the higher points heavy thinning intensities. Percentages represent the unconstrained optimal rotation periods at each respective rate of interest. A dot denoted by, e.g., A(5–7%) shows the optimal solution under legal limits when the rate of interest is 5–7%.

Maximizing the forest rent leads to a longer rotation period than that proposed in the recommendations for a fertile Norway spruce site (Figure 3a). By contrast, when the rates of interest between 1 and 4% or MSY are used as a cutting criterion, the optimal solutions are approximately within the recommended range. Discounting at 5 or 6% leads to an optimal solution, which falls slightly below the legal limits. Rates of interest of 7% or higher lead to clearly illegal optimal rotation periods. An illustration with a less fertile Norway spruce site in Figure 3b shows similar relationships between optimal rotation period and regulations. However, at that site, a rate of interest of 5% already leads to clearly illegal optimal solutions.

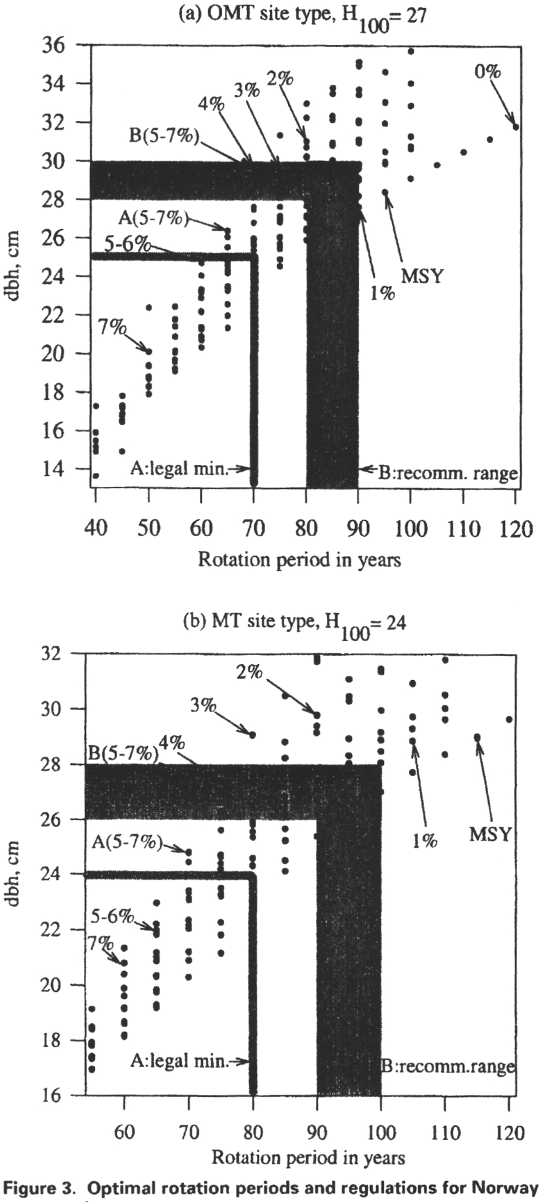


Figure 3. Optimal rotation periods and regulations for Norway spruce cultures.

Both the rotation age and the thinning regime are highly sensitive to the rate of interest. Optimal rotation age decreases as the rate of interest rises. Light thinning intensities turn out to be optimal at low, and heavy thinning intensities at high, rates of interest. When the regulations are binding, they typically lead to high thinning intensity.

Figure 4 shows the results for the three Scots pine site types. Discounting at $r \leq 1\%$ or using MSY as a decision criterion leads to longer rotations than recommended in a fertile Scots pine site type in Figure 4a. At a 2% rate of interest, the optimal solution coincides with a lower boundary for the recommended range, and at a 3% rate with the minimum legal limit. Application of 4% or higher rates of interest leads to illegal optimal rotation periods. Corresponding illustrations on VT (Figure 4b) and CT (Figure 4c) site types reveals that the regulations become gradually stricter, the less fertile the site. When using, for example, a 2% rate of interest, the optimal solution accords with the recommendations for the most fertile MT site, but falls below the recommended range for a less fertile VT site and lies in the minimum legal limit for the least fertile Scots pine site type.

Contrary to the findings on optimal thinning policy for Norway spruce stands, thinning intensity for Scots pine cultures does not turn out to be sensitive to the rate of interest. Light thinning intensities remain optimal given all the rates of interest used, implying that it is profitable to keep a high proportion of the growing stock until the end of the rotation period. Therefore, a Scots pine stand reaches first the age, rather than dbh, condition stated in the regulations. Because of the insensitive thinning regime, economically optimal Scots pine management is in line with the recommendations for a fairly narrow range of the rates of interest and becomes illegal already at quite low rates of interest.

When binding, the regulations alter significantly the optimal thinning strategy for Scots pine cultures. They increase optimal thinning strategy for two reasons. Firstly, with intensive thinning, a stand reaches the dbh condition faster and can consequently be cut earlier. Secondly, by thinning, at least a portion of the capital tied up in the growing stock can be realized sooner.

The northernmost forestry zones include large areas of Scots pine forests. The growth is generally slower, the further north the zone is located. Abundance of Scots pine stands and poor site indexes together with the stricter legal

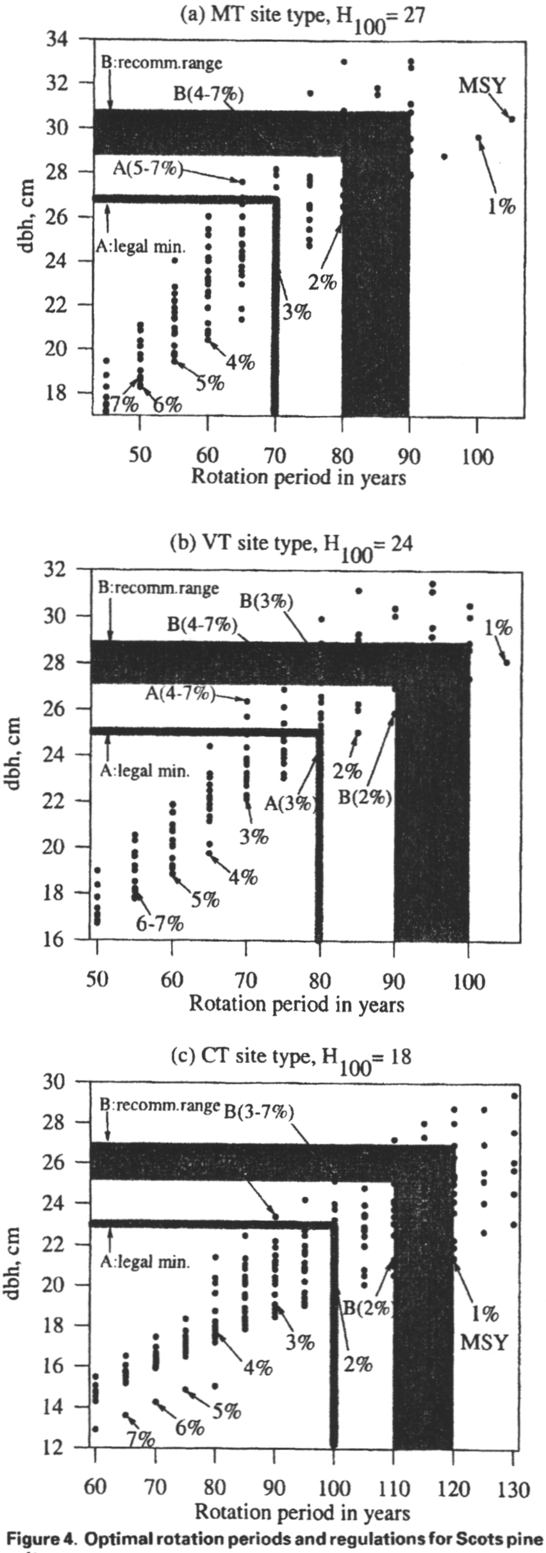


Figure 4. Optimal rotation periods and regulations for Scots pine cultures.

limits[8] imply that the regulations are more binding the further north the zone is located. In the northernmost zone, Lapland, discounting only at very low rates of interest (0 – 1%) exceeds the legal limits, and discounting at $r \geq 2\%$ leads in most cases to illegal rotations.

The 3 m interval H_{100} classification applied in the growth data is more precise than the Cajanderian site type classification applied in regulations. On average, 2–3 H_{100} classes correspond to one Cajanderian site type. Selecting another H_{100} class on the upper or lower edge within each site type instead of the class average proposed by Vuokila and Väliaho (1980) caused at most a $\pm 1\%$ deviation in determining the critical rates of interest before the constraints start to bind.

Economic Loss Because of Regulation

When binding, the regulations cause losses to the forest owner.[9] The relationships between stand age, value of forestland with standing timber, and losses resulting from regulations are illustrated for a common VT Scots pine site (H_{100} = 24) assuming a 4% rate of interest.

The solid line in Figure 5a depicts the development of the maximized value of land including standing timber. When discounting at a 4% rate of interest, the optimal management regime within the simulation material consists of one thinning (30% removal rate) carried out at the age of 45 yr and clearcutting at the age of 65 yr. The dashed and dotted lines illustrate the optimal management regimes that meet the legal requirements, A, and the recommendations, B, respectively. In these cases, the stand is thinned a second time at 65 yr and allowed to grow until age 80 yr to meet the legal requirements and 90 yr to fulfill the recommendations.

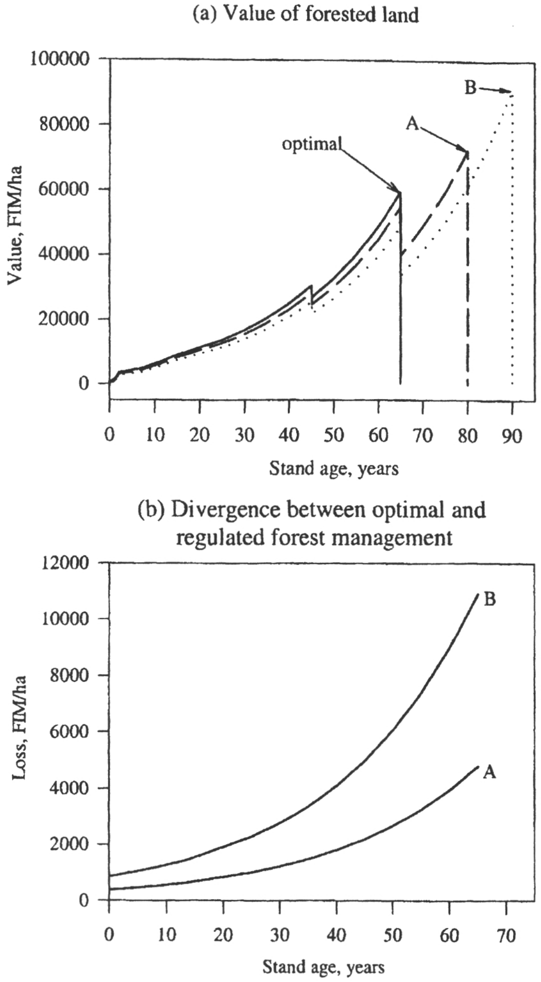
The difference between optimal land values and land values under the regulations are presented in Figure 5b. The losses remain at a quite moderate level in the case of bare land, but increase at a rising rate with stand age. The losses are at a maximum at the optimal clearcutting age. Even though the value of the remaining stock grows fast between 65 and 90 yr, the extended rotations ultimately lead to losses (at the age of 65 yr) of about 4,500 FIM/ha with legal restrictions and 10,500 FIM/ha with recommendations.

Implications for Long-Term Timber Supply

The maximum sustainable yield and the long-term timber supply or mean annual increment, MAI, that results from economically optimal forest management are presented in Figure 6 by assortments of MT Norway spruce and Scots pine sites under optimality, regulations, and recommendations. The MSY criterion is independent of the rate of interest and the production levels are described as straight horizontal lines for the total usable timber, MSY_t, consisting of sawlogs, MSY_s, and pulpwood MSY_p. The solid lines depict total and timber assortment production levels for economically opti-

8 For example, in the northernmost region, the minimum legal age and dbh limits for an H_{100} = 18 Scots pine site are 120 yr and 23 cm while the limits in a corresponding site in Southern Finland (see Figure 4c) are 100 yr and 23 cm (Ministry of Agriculture and Forestry 1997).

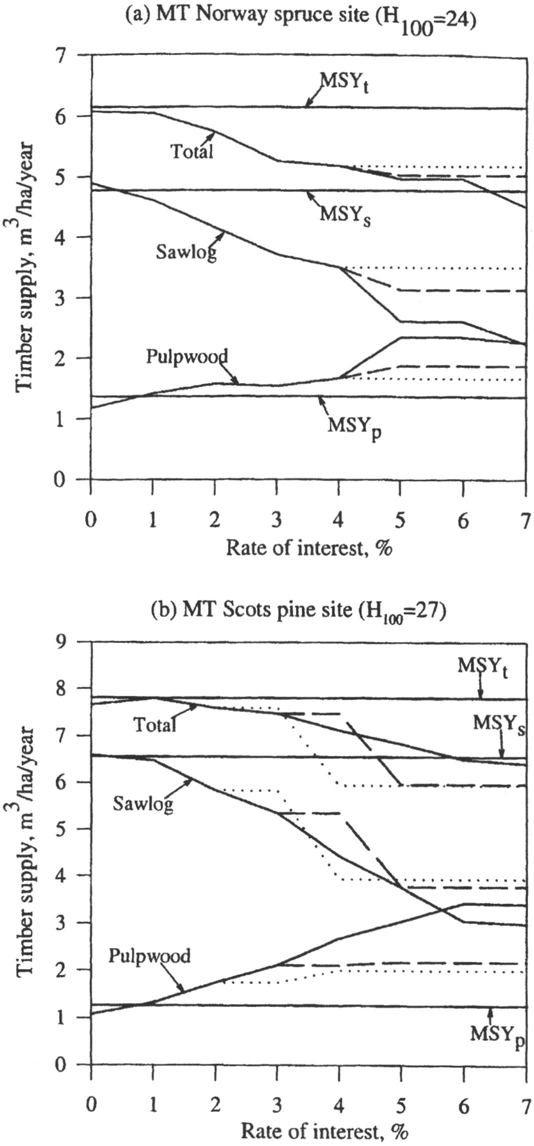
9 There is an implicit assumption that forestland is not converted to other land uses when the rate of interest changes. This is a realistic assumption, especially for stands which are unsuitable for agriculture or grazing and are located in sparsely populated regions.



A: Compliance with legal minimum

B: Compliance with recommendations

Figure 5. Value of forested land and divergence between optimal and constrained management at VT Scots pine site (H_{100}=24), r = 4%.



Note: Dashed lines refer to legal limits
and dotted lines to recommendations

Figure 6. Total, sawlog and pulpwood long-term timber supply as a function of the rate of interest.

mal forest management. Dashed and dotted lines represent the supply levels when the legal limits or recommendations restrict the optimal timber management.

The optimal rotation period produces the maximum sustainable yield when the rate of interest is about 0.5–1.0%. The total timber supply declines with the rate of interest when $r >$ 1%. An increase in the rate of interest leads to lower sawlog supply but higher pulpwood supply. The total timber supply is more sensitive to changes in the rate of interest for Norway spruce (Figure 6a) than for Scots pine (Figure 6b) stands. By contrast, the proportions of produced timber assortments react to changes in the rate of discount more sensitively for Scots pine than for Norway spruce.

The regulations tend to squeeze total and timber assortment production to constant levels once the constraints start to have an effect (Figures 6a and b). The recommendations (dotted lines) start to affect decision-making at lower rates of interest than the legal limits (dashed lines) and consequently their effect on timber production is more significant. The regulations increase total timber supply from Norway spruce stands (Figure 6a) but reduce the total timber supply from Scots pine stands (Figure 6b). Generally, total timber production is increased because of extended rotation in the regulated case. This increase is, however, overwhelmed by the opposite effect of heavy thinning intensity for Scots pine stands (Figure 6b). Regarding timber assortment production levels, the regulations have the same effect for both species: when effective, they increase the supply of sawlog dimensions and reduce the supply of pulpwood, compared to the unconstrained optimum.

Implications for the Structure of Costs and Labor in Forestry Activities

The explicit reason for the state intervention and forest policy is often the aim of increasing working opportunities within forestry. We analyze the effects of regulations as a major policy tool on the structure and level of forestry costs, and then tentatively on the availability of working opportunities within forestry. This is done by evaluating the level of forestry activities and the resulting expenses of thinning, clearcutting, and stand establishment, both in the optimal and the constrained cases, as averages over all site classes. The logging costs (thinning and clearcutting) include both felling and hauling to the roadside. In addition, total costs include an administrational cost component (25 FIM/ha/yr). The average annual costs per hectare over the rotation are shown in Figure 7 for 3% and 6% rates of interest.

At a 6% rate of interest, the regulations lead to slightly higher thinning costs, but on the other hand, lower clearcutting, stand establishment, and silvicultural costs than in the unconstrained case, as presented in Figure 7a for Norway spruce forest. The total costs are about 5% lower when following the legal requirements and about 6% lower under the recommendations compared to the unconstrained optimum. When r = 3% (not shown), the regulations do not restrict optimal forest management in Norway spruce forest and consequently have no effect on the costs.

The regulations lead to increased thinning expenses, but lower clearcutting, stand establishment, and total costs also for Scots pine. The effect is stronger, the higher the applied rate of interest. At r = 3% (Figure 7b) the cost structure is somewhat altered, but the decrease in total costs remains quite small, i.e., 1% for legal limits and 5% for recommendations. With a 6% rate of interest (Figure 7c), the regulations change the cost structure significantly, and reduce the total costs by 11% and 14% compared to the unconstrained optimum.

The changes in the level and structure of costs are consequences of changes in optimal rotation periods and thinning regimes. The extended rotation decreases the level of clearcutting, stand establishment, and silvicultural activities at the aggregate level. For example, the area to be clearcut and regenerated annually decreases by 50% if a 100 yr rather than 50 yr rotation period is applied to a normal forest. Even though the extended rotation period increases harvested volumes (assuming that $s^* < s^*_{MSY}$), the level of clearcutting costs is reduced because of smaller operated areas and unit logging costs. However, the regulations increase thinning activities, especially in Scots pine stands.

Modern harvesting, especially clearcutting, is capital-intensive and highly mechanized. On the other hand, stand establishment and silvicultural operations like planting, weeding, and tending of a young stand still require considerable human input. If there is a positive relationship between forestry expenses and the demand for labor, this suggests that

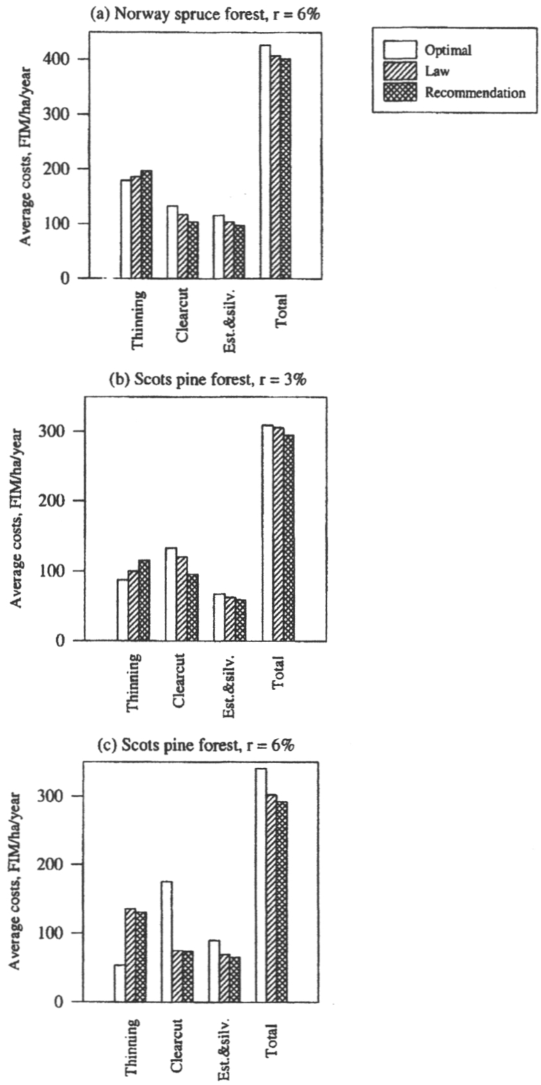


Figure 7. Average annual unit costs computed as an average for all site classes.

the regulations may reduce, rather than increase, working possibilities in forestry. However, these negative effects may be partly offset by the increased thinnings.

Discussion

Comparison with Other Studies

Table 2 gives a comparison of the optimal rotation periods for this study and other recently published studies. The legal limits and recommended rotation ages are repeated in order to facilitate the comparison. The rate of interest in all the studies is 3%. The results of a pioneering study by Nyyssönen (1958) are presented also because they have been claimed to serve as an initial guideline for the regulations. In his study, thinnings were not optimized and the rotation period was determined using the "marginal-growth method" originated

Table 2. Comparison of published results on optimal rotation periods (age in years/average dbh at end of rotation period in cm) for Finnish conifer stands with 3% rate of interest.

	Norway spruce		Scots pine			
Studies	OMT	MT	MT	VT	CT	Peatland
Law 1989–	70/25	80/24	70/27	80/25	100/23	
Recommendation 1989–	80–90/28–30	90–100/26–28	80–90/29–31	90–100/27–29	110–120/25–27	
Nyyssönen 1958	70/–	85/–	70/–	80/–	120/–	
Pesonen & Hirvelä 1992	75–80/–	80–90/–	65–75/–	70–80/–	90–100 /–	
Salminen 1993a	70/–	80/–	55–65/–	65–70/–	—	
Valsta 1992	77/23	—	—	—	—	
Miina 1996	—	—	—	—	—	77–79/–
Vettenranta & Miina 1999	—	70–78/–	70–78/–	—	—	
This study	75/30	80/29	70/24	70/22	90/19	

NOTE: Pesonen and Hirvelä (1992) utilize MELA forestry software and National Forest Inventory sample plot data from four sub-regions in southern Finland. Salminen (1993a) applies dynamic programming and, as in this study, the variable-density whole-stand stand simulators of Vuokila and Väliaho (1980), with and without the ad-hoc thinning response model. Nonlinear optimization with experimental data and individual-tree growth simulators were used by Valsta (1992) for fertile Norway spruce stands, Miina (1996) for Scots pine stands growing on rather infertile drained dwarf-shrub pine mires, and Vettenranta and Miina (1999) for Scots pine and Norway spruce mixtures.

by Duerr and Bond (1952). This rotation criterion deviates from the Faustmann criterion and has been shown to be economically incorrect by Chang (1990) and Klemperer (1996, p. 230).

It is somewhat surprising that after fixing the rate of interest, all the studies yield rather similar rotation periods although they are based on different biological growth models, different optimization techniques, and varying economic data on price and cost parameters. According to all of the studies, the optimal rotation periods are shorter than the recommended rotations for both species and for all site classes. With a 3% rate of interest, the legal limits do not restrict the optimal rotation for Norway spruce. In contrast, for Scots pine, the optimal rotation periods are shorter than the legal minimum requirements and the situation is the clearest in case of infertile Scots pine sites. In previous studies, however, the regulations and the optimal rotation ages cannot be rigorously compared because, except for Valsta (1992), the development of dbh is not reported. If only the rotation period is taken into account, the legal limits coincide with the results of Nyyssönen (1958) excluding the CT site class for Scots pine. We note that Nyyssönen (1958) applied exogenous and frequent thinnings, which may partly explain the differences between his results and the later studies for Scots pine.

In order to test the robustness of the development of dbh in our study, we compare the management regimes of four representative site types with National Forest Inventory (NFI) results (Tomppo et al. 1999a,b) on the average relationship between stand age and dbh derived from three regions in southern Finland (Figure 8).

The development of the dbh in our data is in the same class with NFI-results in early ages. However, at older stands, the thinning from below applied in our growth and yield data and especially the heavy thinning regimes lead to faster development of dbh compared to the inventory data. The deviation is smaller for Scots pine than for Norway spruce. We also expect that part of the deviation follows from the fact that the degree of heterogeneity increases in natural forests with age. Nevertheless, it is unlikely that our growth data markedly underestimates the development of dbh of managed forests and thus overestimates the consequences of the regulations.

As a means of comparison with the Finnish results, a Norwegian study (Solberg and Haight 1991) computes optimal management of Norway spruce for a site comparable to the OMT site class in Table 2. With a 3% rate of discount and applying thinnings from above, they obtain 80 yr as the optimal rotation period. Recent Swedish studies on optimal Scots pine management by Eriksson (1999) and Gong (1999) find optimal rotation periods similar to ours, while the optimal rotation periods found by Zhou (1999) are somewhat shorter.

Table 2 suggests rather clearly that there is a deviation between the results on economically optimal rotation periods and the recommendations, and that the legal limits may restrict the economically optimal rotations, especially for Scots pine. However, some caveats are in order. Most of the studies are based on growth and yield data that describes well-managed, artificially regenerated stands. Although the fraction of these stands is increasing (Finnish Statistical Yearbook of Forestry 1999, p. 108) there is no doubt that the heterogeneity of harvested stands is great, and so one should question the extent to which the study results can be applied, for example, to naturally regenerated or unmanaged stands.

A partial answer is given by the study by Pesonen and Hirvelä (1992), which uses national forest inventory sample plot data but still reports rotation periods similar to other studies. This view is supported by the preliminary computations using a system by Valsta and Linkosalo (1996), which uses sample plot data but still yields optimal rotation periods that contravene the legal limits. Salminen (1993b) studies understocked stands that are initially near the potential clearcutting age, and obtains rotation periods that are close to his 1993a study (Table 2). Although in the light of the existing studies the results shown in Table 2 are quite robust, there is no doubt that more reliable and versatile growth data, especially for stands based on natural regeneration, are needed for optimal forest management studies.

Interpretations

The long-term trend in Finnish (real) stumpage prices, from 1950 to 1999, has resulted in a 0.44% annual increase

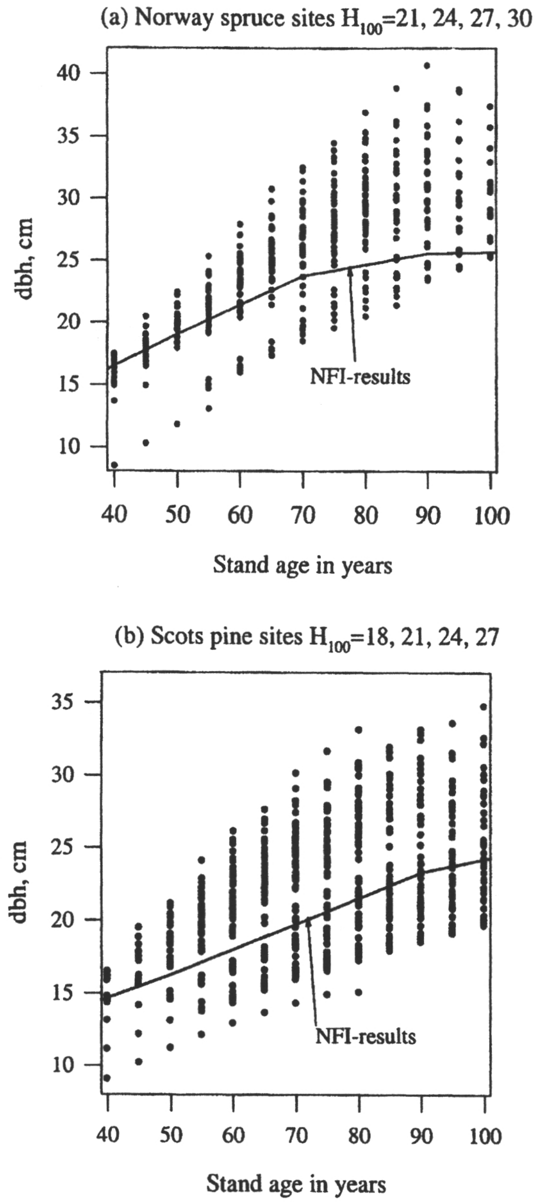


Figure 8. Relationship between average dbh and stand age according to National Forest Inventory (NFI) results (line), comparison with our growth data.

(Finnish Statistical Yearbook of Forestry 1999, p. 150). For the period 1978–1998, the price trend has been approximately flat (Hänninen 1999, p. 35). However, the timber price level fluctuates widely over the business cycle. There is empirical evidence that a large fraction of Finnish forest owners aim to time their timber selling with periods when the price is highest (Kuuluvainen and Tahvonen 1999). Thus, fluctuation of prices may either shorten or lengthen the rotation period compared to the optimal rotation computed using average prices. In such a situation, fixed minimum limits for clearcutting in fact accelerate fluctuations in timber prices. Similar problems are related to the varying rate of interest. We now attempt to identify the driving forces behind the regulations.

Perhaps the most straightforward interpretation is that the legal limits and recommendations reflect the long-term post-World War II (real) interest rate levels up to the capital market liberalization. The average bank lending and borrowing rates of interest for the period 1950–1985 were 1.2 and –2.1% respectively, while the after-tax rate on government bonds was –0.6% for the period 1970–1985. Figures 3 and 4 show that the present and past (Table 1) recommendations are closer in line with rates of interest between 1–2%. Recalling that buying forestland was restricted and allowed mainly to farmers until the end of 1997 and that in the absence of perfect land and capital markets there may not be enough incentives for long-term decision making, some legal minimum limits for rotations may have been justified.

It would be expected that during the period of credit rationing when the bank lending rate of interest was negative, many forest owners faced a constraint on borrowing. Theoretical analysis (Tahvonen et al. 2001) shows that under such circumstances, a rational forest owner applies a rotation period shorter than indicated by the market rate of interest. Thus the legal limits based on information from studies that assume perfect capital markets may have caused additional welfare losses for borrowing constrained forest owners. The present legal limits were set in 1997 and recommendations in 1994, both after the capital market liberalization and after the regime switch in the interest rate level. Thus one must observe that these institutions do not reflect the important changes in the surrounding economic environment.

Another possible explanation is the doubt concerning discounting among Finnish forestry experts and even academic forest scientists. The pioneering study by Nyyssönen (1958) applied the marginal-growth method instead of the Faustmann formula in order to avoid "unrealistic discounting." The government financed National Center for Forestry Extension offers the maximum sustainable yield and the forest rent as a perfectly valid basis for the clearcut timing decision (Tapio 1997, p. 214). The misunderstanding of the capital markets and of discounting is also reflected in the belief that the rate of interest in forest economic calculations depends on the site productivity (Tapio 1997, p. 446). From the economic point of view, this is confusing and difficult to interpret. Note that our results reflect this belief in the sense that the legal limits and recommendations are based on a lower rate of interest in northern Finland, where site productivity is lowest. We note that such beliefs tend to yield the biggest losses in terms of forestry-dependent wealth and income level in those rural regions where the standard of living is low and the rate of unemployment is high. It is also worth noting that taxing capital income and allowing (partial) deduction of interest costs works in the direction of lowering the interest rates and lengthening the rotation periods, compared to social optima.

According to one interpretation, long rotation periods and a high level of standing timber stock are means of avoiding a timber shortage that forestry planners have frequently feared during the period since the World War II. The tendency is to argue that the higher the level of timber production, the better for the whole economy. The behavior of nonindustrial private forest owners and their incentives for discounting cause deviation from this "golden rule" and thus it is natural to regulate their clearcut decisions (Kuusela 1999, p. 241). This is clearly a variant of the maximum sustainable yield argument and has already been commented on elsewhere (e.g., Ohlin 1921, Samuelson 1976). A related idea is that the economically optimal rotation periods cannot be applied since they would yield a lower sawlog/pulpwood production ratio than is presently realized in the markets. This type of planning neglects the determination of timber prices and increased incentives to produce more sawlogs using a longer rotation period if the sawlog price level would be higher. From this point of view, the present legal limits and recommendations may keep the sawlog price at an artificially low level. This yields welfare losses for the sellers of timber but benefits the buyers. This effect may be partly offset by the higher pulpwood price level.

Long rotation periods may be beneficial from the point of view of environmental preferences. Perhaps the legal limits and recommendations reflect these values implicitly. For forest owners with such preferences, the legal limits may not be restrictive. However, it may be questioned whether a general minimum limit for the rotation period is a good instrument for environmental purposes. Since the nontimber values produced by private forest owners are external benefits or public goods (cf. the right of common access), a better alternative may be to subsidize forest management decisions that yield or maintain nontimber benefits. Government officials may also apply a method of competitive bidding games for selecting and buying efficiently the exceptionally high-value natural environments to be totally conserved from timber production (Tahvonen and Salo 1999). In addition to economic efficiency effects, the choice between these policy alternatives has strong effects on how the ultimate resource rent from forests will be distributed between the various interest groups.

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APPENDIX 1. Prices and Costs

The average real unit stumpage prices, p_j, for sawlogs (j = 1) and pulpwood (j = 2) derived from a time series 1983–1997 (Finnish Statistical Yearbook of Forestry 1998, p. 159) are: p_1 = 209.20 FIM/m^3 and p_2 = 127.44 FIM/m^3 for Norway spruce, and p_1 = 267.07 FIM/m^3, and p_2 = 108.12 FIM/m^3 for Scots pine. Quality premium multipliers, α_{ij}, were derived as a function of the diameter at breast height, dbh_i, of the removed trees, expressed in centimeters. Quality premium functions for Norway spruce are:

$$\alpha_{i1} = 0.7107 + 0.1212\, ln(0.3990(dbh_i));$$

$$\alpha_{i2} = 0.9176 + 0.1996\, ln(0.07652(dbh_i)),$$

and for Scots pine:

$$\alpha_{i1} = 0.7589 + 0.008929(dbh_i);$$

$$\alpha_{i2} = -0.00002482(dbh_i)^3 + 0.001632(dbh_i)^2 - 0.01666(\mathrm{dbh}_i) + 0.8573.$$

The absolute effect of logging conditions on harvest revenues, β_{ih}, has been modeled as a function of the average size of the removed trunks in liters, v_i, and the removed total volume, x_i, expressed in m^3. Conditions when $\beta_{ih} = 0$ correspond to average logging cost: 47.90 FIM/m^3 (Finnish Statistical Yearbook of Forestry 1998, p. 180). The equations are, for thinning, (h = 1):

$$\beta_{i1} = (1.413 + 4.005\, ln(0.008908\, x_i) + 4.634\, ln(0.0422\, v_i) - 2622/v_i\,)x_i$$

and for clearcutting (h = 2):

$$\beta_{i2} = (-0.3191 + 1.602\, ln(0.004259\, x_i) + 2.408\, ln(9.579\, v_i) - 2642/v_i)x_i$$

The relationship between unit stumpage price and timber dimensions follows the shape (but with milder slope) of the results for industry's "potential ability to pay" for different dimensions (Ministry of Agriculture and Forestry 1993, p. 57). On the other hand, in our study the relationship between harvest revenue and timber dimensions is clearly stronger compared to the Finnish logging and hauling work tariffs that were applied in actual timber sales until 1991. Compared to other recent rotation studies in Finland, the dependence is about the same class as the scaling used by Salminen (1993a, p. 44–45) and a bit stronger than that applied by Valsta (1992), Salminen (1993b), Miina (1996), and Vettenranta and Miina (1999). Our price functions have been constructed to describe realistically the actual timber markets and are based on various discussions, e.g., with Valsta (pers. comm.) on the dependencies between quality premium, logging costs and stand characteristics. Unit costs, timing of activities, and the probability of occurrence required to establish a well-managed fully stocked young stand are presented in Table A3.

Table A3. Unit costs, timing, and probability of occurrence of stand establishment and silvicultural activities.

Stand establishment or silvicultural activity	Unit cost FIM/ha	Timing year	Probability of occurrence in H_{100} class: Norway spruce 21–27	Norway spruce 30–33	Scots pine 15–21	Scots pine 24	Scots pine 27–30
			(%)				
Clearing	424	0	60	60	75	75	75
Harrowing and scarification	719	0	40	20	75	75	75
Planting	3,286	1	100	100	0	50	100
Sowing (alternative to planting)	902	1	0	0	100	50	0
1st grass weeding	532	2	25	25	20	20	45
2nd grass weeding	532	2	20	20	10	10	35
3rd grass weeding	532	3	15	15	0	0	10
Complementary planting	2,340	4	25	25	0	10	25
1st tending	1,058	7	90	90	80	80	80
2nd tending	1,058	11	60	60	70	70	70
3rd tending	1,058	14	30	30	10	10	10

NOTE: A fixed annual cost (25 Fim/ha/yr) is assumed the same for all site types and consists of forestry fees, services provided by experts, and administration costs. The unit costs are the average costs for the year 1997 excluding value added tax. [Sources: Finnish Statistical Yearbook of Forestry (1998) and Metsäteho (1998, p. 3). Timing of activities and probability of occurrence are partly adapted from Salminen (1993a, p. 41)]

Paper II

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II

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Economics of Forest Thinnings and Rotation Periods for Finnish Conifer Cultures

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In this study simultaneous optimization of thinnings and clear-cutting was investigated. The density-dependent whole-stand model was specified for all relevant Finnish Norway spruce [*Picea abies* (L.) Karst.] and Scots pine (*Pinus sylvestris* L.) site indices and solved by non-linear programming. Sensitivity analysis showed that in some cases and owing to endogenous thinnings the optimal rotation length may increase with the rate of interest and site fertility, and decrease with harvesting cost. The number of thinnings is more sensitive to changes in the rate of interest, logging conditions and site productivity for Scots pine stands than for Norway spruce stands. Economic optimization suggests that for both species the first thinning should be performed later than officially recommended. The last thinning should be heavier than officially recommended, especially at high rates of interest. This increases the optimal rotation length compared with solutions under restricted thinning intensity. *Key words: optimal rotation, optimal thinnings, Picea abies, Pinus sylvestris, rate of interest.*

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INTRODUCTION

Thinning is perhaps the most important intermediate activity in even-aged forest management. From the economic point of view it leads to earlier harvesting revenues and increases the value growth of residual trees. In Finland, about 20% of the domestic timber supply comes from thinnings (Anon. 1993).

The economic theory of timber production is well established (Faustmann 1849, Preßler 1860, Ohlin 1921, Samuelson 1976). However, most economic research on stand-level forest management ignores thinnings. It is through the use of density-free whole-stand growth models that most of the knowledge has been obtained about how the optimal rotation length and long-term timber supply depend on economic parameters such as prices, costs, rates of interest and taxes (see e.g. Chang 1983, 1984, 1998, Johansson & Löfgren 1985, Montgomery & Adams 1995). Such models are also suitable for adding various economic complexities (e.g. Willasen 1998, Tahvonen et al. 2001). However, it is an open question whether the results are robust when any of the intermediate forestry activities are included in the analysis.

Simultaneous determination of interacting thinnings and rotation period is a complex problem. The problem was at first specified as a continuous-time optimal control problem (e.g. Näslund 1969, Schreuder 1971). Clark (1976), Clark & De Pree (1979), Cawrse et al. (1984) and Betters et al. (1991) showed that the optimal solution for thinning is a singular path with a time period where thinning is applied continuously. This theoretical model is intuitively appealing, but the drawback is that thinning forests continuously is unrealistic.

Optimization models with detailed empirical growth specifications are often too complex to be solved analytically. Numerical methods have been used to solve whole-stand models (e.g. Kilkki & Väisänen 1969, Brodie et al. 1978) and increasingly complex individual-tree (e.g. Roise 1986, Valsta 1992), distance-dependent (e.g. Pukkala et al. 1998*b*, Vettenranta & Miina 1999) and stage-structured growth models (e.g. Haight 1987, Solberg & Haight 1991). Increasing the complexity may have improved the reliability of the results, but it has forced researchers to concentrate heavily on numerical solution techniques. Typically, the optimization results are given only for one or a few parameter values, which is not enough for determining the rich relationships between optimal forest management and various economic and biological factors.

Recent Finnish studies on optimal timber management use highly advanced descriptions of growth (e.g. Valsta 1992, Miina 1996, Pukkala et al. 1998*b*, Vettenranta & Miina 1999). These and earlier studies do not, however, explain exhaustively how the char-

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acteristics of optimal harvests (number, timing and intensity of thinnings, and timing of clear-cut) depend on economic parameters, logging conditions or site quality. The main commercial timber species, Norway spruce [*Picea abies* (L.) Karst.] and Scots pine (*Pinus sylvestris* L.), differ in terms of physiology. Still, there are no optimization results on how their management differs for single-species stands. Most earlier formulations have constraints, which may restrict the number, timing or intensity of thinnings without sound economic or ecological justification. In addition, there are few results on timing of the first thinning, even though this was one of the most discussed topics in Finnish forestry during the 1990s.

The objective of this study was to analyse the dependence of optimal thinnings and clear-cutting on various economic parameters and on all relevant site indices in the Norway spruce and Scots pine stands in Finland.

MATERIALS AND METHODS

Stand-level optimization model for thinnings and rotation

According to the classification of Getz & Haight (1989), p. 228–), the present stand-level optimization model is an extended univariate model. The stand-level results can be aggregated to forest level in the absence of spatial adjacency constraints for harvesting (e.g. Barrett et al. 1998), regulatory constraints on age class distribution or varying returns to scale in harvesting technology (e.g. Heaps 1984, Johansson & Löfgren 1985, p. 121) or exogenous timber prices (Salo & Tahvonen 2001). This model is more detailed than is typical in the economics literature on stand-level forest management, but contains less detail than single-tree specifications. The main limitation is that the model does not include classes for tree size. Thus, it is not possible to optimize the type of thinning (from above or below).

The problem is to determine a harvesting regime that maximizes the economic surplus of timber production. The dates of thinnings and clear-cutting and the removal rates of thinnings are chosen such that the value of the bare land is maximized. In order to solve the problem, equations are needed that predict the growth and yield of a forest stand and information on the costs and revenues of forestry activities.

Let J_n denote the value of the bare land under $n-1$ thinnings ($n \geq 1$) where n is the number of harvests (thinnings plus clear-cutting) during the rotation. The parameter w_k, $k = 1, \ldots, m$, is the cost of the kth silvicultural activity (exogenous) at stand age t_k and m is the number of silvicultural activities. The real rate of interest is r and the age of the stand is t. Index j refers to the timber assortment, with $j = 1$ for sawlogs and $j = 2$ for pulpwood. The roadside price of assortment j is $p_j(t_i)$, and $x_j(t_i)$ is the harvested volume of assortment j in the ith harvest when the stand age is t_i. The cost of harvesting activity i is denoted by $C_i(t_i)$, T is the (exogenous) level of annual administrative costs per hectare and τ is the rate of the proportional profit tax. The objective function for maximizing the value of the bare land is

$$J_n = (1-\tau) \times \left\{ \frac{-\sum_{k=1}^{m} w_k \, e^{-rt_k} + \sum_{i=1}^{n} \left[\sum_{j=1}^{2} p_j(t_i) x_j(t_i) - C_i(t_i) \right] e^{-rt_i}}{(1 - e^{-rt_n})} - \frac{T}{r} \right\} \quad (1)$$

The fundamental state variables of the model are the basal area at breast height without bark, $G(t)$, in $m^2\ ha^{-1}$, and the age of the stand, t. $N(t)$ denotes the number of trees per hectare. The basal area after thinning i at stand age t_i is given by

$$G(t_i) = \{G(t_{i-1}) + \Delta G[H(t_{i-1}), G(t_{i-1}), t_{i-1}, t_i]\} \times \left\{1 - \frac{m[N(t_{i-1}), \Delta G(t_i), G(t_{i-1})]}{N(t_{i-1})}\right\}(1-u_i) \quad (2)$$

where $G(t_0) = G_0$ is the initially given basal area, ΔG is the growth of basal area during the period from t_{i-1} to t_i, $H(t_i)$ is the average height of dominant trees at age t_i and m is the number of trees that die during the period from t_{i-1} to t_i. The exact function forms and parameter values for functions such as $H(t_i)$ and ΔG are site index specific and are given in the Appendix.

At any harvest, a proportion u_i of the growing stock is removed. The number of trees after the ith harvest is given by

$$N(t_i) = \{N(t_{i-1}) - m[N(t_{i-1}), \Delta G(t_i), G(t_{i-1})]\}(1-u_i) \quad (3)$$

where $N(t_0) = N_0$ is the given initial number of trees. Note that the evolution of the number of trees depends on the basal area, initial number of trees and thinnings. In eqs (2) and (3) tree mortality is expressed as a function of the number of trees and basal area variables reflecting the relative stand density with respect to the maximum level.

The total harvested volume, x_0, is partitioned into sawlogs, x_1, pulpwood, x_2, and wastewood, x_3. The sawlog volume when harvesting a stand of age t_i is

$$x_1(t_i) = f\{x_0(t_i), F(t_i), H(t_i), \tilde{G}(t_i^-), N(t_i^-), t_i\} \quad (4)$$

where $F(t_i)$ is the form factor, $H(t_i)$ is the dominant height and $\tilde{G}(t_i^-)$ is the basal area including bark just before thinning. The left-hand limit of t_i is denoted by t_i^-. The harvested timber is given in quantities that include bark. See the Appendix for the conversion between basal area with and without bark. The form factor is given by $F(t_i) = F[H(t_i), \tilde{G}(t_i), N(t)]$, i.e. it depends on the number of trees and the fundamental state variables of the model. The form factor represents the ratio between the volume of a tree and a cylinder of equal height and diameter at breast height. The volume of pulpwood at harvest i is given by subtracting the quantities of sawlog and wastewood from the total harvest removal:

$$x_2(t_i) = x_0(t_i) - x_1(t_i) - x_3(t_i) \quad (5)$$

where $x_3(t_i)$ is the volume of wastewood, given by

$$x_3(t_i) = g[x_0(t_i), F(t_i), \tilde{G}(t_i^-), H(t_i), N(t_i^-)]$$

The roadside price for timber assortment j is

$$p_j(t_i) = \alpha_j[\tilde{G}(t_i^-), N(t_i^-)]\,\bar{p}_j \quad (6)$$

where $\bar{p}_j$ is the average unit price per cubic meter of timber assortment j, and α_j is the quality premium. The dependent variables, $\tilde{G}$ and N, reflect the radial dimensions (diameter at breast height) of the harvested trees. The quality premium is typically an increasing function of the diameter. Finally, the harvesting costs, C_i, are specified separately for thinning and clear-cutting as

$$C_i(t_i) = g[x_1(t_i) + x_2(t_i), \bar{v}(t_i)] + \bar{c} \quad (7)$$

where $\bar{c}$ is the fixed harvesting cost, i.e. the cost of transporting harvesting equipment to the logging area. The variable cost g also reflects the scale economies of harvesting. It is strictly concave in the average volume of harvested trees ($\bar{v}$) and harvested volume of commercial timber assortments per unit area ($x_1 + x_2$). The justification is that logging and forest hauling time and cost decrease with larger harvested trees and denser stands (Cooney & Haley 1982). The variable harvesting cost is higher for thinning than for clear-cutting because thinning must be done with greater care in order not to damage the residual trees.

The endogenous variables to be optimized for maximizing eq. (1) are the timing of harvesting activities, t_i, $i = 1, \ldots, n$, and the fraction of total timber volume harvested at each thinning, u_i, $i = 1, \ldots, n-1$. The volume harvested at clear-cut equals what is left after the last thinning plus the subsequent growth. The removal rates of thinnings must satisfy the constraints

$$0 \le u_i \le 1, \quad i = 1, \ldots, n-1 \quad (8)$$

and the sequence of the harvesting dates the constraints

$$t_{\min} \le t_1 \le t_2 \cdots \le t_{n-1} \le t_n \quad (9)$$

where $t_{\min} > 0$ is the lower limit for the first commercial thinning. This is set at the age after the chain of exogenous silvicultural activities have been completed and the trees have reached the minimum requirements for marketable timber assortments.

The maximization problem (1)–(9) is continuous in time and space for a specific number of thinnings. The final solution is obtained by selecting the number of thinnings that gives the highest value for J_n.

Solving eq. (1) with exogenous stand establishment costs and sufficiently high rates of interest leads to negative bare-land values and arbitrarily long rotation periods with low final stand density. It is more profitable to thin a stand to the lowest possible density and then to leave the stand idle than to clear-cut and establish the next (unprofitable) rotation period. Therefore, the solution is always computed for the first rotation only, when it is optimal, and it is assumed that the net present value from subsequent land use is zero. This can be interpreted as a switch to a cheaper regeneration method or to a change in land use.

Data and solution technique

The growth and yield data are based primarily on the growth models of Vuokila & Väliaho (1980). Their extensive study probably gives the most comprehensive description presently available for Finnish conifer cultures. The models are reliable for conventional management regimes and stable single-species stands that have passed the juvenile phase. They cover virtually the whole range of suitable site types for Norway spruce and Scots pine cultures. The models consist of growth equations that are estimated from a large number of temporary sample plots scattered throughout Finland and auxiliary equations for stand structure that are based on a separate set of field measurements from permanent sample plots.

In this study, the original discrete-time (5 yr time-step) equations are converted to continuous time. In addition, they are modified to agree with other

Finnish whole-stand growth models outside the reliable domain. Figure 1 shows annual net growth in basal area with bark for different stand ages on a typical Scots pine site index: $H(100) = 24$. The site index, $H(100)$, reflects the average height of dominant trees at the age of 100 yrs. A high index value denotes a fertile site, and a low index value an infertile site. For high basal areas, a mortality function is added that is proportional to the maximum level characterized by the self-thinning equations of Hynynen (1993). Maximum net growth is attained at the culmination point where the stock is about 80% of the maximum (Assmann 1970, p. 230). For low residual basal areas, the growth function is modified to agree with the models of Nyyssönen & Mielikäinen (1978) and Gustavsen (1992) and to give realistic tree-wise radial increments.

The stand-level models of Vuokila & Väliaho (1980) were initially designed for thinning from below. However, optimization reveals that they entail some inconsistencies. They are modified here to describe thinnings that remove equal proportions of all size classes. Noting that the original thinning equations are estimated from separate field material and that diameter distribution is narrow and centred in homogeneous and well-managed conifer cultures, this modification should not bias the results. In addition, the form factor equations of Gustavsen & Fagerström (1983) and the bark conversion equations adapted from a study by Laasasenaho & Melkas (2001) are used. These equations give similar values but are better suited to optimization than the original equations.

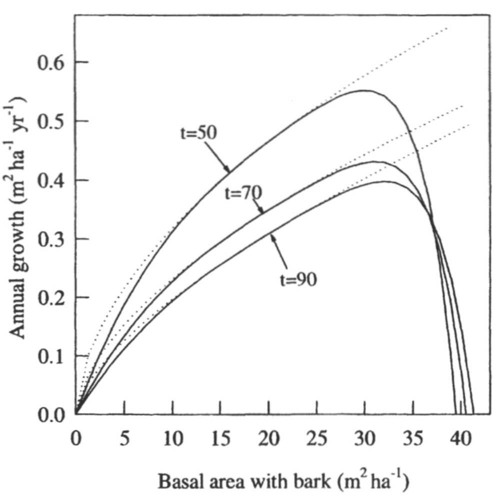

Fig. 1. Annual growth in basal area with bark in a typical Scots pine site, $H(100) = 24$. Solid lines represent modified growth equations and dotted lines the original equations by Vuokila & Väliaho (1980).

The models in Vuokila & Väliaho (1980) assume that all timber that exceeds the minimum dimensions qualifies as sawlog. Some proportion of sawlogs may, however, be disqualified because of quality defects. Sawlog volume adjustment functions are applied that are in line with those used in the Finnish forestry model and management planning tool MELA (Siitonen et al. 1996). The economic data are from Hyytiäinen & Tahvonen (2001). Prices and costs are expressed in real terms. They depend on stand characteristics but remain constant between subsequent rotations. The optimization problems are solved by means of non-linear programming and the Sequential Quadratic Programming algorithms of Matlab software (Coleman et al. 1999). Each calculation is repeated for several randomly and subjectively selected initial guesses to ensure attainment of a global, rather than a local, optimum.

RESULTS

Number of thinnings

Table 1 shows the optimal number of thinnings for different rates of interest and site indices for Norway

Table 1. *Number of thinnings in optimal management regimes for Norway spruce and Scots pine site indices, H(100)*

	Norway spruce					Scots pine					
r	21	24	27	30	33	15	18	21	24	27	30
0.5	2	2	2	3	2	1	2	3	4	6	6
1.0	2	2	2	2	2	2	2	3	3	5	5
1.5	2	2	2	2	2	2	2	2	3	4	4
2.0	2	2	2	2	2	**1**	2	2	2	3	4
2.5	2	2	2	2	1	**1**	**1**	2	2	3	3
3.0	**2**	2	2	2	1	**1**	**1**	2	2	2	3
3.5	**2**	**2**	2	2	2	**1**	**1**	**1**	2	2	2
4.0	**1**	**2**	2	2	2	**0**	**1**	**1**	**2**	2	2
4.5	**1**	**2**	**2**	2	1	**0**	**0**	**1**	**1**	**2**	2
5.0	**1**	**2**	**2**	2	2	**0**	**0**	**1**	**1**	**2**	2
5.5	**1**	**1**	**1**	**2**	2	**0**	**0**	**0**	**1**	**1**	**2**
6.0	**1**	**1**	**1**	**2**	2	**0**	**0**	**0**	**1**	**1**	**2**
6.5	**1**	**1**	**1**	**2**	**2**	**0**	**0**	**0**	**0**	**1**	**1**
7.0	**0**	**1**	**1**	**2**	**2**	**0**	**0**	**0**	**0**	**1**	**1**

Roman: positive bare-land value; bold: negative bare-land value.

spruce and Scots pine. The level of fixed harvesting cost is $\bar{c} = 500$ FIM ha^{-1}. The roman numbers represent the infinite chain of rotation periods ($J^*_{n^*} > 0$) and the bold numbers one clear-cut ($J^*_{n^*} < 0$). The optimal number of thinnings is determined by first solving the problem (1)–(9) with an exogenous number of thinnings ($n = 0, 1, \ldots$) and then selecting the number of thinnings that yields the highest value for the objective function. For example, with a 3% rate of interest and average site quality for Norway spruce [$H(100) = 27$], the maximized bare-land values are $J^*_1 = 3918$, $J^*_2 = 4538$, $J^*_3 = 4568$, $J^*_4 = 4554$, $J^*_5 = 4522$, $J^*_6 = 4490$ FIM ha^{-1}, i.e. it is optimal to thin twice.

The two thinning regime is optimal for most Norway spruce site indices and for most of the rates of interest studied (Table 1). One thinning is optimal for poor sites and high ($r > 4\%$) rates of interest, as well as for the most fertile site at medium rates. In Scots pine cultures, site fertility and rate of interest have much stronger effects on the optimal number of thinnings, which varies between 0 and 6. The number of thinnings increases with improved site quality and decreases with higher rates of interest (except for the most infertile site and 0.5% rate of interest).

Table 2 shows the optimal number of thinnings as a function of the rate of interest and fixed harvesting cost for representative site indices of Norway spruce and Scots pine cultures. The number of thinnings either decreases or remains unchanged in response to increased fixed harvesting cost at any rate of interest and for both species. For a Norway spruce stand, the number of thinnings decreases or remains unchanged in response to an increase in the rate of interest at low ($r < 2.0\%$) and high ($r > 4.0\%$) rates, but increases at certain cost levels at medium rates. For a Scots pine stand, the number of thinnings again reacts much more sensitively to variations in the economic parameters.

Timing of thinnings

During the past decade, postponement of the first commercial thinning has been seen as one of the most serious threats to sustainable forestry in Finland. Postponing the first thinning increases the immediate net harvest revenue. However, excessively high growing density decreases the growth capacity and value of residual trees. According to official silvicultural guidelines (Anon. 1994), the first thinning should be done before the ratio of the length of living crown to total tree height is reduced to less than 60% for Norway spruce stands and less than 40% for Scots pine stands. The recommended moment is when a stand reaches a dominant height of 13–15 m, regardless of species or site type. The recommended initial density is 1600–2000 stems ha^{-1}.

Figure 2 shows the dominant height of trees at first thinning for all Norway spruce and Scots pine site indices as a function of the rate of interest. The stands are thinned late at low and medium rates of

Table 2. *Number of thinnings in optimal management regimes for selected fixed harvesting cost levels*

	Norway spruce $H(100) = 27$						Scots pine $H(100) = 24$					
r	0	500	1000	2000	3000	6000	0	500	1000	2000	3000	6000
0.5	3	2	2	1	1	1	7	4	3	3	2	2
1.0	3	2	2	1	1	1	6	3	3	2	2	1
1.5	3	2	1	1	1	1	5	3	2	2	2	1
2.0	3	2	1	1	1	1	4	2	2	2	1	1
2.5	2	2	2	1	1	1	3	2	1	1	1	1
3.0	3	2	1	1	1	1	3	2	1	1	1	1
3.5	3	2	1	1	1	1	2	2	1	1	1	**0**
4.0	3	2	2	1	**1**	**1**	**2**	**2**	**1**	**1**	**1**	**0**
4.5	**3**	**2**	**2**	**1**	**1**	**1**	**2**	**1**	**1**	**1**	**0**	**0**
5.0	**3**	**2**	**1**	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**	**0**
5.5	**3**	**1**	**1**	**1**	**1**	**0**	**1**	**1**	**1**	**0**	**0**	**0**
6.0	**2**	**1**	**1**	**1**	**1**	**0**	**1**	**1**	**0**	**0**	**0**	**0**
6.5	**2**	**1**	**1**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**
7.0	**2**	**1**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

A maximum of seven thinnings was made available. Roman: positive bare-land value; bold: negative bare-land value.

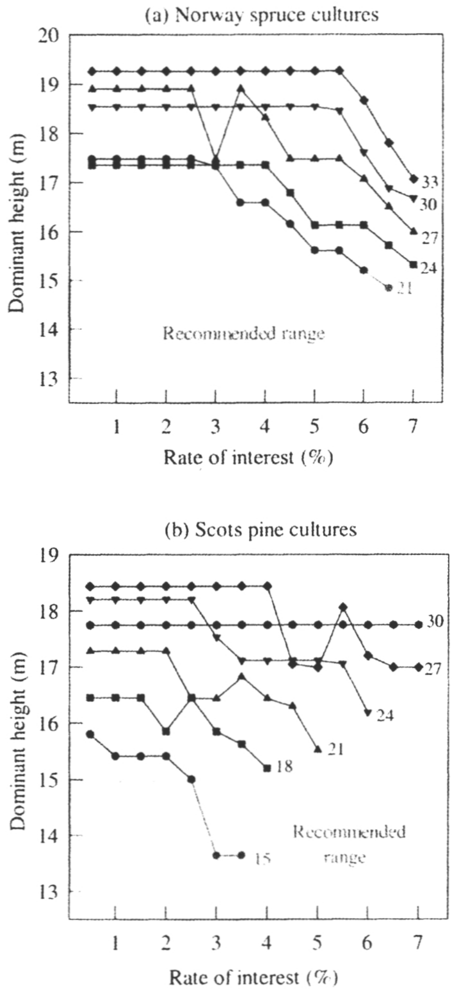

Fig. 2. Dominant height at time of first thinning. Numbers denote site index, i.e. average height of dominant trees at the age of 100 yrs.

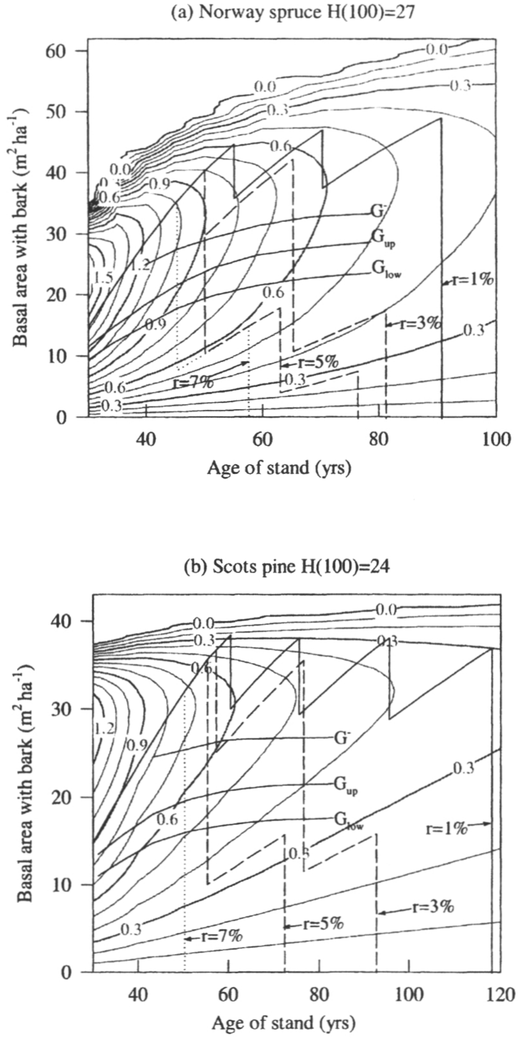

Fig. 3. Development of basal area for optimal management regimes. Iso-growth curves give annual growth of basal area (m^2 ha^{-1}). Recommended basal area before thinning is denoted by G^- and upper and lower limits after thinning by G_{up} and G_{low}.

interest. Compared with the guidelines, the dominant height is typically 4 m higher for Norway spruce sites and 3 m higher for Scots pine sites. Increasing the rate of interest results in an earlier first thinning. The dominant height at first thinning is typically higher on a fertile site than on a poor site.

Figure 3 shows the development of basal area over the rotation period for typical Norway spruce and Scots pines sites for 1, 3, 5 and 7% rates of interest. G^- refers to the recommended basal area before thinning, and G_{up} and G_{low} to the upper and lower limits for the residual stock. The iso-growth curves depict annual growth of the basal area (with bark). The optimal solutions show that the first thinning is always postponed until annual growth of the basal area starts to decline.

Figure 3 also shows how subsequent harvests are distributed over the rotation period. The interval between the last thinning and clear-cut is, in most cases, longer than other intervals. There are also some differences between the two species. For a

Norway spruce stand, increasing the rate of interest shortens the harvest interval. For a Scots pine stand, the harvest intervals remain about the same (15–20 yrs), but the number of thinnings is reduced (see also Tables 1 and 2). In addition, for a Scots pine stand, the interval between the higher number of thinnings increases slightly with age.

If the rate of interest is low ($r = 1\%$), the optimal development of the basal area follows rather closely the level at which annual growth is at a maximum. This holds for both species. Compared with the official guidelines, the optimal basal area is clearly higher. Increasing the rate of interest leads to lower basal areas and growth levels towards the end of the rotation period. The residual basal areas are clearly lower than recommended, especially for Norway spruce.

Thinning intensity

Figure 4 shows the intensity of thinnings as a function of the rate of interest. Intensity of the *i*th thinning is described as the proportion of growing stock, $0 < u_i < 1$, that is removed from the standing timber. For a Norway spruce stand (Fig. 4a), both thinnings are light ($u_i \approx 0.2$) for lower than 1.5% rates of interest. The first thinning remains moderate, but the intensity of the second thinning increases from 0.2 to about 0.7 when the rate of interest changes from 1.5 to 3%. Both thinnings are heavy in the range of 3.5–5.5% rates of interest, leaving only a small proportion of the initial density to grow until the clear-cut. For higher than 5.5% rates of interest, a single, extremely heavy thinning is optimal.

For a Scots pine stand (Fig. 4b), all two to four thinnings are light and about equal in intensity for lower than 2.5% rates of interest. The first thinning remains moderate, but the intensity of the last thinning increases sharply as the rate of interest changes from 2.5% to 3.0%. As the rate changes from 4.0% to 4.5%, the second thinning becomes suboptimal and the intensity of the first thinning increases sharply. For higher than 6.5% rates of interest, the clear-cut is the only harvest.

Rotation period

Figure 5 shows the optimal rotation length as a function of the rate of interest for all Norway spruce (Fig. 5a) and Scots pine (Fig. 5b) site indices. For all site indices and both species, the rotation period becomes shorter when the rate of interest is increased from 1 to 7%. However, the decrease is not monotonical, and for most site indices the length of the optimal rotation may also increase with the interest rate. The rotation period is generally shorter, the more fertile the site. However, at high rates ($r \geq 4\%$), some of the curves cross each other, indicating that a rotation period is shorter on a poor site than on a fertile site.

Figure 6 shows the effects of increased fixed harvesting cost on optimal rotation period at 1, 3, 5 and 7% rates of interest for typical Norway spruce and

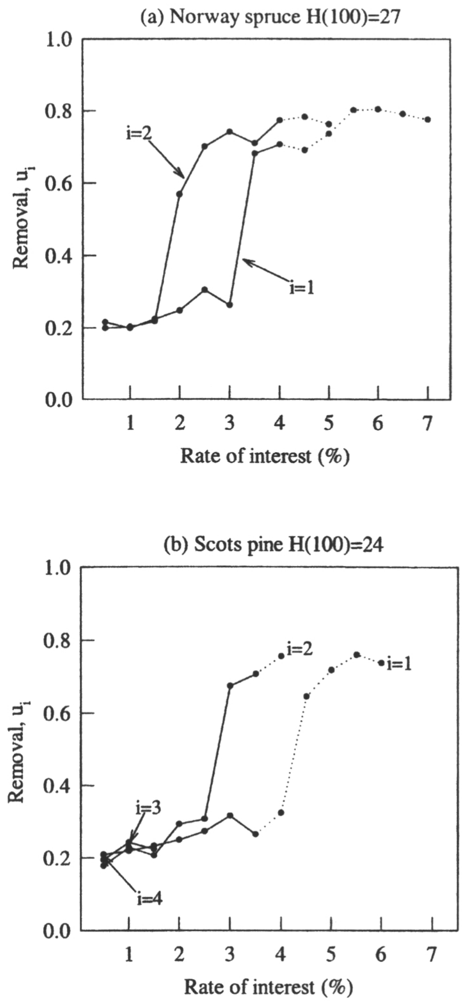

Fig. 4. Optimal thinning intensity as a function of the rate of interest. Solid lines refer to positive and dotted lines to negative bare-land values. The number of the thinning is denoted by *i* and the proportion removed from the growing stock by u_i.

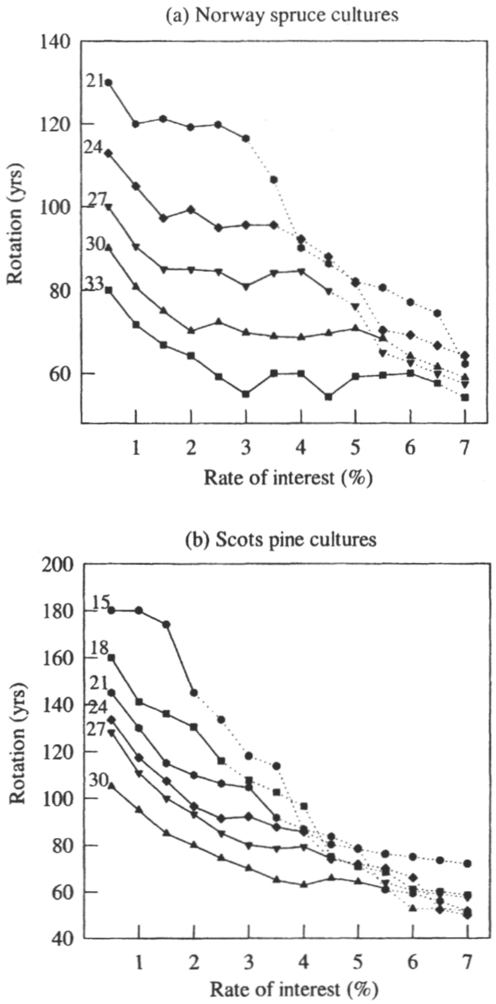

Fig. 5. Optimal rotation length as a function of the rate of interest for Norway spruce and Scots pine sites. Solid lines refer to positive ($J_n^* > 0$) and dotted lines to negative ($J_n^* < 0$) bare-land values. Numbers denote site index, i.e. average height of dominant trees at the age of 100 yrs.

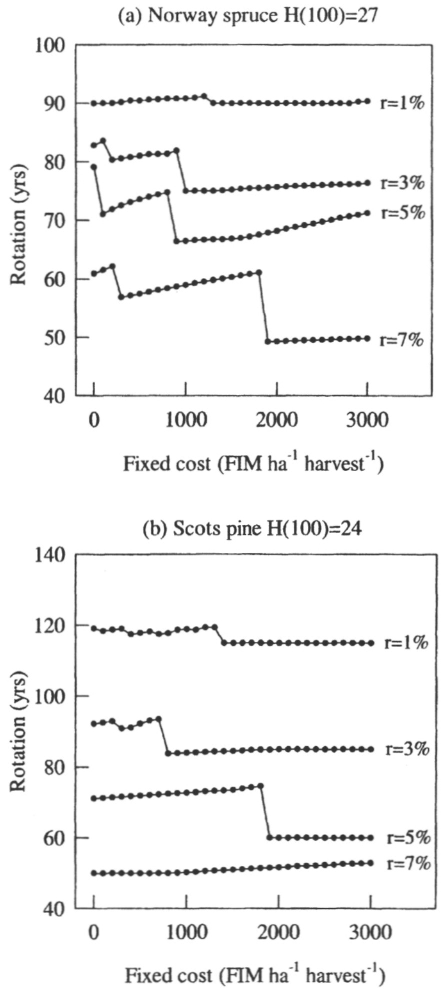

Fig. 6. Effects of fixed harvesting cost on optimal rotation length for 1, 3, 5 and 7% rates of interest.

Scots pine sites. With increasing cost, the optimal rotation length may either increase or decrease. Comparison with Table 2 reveals that downward shifts are always related to decreases in the number of thinnings. The reduction in rotation length is greater, the higher the rate of interest and the smaller the number of initial thinnings. When the number of thinnings remains constant, the optimal rotation increases with cost.

DISCUSSION

Number of thinnings

The optimal number of thinnings is finite (see Tables 1 and 2) because of two characteristics of the harvesting cost function (see explicit specification in the Appendix). First, a fixed harvesting cost decreases the number of thinnings. Secondly, a strictly concave harvesting cost function makes low harvested volumes and hence frequent (or continuous) thinnings suboptimal. If one neglects fixed harvesting cost and linearizes the cost function, increasing the number of thinnings up to seven improves the value of the objective function at any rate of interest and site index and for both species. This is in line with the theoretical model in Clark (1976) and Clark & De Pree (1979), which is characterized by a linear harvesting cost function, zero fixed costs and a singular path solution representing continuous thinning. The criterion of maximizing the sustained yield (MSY) ignores economic factors and thus must yield continuous thinning. This means that forest stands are thinned annually, for example. This is an unrealistic outcome and it is thus possible to argue that in forestry full application of the MSY criterion is impossible.

The specification of harvesting is critical in determining the optimal number of thinnings, as can be observed from Scandinavian studies. Solberg & Haight (1991) use a discrete-time stage-structured growth model for Norway spruce with a linear logging cost function and zero fixed cost. This implies that there exists a sequence of periods with thinning at every time step (Clark 1976, p. 266). Thus, this model cannot reveal the optimal number of thinnings. The same outcome is attained by Wernerheim (1989), who discretizes continuous thinning into 5 yr time steps for Norway spruce stands. Thorsen & Helles (1998) make the strong assumption that thinnings are distributed evenly over the rotation period and find numerous (10) thinnings to be optimal at a 3% rate of interest and fixed harvesting cost of 1000 DKK ha^{-1} for Norway spruce stands in Denmark.

Eriksson (1999) uses a single-tree model and Swedish data for Scots pine stands and demonstrates how the number of thinnings decreases from the highest possible to zero as fixed cost increases from 0 to 3000 SEK ha^{-1}. Three thinnings are optimal at average fixed cost ($\bar{c} = 1000$ SEK ha^{-1}) and 3% rate of interest. Eriksson also shows that the optimal number of thinnings may either increase or decrease when the rate of interest increases from 2.5 to 3.5%. This study gives similar results but, as shown in Table 2, only for Norway spruce.

Despite the importance of thinnings, the existing Finnish studies provide few results concerning the number of thinnings based on optimization without any ad hoc constraints. Like all later Finnish studies, the early study of Kilkki & Väisänen (1969) applies a concave cost function but with zero fixed cost. They report that one to four thinnings are optimal for an initially 50-yr-old Scots pine stand at a 4% rate of interest. The number of thinnings is smaller, the lower the initial density or the higher the harvesting cost and rate of interest.

Valsta (1992) applies individual-tree simulators for fertile Norway spruce cultures. Three thinnings are optimal at 1 and 3% and two thinnings at 5% rates of interest. Pesonen & Hirvelä (1992) simulate management regimes for conifer stands in southern Finland, consider three possible thinning intensities, $u_i \in [0.15, 0.35, 0.50]$ and (being the only previous Finnish study to do so) include fixed harvesting costs (unknown level). They report that on average two or three thinnings is optimal at a 3% rate of interest and only one thinning at a 5% rate of interest, but they do not give detailed results for different sites or species.

Miina (1996), Pukkala et al. (1998*b*) and Vettenranta & Miina (1999) use detailed spatial growth models and a 3% rate of interest. Miina (1996) reports that a single thinning is optimal for a Scots pine stand on an infertile, drained peatland site. In Pukkala et al. (1998*b*) and Vettenranta & Miina (1999), a two-thinning regime is superior to a single-thinning regime in mixed Norway spruce and Scots pine stands of medium fertility, but they do not explicitly report whether additional thinnings were optimal.

The results shown in Tables 1 and 2 are in line with the existing Finnish case study results, but offer new results for a wider range of interest rates, site quality indices and harvesting costs. As an extension to previously presented results, it is shown that the optimal number of thinnings (and rotation period) depends more sensitively on the economic parameters and site quality for Scots pine than for Norway spruce stands. This difference can be explained by a higher quality premium for Scots pine sawlogs and the properties of the growth function. Comparison of the iso-growth curves in Fig. 3 shows that growth decreases more with age for Norway spruce than for Scots pine stands. In addition, thinning leads to a smaller reduction in growth rate for Norway spruce than for Scots pine stands. These factors lead to light and frequent thinnings and long rotation at low rates and a reduced number of thinnings and shorter rotation at high rates of interest for Scots pine.

The recommended thinning regimes (Anon. 1994) are site and species specific, but are not elaborated for different logging or other economic conditions. Depending on the quality of the site, the guidelines lead to two to four thinnings for Norway spruce sites and one to four thinnings for Scots pine sites. The recommended number of thinnings increases with site quality similarly for both species. In the optimal case (see Table 1), in contrast, the number of thinnings is clearly sensitive to site quality only for Scots pine.

Timing of thinnings

The results shown in Figs 2 and 3 indicate that the late first thinning and high growing density at early ages are optimal despite retarded diameter growth and some mortality induced by competition. This is caused by the cost of harvesting, which is strictly concave in the average volume of removed trees. The whole-stand specification does not account for the fact that growth may not recover immediately but may stagnate for some time after thinning of a dense stand (Pukkala et al. 1998*a*, Vettenranta 1999). Accounting for this effect would result in an earlier thinning.

Nevertheless, other optimization studies, using more detailed growth models, also report later first thinnings and higher basal areas before thinning than recommended in the official guidelines (Anon. 1994). According to Valsta (1992), fertile Norway spruce stands are thinned after they have reached 18 m dominant height and 40 m^2 ha^{-1} basal area (over bark) for 1 and 3% rates of interest and 16 m dominant height for a 5% rate of interest. Pesonen & Hirvelä (1992) show that a typical Norway spruce stand is thinned once it has reached 15 m dominant height for 3 and 5% rates of interest. Dense Norway spruce and Scots pine mixtures (about 2000 trees ha^{-1}) of medium fertility are thinned for the first time at ages representing 14–17 m dominant height and 36–42 m^2 ha^{-1} basal area (over bark) for a 3% rate of interest in Pukkala et al. (1998*b*) and Vettenranta & Miina (1999).

The difference implies that either the optimization studies neglect some important benefits of early thinnings or the recommended first thinning comes too early in well-managed conifer stands. A growth and yield study by Hynynen & Arola (1999) investigates the yield and vigour of the residual stock after thinning from below in Scots pine stands of various fertility. They do not give full economic analyses, but conclude that postponing the first thinning until 17.7 m dominant height is not likely to jeopardize future harvesting revenues, provided silvicultural operations have been done on time. Even though the dominant height of the trees clearly exceeds the official recommendations, the minimum criterion for the living crown (40% out of total stem height) is still fulfilled.

Thinning costs exceed clear-cutting costs. This implies that the interval between the clear-cut and the last thinning is longer than the average harvest interval (Clark 1976, p. 269). Similarly, a higher price premium and slower reduction of growth with stand age for Scots pine stands imply that the thinning interval increases with age.

Thinning intensity

Increasing the rate of interest first increases the intensity of the last thinning and then, in reverse order, the intensity of the earlier thinnings (shown in Fig. 4). This can be explained by the characteristics of the growth function (see Figs 1 and 3). The slope with respect to the basal area decreases with age. Hence, heavy thinning leads to a smaller marginal decrease in growth for an old than for a young stand.

Thinning intensity increases sharply when the rate of interest increases above a critical level (see Fig. 4). This is caused by the almost linear shape of the growth function at average basal areas (see Fig. 1). Once the critical rate is exceeded, a small increase in the rate of interest triggers high thinning intensity and low residual basal area.

Thinning removal is not recommended to exceed one-third of the growing stock (Anon. 1994). However, results from the present and other recent optimization studies typically report higher intensities for medium rates of interest. For a typical Norway spruce site and 3% rate of interest, the intensity of the first thinning is 0.26 and that of the second thinning 0.74 (shown in Fig. 4). For Scots pine, the intensity of the first thinning is 0.32 and that of the second thinning 0.67. In spatial studies by Pukkala et al. (1998*b*) and Vettenranta & Miina (1999), the intensities of the optimal two-thinning regime vary from 0.37 to 0.60 for the first thinning and from 0.41 to 0.55 for the second thinning. The intensity of the optimal single-thinning regime is higher and varies from 0.54 to 0.67 (Pukkala & Miina 1997, Pukkala et al. 1998*b*, Vettenranta & Miina 1999). In a study by Valsta (1992), three thinnings and moderate intensities ($0.2 \leq u_i \leq 0.4$) are optimal when the growth models of Ojansuu et al. (1991) are used. The intensity gradually increases with age from first thinning ($u_1 = 0.35$) to last thinning ($u_3 = 0.58$) when the models of Mielikäinen (1985) are used.

Lohmander & Helles (1987) show that for Norway spruce stands in Denmark the probability of windthrow increases with thinning intensity. The risk of destruction is highest immediately after the harvest and declines quickly as residual trees adapt to new conditions. Thorsen & Helles (1998) illustrate that recognition of this risk increases the number of thinnings and decreases thinning intensity. Conversely, recognition of felling and forest hauling damage is likely to decrease the number of thinnings and increase thinning intensity, especially for Norway spruce stands, which are vulnerable to root damage. Clearly, both natural hazards (snow and wind damages) and logging damage are relevant risks for Finnish forests. These factors are ignored in this study and other Finnish optimization studies owing to the lack of applicable models in which the probability of natural hazards or logging damage depends on logging.

Rotation period

The negative relationship between the optimal rotation and the rate of interest is one of the most established results in forest economics (see e.g. Johansson & Löfgren 1985, Montgomery & Adams 1995). The present results (see Fig. 5) suggest that this result may not be robust when the number, timing and intensity of thinnings are determined endogenously. The length of the rotation period may increase when there is a simultaneous increase in the number of thinnings or an increase in thinning intensity.

Kao & Brodie (1980) have shown numerically for Douglas fir and Valsta (1992) for Norway spruce that increasing the exogenously determined number of thinnings extends the rotation period. Additional thinning improves the relative value growth of the residual crop. The present results imply that the number of thinnings generally decreases with increases in the rate of interest. This affects negatively the rotation length. The wider variation in the number of thinnings explains why the rotation length decreases more sensitively with increases in the rate of interest for Scots pine sites than for Norway spruce sites.

However, the results for Norway spruce and the results of Eriksson (1999) for Scots pine stands in Sweden suggest that the number of thinnings may occasionally increase with the rate of interest at medium rates. This typically leads to a longer rotation period. Sharply increased thinning intensity has the same effect as an increase in the number of thinnings. With increased intensity and number of thinnings, the stand volume is lower before the clearcut and the relative value growth is higher, implying a positive effect on rotation length.

Heaps (1981) shows that improvement in growth conditions, e.g. soil type, decreases the optimal rotation length when thinnings are ignored. Similarly, Löfgren (1985) demonstrates that once-and-for-all biotechnological improvements that result in increased growth reduce the optimal rotation length. Löfgren refers to tree species or genetic planting material, but this result applies to site fertility as well. In general, the present numerical results (see Fig. 5) agree with these theoretical results. However, at high rates ($r \geq 4.0\%$), the rotation periods are occasionally shorter on a poor site than on a fertile site. This unexpected result is caused by the smaller number of thinnings, which shortens the rotation period. The lower the number of thinnings initially, the greater the shortening. The negative effect of shifting, for example, from one to zero thinnings outweighs the opposite effect of improved site quality on optimal rotation.

The official recommendations for rotation ages in Finland are specific for species and vegetation site type (Anon. 1994). Using the conversions between dominant height and vegetation site classification by Vuokila & Väliaho (1980, p. 26), the results shown in Fig. 5 comply with the recommendations for 1–4% rates of interest for Norway spruce and for 2–3% rates of interest for Scots pine cultures. For lower rates, the rotations are typically longer, and for high rates shorter than recommended.

In this study, the optimal rotation periods were about 5–20 yrs longer than in other Finnish optimization studies (Valsta 1992, Salminen 1993, Pesonen & Hirvelä 1992, Pukkala et al. 1998*b*, Vettenranta & Miina 1999, Hyytiäinen & Tahvonen 2001). For both species, the deviations are greatest for medium (3–4%) rates of interest and the poorest sites. This is due to the quality premium function and (somewhat controversial) sawlog volume adjustment function used here, as well as the absence of ad hoc constraints on thinnings. The quality premium was previously considered only by Hyytiäinen & Tahvonen (2001) and sawlog adjustment due to quality defects by Pesonen & Hirvelä (1992). In this study, neither the number of thinnings nor their intensity is limited, in contrast to Pukkala et al. (1998*b*), Vettenranta & Miina (1999), Pesonen & Hirvelä (1992) and Hyytiäinen & Tahvonen (2001). The rotation periods are about 0–10 yrs shorter when the sawlog adjustment is removed and about 10 yrs shorter when thinning intensity is limited by $u_i \leq 0.35$ and this constraint is binding.

Increasing the cost reduces the value of the bare land and the growing stock. This results in a positive effect on rotation length. Heaps (1981) shows that an increase in transport cost (in other words, in fixed harvesting cost) lengthens the rotation, when thinnings are ignored. Figure 6 shows that this result holds as long as the number of thinnings remains constant. However, a fixed harvesting cost reduces the number of thinnings. Once the number of thinnings decreases, its negative effect is likely to outweigh the positive effect of increased fixed harvesting cost on rotation period. Eriksson (1999) reports an identical result.

Conclusions

These results imply that the optimal management regime is highly sensitive to the level and structure of harvesting costs. Therefore, the location and size of the uniform forest management units or logging areas affect harvesting. Optimal rotation periods are shorter and thinnings are less frequent but heavier in small and scattered management units than in large and easily accessible units.

The results entail three deviations from the present understanding of how rotation period depends on economic factors and site quality. First, the optimal rotation length does not decrease monotonically with the rate of interest. Secondly, an improvement in site fertility does not always decrease the optimal rotation length. Thirdly, an increase in costs, if directed to fixed harvesting cost, is likely to decrease rather than increase the optimal rotation length. Analogous deviations are reported by Jackson (1980) and Chang (1983). They show theoretically that the rotation length may increase as a consequence of an increase in the rate of interest, with endogenous planting density.

According to the results, variation in the economic parameters, with exogenous stand establishment cost, leads to negative land values. This implies a need for models where stand establishment is endogenous and continuous and which can take into account different methods of regeneration (e.g. natural regeneration).

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APPENDIX A

Dominant height

Average height of dominant trees, $H(t)$, is independent of stand management, and is specified as: $H(t) = \gamma_1 - \gamma_2 e^{-\gamma_3 t}$. Expressed in simpler form, these height curves follow very closely the original formulations of Vuokila & Väliaho (1980). Parameter values are as follows:

H (100)	Norway spruce γ_1	γ_2	γ_3	Scots pine γ_1	γ_2	γ_3
33	38.012	45.581	0.02221			
30	35.035	43.403	0.02150	34.451	40.271	0.02200
27	32.487	40.993	0.02008	31.634	37.406	0.02084
24	30.030	38.495	0.01851	28.870	34.501	0.01957
21	27.719	35.924	0.01673	26.140	31.523	0.01813
18				23.480	28.492	0.01647
15				20.974	25.481	0.01450

Basal area

Increase in basal area, ΔG, is specified for Norway spruce by

$$\Delta G = \nabla t G[-0.00099776 + 32.567 H^{-0.5} g^{-0.6} t^{-0.9} - 317.36(H^{-0.5} g^{-0.6} t^{-0.9})^2]$$

and for Scots pine by

$$\Delta G = \nabla t G[-0.0089537 + 0.000019509 H(100)^2 + 7.3026 H^{-0.3} g^{-0.4} t^{-\delta} + 3637.8(H^{-0.3} g^{-0.4} t^{-\delta})^3]$$

where $\delta = 0.8 + H(100)/175$ and $g = G$ for $G \geq G'$. The level of G' is 17 for Norway spruce and 12 for Scots pine. For low basal areas ($G < G'$) the original equations of Vuokila & Väliaho (1980) were modified by setting $g = G + 3[1 - \sin(0.5\pi G/G')]$. The cumulative growth between subsequent harvests, $[t_i, t_{i+1}]$ for $i = 1, \dots, n-1$, is computed first for the maximum number of 5 yr periods: $t_i + \nabla t y \leq t_{i+1} \mid \nabla t = 5$, where y is an integer. The last time step for the period from t_i to t_{i+1} is $\Delta t = t_{i+1} - 5y - t_i \leq 5$.

The basal area over the bark, $\tilde{G}$, is calculated from $\tilde{G} = N\pi(0.005\tilde{D})^2$, where $\tilde{D}$ is average diameter (cm) at breast height over the bark and is given by $\tilde{D} = \exp\{\gamma_1 + \gamma_2 \ln[200G^{0.5}(\pi N)^{-0.5}]\}$. The parameters are derived from a study by Laasasenaho & Melkas (2001): $\gamma_1 = 0.1397$ and $\gamma_2 = 0.9762$ for Norway spruce, and $\gamma_1 = 0.09796$ and $\gamma_2 = 1.0034$ for Scots pine.

Mortality

Mortality is specified according to the self-thinning equations of Hynynen (1993): $N^{\max} = \exp\{\gamma_1 + \gamma_2 \ln[H(100)] + \gamma_3 \ln(\tilde{D})\}$, where $N^{\max}$ represents the site-specific maximum number of stems. The parameter values for Norway spruce are $\gamma_1 = 10.059$, $\gamma_2 = 0.446$, $\gamma_3 = -1.354$; and for Scots pine $\gamma_1 = 11.213$, $\gamma_2 = 0.489$, $\gamma_3 = -1.855$. Mortality is proportional to $N^{\max}$ and is specified as $m = N[\Delta G(N/N^{\max})^{12}]/[\Delta G(N/N^{\max})^{12} + G]$.

Harvested volumes

The total volume from harvest i is given by $x_o(t_i) = u_i F(t_i)\tilde{G}(t_i^-)H(t_i)$, where u_i is the fraction harvested. The equations for the form factor, F, are from Gustavsen & Fagerström (1983): for Norway spruce, $F = 0.42844 - 0.04526[3.13 - \ln(\tilde{d})]^{1.3}$, where $\tilde{d} = \tilde{D} \mid \tilde{D} \le 22.8$ and $\tilde{d} = 22.8 | \tilde{D} > 22.8$, and for Scots pine, $F = 0.24595 + 0.06366[3.23 - \ln(h)]^{2.5} + 0.07067 \ln(\tilde{D})$, where $h = H \mid H = 25.2$ and $h = 25.2 \mid H > 25.2$.

Sawlog volume for Norway spruce is given as $x_1(t_i) = 0.95x_0(t_i)[1 - \exp(1.254 - 4492\bar{v} - 0.1081H + 0.002757H^2)]\omega(t_i)$, where $\bar{v}$ is the average volume of the stem in m^3, $\bar{v}(t_i) = F(t_i)\tilde{G}(t_i^-)H(t_i)N(t_i^-)^{-1}$, and $\omega(t_i)$ represents the proportion of sawlog removal that fulfils both the dimension and quality requirements for commercial sawlogs. It is specified as $\omega(t_i) = -0.00004286\tilde{D}^2 + 0.008036\tilde{D} + 0.6211$. Wastewood removal from harvest i is given by $x_3(t_i) = 0.01x_0[-0.1045 + 306.394/(1000\bar{v} - 3.76)]$. The corresponding functions for Scots pine are $x_1(t_i) = 0.96x_0(t_i)[1 - \exp(0.4249 - 5129\bar{v} - 0.003751t_i)]\omega(t_i)$, $\omega(t_i) = 0.0001145\tilde{D}^3 - 0.01200\tilde{D}^2 + 0.4081\tilde{D} - 3.634$ for $\tilde{D} < 40$ and $x_3(t_i) = x_0[-0.001386 + 0.003126(0.8333\bar{v} - 0.008183)^{-0.9}]$.

Economic data

The proportional profit tax rate is $\tau = 0.29$. The annual administrative cost is $T = 25$ FIM ha^{-1}. The average roadside prices are $\bar{p}_1 = 257.10$ and $\bar{p}_2 = 175.34$ FIM (m^3)$^{-1}$ for Norway spruce, and $\bar{p}_1 = 314.97$ and $\bar{p}_2 = 156.02$ FIM (m^3)$^{-1}$ for Scots pine sawlogs and pulpwood, respectively. The quality premium multipliers are $\alpha_1 = 0.7107 + 0.1212 \ln(0.3990\tilde{D})$ for Norway spruce sawlogs, $\alpha_2 = 0.9176 + 0.1996 \ln(0.07652\tilde{D})$ for Norway spruce pulpwood, $\alpha_1 = 0.7589 + 0.008929\tilde{D}$ for Scots pine sawlogs and $\alpha_2 = -0.00002482\tilde{D}^3 + 0.001632\tilde{D}^2 - 0.0167\tilde{D} + 0.8573$ for Scots pine pulpwood. The harvesting costs are specified as

$$C_i = \overbrace{(x_1 + x_2)}^{\text{removal}}\left\{\gamma_1 + \overbrace{\gamma_2 \ln[\gamma_3(x_1 + x_2)]}^{\text{removal ha}^{-1}} + \overbrace{\gamma_4 \ln(\gamma_5\bar{v}) + \gamma_6\bar{v}^{-1}}^{\text{average volume of removed tree}}\right\} + \bar{c}$$

The fixed harvesting cost, $\bar{c} = 500$ FIM ha^{-1}, is the same for both thinning and clear-cutting. The variable cost is found by multiplying the removed volume of usable timber assortments $(x_1 + x_2)$ by the unit variable cost, $\{\bullet\}$, which is specified as a function of harvested volume per area unit and average volume of removed trees. The parameters for thinning $(i < n)$ and clear-cut $(i = n)$ harvests are:

	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
$i<n$	46.49	−4.005	0.008908	−4.634	42.2	2.622
$i=n$	48.21	−1.602	0.004259	−2.408	9579	2.642

for $C_i < 0.975C_n \mid i < n$. The cost parameters for stand establishment and silvicultural activities are presented in Hyytiäinen & Tahvonen (2001).

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Paper III

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III

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Maximum Sustained Yield, Forest Rent or Faustmann: Does it Really Matter?

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The efficiency gains when harvests are determined using the Faustmann approach instead of Maximum Sustained Yield (MSY), Forest Rent or some silvicultural rules were studied. The analysis was based on a variable-density stand growth model and computations over all admissible combinations of initial stand age and basal area for Finnish Norway spruce and Scots pine sites. Following MSY or Forest Rent led to a major reduction in the economic value of especially highly stocked stands. Finnish silvicultural recommendations (an example of silvicultural rules) encouraged production of higher than optimal quality timber. Applying Forest Rent, MSY or silvicultural recommendations led to respective reductions of 63%, 30% and 13% in the value of a typical conifer forest at a 4% rate of interest. The results also showed that an increase in the rate of interest may lengthen the ongoing rotation if the initial state falls outside the optimal path initiated at bare land. *Key words: Faustmann rotation, forest policy, value of forest land, understocked forest stand.*

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INTRODUCTION

The debate as to the appropriate criterion for determining optimal forest rotation is an old one (Faustmann 1849). Various approaches to determine optimal rotation are compared in widely cited studies by Gaffney (1957) and Samuelson (1976). The same theme is continued in a well-known book by Johansson & Löfgren (1985). After these studies, the superiority of the Faustmann (1849) approach over other approaches based on, for example, the Maximum Sustained Yield (MSY) or the "Forest Rent" doctrines has become perhaps the most established result in forest economics. However, despite a vast amount of theoretical research based on the Faustmann approach it is more or less an open question as to how large are the efficiency gains from determining the harvests using the Faustmann formula instead of the other criteria.

According to Judd (1999, p. 7), the economic literature includes numerous optimization models that may offer only negligible efficiency gains compared with simple "rules of thumb" or models requiring less information. Thus, if following the MSY criterion (i.e. maximizing the mean annual increment), for example, typically causes only minor economic losses, it can be claimed that the Faustmann approach, although theoretically correct, is less important from the point of view of practical forest management.

Recently, Möhring (2001) wrote that, although the land rent theory was originally developed in Germany, practical forestry is nowadays guided by silvicultural and ecological models that aim to guarantee sustainability instead of economic efficiency. Where economic reasoning has been applied, it has been based on the aim of maximizing the average annual net income, i.e. the "Forest Rent". This corresponds to the Faustmann model with zero rate of interest. Since the Forest Rent typically leads to longer rotation periods and higher stocking levels, it has been considered to support sustainability and multiple use of forest resources better. Such arguments, however, neglect the possibility of incorporating non-timber benefits explicitly in forest economic models, as has been done in many studies since Hartman (1976).

The current understanding of forestry experts and many forest scientists is that using MSY or Forest Rent as a policy guideline will yield an economic outcome that is close enough to optimal outcome. In Finland, this view is prevalent in many forest policy and extension documents. Examples of these are official silvicultural guidelines (Tapio 2001) and an influential series of manuals for private forest owners published since 1907 (see e.g. Tapio 1997).

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The aim of this study was to produce a systematic picture of the magnitude of economic loss if MSY, Forest Rent or some easy-to-follow silvicultural rule is applied, instead of an economic criterion, to determine harvests in even-aged forest stands. Current knowledge of this issue is rather limited and somewhat narrow in scope. Hann et al. (1983), Haight & Monserud (1990), Solberg & Haight (1991) and Valsta (1990, 1992) report that some deviations in a single decision variable, such as rotation length, may yield only a minor reduction in bare land value, particularly if the remaining endogenous variables are free to adjust. In contrast, Riitters et al. (1982) show that following MSY or Forest Rent instead of the Faustmann approach leads to a large reduction in maximized bare land value (25% and 37%, respectively) in Douglas fir [*Pseudotsuga menziesii* (Mirb.) Franco] stands. As examples of silvicultural rules of thumb, the Finnish silvicultural recommendations have been shown to lead to 20–30% economic losses in fertile Norway spruce stands (Valsta 1992) and Norway spruce and Scots pine mixtures (Vettenranta & Miina 1999).

All of the above-mentioned results consider the value of bare land. There are considerably fewer results on the value of forested land for initial states along the optimal path and, in particular, outside the optimal path starting from bare land. An early study by Kilkki & Väisänen (1969) reports that the biggest losses from deviating from the optimal path are incurred through overdensity. Lämås et al. (1996) use Swedish sample plot data for mature stands and show that the loss from extended rotation increases with initial volume and stand age. For example, lengthening the rotation from 95 to 110 yrs in a typical Scots pine site (with initial volume of 250 $m^3 ha^{-1}$) leads to a loss of about 550 € ha^{-1} at 3% rate of interest, which is about 13% of the maximized stand value.

In this study, the economic value of the two major Scandinavian timber species, Norway spruce [*Picea abies* (L.) Karst.] and Scots pine [*Pinus sylvestris* L.), is investigated. The results are presented separately for bare land and a large number of initial states in forested stands. For each initial state, the maximized forest values are compared with the present value of net forest income obtained if the harvests are determined by applying MSY, Forest Rent or harvesting rules stated in the Finnish silvicultural recommendations. One contribution of this paper, compared with earlier studies, is that it presents a systematic picture of the economic value of these non-optimal solutions for forested land covering all admissible combinations of initial stand age and basal area.

The results were computed using a modified version of the variable-density stand growth model presented in Hyytiäinen & Tahvonen (2002). This earlier study focused on comparative static analysis of a stand-level optimization model. The present study focuses on examining the efficiency gains that can be obtained by following economic principles in forestry.

MATERIALS AND METHODS

Stand growth model

The state variables of the variable-density stand growth model are the basal area at breast height without bark, $G(t)$, the age of the stand, t, and the number of trees per hectare, $N(t)$. Basal area reflects stock size and is expressed as the total cross-sectional area of the trees measured at breast height ($m^2 ha^{-1}$). $N(t)$ reflects the density of the growing stock. Let index i be the number of harvest during the rotation period, with $i = 1, \ldots, n-1$ for thinnings and $i = n$ for clear-cutting. The basal area after thinning i at stand age t_i can be expressed as

$$G(t_i) = \{G(t_{i-1}) + \Delta G[H(t_{i-1}),\ G(t_{i-1}),\ t_{i-1}, t_i]\} \times \left\{1 - \frac{m[N(t_{i-1}), \Delta G(t_i), G(t_{i-1})]}{N(t_{i-1})}\right\}(1 - u_i) \quad (1)$$

where $G(t_0) = G_0$ is the given initial basal area and ΔG is the growth of basal area during the period from t_{i-1} to t_i. $H(t_i)$ is the average height of dominant trees at age t_i and m is the number of trees that die during the period from t_{i-1} to t. At any harvest, a proportion u_i of the growing stock is removed. The number of trees after the ith harvest is given by

$$N(t_i) = \{N(t_{i-1}) - m[N(t_{i-1}),\ \Delta G(t_i),\ G(t_{i-1})]\} \times (1 - u_i) \quad (2)$$

where $N(t_0) = N_0$ is the given initial number of trees. In eqs (1) and (2) tree mortality is expressed as a function of the number of trees and basal area variables reflecting the relative stand density with respect to the maximum level. The harvested proportion, u_i, is the same for the number of trees and basal area, implying that thinning removes an equal proportion of trees of each size class.

The total harvested volume from thinnings and clear-cutting, x_0, is partitioned into sawlogs, x_1, pulpwood, x_2, and wastewood, x_3. The sawlog volume from harvesting a stand of age t_i is

$$x_1(t_i) = f\{x_0(t_i),\ F(t_i),\ H(t_i),\ \tilde{G}(t_i^-),\ N(t_i^-)\} \tag{3}$$

where $F(t_i)$ is the form factor of the trees and $\tilde{G}(t_i^-)$ the basal area, including bark, before thinning. The form factor defines the ratio between the volume of a tree and the volume of a cylinder that has the same height and diameter at breast height (Assmann 1970, pp. 64 ff.). t_i^- denotes the left-hand limit of t_i. The volume of pulpwood at harvest i is found by subtracting the quantities of sawlog and wastewood from the total harvest removal:

$$x_2(t_i) = x_0(t_i) - x_1(t_i) - x_3(t_i) \tag{4}$$

where $x_3(t_i)$ is the volume of wastewood, given by

$$x_3(t_i) = g[x_0(t_i),\ F(t_i),\ H(t_i),\ \tilde{G}(t_i^-),\ N(t_i^-)].$$

The roadside price for timber assortment j is

$$p_j(t_i) = \alpha_j[\tilde{G}(t_i^-),\ N(t_i^-)]\bar{p}_j \tag{5}$$

where $\bar{p}_j$ is the average unit price per cubic metre of timber assortment j, and α_j is the price premium. The average unit prices $\bar{p}_j$ are market determined (for details see Hyytiäinen & Tahvonen 2001) and exogenous to sellers. The dependent variables of the price premium equation, $\tilde{G}$ and N, reflect the radial dimension (diameter at breast height) of harvested trees. Roadside prices typically increase with tree diameter. Finally, the harvesting costs, C_i, are specified separately for thinnings and clear-cutting as

$$C_i(t_i) = g[x_1(t_i) + x_2(t_i),\ v(t_i)] + \bar{c} \tag{6}$$

where $\bar{c}$ is the fixed harvesting cost, i.e. the cost of transporting harvesting equipment to the logging area. The variable cost function also reflects the scale economies of harvesting. It is strictly concave in average volume of harvested trees (v) and harvested volume of commercial assortments ($x_1 + x_2$) per unit area. All of the exact site index and species-specific parameter values for eqs (1)–(6) are presented in Hyytiäinen & Tahvonen (2002).

The removal rates of thinnings must satisfy the constraints

$$0 \leq u_i \leq 0.45, \quad i = 1, \ldots, n-1 \tag{7}$$

and the sequence of harvesting dates the constraints

$$t_{\min} \leq t_1 \leq t_2, \ldots, \leq t_{n-1} \leq t_n \tag{8}$$

where $t_{\min} > 0$ is the lower limit for the first commercial thinning. This is set at the age after the chain of exogenous silvicultural activities has been completed and the trees have reached the minimum requirements for merchantable timber assortments.

Lohmander & Helles (1987) show that heavy and frequent thinnings increase the probability of windthrow. Unfortunately, there are no Finnish models that would account for the effects of logging on the probability of windthrow at stand level. In the present study, this risk is taken into account by setting a 0.45 upper limit for thinning intensity in (7). This constraint is based on numerous discussions, expert opinions and studies by, for example, Peltola et al. (1999) and Laiho (1987). It was also found that allowing higher thinning intensities would only slightly increase the maximized value of the objective functions.

Criteria for thinning and clear-cutting

The objective function for maximizing the value of bare land, J_n, under $n-1$ thinnings is

$$J_n = (1-\tau) \left\{ \frac{-\sum_{k=1}^{m} w_k e^{-rt_k} + \sum_{i=1}^{n}\left[\sum_{j=1}^{2} p_j(t_i)x_j(t_i) - C_i(t_i)\right] e^{-rt_i}}{(1-e^{-rt_n})} - \frac{T}{r} \right\} \tag{9}$$

where the parameter w_k, $k = 1, \ldots, m$, is the cost of the kth silvicultural activity and m is the number of silvicultural activities. The cost and number of silvicultural activities are exogenously given, and depend on species and site index. The real rate of interest is denoted by r. T is the (exogenous) level of annual administrative costs per hectare, consisting of silvicultural fees and other fixed costs. The rate of the proportional profit tax is denoted by τ.

The problem is to maximize the discounted post-tax net revenues from an infinite chain of rotation periods (9), subject to feasibility constraints (7) and (8) and given the relationships defined by eqs (1)–(6). Equations for stand dynamics (1) and (2) are used to simulate stand development over age. Equations for harvested volumes (3) and (4), prices (5) and harvesting cost (6) are needed to compute the net harvest revenues. The endogenous variables are the timing of harvesting activities, t_i, $i = 1, \ldots, n$, and the fraction of total timber stock harvested at each thinning, u_i, i = 1, …, $n-1$. The problem is solved first for different numbers of thinnings, $n = 0, 1, \ldots$. The final solution is obtained by selecting the number of thinnings that generates the highest value for bare land.

Plantation forestry yields negative bare land values for high rates of interest because of high stand establishment costs. In such cases, a rational forest owner applies less costly stand establishment methods (e.g. natural regeneration) or converts the stand to other, more profitable, land use. It is assumed that in such cases the value of subsequent land use is zero.

The objective function for maximizing the value of a forested stand, J_n^f, with any initial state (t_0, G_0, N_0) is

$$J_n^f = (1-\tau)\left\{\sum_{i=1}^{n}\left[\sum_{j=1}^{2} p_j(t_i)x_j(t_i) - C_i(t_i)\right] \times e^{-r(t_i-t_0)} - T\frac{e^{r(t_i-t_0)}-1}{re^{r(t_i-t_0)}}\right\} + J^* e^{-r(t_n-t_0)} \tag{10}$$

where $t_0 > 0$ and J^* denotes the maximized bare land value from eq. (9). It is assumed that harvests that maximize bare land value are perpetuated indefinitely after clear-cutting of the first timber stock. Note that the optimal harvests for the first (ongoing) rotation period are often different from the optimal harvests of all the following rotation periods (see Faustmann 1849, Getz & Haight 1989, pp. 263 ff.).

Forest Rent is an old forestry doctrine that is still predominant, for example in German forestry (see Möhring 2001). It deviates from the economic criterion, since the rate of interest is assumed to be zero. For bare land, the objective function for Forest Rent is

$$FR_n = (1-\tau) \times \left[-\sum_{k=1}^{m} w_k + \sum_{i=1}^{n}\left(\sum_{j=1}^{2} p_j(t_i)x_j(t_i) - C_i(t_i)\right) - Tt_n\right] \times (t_n)^{-1} \tag{11}$$

When $r \to 0^+$, the bare land solution obtained with the economic criterion (9) is the same as the bare land solution obtained with the Forest Rent criterion (11) as shown, for example, by Clark (1976, p. 272). In the present study, this principle is followed when computing the Forest Rent solutions for forested land. Thus, if the initial state is other than bare land, the Forest Rent solution is computed by using (10) and a negligibly low rate of interest.

The origin of the MSY criterion can be traced to mid-eighteenth century Germany. The commitment to maximum even flows of timber volume followed from closed local economies, limited timber markets and fear of timber famine, according to Hyde (1980, p. 197). However, despite the changed economic environment, MSY has been prevalent in forest policies in many countries. For example, the US Forest Service has been reported to apply MSY when choosing rotation ages for its commercial timber production areas (Gaffney 1957, p. 62, Klemperer 1996, p. 221).

MSY is used to maximize the long-term production of usable timber assortments. For bare land, the optimality criterion reduces to

$$\mathrm{MSY} = \left(\sum_{i=1}^{n}\sum_{j=1}^{2} x_j(t_i)\right)(t_n)^{-1} \tag{12}$$

For forested stands, the MSY solution is computed using eq. (10) with zero harvesting, administration and stand establishment costs. A negligibly low rate of interest is used to approximate the zero cost of capital. The prices for both commercial timber assortments are set equal to unity.

The MSY criterion leads to an infinite number of thinnings or continuous thinning over a certain period, as shown by Clark & De Pree (1979) and Hyytiäinen & Tahvonen (2002). However, owing to positive fixed harvesting cost, an infinite number of thinnings would lead to infinitely large negative forest values for stands that are managed according to the MSY solutions. When computing the MSY solutions, the number of thinnings is thus limited to six ($n = 7$). The justification is that the gain in yield from additional thinnings, though non-zero, is negligible.

As examples of silvicultural rules of thumb, the economic efficiency of the Finnish silvicultural recommendations given by the government-funded forestry extension organization Tapio (2001) were assessed. The silvicultural recommendations are not legally binding, but are widely followed by private forest owners in Finland. The silvicultural recommendations give explicit guidelines for stand establishment, silvicultural activities and harvesting; define the basal area that triggers thinnings; and specify upper and lower bounds for residual basal area after thinning. The recommended thinning intensity is between 0.2 and 0.33. A forest stand may be clear-cut once either the average tree diameter or the stand age has reached its recommended range.

Data and computations

The data are explained in detail in Hyytiäinen & Tahvonen (2002) and are only briefly summarized here. For growth and yield equations (1)–(4) the data are primarily adapted from the study by Vuokila & Väliaho (1980). Their models cover virtually all relevant Finnish Norway spruce and Scots pine site indexes. The models are based on extensive fieldwork. They describe growth for conventional stand management and stable single-species stands that have passed the juvenile phase.

For optimization purposes, the equations of Vuokila & Väliaho (1980) were complemented outside their reliable domain. For low residual basal areas, the growth function is modified to agree with the models of Nyyssönen & Mielikäinen (1978) and Gustavsen (1992). For high basal areas, a mortality function was added. Mortality is proportional to the maximum basal area level characterized by the self-thinning equations of Hynynen (1993). The form factor equations were from Gustavsen & Fagerström (1983). The sawlog volume adjustment equations were adapted from Siitonen et al. (1996). The economic data were mainly adapted from Hyytiäinen & Tahvonen (2001).

The non-linear programming algorithms of Matlab software were used to solve the harvesting problem with the economic criterion, MSY and Forest Rent. Each optimization was repeated for various initial guesses and different numbers of thinnings, to find the global optimum. The calculations were repeated for five Norway spruce and six Scots pine sites. For the economic criterion, the calculations were done using rates of interest between 1 and 7%. The economic value of stands when the harvests are determined by applying Forest Rent, MSY or Finnish silvicultural recommendations, were calculated using eq. (9) for bare land and eq. (10) for forested land.

RESULTS

Maximized value of conifer cultures

Table 1 shows the maximized value of bare land for different rates of interest and Norway spruce and Scots pine sites. The site quality index, $H(100)$, gives the average height of dominant trees at age 100 yrs. The value of bare land decreases steeply with the rate of interest. With exogenous stand establishment, the maximized value of bare land remains positive only if the rate of interest was below 2–7%, depending on species and site.

Maximizing eq. (10) gives the value of forested stand. In Fig. 1a and c, the contours show the maximized value of forested land ($J^{f}*$) for any admissible initial state (i.e. any feasible combination of stand age and basal area) at 4% rate of interest on typical Norway spruce and Scots pine sites, respectively. The contours are based on computations from an evenly distributed grid of about 300 initial states for t_0 and $G(t_0)$. The initial value for the third state variable (i.e. number of trees), $N(t_0)$, is determined by assuming preceding commercial thinnings that depend on initial stand age and basal area. For example, stands with low initial basal area are assumed to have been thinned more heavily than stands with high initial basal area.

The value of the forested land typically increases with initial stand age and basal area. This is caused by greater volume of merchantable timber assortments, reduced unit harvesting costs and a price premium for high-dimension timber. The value of forested land decreases with the rate of interest, but less sensitively (in relative terms) than the value of bare land. For example, for Norway spruce $H(100)=27$ site and initial stand age 50 yrs and basal area 20 m^2 ha^{-1}, the values of forested land are 7616, 3932 and 3190 € ha^{-1} for 2, 4 and 6% rates of interest, respectively.

In Fig. 1b and d, the optimal solutions are shown for bare land (solid curve) and two initial states where $t_0>0$. The initial states for the two exemplary forested stands are denoted by filled circles, and optimal development of stand basal area over the first rotation period by dotted and dashed curves. For a typical Norway spruce stand (Fig. 1b), basal area is reduced towards the end of the rotation period by frequent thinnings. For a typical Scots pine stand (Fig. 1d), the growth response to thinning is weaker, and thus there is only one thinning for high initial basal areas and no thinnings for low initial basal areas. The initial state also affects the length of the ongoing rotation. For a 4% rate of interest, a decrease in the initial basal area typically lengthens the rotation period, but may also shorten it if the number of thinnings is reduced simultaneously.

Losses from Forest Rent

Table 2 shows the true, discounted value of bare land (€ ha^{-1}) when the forest management activities are determined using the Forest Rent approach. The results are shown for different rates of interest and Norway spruce and Scots pine site indexes. The economic loss following from application of Forest Rent solutions instead of economically optimal solutions (see Table 1) is presented in parentheses. The Forest Rent approach leads to a major decrease in bare land value for all non-zero rates of interest. Forest Rent solutions render the value of bare land negative at interest rates 0.5–2 percentage points lower than in the economically optimal solutions.

Table 1. *Maximized unconstrained value of bare land (€ ha^{-1}) at selected rates of interest and Norway spruce and Scots pine sites*

	Site index, $H(100)$						
Rate of interest	33	30	27	24	21	18	15
Norway spruce							
1%	18 495	13 633	10 145	7 060	4 486		
2%	6 071	4 096	2 700	1 561	642		
3%	2 554	1 495	760	200	0		
4%	1 095	463	28	0	0		
5%	384	0	0	0	0		
6%	3	0	0	0	0		
7%	0	0	0	0	0		
Scots pine							
1%		18 047	12 606	8 280	4 813	2 504	796
2%		5 393	3 253	1 938	940	249	0
3%		1 987	894	410	76	0	0
4%		657	33	0	0	0	0
5%		45	0	0	0	0	0
6%		0	0	0	0	0	0
7%		0	0	0	0	0	0

In Fig. 2a, the contours show the losses from Forest Rent for any feasible combination of initial stand age and basal area for a Norway spruce stand of average fertility at 4% rate of interest. The development of basal area over stand age is given for optimal (solid lines) and Forest Rent (dashed lines) solutions for bare land and two other exemplary initial states in Fig. 2b.

In absolute terms, the economic losses are highest for stands that have already passed the economic maturity age and have large basal area. For a 4% rate of interest, the losses amount to up to 6170 € ha^{-1} (about 50% of maximized stand value) for initial stand age 85 yrs and basal area 42.5 m^2 ha^{-1}. This stand is economically mature for clear-cutting, but application of the Forest Rent criterion leads to extended rotation (about 30 yrs) and a light thinning that keeps the level of basal area high until the end of rotation (compare optimal and Forest Rent paths in Fig. 2b).

The losses from Forest Rent typically increase with the rate of interest. For 2% rate of interest, the losses amount to up to 3140 € ha^{-1} (21% of maximized stand value), and for 6% rate of interest up to 8680 € ha^{-1} (70% of maximized stand value). For Scots pine sites, the losses are even higher, but are caused by excessively long Forest Rent rotation, which may exceed the reliable domain of the growth data.

High relative losses are associated with young stands with low basal areas when applying the Forest Rent criterion. This is caused by premature clear-cutting. For example, a stand with initial age 60 yrs and basal area 5 m^2 ha^{-1} is clear-cut immediately according to the Forest Rent criterion, but grown until 78 yrs if the economic criterion and a 4% rate of interest are applied (Fig. 2b).

Fig. 3 shows the clear-cutting curves for Forest Rent, MSY and economic criterion at 2, 4 and 6% rates of interest on typical Norway spruce and Scots pine stands. Immediate clear-cutting is optimal for all initial states on the right-hand side of the clear-cutting curve and retaining of the growing stock for all initial states on the left-hand side of the curve. Comparison of the curves reveals that an increase in the rate of interest increases the length of the first, ongoing, rotation for some initial states. The stands that fall on the right-hand side of the 0% (Forest Rent) curve and on the left-hand side of the 2% curve, for example, are clear-cut immediately at 0% rate of interest, but retained at 2% rate of interest.

Losses from Maximum Sustained Yield

Table 3 shows the economic value of bare land under MSY solutions and the resulting economic loss. With six thinnings, the MSY solutions lead to major economic losses and negative bare land values at interest rates 0.25–1 percentage points lower than optimal solutions.

For a forested Norway spruce stand of average fertility, MSY solutions lead, on average, to 20% relative losses (Fig. 4a). In absolute terms, the losses

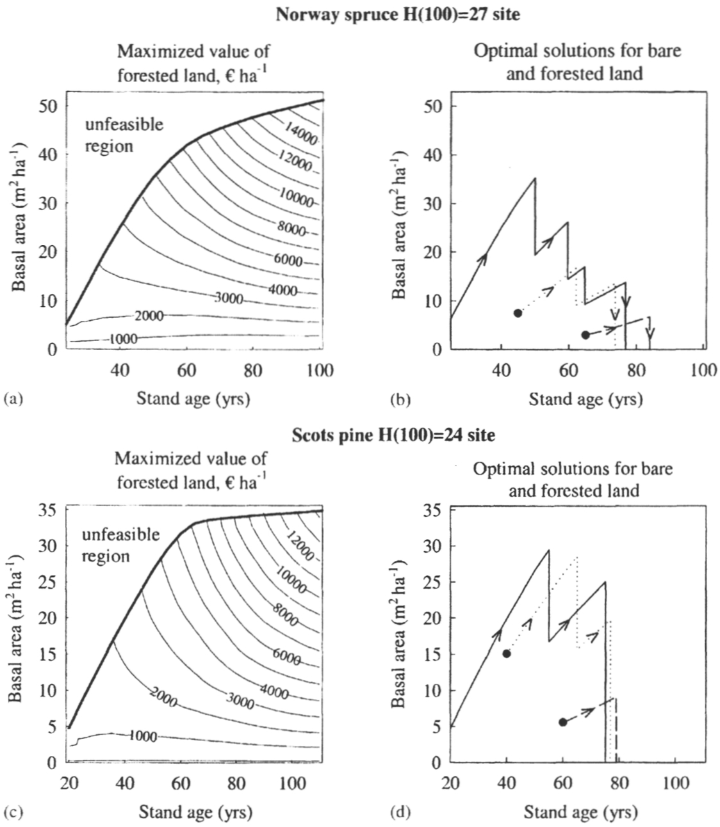

Fig. 1. Maximized value of forested land for typical Norway spruce and Scots pine sites at 4% rate of interest, and optimal paths for bare (solid curve) and forested (dotted and dashed curves) land.

Table 2. *Value of bare land under Forest Rent solutions and resulting loss (in parentheses) (€ ha^{-1})*

	Site index, $H(100)$						
Rate of interest	33	30	27	24	21	18	15
Norway spruce							
1%	17 646 (849)	12 949 (684)	9 657 (488)	6 666 (394)	4 144 (343)		
2%	5 105 (966)	3 347 (749)	2 144 (557)	1 127 (434)	314 (329)		
3%	1 687 (866)	849 (645)	290 (471)	−140 (340)	−459 (459)		
4%	401 (694)	−33 (496)	−320 (348)	−515 (515)	−649 (649)		
5%	−144 (527)	−380 (380)	−537 (537)	−630 (630)	−690 (690)		
6%	−387 (390)	−520 (520)	−614 (614)	−660 (660)	−688 (688)		
7%	−498 (498)	−575 (575)	−636 (636)	−661 (661)	−674 (674)		
Scots pine							
1%		17 248 (799)	11 961 (645)	7 716 (564)	4 370 (443)	2 184 (320)	616 (180)
2%		4 517 (875)	2 591 (662)	1 406 (532)	576 (365)	11 (238)	−331 (331)
3%		1 272 (715)	409 (485)	45 (365)	−146 (223)	−338 (338)	−432 (432)
4%		137 (520)	−282 (316)	−343 (343)	−322 (322)	−397 (397)	−428 (428)
5%		−316 (361)	−533 (533)	−463 (463)	−365 (365)	−398 (398)	−408 (408)
6%		−510 (510)	−628 (628)	−499 (499)	−370 (370)	−385 (385)	−389 (389)
7%		−595 (595)	−663 (663)	−504 (504)	−363 (363)	−370 (370)	−372 (372)

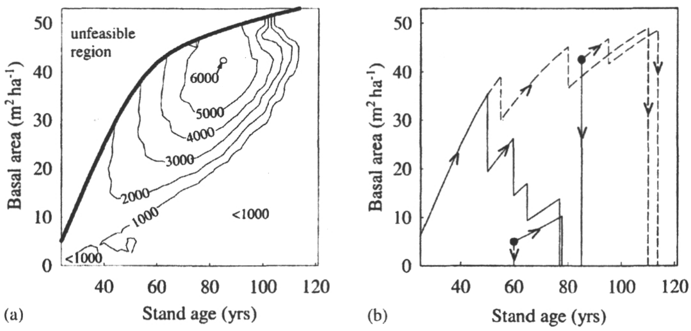

Fig. 2. (a) Economic loss from Forest Rent solutions and (b) comparison between Forest Rent (dashed line) and optimal (solid line) solutions for Norway spruce $H(100) = 27$ site at 4% rate of interest.

are highest (up to 3450 € ha^{-1} or 31% of stand value) for economically mature stands with large basal area. The losses follow mainly from extended (about 20 yrs) rotation (Fig. 4b). For stands with high initial basal area, thinnings are lighter and more frequent than with the economic criterion (compare optimal and MSY paths for bare land in Fig. 4b). High relative losses (30–40% decrease in stand value) are realized for economically premature stands that are clear-cut immediately when MSY criterion is applied (compare optimal and MSY paths for initial stand age 40 yrs and basal area 5 m^2 ha^{-1} in Fig. 4b).

Losses from silvicultural rules of thumb

Table 4 shows the value of bare land and respective losses when conifer stands are managed according to Finnish silvicultural recommendations given for southern Finland. Following the recommendations reduces bare land value considerably, especially at low rates of interest and on fertile sites. The recommended solutions lead to negative bare land value at interest rates 0–1 percentage points lower than optimal solutions.

Given a 3% rate of interest and typical Scots pine site, the recommendations yield, on average, 200 € ha^{-1} or 5% losses (Fig. 5a). The losses are greater, amounting to up to 970 € ha^{-1} or 12% of stand value for economically mature stands that have not yet reached the recommended rotation age and have high basal area.

Value of a representative forest

Finally, market prices for forest estates and the value of a typical conifer forest under four different management policies are compared in Fig. 6. The results, which are indicative, are given for a representative conifer forest in southern Finland. The forest consists of average proportions of Norway spruce (35.7%) and Scots pine (64.3%) stands of average volume and site quality (Finnish Forest Research Institute 2000, pp. 54, 64). The average volume for a Norway spruce-dominated stand (172 m^3 ha^{-1}) is represented by the initial stand age of 50 yrs and basal area 20 m^2 ha^{-1} on an $H(100) = 27$ site. For Scots pine, the average volume (101 m^3 ha^{-1}) is represented by an initial stand age of 45 yrs and basal area 12.5 m^2 ha^{-1} on an $H(100) = 24$ site. In year 2000, the average market

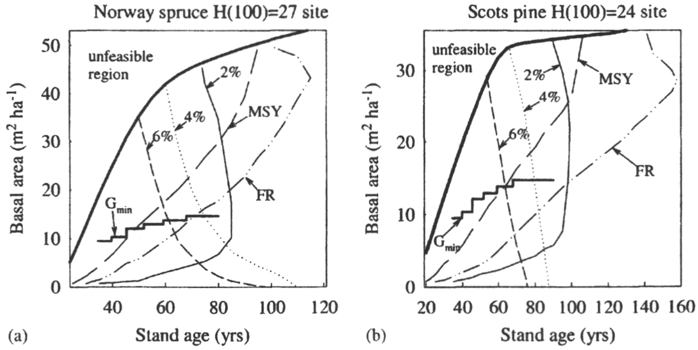

Fig. 3. Clear-cutting curves for Maximum Sustained Yield (MSY), Forest Rent (FR) and economic criterion (2, 4 and 6% rates of interest) compared with legal minimum residual basal area (G_{min}).

Table 3. *Value of bare land under Maximum Sustained Yield solutions and resulting loss (in parentheses) (€ ha^{-1})*

	Site index, $H(100)$						
Rate of interest	33	30	27	24	21	18	15
Norway spruce							
1%	18 187 (308)	13 349 (283)	9 879 (266)	6 833 (227)	4 240 (246)		
2%	5 810 (261)	3 838 (258)	2 388 (312)	1 291 (270)	389 (254)		
3%	2 211 (343)	1 185 (310)	436 (324)	−52 (252)	−430 (430)		
4%	738 (357)	163 (300)	−247 (275)	−477 (477)	−641 (641)		
5%	57 (327)	−275 (275)	−505 (505)	−616 (616)	−689 (689)		
6%	−274 (277)	−468 (468)	−602 (602)	−655 (655)	−689 (689)		
7%	−438 (438)	−551 (551)	−633 (633)	−659 (659)	−675 (675)		
Scots pine							
1%		17 795 (252)	12 402 (204)	8 060 (221)	4 521 (292)	2 293 (211)	604 (192)
2%		5 301 (91)	3 157 (96)	1 847 (91)	862 (78)	176 (74)	−276 (276)
3%		1 801 (186)	733 (162)	288 (122)	−4 (81)	−266 (266)	−413 (413)
4%		440 (216)	−128 (161)	−231 (231)	−267 (267)	−372 (372)	−423 (423)
5%		−154 (198)	−465 (465)	−415 (415)	−346 (346)	−390 (390)	−407 (407)
6%		−426 (426)	−599 (599)	−478 (478)	−364 (364)	−383 (383)	−389 (389)
7%		−552 (552)	−650 (650)	−495 (495)	−362 (362)	−370 (370)	−372 (372)

price was 2110 € ha^{-1} (with standard deviation of 1370 € ha^{-1}) for forest estates in southern Finland (National Land Survey of Finland 2001, p. 52). Here, these prices are generalized to apply to conifer forests, which account for 88.5% of all Finnish forests (Finnish Forest Research Institute 2000, p. 64).

Fig. 6 shows that the value of a representative conifer forest, when managed according to Finnish silvicultural recommendations, is well in line with market prices with a rate of interest between 3 and 6%. However, the value of forest property is sensitive to management policy. For a 4% rate of interest, the maximized stand value using the Faustmann approach is 2756 € ha^{-1}. For Forest Rent, MSY and recommended solutions, the forest values are 1033, 1918 and 2411 € ha^{-1}, respectively. Compared with the unconstrained maximum, application of Forest Rent, MSY and silvicultural recommendations leads to 63, 30 and 13% reductions in forest value, respectively.

DISCUSSION

Interpretation of the results

Rates of interest of 4 and 5% are high enough to lead to zero bare land value on average Scots pine and Norway spruce sites, respectively, as shown in Table 1. However, part of the stand establishment and silvicultural costs can be considered variable rather than fixed, even in plantation forestry. Reductions from conventional planting density are likely to become optimal, especially at low and zero bare land values, as illustrated for Scandinavian Norway spruce stands by Solberg & Haight (1991) and Valsta (1992). This implies that, with higher rates of interest, the bare

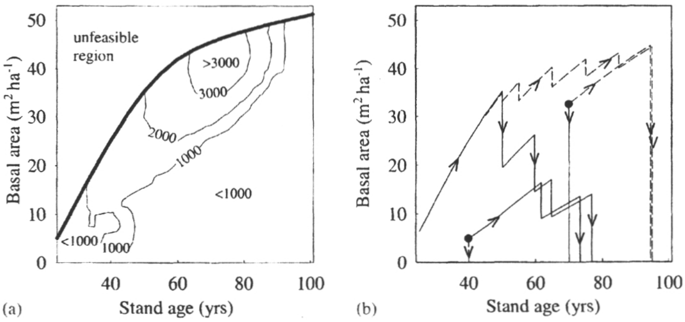

Fig. 4. (a) Economic loss from Maximum Sustained Yield (MSY) solutions, and (b) comparison between MSY (dashed line) and optimal (solid line) solutions for Norway spruce $H(100) = 27$ site at 4% rate of interest.

Table 4. *Value of bare land under recommended harvests and resulting loss (in parentheses) (€ ha^{-1})*

	Site index, $H(100)$						
Rate of interest	33	30	27	24	21	18	15
Norway spruce							
1%	15 399 (3 096)	11 988 (1 645)	9 097 (1 048)	6 131 (929)	3 727 (759)		
2%	4 858 (1 213)	3 536 (559)	2 450 (251)	1 358 (203)	552 (90)		
3%	1 829 (724)	1 147 (348)	604 (156)	91 (109)	−265 (265)		
4%	602 (493)	205 (258)	−103 (131)	−365 (365)	−541 (541)		
5%	33 (351)	−215 (215)	−403 (403)	−542 (542)	−635 (635)		
6%	−250 (253)	−411 (411)	−534 (534)	−610 (610)	−662 (662)		
7%	−396 (396)	−505 (505)	−589 (589)	−632 (632)	−662 (662)		
Scots pine							
1%		13 872 (4 175)	9 596 (3 010)	6 470 (1 811)	3 753 (1 060)	2 031 (473)	410 (385)
2%		4 216 (1 176)	2 619 (634)	1 589 (349)	807 (134)	180 (69)	−255 (255)
3%		1 459 (528)	671 (223)	287 (122)	44 (32)	−236 (236)	−386 (386)
4%		356 (301)	−81 (114)	−184 (184)	−217 (217)	−350 (350)	−408 (408)
5%		−146 (191)	−405 (405)	−371 (371)	−312 (312)	−376 (376)	−400 (400)
6%		−389 (389)	−551 (551)	−446 (446)	−343 (343)	−375 (375)	−386 (386)
7%		−511 (511)	−615 (615)	−473 (473)	−349 (349)	−365 (365)	−371 (371)

land values shown in Table 1 may be underestimates. Unfortunately, models enabling reliable analysis of varying stand establishment and silvicultural effort are not available.

The clear-cutting curves shown in Fig. 3 indicate that juvenile stands with low enough basal area are optimal to be replaced immediately by a new (fully stocked) tree generation when Forest Rent or the economic criterion with low (2%) rate of interest is applied. MSY neglects the positive effects of increased timber dimensions on the unit value of commercial timber, and thus generates premature clear-cutting for even higher stocking levels than does Forest Rent. However, the rotation period may be longer for low than high initial basal areas at 4% and 6% rates of interest. This is due to high investment costs for stand establishment. Retaining low growing stocks becomes optimal because of high (relative) value growth in existing stock and small or zero benefits from subsequent rotations.

It is an established result in forest economics that maximizing Forest Rent cannot yield a shorter rotation than the Faustmann approach with positive rate of interest (e.g. Johansson & Löfgren 1985, Montgomery & Adams 1995). However, this classical result is based on the use of density-free whole stand models, and assumes bare land as the initial state. These assumptions can be relaxed by using a variable-density model and allowing for any initial state. The present results give examples of initial states where application of the Forest Rent criterion leads to immediate clear-cutting, while application of the economic criterion with positive rate of interest leads to continued growth of the stand (Fig. 3).

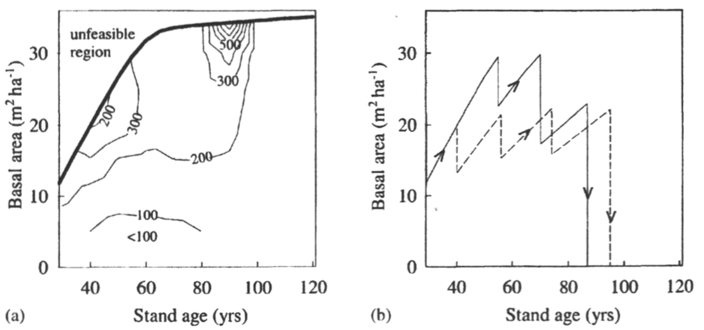

Fig. 5. (a) Economic loss from silvicultural recommendations and (b) comparison between recommended (dashed line) and optimal (solid line) solutions for Scots pine $H(100) = 24$ site at 3% rate of interest

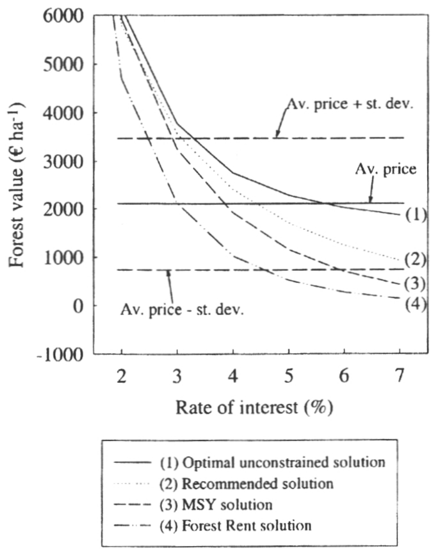

Fig. 6. Value of a typical conifer forest under various management policies, as a function of the rate of interest and comparison with market prices. MSY: Maximum Sustained Yield.

Kilkki & Väisänen (1969) present clear-cutting curves for average Scots pine site at a 4% rate of interest, but do not extend the analysis to low volume stands. Salminen (1993) uses growth models specified for "understocked" Norway spruce and Scots pine stands. He presents clear-cutting curves for 1% and 3% rates of interest, but only for low stocking levels. The results shown in Fig. 3 accord with these studies, but clear-cutting curves are presented for all admissible combinations of initial stand age and basal area, and various rates of interest.

The losses from MSY are typically smaller than the losses from Forest Rent (compare Tables 2 and 3 and Figs. 2a and 4a). The reason is that the MSY rotation is shorter than the Forest Rent rotation as long as stand establishment costs are positive (see Binkley 1987). According to the results for Scots pine (Table 3), the losses from MSY are smallest for the 2% rate of interest. This is due to the fact that the economically optimal rotation length is about the same as the MSY rotation.

The recommended harvests accord best with the optimal harvests and lead to the smallest (relative) losses at 1–2% rates of interest for Norway spruce sites and at 2–3% rates of interest for Scots pine sites (Table 4). The losses follow partly from non-optimal rotation, as shown by Hyytiäinen & Tahvonen (2001). In addition, the losses follow from recommended thinnings that are started earlier and aim at producing higher dimension timber than optimal (Fig. 5b). Valsta (1982) reports analogously that an earlier version of the Finnish silvicultural recommendations accords best with the optimal solution for 2–3% rates of interest for fertile Norway spruce stands.

Timber prices may not be constant, as assumed in this study. Under fluctuating prices, forest owners tend to sell timber when the price is highest, as shown, for example, by Kuuluvainen & Tahvonen (1999). Monotonic changes in prices may either increase or decrease optimal harvesting ages compared with solutions obtained with constant price level (Hyde 1980, p. 69, Newman et al. 1985), and may therefore either decrease or increase losses from application of silvicultural rules of thumb compared with the results shown in Table 4 and Fig. 5a.

The market prices for forest estates may contain non-timber and *in situ* benefits in addition to benefits from timber production. Moose density may, for example, have a positive effect on the market value of forest land (Aronsson & Carlén 2000). In this respect, the average equilibrium price in Fig. 6 is low and may reflect, for example, capital market imperfections (see Tahvonen et al. 2001), thin markets for forest land, or the fact that the estates sold may consist of younger and less stocked stands than the average estates. Moreover, it is not known in empirical terms what management policies the forest owners are actually using.

Policy implications

Results in Valsta (1992), Vettenranta & Miina (1999) and this study suggest that the Finnish silvicultural recommendations overemphasize production of high-quality timber and reduce forest owners' economic surplus. Some documents (e.g. Ministry of Agriculture and Forestry 1993, p. 57) suggest that the forest industry might be willing to pay more for high-quality versus inferior-quality timber than is realized now in the timber market. However, it may be assumed that the present pricing policy with low price premium for high-dimension sawlogs will be maintained if the forest owners produce high-quality timber irrespective of the economic incentives.

The legal restrictions on minimum residual basal area are closer in line with the MSY and Forest Rent than with the economically optimal solutions (Fig. 3). The statutory provisions (see Niskala 2000) require that stands where basal area falls below the legal minimum (G_{min}) be clear-cut and regenerated if it is "economically more appropriate than retaining of the existing trees". The present results, however, imply that clear-cutting of young stands is hardly ever rational. For average market rates of interest, clear-cutting of young stands is optimal only on the most fertile sites and very low basal areas, distinctly below the legal minimum.

Most boreal forest sites regenerate after clear-cutting, even without planting or any other stand establishment efforts. Hence, leaving forest stands unmanaged after clear-cutting may not jeopardize the sustainability of forestry. However, the species composition and stand structure may be undesirable in naturally born stands. Regeneration lag delays future harvests and reduces the long-term timber supply and annual net incomes. This is not acceptable under the Forest Law (see Niskala 2000), nor is it an option in the Finnish silvicultural recommendations (Tapio 2001). Costly soil preparation, planting and tending is practically the only option to guarantee that the recommended targets for stand establishment are achieved on most forest soils in Finland. In this light, legislation and the Finnish silvicultural recommendations reflect MSY or Forest Rent rather than the economic criterion for higher rates of interest.

Conclusions

The results suggest that relying on the MSY or Forest Rent instead of the economic criterion leads to major losses for Finnish forest owners. Following the silvicultural rules of thumb may also lead to substantial losses in conifer cultures, especially if the initial state is off the optimal path starting from bare land. However, it is clear that research results on the absolute magnitude of losses following from application of a non-optimal decision criterion are always case specific and depend on descriptions of growth, harvesting technology and economic parameters. It would be interesting to study whether the results on efficiency gains from following economic principles are altered if some important elements of the optimization model (e.g. prices or rates of interest) are stochastic. Another interesting theme would be to study the interaction of biological and economic relationships by combining more sophisticated descriptions of stand growth to economic optimization. One promising possibility is to describe stand growth using detailed process-based forest models (e.g. Mäkelä 1997, Johnsen et al. 2001) that have been under fast development and are based on sound ecophysiological theories on tree structure and growth.

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Paper IV

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IV

Optimum juvenile density, harvesting and stand structure in even-aged Scots pine stands

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ABSTRACT

The economics of timber production is studied using empirical data for various juvenile densities and a distance-independent individual-tree growth model specified for Finnish Scots pine (*Pinus sylvestris* [L.]) stands. Our results imply that explicit inclusion of quality effects on prices increases optimum juvenile density. In addition, quality effects make economic surplus of forestry more sensitive to variations in juvenile density. Optimum thinnings remove trees that have no prospects for high relative value growth. Optimum thinnings are selective thinnings that remove inferior-quality trees, some of the smallest trees and, contrary to conventions, those of the largest trees that fulfill the sawlog dimensions. Thinnings removing trees strictly above and/or below certain tree diameters homogenize the remaining growing stock by narrowing the diameter distribution. Homogeneous stand structure is economically favorable at final clearcutting because the stand can be clearcut promptly after the majority of trees have reached sawlog dimensions and have experienced the last high peak in value growth.

Keywords: forest economics, individual-tree growth model, timber quality, thinning type.

INTRODUCTION

Economically, the key questions of timber production concern how much to invest in the initial stock, when to thin and clearcut, and which trees to remove in thinnings. The theoretical framework for solving the even-aged management problem has been known for a long time (Faustmann 1849). Traditionally, economic studies on stand-level forest management have concentrated on analyzing the factors of optimum rotation. The classical rotation problem can be solved using univariate growth models, where stand development follows a predetermined trajectory over stand age (Getz and Haight 1989, p.228). Such models are easy to construct from normal or empirical growth and yield tables (Davis et al. 2001, p.190-194). However, their application is limited to solving optimum rotation with exogenously given prior management.

In recent decades, there has been increasing interest in including thinnings and silvicultural activities in stand-level models. Numerical solution of complex models has become possible with the availability of advanced computer technology and powerful solution algorithms. In addition, successful solution of increasingly detailed versions of the stand management problem has required the development of growth models that are compatible with optimization and which project reliably the consequences of different silvicultural and harvesting options.

The first studies capable of determining optimum timing, number and intensity of thinnings used variable-density whole-stand models (e.g. Amidon and Akin 1968, Kilkki and Väisänen 1969, Risvand 1969, Brodie et al. 1978, Brodie and Kao 1979, Kao and Brodie 1980). These models contain a few state variables (such as basal area, volume or number of trees) that evolve over stand age due to growth, mortality and harvesting. Variable-density whole-stand models are still popular in forest economics computations, and enable many extensions. The main limitation of these models is that they apply only to forest stands where the economic value of timber or aggregate growth does not depend on the distribution of trees by size classes.

Optimum type of thinning (i.e. which trees to remove in thinnings) and development of stand structure can be determined using stage-structured or individual-tree growth models. In stage-structured models (e.g. Haight 1987, Solberg and Haight 1991) trees are grouped into size classes typically characterized by tree diameter. Growth is described as transition from one class to another. Individual or single-tree models are probably the most detailed specifications currently used in stand-level optimization. They describe growth and mortality separately for each tree in a stand or sample plot. Growth and mortality are specified as functions of stand density variables when using distance-independent models (e.g. Roise 1986, Yoshimoto et al. 1990, Haight and Monserud 1990b, Valsta 1992a,b, Eriksson 1994, Wikström 2001). When using a distance-dependent or spatial model (e.g. Vettenranta 1996, Pukkala et al. 1998b, Pukkala and Miina 1998, Vettenranta and Miina 1999), growth depends explicitly on location and size of an individual tree relative to its neighbors.

Forest stands are conventionally established at high initial density and spaced and thinned in order to improve the quality of stock trees. However, the existing models neglect quality factors (such as knottiness or proportion of juvenile wood) and their effects on timber prices. Consequently, the magnitude of quality effects on optimum initial density or harvesting remains an open question. Explicit inclusion of quality effects in the optimization model is vital, especially for timber species with markets for several end uses. This is the case for many slowly growing conifers that can be used for sawmilling, plywood or other ”value-added” products (see e.g. Barbour and Kellogg 1990).

Determination of optimum initial density is a challenging task because the probability distribution for success in establishing a forest stand is wide. Compared to older established stands, development of a seedling stand depends more sensitively on site properties, natural hazards, and competition with weeds and other tree species. However, there is a lack of empirical models that reliably predict stand development in the early ages. In addition, the existing results on optimum initial density are somewhat mixed for conifer species. Most earlier studies (e.g. Haight and Monserud 1990b, Teeter and Caulfield 1991, Solberg and Haight 1991, Valsta 1992b, Haight 1993) suggest that a conifer stand should be established at high initial density to enable returns from thinnings. Some studies (Gong 1998, Zhou 1998, 1999), in contrast, suggest that a conifer stand should be established directly at the final clearcut density.

Numerical studies have traditionally concentrated more on developing optimization techniques (e.g. Roise 1986, Haight and Monserud 1990a, Yoshimoto et al. 1990) or specification of stand structure and harvest control (e.g. Pukkala and Miina 1998, Wikström 2001) than on elaborating the actual results. Compared to theoretical studies, most numerical studies lack clear and intuitive interpretation of the results. For example, one of the main lessons of this line of research is that thinning trees from above or from the both ends of the diameter distribution is superior to conventional thinning from below. Many North American and Scandinavian studies produce this result for monocultures (Solberg and Haight 1991, Valsta 1992a,b, Eriksson 1994) as well as for mixed conifer stands, with in-growth (Haight 1987, Haight and Monserud 1990b) or without in-growth (Vettenranta 1996, Pukkala et al. 1998b, Pukkala and Miina 1998, Vettenranta and Miina 1999). However, only a few earlier studies have focused on interpreting this result. Hence, we still lack a broad understanding of the reasons behind optimum thinnings.

This study aims to contribute to the stand-level numerical optimization literature by including the effects of timber quality on timber assortment prices in a distance-independent individual-tree growth model. The economics of timber production is studied using empirical data for various juvenile densities and a distance-independent individual-tree growth model specified for Finnish Scots pine (*Pinus sylvestris* [L.]) stands. Juvenile density is the density of an established young stand before the first commercial thinning. Our specification of timber quality is based on a negative relationship between early growth (ring width at tree pith) and the proportion of high-quality timber. Quality effects have been included in stand-level optimization earlier by Zhou (1999), but his whole-stand growth model does not enable rigorous analysis of thinnings.

This paper is organized as follows. The second section presents the general structure of the optimization model. The data and computations are explained in the third section. The fourth section compares various juvenile densities, and the fifth section gives results on the optimum harvests. The results are discussed further in the final section.

OPTIMIZATION PROBLEM

Next, we present the optimization problem for even-aged forest stands when stand development is projected with an individual-tree growth and yield model. Stand structure is described with n trees. Each tree is characterized by l variables reflecting its current dimensions (diameter, height etc.) and an expansion factor representing the number of trees of its kind per hectare or other area unit. The tree vectors evolve over time due to growth, mortality and harvesting. The vectors are updated after every growth prediction and harvest.

Let $\mathbf{Z}_{t_u}$ be an $(n x l)$ matrix describing the stand state before the u^{th} harvest at stand age t_u, $u = 1,\ldots,k$. Define the n-dimensional vector $\mathbf{h}_u$ as the ratio of trees removed in the u^{th} thinning, $u = 1,\ldots,k-1$. In the final (k^{th}) harvest all the remaining trees are removed. The aim is to choose the initial state of the stand $\mathbf{Z}_{t_0}$, the timing of the thinnings and clearcutting t_u, $u = 1,\ldots,k$, and the intensity of thinnings $\mathbf{h}_u$, $u = 1,\ldots,k-1$, so as to maximize the (post-tax) value of bare land V,

$$V = \frac{\sum_{u=1}^{k}\left\{\left[\sum_{i=1}^{n}\sum_{j=1}^{m} p_{uij}(\mathbf{Z}_{t_u})\, g_{uij}(\mathbf{Z}_{t_u},\mathbf{h}_u)\right] - C_u(\mathbf{Z}_{t_u},\mathbf{h}_u)\right\} b^{t_u} - W(\mathbf{Z}_{t_0})}{1 - b^{t_k}}(1-\tau). \quad [1]$$

Gross harvest revenues of the u^{th} harvest are summed over n trees and m timber assortments as products of road-side prices p_{uij} (€/m^3) and harvested volumes g_{uij} (m^3 ha^{-1}). The net harvest revenues are attained by subtracting the "stump-to-truck" harvest cost, C_u, consisting of felling and on-site-transport costs. The net harvest revenues are discounted to the beginning of the rotation period by a factor $b = (1+r)^{-1}$, where r is the real rate of interest. The sum of the (discounted) stand establishment and silvicultural costs is denoted by W, and the profit tax rate by τ.

The value of bare land in [1] is maximized subject to the stand dynamics that define the stand state just before the harvest at age t_u. This is given as a function of stand state before the previous harvest ($\mathbf{Z}_{t_{u-1}}$), intensity of the previous harvest ($\mathbf{h}_{u-1}$), and time difference between present (t_u) and previous harvest (t_{u-1}):

$$\mathbf{Z}_{t_u} = f(\mathbf{Z}_{t_{u-1}}, \mathbf{h}_{u-1}, t_u - t_{u-1}) \qquad \forall u = 1,\ldots,k. \quad [2]$$

Additional constraints ensure that the harvests are in the correct order:

$$t_u \le t_{u+1} \ \forall\ u = 1,\ldots,k-1, \quad [3]$$

the initial state belongs to the feasible set of initial states:

$$\mathbf{Z}_{t_0} \in \mathcal{Z}_{t_0}, \quad [4]$$

and all n elements in the harvest control vectors are feasible:

$$0 \le \mathbf{h}_u(i) \le 1 \qquad \forall u = 1,\ldots,k-1 \qquad \forall i = 1,\ldots,n. \quad [5]$$

The harvesting problem [1]-[5] is solved for different numbers of thinnings in order to find the optimum number of thinnings and global maximum.

DATA AND COMPUTATIONS

Individual-tree model

The optimization model is specified for Finnish Scots pine stands. The growth equations for [2] are based on extensive data sets from permanent sample plots of National Forest Inventory and are explained in detail in Hynynen et al. (2002). These difference equations predict stand growth at 5-yr periods. For shorter periods, growth is interpolated linearly. The state variables are stand age (t), tree basal area (ba), crown ratio (cr), and the expansion factor expressing the number of trees that the particular tree represents in the stand. In addition, tree height (th) is given initial value that is updated at every time step. Its development is based on a general age-dependent pattern, but depends also on the relative tree size and stand density.

The basal area growth, $\triangle ba$, is defined as a function of tree variables (basal area and crown ratio) and stand variables reflecting mean height of the dominant trees (H_{dom}), time period after the previous thinning ($THIN$), density of trees larger than the object tree ($RDFL$), stand density (RDF), dominant height site index (SI), fertility class (SC), temperature sum (TS) and a factor calibrating the level of growth for a specific stand (β):

$$\triangle ba = f(ba, cr, H_{dom}, THIN, RDFL, RDF, SI, SC, TS, \beta). \qquad [6]$$

The growth calibration factor is used only if there exists field measurement data against which the model predictions can be compared. Some parameter values used in the present study are different from those given in Hynynen et al. (2002), because some of the equations are based on earlier measured data sets. These differences, however, have only minor effects on growth predictions. The major limitation of the original equation is that it is designed for a narrow range of conventional thinning intensities varying between 15 and 35%. It does not account for the effect of thinning intensity on growth after thinning. In order to render the growth model valid also for extreme aggregate thinning intensities ($\gamma<0.15$ or $\gamma>0.35$), the original equation is modified by adding a multiplier, $\omega=1.11\,\gamma^{1.81}(0.0129+\gamma^{1.81})^{-1}$, to the $THIN$ variable accounting for the growth response to thinning. This modification, which is based on Pukkala et al. (1998a) and Hynynen (1995), reduces growth response at very low intensity thinnings.

The number of trees decreases with stand age, due to age-dependent mortality and self-thinning. Mortality depends on tree basal area, stand basal area, aggregate basal area of other trees larger than the object tree and site index. Self-thinning induced by high within-stand competition is realized only when the aggregate stand basal area reaches the maximum level.

The commercial timber assortments for Scots pine are sawlogs (j = 1) and pulpwood (j = 2). Timber volumes by assortment depend on tree basal area and height (Laasasenaho 1982) and quality of timber. A negative relationship between tree growth rates at young ages and the quality of final sawn goods is an established result in the timber technology literature (see e.g. Barbour and Kellogg 1990). Fast early growth increases knottiness and extends juvenile core around tree pith (Haygreen and Bowyer 1996, p.277-). This relationship can be accommodated for individual-tree simulators that maintain the identity of the trees over the rotation period. In our model, each tree is assigned a timber quality index (q), which gives the proportion of timber that fulfills the quality requirements for sawlogs. The index is assigned when a tree's diameter measured at breast height exceeds 10 cm. The index is specified as: $q = \max(1 - 1.2\alpha, 0)$, $\alpha \geq 0$, where α is the average annual ring width (in cm) within a 5-cm radius of the pith at breast height. Average annual ring width is determined by dividing tree radius (below bark) at breast height by the number of years since the tree reached breast height. This specification is based on Finnish timber technology literature for Scots pine (e.g. Uusvaara 1981, Halinen 1985).

All commercial inferior-quality timber is valued as pulpwood. Our preliminary computations revealed that the inferior-quality trees are optimally removed at the earliest possible thinning once the trees reach pulpwood dimensions. However, it is difficult to estimate visually the quality of timber from young standing trees. Hence, it may not be possible to select all inferior-quality trees already at the first thinning. In our model, it is assumed that thinning removal may consist entirely of inferior-quality trees if their proportion in growing stock before thinning is higher than 50%. Otherwise, the proportion of inferior-quality trees in removal is at most twice the proportion in growing stock.

The roadside unit price for sawlogs (€/m^3) is given as: $p_{ui1} = \max(-13.03 + 57.22(d_{ui} - 19)^{0.14}, p_{ui2})$, where d_{ui} is the diameter of the ith tree at breast height (in cm) in the uth harvest. The quality premium is from Valsta (2001). The unit price for pulpwood is constant: $p_{ui2} = 26.24$ €/m^3. The harvesting cost is given as $C_u = f\left[\sum_{i=1}^{n} g_{ui1}, \sum_{i=1}^{n} g_{ui2}, (\mathbf{n}_{t_u})^T \mathbf{h}_u\right] + c$, where c is the fixed harvesting cost. The variable harvesting cost depends on harvested volumes (by assortment) and the total number of trees removed. It is given separately for thinning and clearcutting. The equation is based on detailed work productivity functions for mechanized cutting (Nordic "cut-to-length" system) and the forest haulage given in Kuitto et al. (1994). The unit costs in mechanized cutting and haulage are 67.28 and 47.09 € hr^{-1}. The average haulage distance is 250 m, and the fixed harvesting cost is 100 € ha^{-1}.

The stand establishment cost parameters are adapted from Hyytiäinen and Tahvonen (2001), and are specified for a Scots pine site where artificially established stock is replenished with a large number of naturally born seedlings. A sown stand can be established at constant cost for any juvenile density. Cost of spacing of a dense stand is approximately independent of the resulting density. The cost W is a sum of discounted expenses from land preparation (206 € ha^{-1} in yr 0), sowing (167 € ha^{-1} in yr 1), slashing (218 € ha^{-1} in yr 6), and juvenile spacing (218 € ha^{-1} in yr 11). The profit tax rate is $\tau = 0.29$.

The forest skid roads are established at the first thinning. This is accounted for by setting a lower bound of 0.16 in [5] for the first thinning. The most general approach to determine the optimum type and intensity of thinning is to define thinning rates for each tree. This would however lead to an excessively high number of optimized variables. In order to keep the problem solvable, the number of optimized variables can be reduced e.g. by grouping (Wikström 2001). In this study the diameter range between the smallest and largest trees is divided into 1-5 sections of equal width. Thinning rates are optimized for the minimum and maximum diameters and for each diameter between these sections. For other diameters, the rates are interpolated linearly (see detailed description of thinning type definition in Valsta 1992a). Thus, the number of optimized variables defining the type of each thinning varies from 2 to 6 (the default used in the computations is 3).

Data for initial states

Predicting stand development reliably in young stands is difficult. Thus, rather than relying on existing models, we use empirical data from permanent sample plots to describe the effects of various juvenile densities on stand structure of an established stand. The inventory data for initial states are from six large sample plots (0.15-0.55 ha) from a juvenile (18-yr-old) Scots pine stand located in Southern Finland (see details in Varmola 1982, stand number 13). One plot is unspaced and five plots are spaced at age 11 yrs for juvenile densities varying between 600 and 3000 trees ha^{-1}. The unspaced plot has grown undisturbed after slashing in yr 6. The stand grows on a *Vaccinium*-type site which is typical for Scots pine. The initial stock consists of both naturally born and sown trees.

A large number of trees ($47 \leq n \leq 50$) were selected from each treatment to attain a detailed description of the stand structure. Figure 1c shows the initial diameter frequency distributions. The distributions are bell-shaped for spaced stands, and reverse J-shaped for the unspaced stand. Tree height (Figure 1a) is calculated using Näslund's equation, $th = 1.3 + d^2(a + bd)^{-2}$, and the crown ratio (Figure 1b) using the linear equation $cr = a + b\ d$. The parameters a and b are fit to each plot using data from sub-samples of the trees.

The sample plots belong to a permanent thinning experiment, which has been monitored for over 30 yrs. We used later data (at 36 yrs age) from the same plots to determine the calibration factor ($\beta = 0.7$) for tree basal area growth [6].

Computations

We use the "Stand Management Assistant" software developed by Valsta and Linkosalo (1996) to solve the harvesting problem [1]-[5] for six initial states, various harvest specifications, and all rates of interest that yield positive bare land value. This computer framework combines stand-level simulation and numerical optimization. The optimization algorithm is based on Hooke and Jeeves' (1961) direct search method, which has proved successful in solving complex non-differentiable forestry problems (see e.g. Haight and Monserud 1990a). The modifications to the original algorithm are based on Osyczka (1984, p.157), and are explained in detail in Valsta (1992b).

OPTIMUM JUVENILE DENSITY

Table 1 shows maximized (post-tax) bare land values computed for six juvenile densities and all rates of interest that yield positive bare land value. At 0% rate of interest, the values represent maximized annual net revenues. The juvenile density of 2245 trees ha^{-1} gives the highest bare land value for all rates of interest. Bare land value of an unspaced stand (8327 trees ha^{-1}) is distinctly lower because of high mortality and reduced tree growth. High within-stand competition diminishes tree crowns and reduces radial growth (see Figure 1). Nor is spacing a Scots pine stand directly at final density for clearcutting (591 trees ha^{-1}) economically attractive. Low juvenile density reduces thinning revenues and degrades timber quality because of rapid growth at young ages.

Our results apply directly to sown stands, where virtually any juvenile density

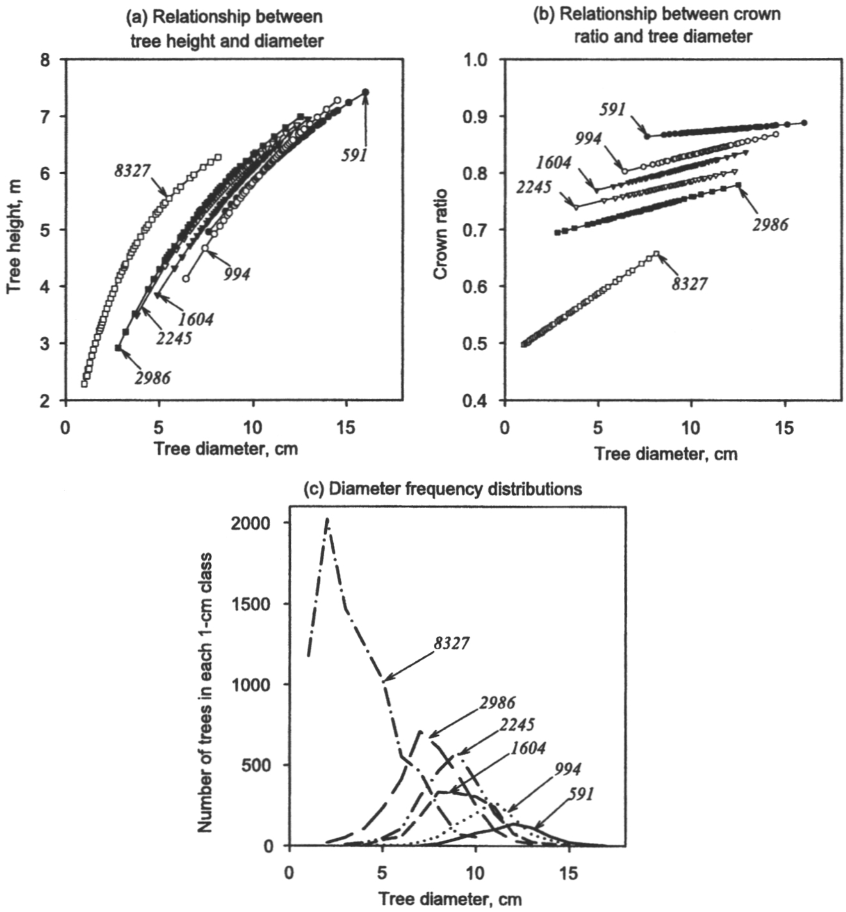


Figure 1. Description of six initial states.

Note: Numbers in italics denote juvenile density.

Table 1. Maximized (post-tax) value of bare land, € ha^{-1}.

Rate of interest	Juvenile stand density, trees ha^{-1}					
	591	994	1 604	2 245	2 986	8 327
0 %	102	144	195	212	208	173
1 %	5 997	8 725	11 088	12 499	11 996	8 713
2 %	1 743	2 774	3 511	3 903	3 627	2 354
3 %	624	1 095	1 433	1 632	1 448	798
4 %	148	419	587	692	581	231
5 %	–	91	183	244	170	–
6 %	–	–	–	2	–	–

can be attained at a constant stand establishment cost. For planted stands with no or little in-growth, optimum planting density is likely to be lower because the stand establishment cost is typically an increasing function of planting density. According to Solberg and Haight (1991) and Valsta (1992b), optimum planting density for Norway spruce (*Picea abies* [L.] Karst.) sites varies between 1400 and 2350 trees ha^{-1} at 3% rate of interest, but decreases steeply with the rate of interest. Our results imply analogously that optimum juvenile density is likely to decrease with the rate of interest, but clearly less sensitively due to fixed stand establishment cost. Comparison of bare land values for juvenile densities of 1604, 2245 and 2986 ha^{-1} in Table 1 implies that the optimum juvenile density is likely to decrease from about 2500 to about 2100 trees ha^{-1} as the rate of interest increases from 0 to 5%.

Figure 2 compares maximized bare land values for various juvenile densities at 3% rate of interest with and without inclusion of quality effects on timber prices. Neglecting the quality effects leads to overestimation of sawlog yield and bare land value, in particular in sparse stands. This reduces optimum juvenile density. Without quality effects, the juvenile density giving the highest bare land value is only 994 trees ha^{-1}.

Explicit inclusion of quality effects in optimization makes maximized bare land value more sensitive to variations in juvenile density, as shown in Figure 2. The optimum is clearly around the juvenile density of 2245 trees ha^{-1}. Without quality, by contrast, there is a large number of near-optimum solutions across the juvenile densities of 1000-2500 trees ha^{-1}. Earlier studies that neglected the quality effects report similarly that the response surface around the optimum is flat for a large range of initial densities. Solberg and Haight (1991) show that up to about ±25% deviation from the optimum Norway spruce planting density leads to a less-than-2% decrease in bare land value. Haight and Monserud (1990b) report less-than-10% changes in bare land value across planting densities of 375-1500 trees ha^{-1} in white pine (*Pinus monticola* Dougl. ex D. Don) plantations with a lot of in-growth of several commercial species. Teeter and Caulfield (1991) and Taylor and Fortson (1991) also report wide ranges of near-optimum planting densities for loblolly pine (*Pinus taeda* L.) plantations in southern USA.

Our results suggest that a Scots pine stand should be established at high enough juvenile density to enable positive net revenues from several thinnings. Inclusion of quality effects in the individual-tree model makes thinnings more advantageous. However, Haight and Monserud (1990b), Teeter and Caulfield (1991), Solberg and

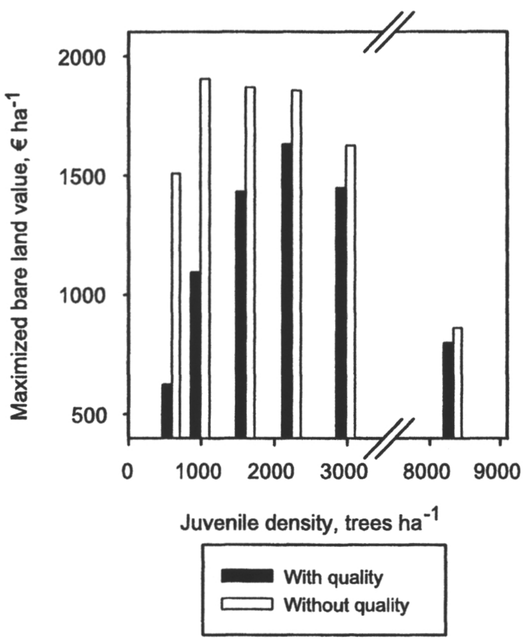


Figure 2. Maximized bare land value with and without timber quality at 3% rate of interest.

Haight (1991), Valsta (1992b) and Haight (1993) find high initial density and several thinnings optimal even without considering quality factors other than tree dimensions.

In contrast, Gong (1998) and Zhou (1998, 1999) suggest that Scots pine stands should be established directly at the final density for clearcutting. This result is due partly to expensive harvesting and low timber productivity on infertile sites in northern Sweden. Nevertheless, it is also possible that the whole-stand growth and yield model used by Gong (1998) and Zhou (1998, 1999) does not fully account for the positive quality and quantity effects of thinnings. In their model, thinnings may increase the sawlog proportion of the harvested timber only slightly because of insensitive timber assortment functions. Hence, it is implicitly assumed that the diameter distribution remains wide irrespective of thinnings. In contrast, Eriksson (1999) uses an individual-tree model specified for similar Swedish Scots pine sites. He shows that thinning at least once remains optimal up to 200-250 € ha^{-1} fixed harvesting cost levels, while Gong (1998) and Zhou (1998, 1999) find the no-thinning option to be consistently optimal already at 180 € ha^{-1} fixed cost.

OPTIMUM HARVESTS

Number, timing and intensity of thinnings

Thinnings bring in some intermediate revenue before clearcutting, and improve the (relative) value growth of the residual stock. Table 2 shows that thinning is advantageous at 2-4% rates of interest even in the sparsest stand (591 trees ha^{-1}), which is intended to be spaced directly at the final density. In denser stands, it is optimal to thin several times during the rotation. The optimum number of thinnings increases with juvenile density, but is less sensitive to changes in the rate of interest.

Figure 3 shows that it is critical to thin at least once during the rotation. Additional thinnings increase bare land value of dense stands, but only marginally. The response surface around the optimum number of thinnings is flat as far as the remaining decision variables are adjustable. For conventional juvenile densities (1604, 2245 and 2986 trees ha^{-1}), bare land values deviate less than 23 € ha^{-1} (2.1%) from the maximum when the number of thinnings is varied from two to six. However, higher fixed harvesting cost decreases the optimum number of thinnings (Eriksson 1999, Hyytiäinen and Tahvonen 2002) and increases the losses from nonoptimally high number of thinnings in distant and small stands that are costly to access.

Table 2. Optimum number of thinning.

Rate of interest	Juvenile stand density, trees ha^{-1}					
	591	994	1604	2245	2986	8327
0 %	0	2	2	3	4	5
1 %	0	1	2	2	4	6
2 %	1	1	3	4	4	4
3 %	1	1	3	3	3	4
4 %	1	2	3	3	3	2
5 %	–	2	2	3	3	–
6 %	–	–	–	3	–	–

Note: The maximum number of thinnings in this calculation is 6.

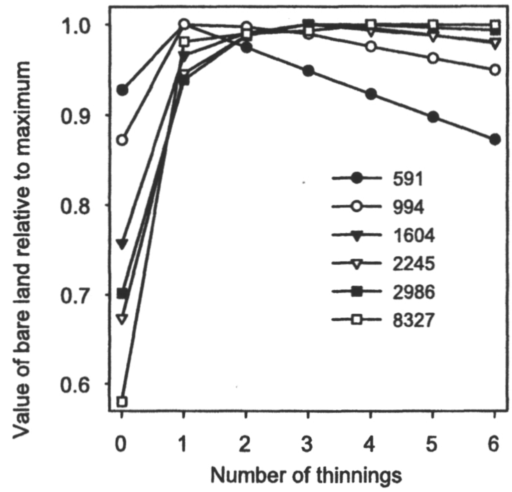


Figure 3. Bare land values with 0-6 thinnings relative to the maximum at 3% rate of interest.

Figure 4 shows how the stand basal area develops over stand age in optimum regimes for three juvenile densities. The optimum regime for the juvenile density giving the highest bare land value (2245 trees ha^{-1}) is characterized by early thinning and efficient utilization of the growing space at early ages. The first thinning is carried out just before the stand reaches the critical basal area when the trees start to die because of self-thinning. By this time, the removed trees have reached commercial dimensions and enable positive (1272 € ha^{-1}) net harvest revenue. The first thinning is heavy compared to the later thinnings. This is caused by the large number of inferior-quality trees that are removed at the earliest possible thinning.

The growing stock in the unspaced stand (8327 trees ha^{-1}) reaches the self-thinning limit at about 30 yrs age. However, the first thinning is postponed up to 48 yrs of age because earlier thinning would yield a high logging cost and small removal of commercial timber assortments. Late first thinning leads to high mortality and slow diameter growth. The subsequent thinnings after a heavy first thinning are lighter and lead to gradually decreasing stock toward the end of rotation.

There is only one very heavy thinning in the optimum regime for low (994 trees ha^{-1}) juvenile density. Note, however, that such a heavy thinning may expose a forest stand to windthrow damage. Unfortunately, models taking into account the effects of thinning on the probability of windthrow are not available. Accounting for the increased probability of windthrow is likely to lead to a lighter first thinning and an increased proportion of inferior-quality trees in later thinnings.

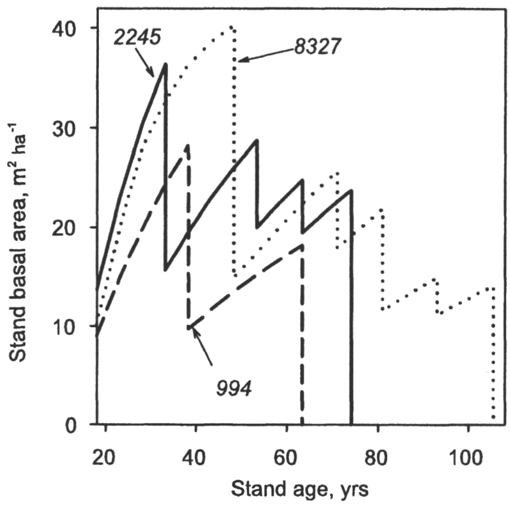


Figure 4. Optimum solutions for three juvenile stand densities at 3% rate of interest.

For very low (0-1%) rates of interest, the first thinning is carried out at the first possible moment (19 yrs). Early thinning yields some commercial timber (small-dimension pulpwood), but the net harvest revenue is negative due to high harvesting cost. For low rates of interest, such investment is rational, as it increases the future stand value. For higher rates of interest, optimum first thinning is carried out later, but there is a local maximum for solutions with immediate thinning (cf. Roise 1986).

Type of thinning

Figures 5a-d show the proportion of trees removed in optimum thinnings for four juvenile densities and 3% rate of interest. The consecutive trees in the list are connected with a line for each thinning. The thinning type is defined with three optimized variables that define thinning rates for minimum, average and maximum diameters. Thinning rates for other diameters are interpolated. The average-sized tree is illustrated by a corner in the curve. Juvenile spacing and the resulting reduced within-stand competition narrows the tree-height frequency distribution. Hence, virtually all trees belong to upper canopy layer trees and can be classified as either dominant or codominant trees in spaced stands. In an unspaced stand, by contrast, trees can be classified more clearly as upper canopy layer trees, intermediate trees and suppressed trees.

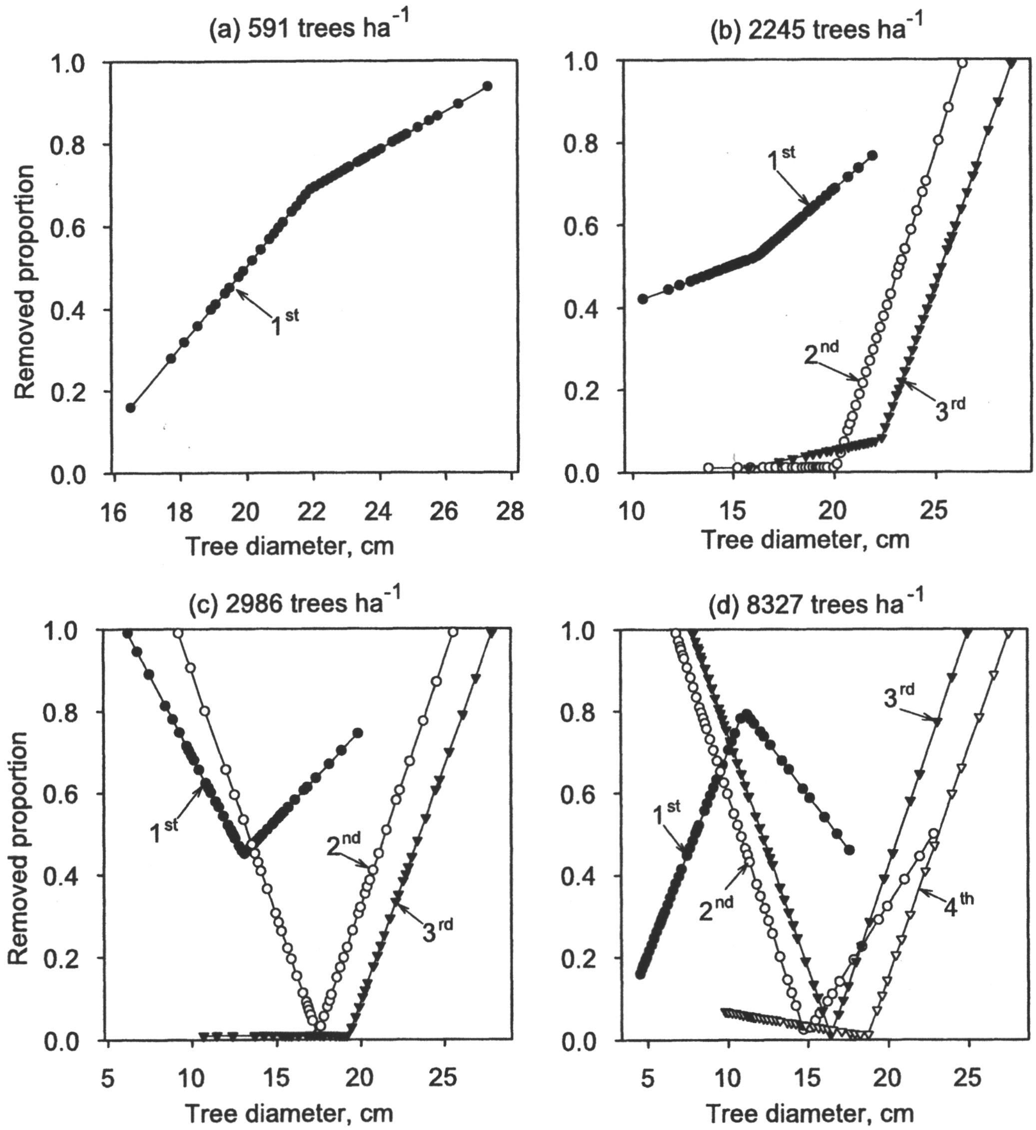


Figure 5. Removal rates in optimum thinnings (1st, 2nd,...) at 3% rate of interest.

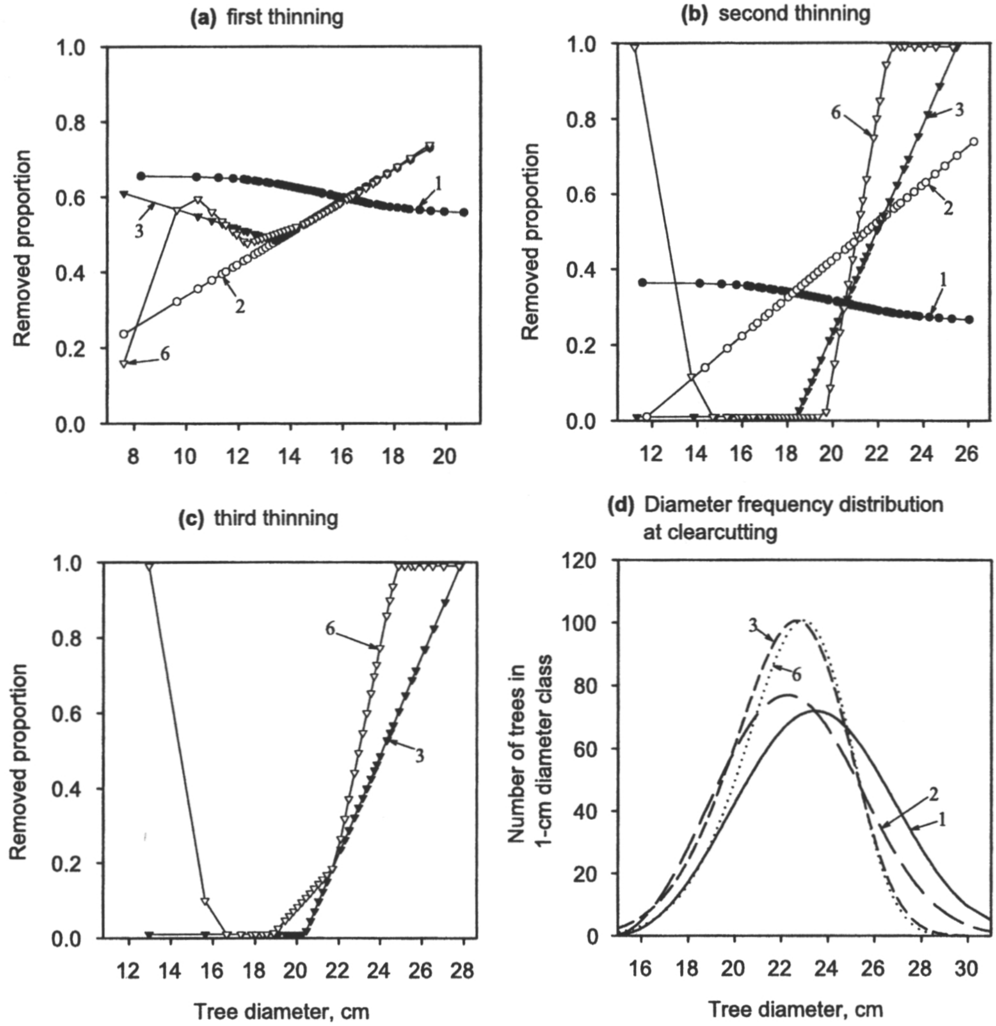
Sparse stands are typically thinned from above and dense stands from both ends of the diameter distribution. The first thinning is aimed at speeding up the development of the most vigorous high-quality trees by removing inferior-quality trees from all canopy layers. In sparse stands, thinning rates are highest for the large trees because of the high proportion of inferior-quality trees. In dense stands, some additional small trees are also removed in the first thinning. For an unspaced stand (8327 trees ha^{-1} in Figure 5d), the first thinning removes particularly intermediate trees because the suppressed trees have not yet reached commercial dimensions and the majority of the dominant trees are of high quality.

Figures 6a, b and c show how the number of optimized thinning type variables used to define each thinning changes the type of the first, second and third thinning, respectively. Two thinnings are optimal with one or two optimized thinning type variables and three thinnings with three or six variables. In the one-variable case, the slope of the curve over tree diameter is predetermined such that a slightly larger proportion of small versus large trees is removed. The first thinning removes especially inferior quality trees over all tree diameters. The types of the later thinnings, however, depend crucially on the number of optimized thinning type variables.

Applying the most flexible thinning type definition (with six optimized thinning type variables), the later thinnings are selective thinnings that remove all trees above and below certain tree diameters, and retain all the average-sized trees. A tree is removed in thinning if its removal and the resulting accelerated growth of its neighbors contribute more to the net present value of a stand than its continued growth. The dominant trees are removed soon after they reach minimum sawlog dimensions, despite a positive price premium for sawlogs.

The number of thinning type variables also affects the final diameter frequency distribution, as shown in Figure 6d. The most flexible thinning type definition leads to a sharply peaked diameter frequency distribution. In addition, increasing the number of thinning type variables from one to three slightly decreases the mean diameter of the clearcut stock.

Careful consideration of which trees to remove in thinning increases the economic surplus of forestry, particularly for dense stands that are thinned several times during the rotation. Figure 7 shows that flexible thinnings may increase maximized bare land value up to 14% compared to a given type of thinning



Note: The final diameter frequency distributions in (d) are fit ro four-parameter Weibull-distribution. The final density of the stand is 2245 trees ha^{-1} and the rate of interest is 3%.

Figure 6. Effects of the number of optimized thinning type variables (1, 2, 3 and 6) on optimal thinnings and final diameter frequency distribution.

(one-variable case). Increased flexibility enables thinnings that remove trees strictly below or above certain diameter limits. This is beneficial for several reasons. Firstly, dominant trees can be removed in thinning soon after they have reached sawlog dimensions and the last high peak in value growth. Secondly, removing small trees in thinning advances clearcutting. Thirdly, homogeneous stand structure enables low average diameter at clearcutting (see Figure 6d). This reduces the capital costs from retaining the largest stock trees. Wikström (2001) shows analogously that bare land value may increase up to 10-20% when the number of decision variables is increased, but does not show how the harvests are altered.

Our results and earlier studies suggest that it is rational to remove those trees that have no prospects for a high value growth rate. However, optimum tree selection and timing of thinnings depend heavily on the applied price and timber assortment functions. Earlier studies suggest that the dominant trees may be removed in thinning if they have reached minimum commercial dimensions (Haight 1987) or if the butt log has reached a certain value (Eriksson 1994). Suppressed trees may be removed if they would not reach favorable dimensions before the end of rotation (Eriksson 1994) or if their removal speeds up development of other subcommercial trees

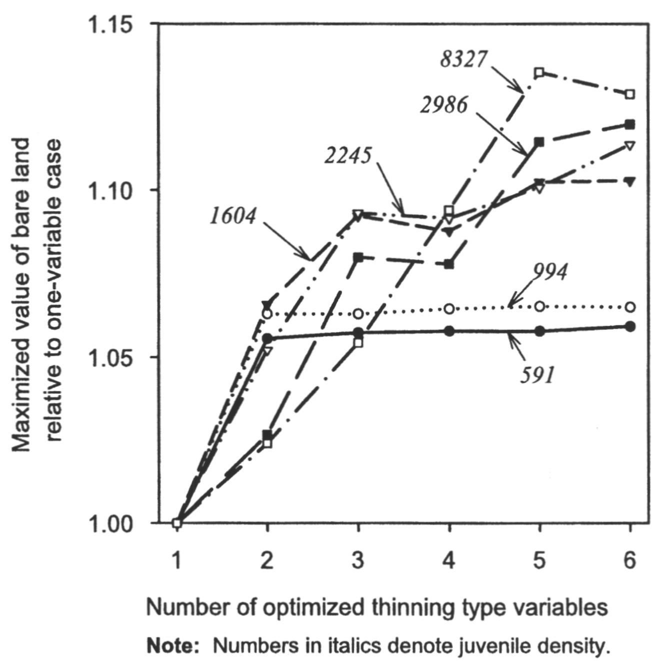


Figure 7. Bare land values as function of number of optimized thinning type variables at 3% rate of interest.

(Haight 1987). Inclusion of natural regeneration may induce removal of planted trees from above in order to create growing space for volunteer trees (Haight and Monserud 1990b).

Earlier Finnish optimization studies for Scots pine suggest that thinning from above is superior to thinning from below (Vettenranta 1996, Pukkala and Miina 1998). This result is supported by growth and yield studies of Vuokila (1970, 1977) and Mielikäinen and Valkonen (1991). They use empirical data from permanent thinning experiments to compare yields from stands thinned from above and below. They show that thinning from above yields about the same or higher volumes of commercial timber during the exogenously given rotation period. However, discounting at a positive rate of interest makes thinning from above economically more advantageous.

Thinning from above or from both ends of the diameter distribution is reported to be optimal also for other pines such as lodgepole pine (*Pinus contorta* Doug. ex Loud.) (Haight et al. 1985), ponderosa pine (*Pinus ponderosa* Laws.) (Haight 1987) or white pine (Haight and Monserud 1990b) and more shade-tolerant species like Douglas-fir (*Pseudotsuga menziesii* [Mirb.] Franco) (Roise 1986) or Norway spruce (Solberg and Haight 1991, Valsta 1992a, b, Eriksson 1994). Thinning from below may be superior at the first commercial thinning (Haight et al. 1985) or precommercial thinning (Roise 1986) or if production of cubic meters is maximized instead of economic surplus of forestry (Haight and Monserud 1990b, Valsta 1992b).

Contrary to the above studies, Arthaud and Klemperer (1988) consistently find thinning from below economically superior to thinning from above in loblolly pine plantations. An obvious reason for this result is that loblolly pine is a species that is highly intolerant of shade. However, Arthaud and Klemperer (1988) include only four extreme thinning type options in their dynamic programming code. More flexible thinning options could change the results.

The few research results available for hardwoods imply that thinning from above may not be advantageous for light-demanding broad-leaved species. Vuokila (1977) shows that thinning from above leads to reduced timber production in boreal silver birch (*Betula pendula* Roth) stands. Rautiainen et al. (2000) find thinning from below optimal for tropical sal (*Shorea robusta* Gaertn. f.) stands.

As an extension to the earlier stand-level optimization literature, we show that careful tree selection in thinning may simultaneously raise high immediate harvest revenues and lead to economically advantageous structure of the residual stock. The first thinning removes suppressed and inferior-quality trees after the unit price of timber has reached the top level. The later thinnings remove in particular valuable sawlog trees, but also increase the value growth rate of the residual stock by homogenizing the stand structure.

Rotation period

Figure 8 shows length of optimum rotation period for all analyzed juvenile stand densities and the rates of interest that yield positive bare land value. The optimum solutions are described by average tree diameter (weighed by tree basal area) and stand age at end of rotation period, in order to enable comparison with the minimum legal restrictions in the Finnish Forest Act (Ministry of Agriculture and Forestry 1997). The solutions for various rates of interest are connected with a line for each juvenile stand density.

The optimum rotation length is 74 yrs for a 3% rate of interest and the juvenile density giving the highest bare land value (2245 trees ha^{-1}). This rotation is illegally short, but is well in line with other Finnish results given for similar sites. Optimum length of rotation period varies between 65 and 80 yrs for Scots pine stands (Pesonen and Hirvelä 1992, Salminen 1993, Miina 1996, Hyytiäinen and Tahvonen 2001) and Scots pine and Norway spruce mixtures (Vettenranta 1996, Pukkala and Miina 1998,

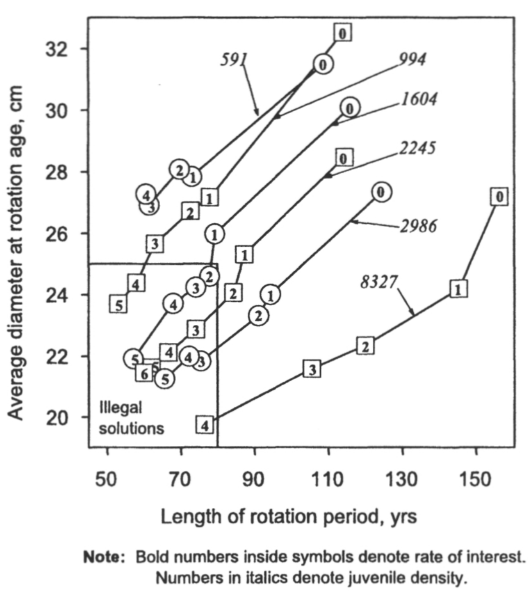


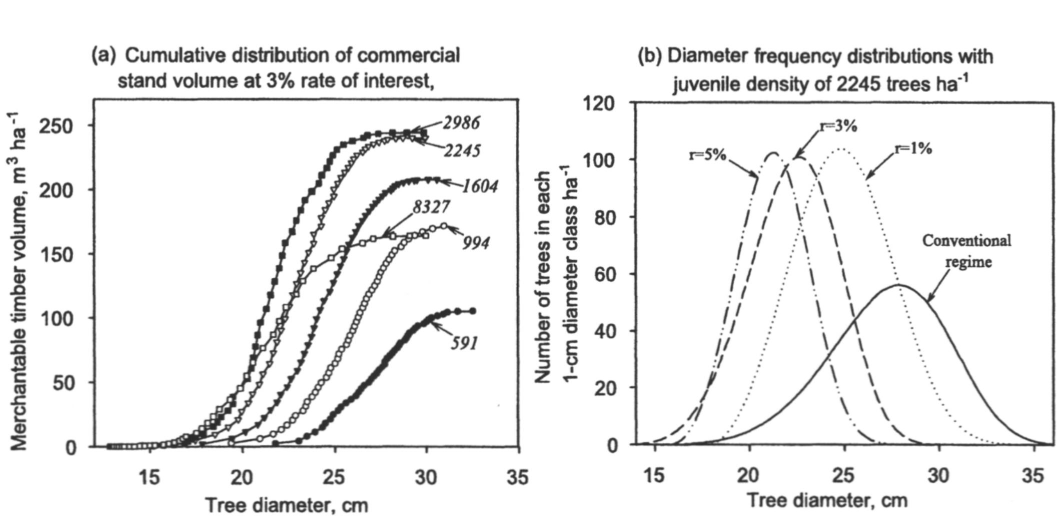
Figure 8. Optimum length of rotation period and average diameter at rotation age.

Pukkala et al. 1998b, Vettenranta and Miina 1999). Pukkala and Miina (1998) and Hyytiäinen and Tahvonen (2002) report slightly longer optimum rotation periods (80-95 yrs) for pure Scots pine stands.

Figure 8 shows that the length of optimum rotation period is sensitive to variations in both the rate of interest and juvenile density. An increase in the rate of interest reduces the length of rotation period and average diameter at clearcutting. An increase in the juvenile stand density lengthens the rotation period but reduces the final diameter. Long rotations in dense stands are due to high within-stand competition, which reduces diameter growth of individual trees and postpones the date when the trees reach commercial dimensions.

Figure 9a shows how commercial stand volume is distributed over tree diameter at the end of the rotation period for six juvenile stand densities at 3% rate of interest. Individual trees are denoted by symbols that are connected with a line to represent each juvenile density. The curves are cumulative, and the symbol representing the largest tree gives the total stand volume.

The clearcut timber volume increases with juvenile stand density in optimum solutions, with the exception of the unspaced stand (8327 trees ha^{-1}). The proportion of trees that fulfill the dimension requirements for sawlog trees (*d*>19.1 cm) is high for all analyzed densities but decreases modestly with juvenile stand density. The



Note: In Figure 9b, the actual distributions are fit to 4-parameter Weibull distribution.

Figure 9. Stand states at rotation age.

slope of the cumulative curve is steeper for high stand densities, implying a more centered frequency distribution. The juvenile density yielding the highest bare land value (2245 trees ha^{-1}) is characterized by high final volume, homogeneous stand structure and a large proportion of trees that have just recently reached sawlog dimensions. The diameter frequency distributions are symmetric for spaced stands but somewhat skewed for the unspaced stand (8327 trees ha^{-1}). This implies that the optimum thinnings retain average-sized trees for spaced stands and a proportion of the largest trees for unspaced stands.

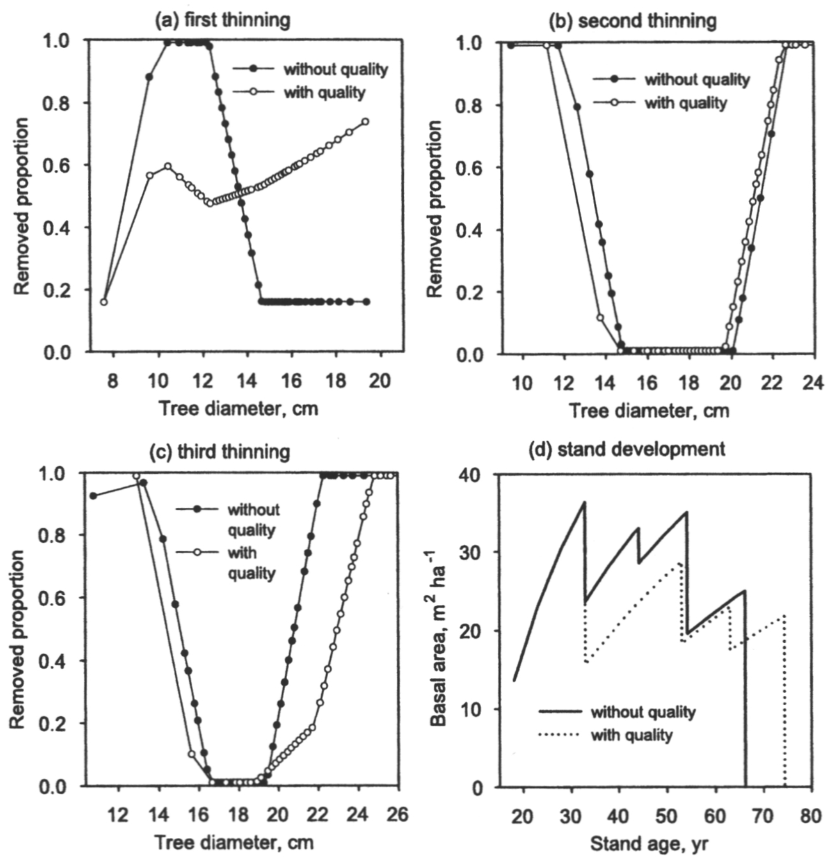
Figure 9b shows the frequency diameter distributions at the end of conventional and optimum (1, 3 and 5% rates of interest) rotations with juvenile density of 2245 trees ha^{-1}. The conventional management regime is designed to accord with the official guidelines (Tapio 2001). The recommended thinning removes a slightly higher proportion of small than large trees. To better illustrate the differences, the actual distributions are fit to a four-parameter Weibull-distribution. Increasing the rate of interest leads to lower final volume, a narrower and more highly-peaked frequency distribution, and lower mean diameter. Compared to optimum solutions, the final frequency distribution of the conventional regime is much wider and has a higher mean diameter.

The optimum solutions for various rates of interest and juvenile densities are characterized by a high proportion of sawlog trees at clearcutting. This is natural because of the large difference in timber product (sawlogs and pulpwood) prices. There is a high peak in the value growth of an individual tree when the tree's diameter grows from about 19 to 21 cm, and the lower part of the stem reaches sawlog dimensions. For higher diameters, the increase in sawlog unit price and the stem's sawlog proportion smoothly declines. In addition, by this age, the tree has already passed the most vigorous growth phase. Consequently, the value growth of the homogeneous stand declines steeply after the majority of trees have reached sawlog dimensions, and it soon becomes optimal to clearcut the stand. Valsta (1992b) obtains a similar result for Norway spruce, but argues that early clearcutting may be caused by neglect of the price premium for high-dimension sawlogs. However, our results imply that recognition of the price premium extends the optimum rotations only slightly.

Impacts of timber quality on harvesting

Neglecting the effects of timber quality on timber assortment prices changes particularly the type of the first thinning as shown in Figure 10a. Without quality, the first thinning removes all commercial trees below a certain tree diameter. Such thinning is targeted to promote growth of the dominant trees and to enable earlier clearcutting (see Figure 10d).

Removing trees from below at the first thinning is optimal also in Haight et al. (1985) and Roise (1986). However, inclusion of timber quality makes it more profitable to remove first the inferior-quality trees rather than narrowing the diameter distribution already at the first thinning. The later thinnings (Figures 10b and c) remove trees strictly below and above certain diameters, thus homogenizing the stand structure irrespective of timber quality.



Notes: Juvenile density is 2245 trees ha^{-1}, rate of interest 3%, number of optimized thinning type variables six.

Figure 10. Optimum thinnings and stand development with and without quality effects.

Explicit inclusion of timber quality increases the intensity of the first thinning and extends the rotation length (Figure 10d). Depending on harvest specification and juvenile density, the optimum rotation may be lengthened by 0-10 yrs at a 3% rate of interest (not shown). Neglecting timber quality, by contrast, leads to lighter first thinning and heavier last thinning.

CONLUSIONS

The optimum management strategy for Scots pine stands, according to our results, is as follows: Establish the stand to the juvenile density (about 2100-2500 trees ha^{-1}) that utilizes efficiently the growing space before the first thinning. At the first thinning, remove all the inferior-quality trees and a proportion of the small commercial trees in favour of the most vigorous trees. In the later thinnings, remove systematically all sawlog trees and a proportion of the smallest commercial trees. Clearcut the stand once the majority of the remaining trees have reached sawlog dimensions.

As a policy-relevant issue, we note that the optimal solutions are inconsistent with Finnish forestry legislation. Optimum length of rotation period is typically shorter than allowed when 3% or higher rate of interest is applied. In addition, the Forest Act (Ministry of Agriculture and Forestry 1997) requires retaining of the upper canopy layer trees in thinning. This may be interpreted as a normative statement to remove trees from below in forestry extension and practical forest management. On the contrary, our results suggest that selective thinnings removing valuable sawlog trees are economically favorable at least for well-managed and homogeneous stands.

Removing trees systematically above a certain diameter limit in thinning may not be rational if the trees are irregularly arranged in a stand. In this case, it may be rational to remove first those dominant trees that overtop small trees. Lone sawlog trees may be retained longer. However, the spatial tree selection problem can be explicitly considered only by using a distance-dependent growth model (see Pukkala and Miina 1998).

Stands that are thinned from above are poorly represented in the data from which the growth equations are derived (Hynynen et al. 2002). Thus, the optimum solutions are outside the most reliable range of applicability. The validity of our results depends on how the growth of codominant trees responds to the removal of dominant trees. Because Scots pine is a pioneer species, it is assumed to tolerate shade poorly. However, Vuokila (1970, 1977) and Mielikäinen and Valkonen (1991) present empirical evidence that removal of the overstory increases significantly the diameter growth of the residual codominant and intermediate trees. Another argument against thinning from above is that it damages residual trees more than does thinning from below. However, Jäghagen and Lageson (1996) compared the residual stocks of Swedish Scots pine stands after thinning from above and thinning from below, and found no difference in the frequency of trees with logging damage.

Our results and the earlier literature suggest that thinning from above is economically attractive for many shade-tolerant and also some pioneer conifer species. This is in contrast to forest management practice in many countries. However, additional economic research using varying initial states, distance-dependent growth models, and more detailed specifications of timber quality are needed to verify and generalize this result.

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Paper V

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V

Connecting a process-based forest growth model to stand-level economic optimization

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Olli Tahvonen and Juhani Taipale

ABSTRACT

This study extends the economic literature on forest stand management by applying a process-based, rather than empirical, stand growth model. The economics of timber production is investigated using a distance-independent individual-tree process model specified for pure Scots pine (*Pinus sylvestris* L.) stands. Stem taper and crown morphology information are used for bucking the harvested trees into several roundwood categories according to quality and dimension requirements applied in the Finnish timber markets. Explicit inclusion of causality and timber quality in stand-level economic optimization generates a set of new results. Economic optimization decreases biomass production but increases roundwood production, compared to undisturbed stands. Optimal rotation length is insensitive to changes in the rate of interest beyond 4% due to nonmonotonic value growth. Better quality attributes and higher productivity in resource use are partial reasons for favoring lower canopy trees in optimal thinnings. The first thinnings are light, irrespective of the rate of interest, due to their favorable feedback effects on the quality of residual trees. Production of the highest-grade roundwood is rational only at rates of interest lower than those prevailing in the capital markets. An example of two optima representing distinct timber management strategies is shown.

INTRODUCTION

Economic questions of forestry can be studied with optimization models that contain descriptions of forest stand growth, harvesting technology, and economic parameters. Stand growth models must include those attributes of tree populations that best reflect the value of the stand. In addition, they must reliably predict development of stand value over time and in response to different management activities and disturbances. Models representing varying degrees of detail have been used to study various economic aspects of forestry (for a review, see Getz and Haight 1989).

The common feature of all earlier economic studies of forestry is that the description of growth is based on empirical models. In empirical models, the equations predicting stand or tree growth are estimated statistically from the available field measurement data. Statistical treatment of tree and stand data is a challenging task due to e.g. multicollinearity problems. Well-established empirical growth models produce reliable predictions of growth and yield, but only as long as they are applied to silvicultural, ecological and climatic conditions that fall within the range of conditions for which the model is built and estimated. Limited range of application becomes a problem in economic optimization, because the optimal solutions do not always fall within conventional forest management practices.

Process-based models provide an alternative approach to describe stand growth. They are based on well-founded forest ecological theories (see e.g. Bossel 1991, Hari 1999) and describe the fundamental ecophysiological processes that drive accumulation of biomass in different tree compartments: bole, branches, roots and foliage. Growth is determined by the balance between carbon gained through photosynthesis and lost through respiration and senescence. Process-based models typically consist of a number of submodels which together create a complex hierarchical structure analogous to real systems. As a consequence, process-based models explicitly include the causal relationships that drive the growth and development of trees within stands.

Originated from the need to integrate complex layers of biological information, process-based models have seen a rapid development during the last 20-30 years (Mäkelä et al. 2000a, Johnsen et al. 2001). They have contributed to the literature particularly by deepening our understanding of how forest ecosystems function. Recently, several suggestions have been forthcoming on the use of process-based models in forest economics and management research, due to their highly promising but still underutilized capabilities (e.g. Korzukhin et al. 1996, Battaglia and Sands 1998, Mäkelä et al. 2000a, Johnsen et al. 2001).

Tree process models have notable advantages over empirical models in studying the economics of timber production. The explanatory power of process models is high due to hierarchical structure and description of the fundamental growth processes. Therefore, process models can produce better justified predictions of growth in conditions from where empirical data do not yet exist (see e.g. Bossel 1991, Korzukhin et al. 1996, Battaglia and Sands 1998). As another consequence of causality, process models explicitly include the feedback effects of stand management (thinnings, silvicultural activities) on tree and stand characteristics. Detailed description of crown morphology and stem taper enables accurate determination of both quantity and quality of commercial timber.

Including the timber quality defects in the economic analysis of forestry remains a challenge. As first attempts, Haight et al. (1995) account for the negative effects of storm damage risk, Zhou (1999) the effects of initial spacing, Möykkynen and Miina (2002) the butt rot effects, and Hyytiäinen et al. (2003) the effects of juvenile growth on timber quality. However, even the most sophisticated empirical models are incapable of providing enough information to fully account for feedback effects of thinnings and other silvicultural activities on branchiness and other quality attributes.

The main objective of the present study is to link a detailed forest process model with a stand-level optimization model. This is of particular interest because, to our knowledge, process-based growth models have not been previously integrated into economic optimization. Another, more pragmatic, objective is to elaborate the underlying economic and ecological determinants of optimal solutions when detailed descriptions of timber quality and roundwood categories are explicitly included in the optimization model. A distance-independent individual-tree process model (Mäkelä 1997, 2002, Mäkelä and Mäkinen 2003) is used to project growth of trees in pure Scots pine stands. This model has been tested extensively with empirical data (see e.g. Mäkelä et al. 2000b) and is directly amenable to economic analysis of harvesting and silvicultural activities. Stem taper and crown morphology information provided by the model are used for bucking the harvested trees into roundwood categories according to prevailing dimension and quality requirements applied in the

Finnish timber markets.

The stand-level optimization model containing a process-based description of growth and population dynamics is presented next. Then, results on biomass and roundwood production in optimally managed and undisturbed stands are shown and discussed. Sensitivity analyses for different rates of interest and roundwood prices are conducted in order to analyze the underlying economic and ecological determinants of optimal harvests and long-run timber supply. An example of multiple optima is shown. Finally, the optimization results from a process model are compared with results from a well-validated empirical model.

MATERIALS AND METHODS

Optimization model based on process-based description of growth

Stand, tree, whorl and branch submodels form a complex hierarchical structure in a process-based stand growth model by Mäkelä (1997, 2002) and Mäkelä and Mäkinen (2003). This model falls into the category of distance-independent individual-tree models (see e.g. Getz and Haight 1989, p. 239). The stand is made up of n size classes each containing a varying number of similar trees. The trees are of the same age and are randomly distributed over the land area. The interaction of trees follows from shading and crown coverage. Shading reduces photosynthesis and higher crown coverage increases the rate of tree mortality and crown rise.

Given any state defined by the state variables at stand, tree, whorl and branch levels (see Table 1), the model determines the stand level rate of photosynthesis measured in terms of dry weight of carbon. This is distributed among the tree size classes on the basis of foliage biomass, crown size and relative vertical location of the crown. The net output of each tree is the difference between photosynthesis and respiration. The net growth of each biomass compartment equals the difference between carbon allocated to the given compartment and senescence. Senescence of foliage and fine roots is proportional to live biomass. The stem consists of heartwood and active sapwood. Sapwood is made up of pipes that serve as transport channels for nutrients and water from fine roots to foliage. In the progress of tree growth, the lowest branches die and the crown base rises, with the consequence that the related active sapwood turns into unused pipes or heartwood.

Table 1. State variables of the tree module.

Variable	Definition	Unit
T_{1k}	Foliage mass	dry weight in kg
T_{2k}	Branch sapwood mass	dry weight in kg
T_{3k}	Branch heartwood mass	dry weight in kg
T_{4k}	Stem sapwood mass	dry weight in kg
T_{5k}	Stem heartwood mass	dry weight in kg
T_{6k}	Transport root sapwood mass	dry weight in kg
T_{7k}	Fine root mass	dry weight in kg
T_{8k}	Crown length	m
T_{9k}	Height of crown base	m
T_{10k}	Stem sapwood area at crown base	m^2
T_{11k}	Tree basal area measured under bark at breast height	m^2

Carbon produced in photosynthesis is allocated among the biomass compartments by applying several structural relationships. First, pipe theory (Shinozaki et al. 1964) yields a ratio between sapwood area at crown base and foliage biomass. Second, an environment-dependent ratio between fine root and foliage mass is assumed (Davidson 1969, Vanninen and Mäkelä 1999). Third, crown surface area is an exponential function of foliage mass (Mäkelä and Sievänen 1992). Fourth, mean branch length is a linear function of crown length. Finally, the rise of the crown base is determined by the competitive status of the tree.

The tree module yields the annual growth of the biomass and dimensional variables for any given vertical structure of the tree. The whorl module obtains the annual growth as an input and updates the vertical and internal structure of the stem and branches and computes the foliage biomass, sapwood and heartwood areas and branch length over the whorls. Given the updated structure of the tree and the updated stand level characteristics, the tree module then determines the growth of the tree for the next period.

The branch module provides the initial number and size of branches in new whorls in addition to the rate of branch death within the crown. In addition, the model includes an empirically based prediction when a dead branch is shed and wood without branches is formed. The branch module obtains input from the whorl module but there is no feedback from branch module to whorl module.

For purposes of economic optimization, we describe the model by Mäkelä (1997, 2002) and Mäkelä and Mäkinen (2003) explicitly as a dynamic system, i.e. in terms of the state variables and their development over time (see Table 1). Let t denote the age of the stand and t_i, $i = 1,\ldots,m$, its age at the i^{th} harvest over rotation. Let $\mathbf{N} = (N_1,\ldots,N_n)$ denote the number of trees in the n size classes. The tree module state variables specified separately for each size class k are given by $\mathbf{T}_k = (T_{1k},\ldots,T_{uk})$, $k = 1,\ldots,n$, where u is the number of tree level state variables. The whorl state variables are specified separately for each whorl in each size class and are denoted by $\mathbf{W}_{jk} = (W_{1jk},\ldots,W_{yjk})$, $j = 1,\ldots,g$, $k = 1,\ldots,n$, where y and g are the number of whorl state variables and the number of whorls respectively. Finally, each branch is described by l states: $\mathbf{B}_{djk} = (B_{1djk},\ldots,B_{ldjk})$, $d = 1,\ldots,s$, $j = 1,\ldots,g$, $k = 1,\ldots,n$, where s is the number of branches in each whorl. Thus, the stand is described by $n+nu+ngy+ngsl$ state variables. In the economic computations of this study n=10, u=11, g=60-140, y=3, s=5, l=2, i.e. the number of state variables varies between 7920 and 18320 in addition to stand age t.

Between thinnings, the time derivative of each of the tree level state variables depends on the number of trees in all the age classes and on the state of each tree as described by the tree and whorl module state variables. Let $\mathbf{T}$, $\mathbf{W}$ and $\mathbf{B}$ denote functions that define the relationships between the state variables. The development of the tree level state variables can be described in a general form as

$$\frac{d\mathbf{T}_k(t)}{dt} = \mathbf{T}[\mathbf{T}_1(t),\ldots,\mathbf{T}_n(t),\mathbf{W}_{1k}(t),\ldots,\mathbf{W}_{gk}(t),\mathbf{N}(t)],\ k = 1,\ldots,n, \qquad [1]$$
$$for\ t \in [t_{i-1},t_i),\ i = 1,\ldots,m.$$

The development of the state variables of a whorl depend on the whorl module state variables over all the whorls and on the tree module state variables:

$$\frac{d\mathbf{W}_{jk}(t)}{dt} = \mathbf{W}[\mathbf{W}_{1k}(t),\ldots,\mathbf{W}_{gk}(t),\mathbf{T}_k(t)],\ j = 1,\ldots,g,\ k = 1,\ldots,n. \qquad [2]$$

The development of the branch module state variables depend on the branch and whorl module state variables:

$$\frac{d\mathbf{B}_{djk}(t)}{dt} = \mathbf{B}[\mathbf{B}_{djk}(t),\mathbf{W}_{jk}(t)],\ d = 1,\ldots,s,\ j = 1,\ldots,g,\ k = 1,\ldots,n. \qquad [3]$$

The number of trees in each size class depends on mortality and harvesting activity. The rate of mortality depends on stand density, which is measured by crown coverage summed over all trees and size classes. This reflects the effects of shading on photosynthesis. Crown coverage per tree is computed using crown length and a specific factor for crown shape (Mäkelä 2002). Thus, between thinnings, the number of trees evolves according to

$$\frac{dN_k(t)}{dt} = U[\mathbf{N}(t), \mathbf{T}_1(t), \ldots, \mathbf{T}_n(t)], \ t \in [t_{i-1}, t_i), \ k = 1, \ldots, n. \qquad [4]$$

Thinnings are discontinuities in the number of trees in each size class. The number of trees per size class after thinning t_i equals the difference between number of trees before thinning and number of trees harvested at the i^{th} thinning. In addition, thinnings cannot exceed the number of trees available. Thus we obtain

$$N_k(t_i) = N_k(t_i^-) - h_{k,t_i} \geq 0, \ i = 1, \ldots, m-1, \ k = 1, \ldots, n. \qquad [5]$$

The initial number of trees at t_0 are given by

$$N_k(t_0) = N_{k,t_0}, \ k = 1, \ldots, n. \qquad [6]$$

Accurate description of stand growth and its characteristics requires a high enough number of tree size classes. This may, however, yield an excessive number of optimized variables. To deal with this, we apply a set of piecewise linear restrictions between numbers of trees thinned in the different size classes (cf. Valsta 1992). Given any state of the stand, there exists some diameter distribution over the size classes. In Figure 1 the diameter difference between smallest and largest size class is divided into three segments of equal length: $\tilde{x}_j = x_1 + (x_{10} - x_1)(j-1)/3, \ j = 1, \ldots, 4$, where $\tilde{x}_j$, $j = 1, \ldots, 4$, denotes the boundaries of each segment. Let $\gamma_j \in [0,1]$, $j = 1, \ldots, 4$ satisfy the system

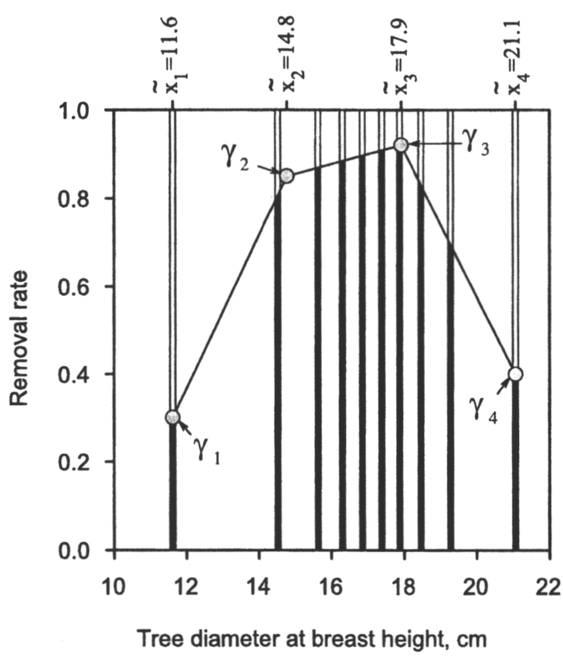
$$\begin{aligned} a_1 \tilde{x}_1 + b_1 &= \gamma_1, \\ a_1 \tilde{x}_2 + b_1 &= \gamma_2, \\ a_2 \tilde{x}_2 + b_2 &= \gamma_2, \\ a_2 \tilde{x}_3 + b_2 &= \gamma_3, \\ a_3 \tilde{x}_3 + b_3 &= \gamma_3, \\ a_3 \tilde{x}_4 + b_3 &= \gamma_4, \end{aligned} \qquad [7]$$

where a_j and b_j, $j = 1,2,3$ are constants. Clearly, choosing γ_j, $j = 1,\ldots,4$ determines a_j and $b_j, j = 1,2,3$. The number of trees harvested in each size class is then determined by

$$h_{k,t_i} = N_k(t_i^-)(a_j x_k + b_j), \text{ given } x_k \in [\tilde{x}, \ \tilde{x}_{j+1}\ ], \ j = 1,2,3, \ k = 1,\ldots,n. \qquad [8]$$

Thus, the choice of $\gamma_j \geq 0$, $j = 1,\ldots,4$ determines h_{k,t_i}, $k = 1,\ldots,n$, for thinnings $i = 1,\ldots,m-1$. All the residual trees are removed in the final harvest: $h_{k,t_m} = N_k(t_m^-)$, $k = 1,\ldots,n$.

We note that there are various alternative specifications to reduce the number optimized variables in thinnings (Wikström 2001). We apply the above specification partly because it leads to realistic outcomes from the point of view of existing thinning technology and partly in order to maintain comparability with respect to some existing empirical models (Valsta 1992, Valsta and Linkosalo 1996).



Note: Four optimized variables ($\gamma_1, \gamma_2, \gamma_3, \gamma_4$) and piecewise linear restrictions are used to determine removal rates for different size classes.

Figure 1. Illustration of thinning at age 38 yrs removing smaller proportions of small and large than average-sized trees.

Stem taper and information about the vertical distribution of branches are used for bucking tree boles into sections representing various roundwood categories. The harvested timber is divided into six commercial roundwood categories R_{vk}, $v = 1,\ldots,6$, and residual wastewood R_{7k}. Sawlogs are divided into superior-quality butt logs ($v = 1$), ordinary butt logs ($v = 2$), middle logs ($v = 3$) and top logs ($v = 4$). Pulpwood is divided into roundwood that is unsuitable for sawlogs because of quality defects ($v = 5$) and pulpwood from tree tops ($v = 6$). The harvested volumes depend on the state variables of the tree, whorl and branch modules and the number of trees harvested:

$$R_{vk} = R_v[\mathbf{T}_k(t_i), \mathbf{W}_{1k}(t), \ldots, \mathbf{W}_{gk}(t), \mathbf{B}_{11k}(t), \ldots, \mathbf{B}_{s1k}(t), \ldots \mathbf{B}_{1gk}(t), \ldots, \mathbf{B}_{sgk}(t),\ h_{k,t_i}], \quad v = 1,\ldots 7,\ k = 1,\ldots,n. \qquad [9]$$

The dimension and quality requirements for various roundwood categories are described in the Appendix.

The logging costs depend on the number of trees harvested and sawlog, pulpwood and wastewood volumes;

$$C_{t_i} = C\left[\sum_{k=1}^{n} h_{k,t_i}, \sum_{v=1}^{4}\sum_{k=1}^{n} R_{vk}(t_i), \sum_{v=5}^{6}\sum_{k=1}^{n} R_{vk}(t_i), \sum_{k=1}^{n} R_{7k}(t_i)\right] + \overline{C},\ i = 1,\ldots,m, \qquad [10]$$

where C_{t_i}, $i=1,\ldots,m-1$, denote the logging costs for thinnings, C_{t_m} the logging costs for clearcutting, and $\overline{C}$ fixed logging costs. We apply a logging cost function that is based on detailed empirical field data concerning the productivity of logging. The details are given in the Appendix.

The system [1]-[10] forms the constraints for the problem of maximizing the present value of economic surplus over an infinite chain of similar rotations. Let p_v, $v = 1,\ldots,6$, denote the prices of commercial roundwood categories and r the real market rate of interest. Stand establishment cost is denoted by S. It may depend on the initial number of trees. The objective function for maximizing the present value of economic surplus over infinite horizon takes the form

$$\max_{\substack{\{N_{t_0}, m, t_i,\ i=1,\ldots,m, \\ \gamma_{jt_i}, j=1,\ldots,4, \\ i=1,\ldots,m-1\}}} V = \left[\sum_{i=1}^{m} e^{-rt_i}\left(\sum_{v=1}^{6} p_v \sum_{k=1}^{n} R_{vk}(t_i) - C_{t_i}\right) - S(N_{t_0})\right](1 - e^{-rt_m})^{-1}. \qquad [11]$$

Data and computations

The process-based individual tree growth model by Mäkelä (1997, 2002) and Mäkelä and Mäkinen (2003) is parameterized for pure stands of Scots pine (*Pinus sylvestris* L.) growing on *Vaccinium*-type sites (Cajander 1949) in Fenno-Scandia. The model allows for any initial stand age or state. In order to maintain comparability with earlier studies and other models, an 18-yr old even-aged stand with initial density of 2250 trees ha^{-1} is selected as the initial state. A stand structure is described with 10 mean trees, each representing a size class. Initial tree heights vary between 4.36 and 6.85 m, tree diameters between 5.3 and 12.3 cm (measured over bark at breast height), and height of crown base between 1.08 and 1.35 m. Each mean tree represents an equal initial number of trees. The data is obtained from a permanent thinning experiment (plot number 13 in Varmola 1982) located in Southern Finland. The cost estimates for establishing this stand are given in the Appendix.

The objective function surface is highly nonmonotonic due to annual variation in tree growth and discontinuous shifts of timber volume between roundwood categories (cf. Martin and Ek 1981). However, with sufficiently long step lengths, the small annual variation in foliage mass, and consequently in growth, does not hamper the search for a global maximum. Multiples of 1 yr were found to be suitable step lengths for determining timing of harvests, and 0.04 for determining thinning rates. In contrast, discontinuous shifts of timber volume between roundwood categories and the consequent jumps in timber value lead to the existence of several local maxima. This is illustrated in Figure 2, which shows bare land value for 4% rate of interest as a function of a variable defining the proportion of residual codominant trees in the last fourth thinning at age 57 yrs and the timing of the final clearcutting when all the remaining 19 endogenous variables are given optimal values.

A direct search method by Hooke and Jeeves (1961) is used to solve the timber harvesting problem [1]-[11] for different numbers of thinning up to five. This method has proved to be computationally efficient in solving similar surface structures (Roise 1986a). In order to find the global maximum, each computation was repeated for 30 or more stochastically determined and several subjectively selected initial guesses (cf. Haight and Monserud 1990a). The computer execution time for determining one local maximum from a single initial guess varied from 3 to 25 minutes depending on the numbers of thinnings and endogenous variables.

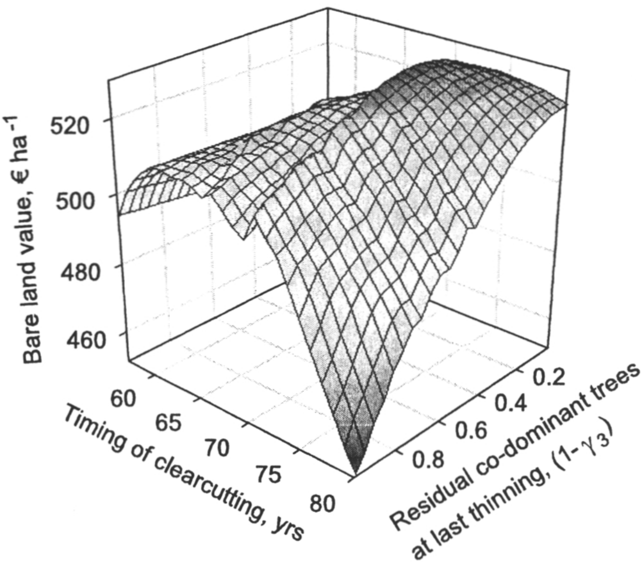


Figure 2. Illustration of objective function response surface: value of bare land at 4% rate of interest as a function of one thinning rate variable and timing of clearcutting.

RESULTS AND DISCUSSION

Effects of economic activity on biomass and timber production

As a side product, economic optimization offers a rigorous test for the growth model. A large number of different stand management alternatives is explored in the search for the global maximum. Ideally, the growth predictions behind the optimal solutions can be tested against empirical data. Unfortunately, this is rarely possible because the optimal solutions often fall outside the conventional forest management practices. Hence, the first step in evaluating the reliability and validity of the optimization results is to check that the model outcomes are logical and consistent with existing economic and ecological theories.

The effects of economic activity on biomass and commercial timber production are analyzed first. Figure 3 compares total biomass production (net production after respiration) and the proportion of biomass allocated to tree boles in an undisturbed stand and optimally (at zero rate of interest) managed stand. The results imply that economic activity decreases the level of biomass production. Reduction from the full biomass production capacity follows from thinnings that lead to temporary reductions from the full crown cover. Light thinnings are however rational because

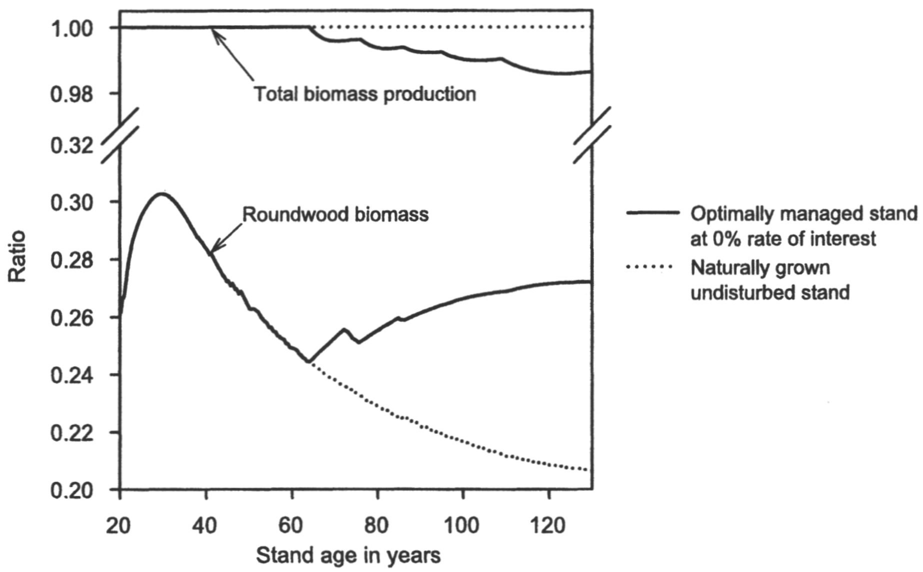


Figure 3. Accumulated biomass and roundwood production in optimally managed stand (with 0% rate of interest) and undisturbed stand.

they increase the quantity and quality of roundwood production. Through thinning it is possible to salvage those commercial trees that would otherwise be lost through natural mortality. Light thinnings also increase the unit value of the residual trees by stimulating diameter growth.

Optimal timber harvesting

Figure 4 shows harvest removals by roundwood category for optimal solutions at 1, 3 and 5% rates of interest. At a 1% rate of interest (Figure 4a), the implicit aim of timber management is to produce high quantity of high-grade timber. The thinnings are light and remove trees after they have reached sawlog dimensions. High growing density reduces branch growth, accelerates lifting of the crown base and speeds up shedding of the dry branches, thus producing high-grade final stock. The stand is clearcut promptly after the first logs in the remaining trees qualify as superior quality butt logs.

At higher, 3 and 5%, rates of interest (Figures 4b and c), optimal harvests are targeted to produce less-valued sawlogs with shorter rotation. A higher rate of interest leads to earlier and heavier thinnings. However, the first two thinnings remain moderate in intensity irrespective of the rate of interest because of their

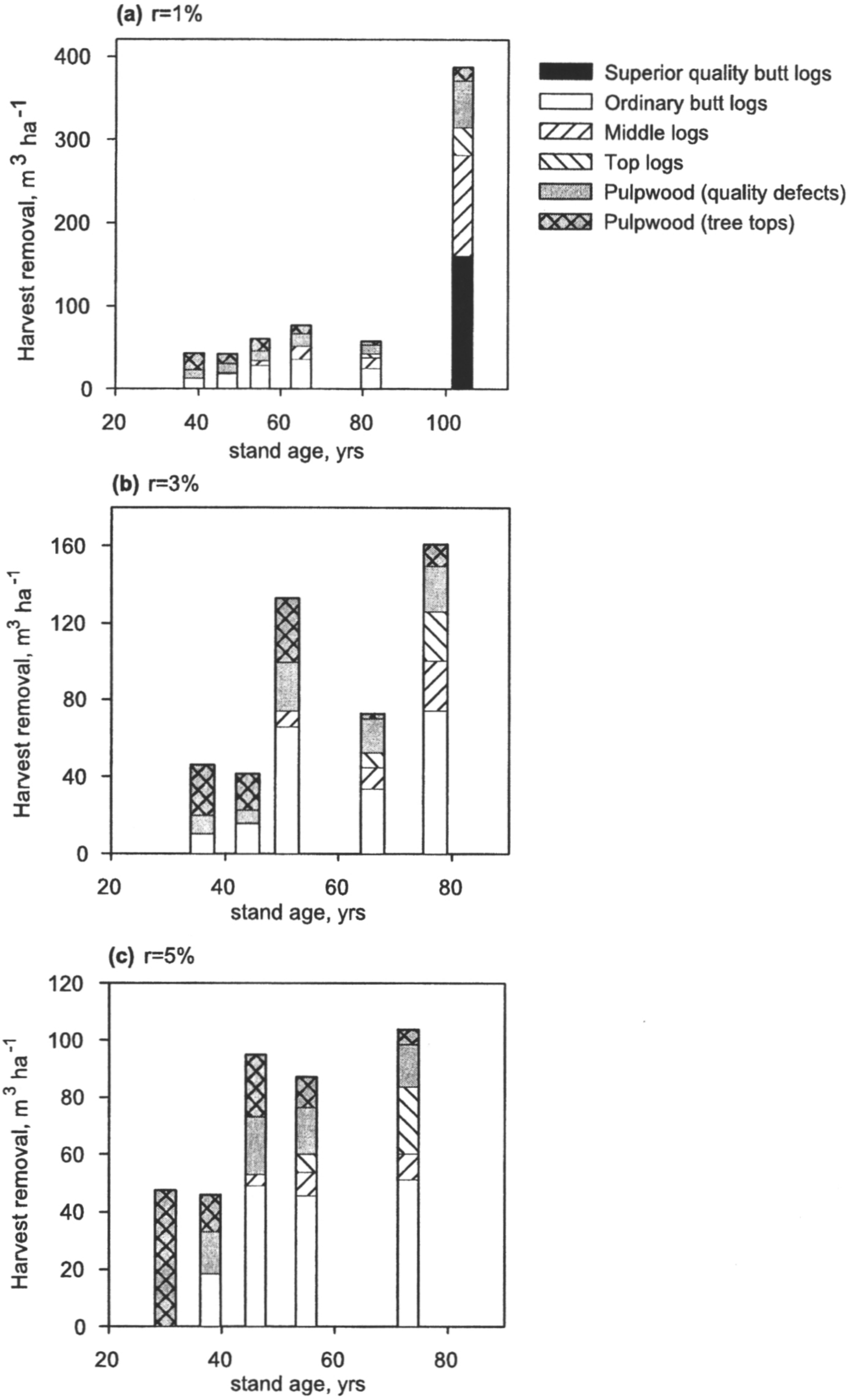


Figure 4. Harvest removals by roundwood category: optimal solutions.

favorable effects on the quality of standing timber. High juvenile density reduces branch growth along the butt log, which is economically the most important part of the tree bole. Butt log represents a high proportion (35-60%) of the total commercial timber volume, and it can reach a much higher unit value than the upper logs (see Appendix). Hence at 3% and higher rates of interest, heavy thinnings become optimal after the crown bases of the residual trees have risen above the butt logs, and may affect branch growth only along the second and upper logs.

Earlier stand-level optimization studies employ empirically validated bucking equations or data from timber utilization studies to define the relationship between tree attributes and stand value (see e.g. Roise 1986b, Solberg and Haight 1991, Valsta 1992, Haight et al. 1995, Wikström 2001). However, only a few tree state variables (typically diameter at breast height and tree height) are used to determine the value of trees. Therefore, even the most sophisticated empirical models do not fully include the feedback effects of silvicultural activities and harvesting on timber quality and stand value (see e.g. Battaglia and Sands 1998). With process-based description of growth, in contrast, it is possible to explicitly account for the causal effects of thinnings on stem taper, quality attributes and, consequently, on value growth of the residual stand.

Long-run timber supply and optimum rotation length

Figure 5a shows the structure and level of long-run timber supply as a function of the rate of interest. The proportion of the upper sawlogs (middle and top logs) decreases, and the proportion of lower-value pulpwood increases with the rate of interest. Superior-quality butt logs are produced only at low, 0-1%, rates of interest. This is an interesting result because it suggests that production of the highest-grade pine timber is not rational. Long run average rates of interest (Hyytiäinen and Tahvonen 2001) and market prices of forest estates (Hyytiäinen and Tahvonen 2003) suggest that the majority of forest owners are likely to apply higher, 2-6%, rates of interest.

Figure 5b shows optimal rotation length as a function of the rate of interest. Optimal rotation length decreases for 0-4% rates of interest, but remains the same thereafter. According to the forest economics literature, the relationship between rate of interest and optimal rotation length is negative (e.g. Clark 1976, Chang 1983, Johansson and Löfgren 1985). This classical result originates from application of density-free stand growth models and constant timber prices that do not depend on

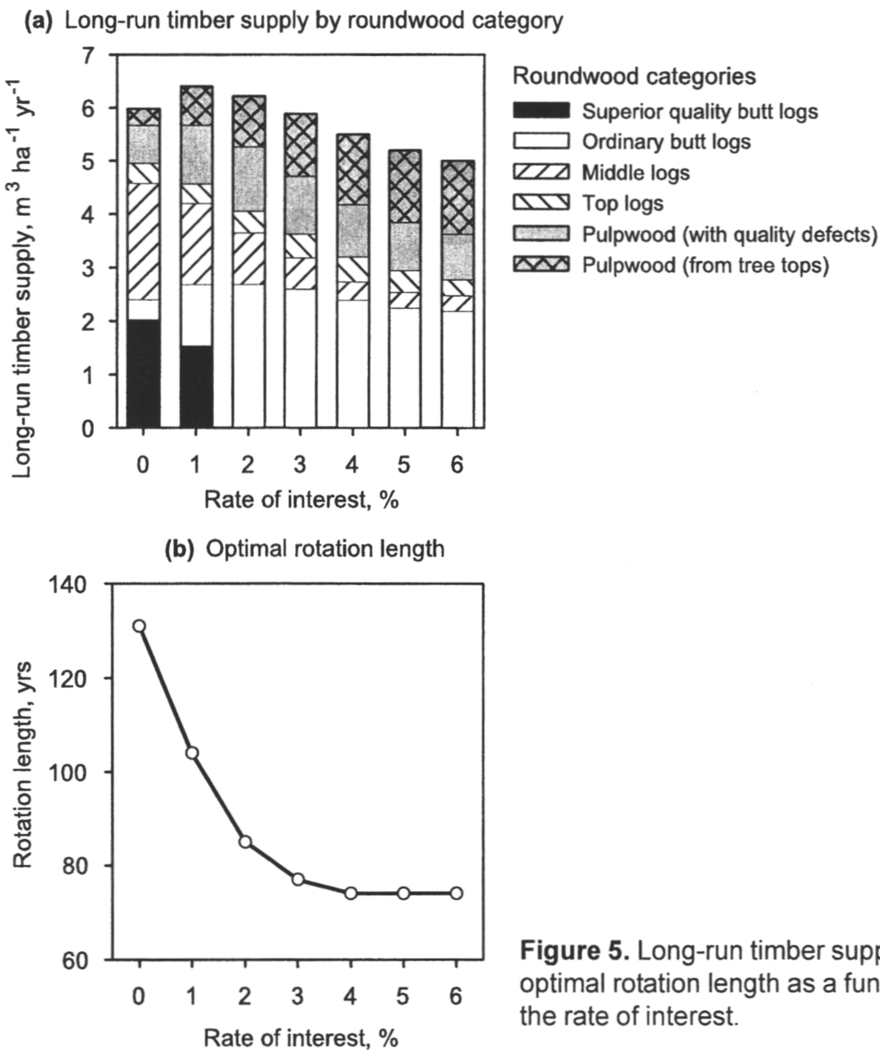


Figure 5. Long-run timber supply and optimal rotation length as a function of the rate of interest.

stand characteristics. Using a continuously differentiable growth function leads to smooth changes in optimal rotation length in response to once-and-for-all changes in the economic parameters.

In this study, insensitivity of optimal rotation length for 4% and higher rates of interest follows from nonmonotonic value growth. Owing to the large difference between typical butt log and pulpwood prices, trees are optimally removed only after the first log has reached the sawlog dimensions. Shorter than 74 yr rotations would severely reduce the proportion of sawlog yield from the final cut for the analyzed stand. It would be possible to accelerate diameter growth and appearance of sawlog categories by starting the thinnings earlier. However, this does not become optimal due to higher harvesting costs and negative effects of early thinning on timber quality. Thus, for higher than 4% rates of interest, the demands of a higher rate of interest are met by increasing thinning intensity and decreasing the level of the final stock rather than reducing rotation length (see Figure 4c).

Optimal type of thinnings

A solution maximizing the sustained yield of commercial roundwood (MSY) is next shown as the first step to analyze the determinants of the optimal type of thinning. Figure 6 shows numbers of harvested and residual trees by size classes for the MSY solution. Thinnings are light and remove larger proportions of small and large than average-sized trees. Such thinnings lead to a smooth bell-shaped diameter distribution of the final stock. Similar results have been obtained e.g. by Haight and Monserud (1990b) and Valsta (1992) using empirical growth models. However, application of a process model in optimization provides a new basis for interpreting and understanding the reasons behind this result.

Large dominant trees accumulate more biomass than suppressed trees because their crowns are larger and more exposed to sunlight (Mäkelä 2002, see also Assmann 1970 p. 117-). The majority of the largest dominant trees are, however, removed in thinnings because they shade and reduce the growth of all the other trees. Another reason for favoring trees that are initially at codominant or intermediate canopy layers is that they are more productive in resource use than the dominant trees. The rate of leaf-specific photosynthesis and the proportion of biomass production allocated to tree bole increase with competition (Mäkelä 2002). The smallest suppressed trees are shaded by all the other trees in a fully stocked stand, and are removed due to small absolute growth.

Figure 7 shows optimal types of thinnings using the economic criterion and a 3% rate of interest. Optimal thinnings are selective thinnings that remove particularly the largest trees in favour of the thinner trees with better quality attributes. Systematical thinning from above leads to the final stock that consist mainly of the smallest trees of the initial stock.

Removing the largest dominant trees in thinnings is economically favorable. Dominant trees have larger volume and consist of higher-valued roundwood categories than do suppressed trees. Logging costs decrease, unit value of harvested timber increases and, as a result, net thinning revenues increase with dimensions of the removed trees. Thinning from above may also increase the value growth of

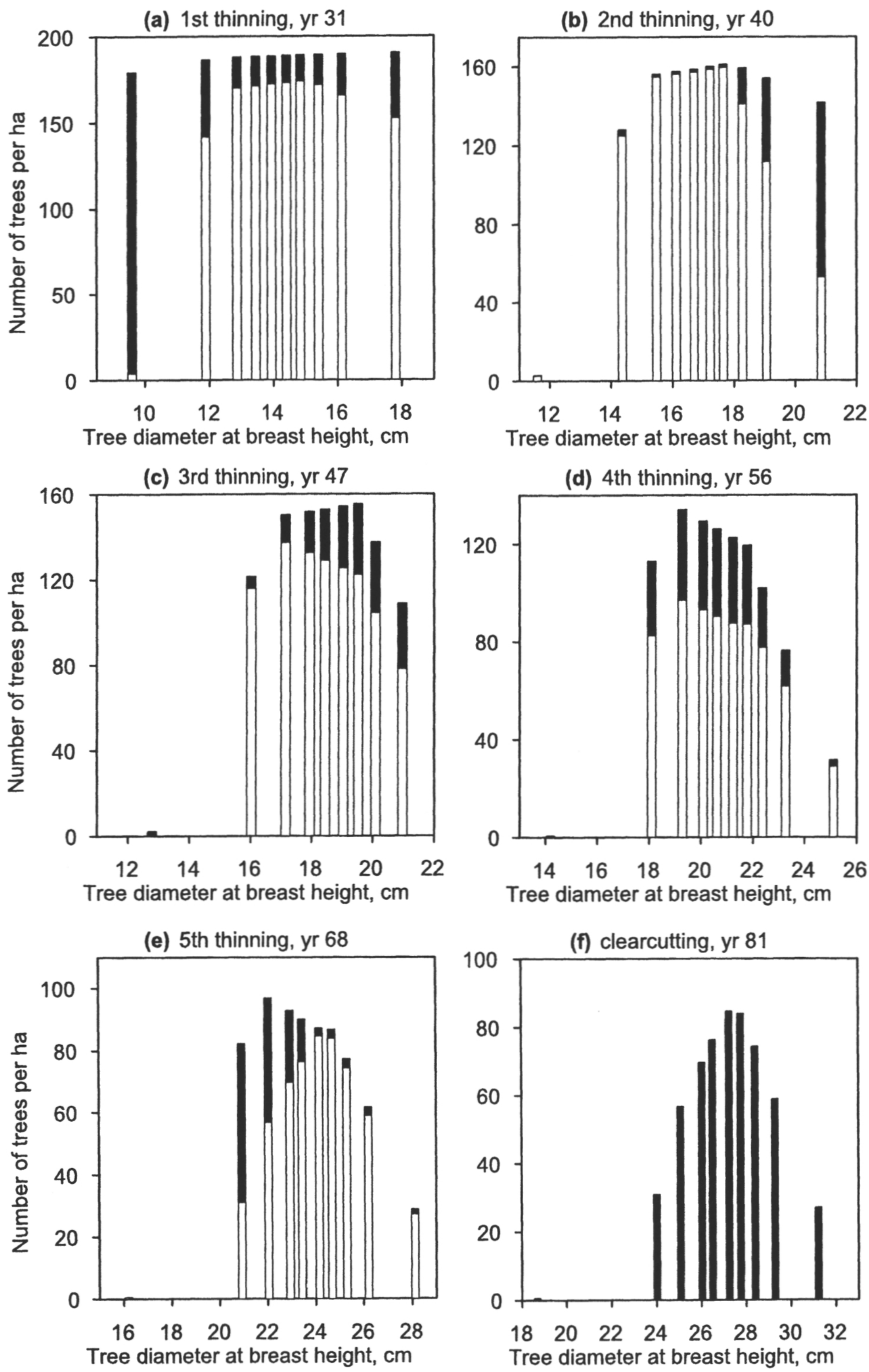


Figure 6. Numbers of cut (black bars) and residual (white bars) trees by size class in MSY harvests.

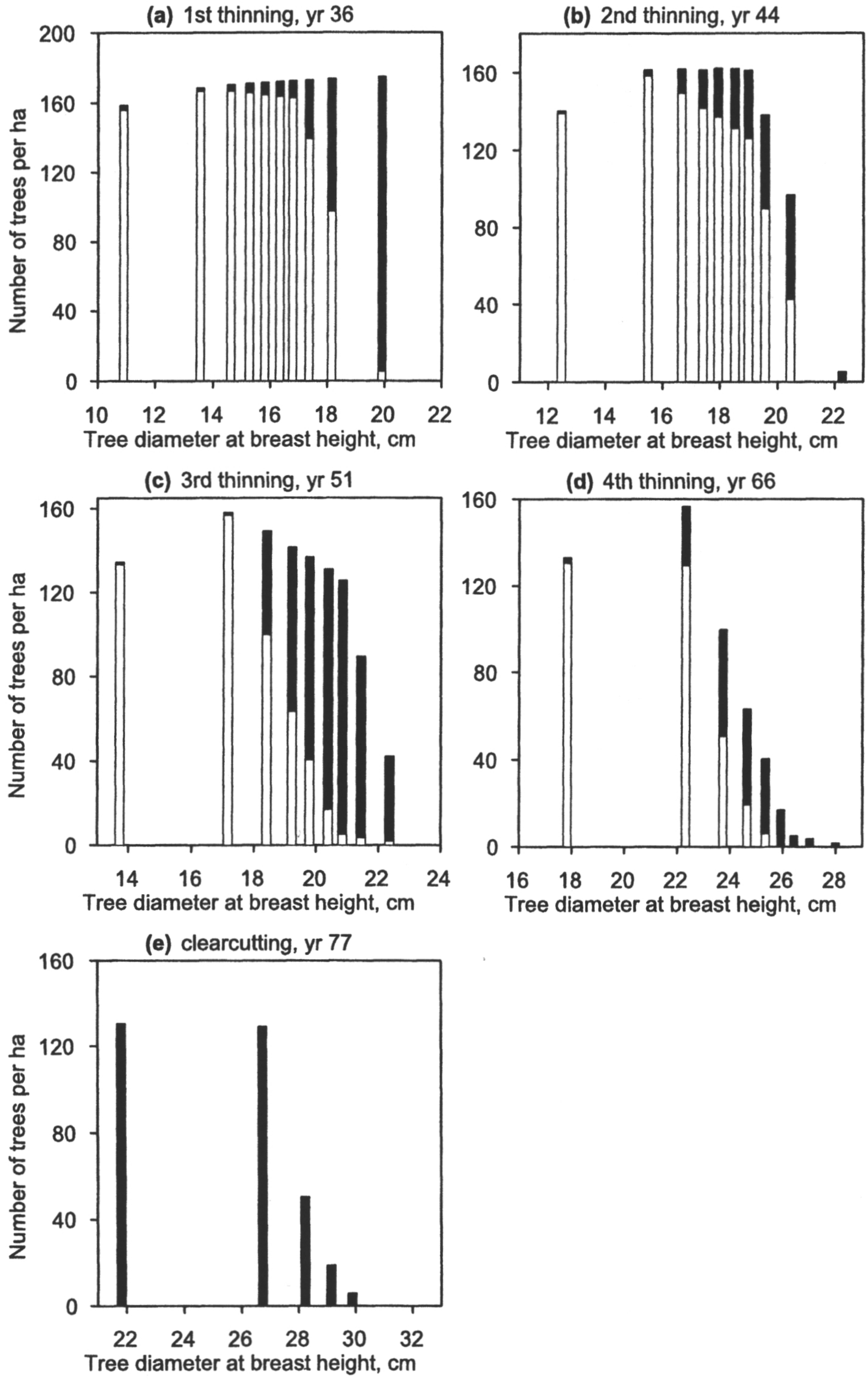


Figure 7. Numbers of cut (black bars) and residual (white bars) trees by size class in optimal harvests at 3% rate of interest.

residual trees by accelerating the appearance of higher-valued roundwood categories. Trees that have undergone a suppressed position at early ages have better quality attributes (thinner living branches, smaller crown ratio and longer branchless section) and can reach higher unit value than dominant trees.

Growth and yield studies for Scots pine (Vuokila 1977, Mielikäinen and Valkonen 1991) suggest that the type of thinning has rather small effects on the long-run timber yield. Our results support this conclusion. Optimal thinnings that are constrained to remove equal proportions of trees from all size classes lead to only 0.3% reduction from the maximized level of long-run timber supply obtained with flexible thinnings. In contrast, type of thinning becomes important when economic factors are taken into account. With a 3% rate of interest, optimal thinnings removing equal proportions of trees from all tree classes lead to about a 15% reduction from the maximized bare land value obtained with flexible thinnings.

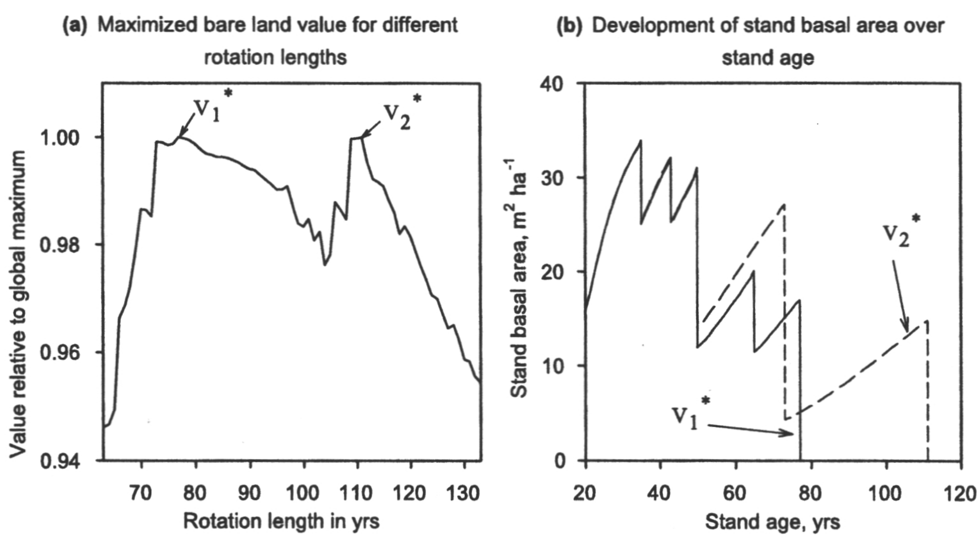
The majority of the economic studies report thinning from above to be superior in even-aged conifer monocultures (e.g. Roise 1986b, Solberg and Haight 1991, Eriksson 1994, Valsta 1992). In addition to solid economic reasons, this result is based on the model outcome that the residual trees respond positively to increased growing space. However, Scots pine is a light-demanding pioneer species and the suppressed trees with the poorest crowns may have only limited potential to react to thinnings (Assman 1970, p. 323-). With distance-independent individual tree models, the attributes of mean trees are typically averages across all trees within each size class. Therefore, it is possible that the growth predictions for mean trees severely overestimate the vigor of those trees that have the poorest crowns within each class. Optimal types of thinnings can be studied more rigorously with detailed spatial growth models.

Existence of multiple optima

The objective function response surface has two distinct peaks that follow from the wide price difference between ordinary and superior-quality butt logs. The first peak is characterized by clearcutting the stand after the residual trees have reached sawlog dimensions, and some self-pruning of dry branches has occurred at tree butt. The second peak involving longer rotation is characterized by clearcutting the stand after all the residual trees of the final stock have reached the quality and dimension requirements for superior-quality butt logs. Which of these two local maxima is the global maximum is sensitive to economic parameters. With default prices, the second peak, involving longer rotation, is optimal for 0-1% rates of interest, and the first peak, involving shorter rotation, for 2% and higher rates of interest.

We performed a sensitivity analysis at 3% rate of interest in order to find a threshold price for superior-quality butt log when longer rotation becomes optimal. At p_1=96.24 € / m^3 roadside price level, the two peaks are equally high, and two optimal solutions exist. This is illustrated in Figure 8a showing a double-peaked projection of the maximized bare land value over exogenously given rotation length. Figure 8b shows the development of stand basal area over stand age for optimal solutions requiring short (V_1^*) and long rotation (V_2^*). The solutions differ with respect to the last thinnings and rotation length. A very small number of trees (about 140 trees ha^{-1}) are retained after the last thinning and are grown until they reach the dimension and quality requirements for superior-quality butt logs with solution V_2^*.

Figure 8a shows that rotation lengths between 68 and 124 yrs lead to smaller than 3% decreases from maximum bare land value. This result supports the earlier finding that deviation from optimum rotation may lead to minor losses (e.g. Haight and Monserud 1990b, Solberg and Haight 1991, Valsta 1992), but only if thinnings are readily adjustable. Optimal and near-optimal solutions are characterized by



Note: A 3% rate of interest and 96.24 €/m^3 roadside price for superior quality butt log are used.

Figure 8. Illustration of two optimal solutions.

harvests in which trees are removed that have recently experienced a peak in value growth (due to upgrading of timber volume to higher-valued categories) and major increases in unit price are not in the offing. However, several distinct harvesting strategies can meet this requirement and serve as good candidates for the optimum. For example, a strategy focusing on quick production of a large quantity of high-grade timber (final stand basal area 34 m^2 ha^{-1} with 98 rotation length) through light thinnings leads to only about a 3% decrease from the maximum bare land value.

The objective function response surface is often nonmonotonic for optimization models accounting for thinnings and containing detailed descriptions of stand growth and harvesting technology. Multiple local maxima may originate from variation in timing, type and number of thinnings (Roise 1986b, Haight and Monserud 1990a, Valsta 1992, Hyytiäinen et al. 2003) or from discrete changes in stand value associated with size class transition (Martin and Ek 1981). However, examples of multiple optima involving distinct management strategies have not been given earlier.

Comparison of optimal solutions from process-based vs. empirical model

Finally, the optimal solutions from process and empirical models are compared. The same initial stand data, harvesting cost model, definition of thinnings and economic parameters are used for both models. The individual tree growth model by Hynynen et al. (2002) is taken as example of a detailed and well-validated empirical growth model. The model version used here was originally developed for the 1996 version of the Finnish long-term forest planning tool MELA, and is based on extensive sets of field data from the Finnish national forest inventory. The simulation-optimization program developed by Valsta and Linkosalo (1996) is used to obtain optimal solutions.

The MSY solutions are shown in Figure 9a to compare two distinct descriptions of the same biological production process. Optimal harvests and stand development are somewhat different, but logical for both models. The level of stand basal area is kept at the highest productive level throughout the rotation period. The maximum number of light thinnings (four for empirical and five for process model) are timed and scaled to capture only those trees that would otherwise die due to self-thinning. The stand is clearcut at the age at which the mean annual increment culminates.

The MSY solution implies longer rotation and lower stand basal area levels for the process model than for the empirical model. These differences mainly follow from distinct specifications of tree mortality. In the empirical model, Reineke's equation (1933) defining the maximum level of stand basal area is used to specify competition-induced mortality. In the process model, on the contrary, self-thinning starts already with crown closure. It is possible that the empirical model underestimates and the process model overestimates mortality at high stocking levels. Empirical results for conifer species (Drew and Flewelling 1977, 1979, Buford and Hafley 1985) suggest that competition-induced mortality starts smoothly some time after crown closure but clearly before the maximum level for stocking is reached.

Figures 9b, c and d show optimal solutions for 1, 3, and 5% rates of interest, respectively. Increasing the rate of interest has similar effects on optimal solutions for both models: optimal rotation shortens and basal area of final stock decreases. However, there are some striking differences that partly follow from distinct specifications of roundwood categories and timber quality. With the empirical model, the proportion of inferior-quality trees increases with ring width at tree pith for each

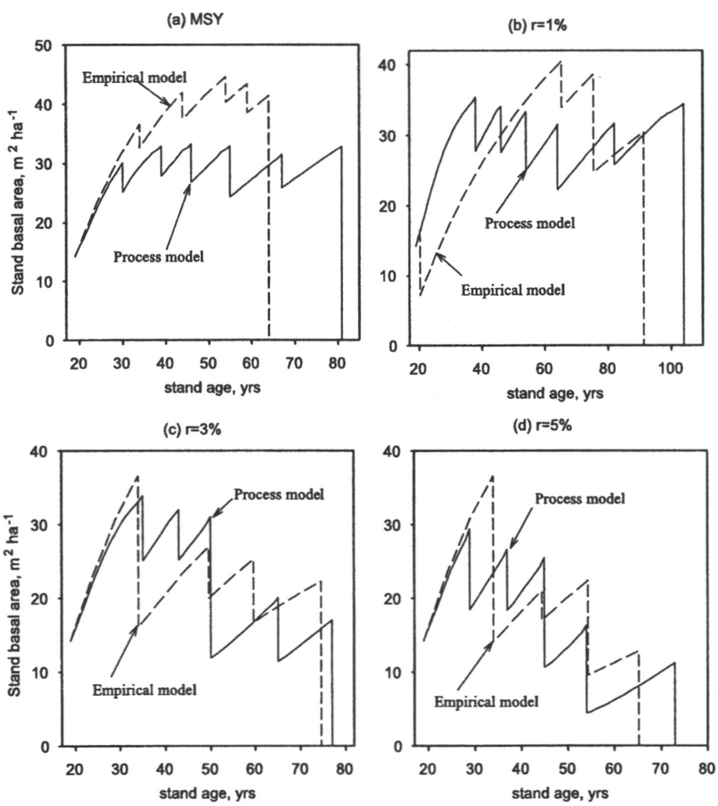


Figure 9. Comparison of optimal solutions from process-based and empirical models.

class (for details, see Hyytiäinen et al. 2003). Such a predetermined cull factor describes the established relationship between early growth and timber quality (see e.g. Haygreen and Bowyer 1996, p. 280), but neglects time-dependence and causal effects of thinnings on quality attributes.

With the empirical model, heavy first thinnings are rational because they enable removal of inferior-quality trees that have no prospects to be ranked as higher-valued sawlog trees. At 1% rate of interest (Figure 9b), the first thinning is conducted early despite high harvesting costs and virtually zero net harvest revenues. Such a harvest can be regarded as an investment in the growing stock as it improves the quality and increases the growth of the residual stock. With the process model, on the contrary, an early thinning is not rational because it would have devastating effects on the quality of the residual trees.

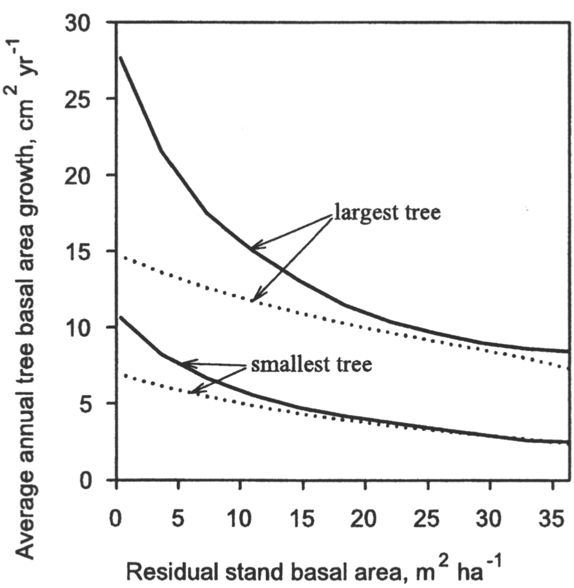
The two growth models produce roughly similar predictions of basal area growth for trees under closed canopy. For sparse stands, however, the process model implies faster growth (see Figure 10). This leads to lower final stocking, a greater number of thinnings and longer rotation for the process model, when 3% and higher rates of interest are applied (see Figures 9c and 9d). Different growth responses to thinning also affect the optimal type of thinning. With the process model, optimal thinnings remove trees systematically from above (see Figure 7). With the empirical model, optimal thinnings remove trees from the both ends of the diameter distribution.

It is difficult to assess which of the two models forecasts growth more reliably for sparse stands. The empirical model is used beyond its most reliable range of application, because measured data is scarce for low stand basal areas. Nevertheless, it has not been verified that the process models generates accurate predictions of diameter growth for trees that face little or no competition.

Conclusions

It has been argued that demanding data requirements and a high degree of resolution may reduce generality and precision of process model outcomes (Mohren and Burkhart 1994, Vanclay 1995, Korzukhin et al. 1996, Battaglia and Sands 1998, Johnsen et al. 2001). Our computations and comparisons however suggest that after several cycles of theory formation, model calibration and empirical validation (Mäkelä 1986, Mäkelä 1997, Mäkelä and Vanninen 2001, Mäkelä 2002, Mäkelä and Mäkinen 2003), a process model can produce predictions as precise as those of empirical models in conditions for which data exists. In addition, a process model produces better justified estimates of growth in conditions in which empirical data are scarce or lacking. However, it is clear that continuous efforts are needed to increase the predictive capability of process models. Our computations also reveal that with modern computers computational complexity is no longer an invincible obstacle to solving process-based optimization models.

Process models have huge potential in economics and management sciences (e.g. Johnsen et al. 2001). In addition, economic optimization can be used as one tool for efficiently identifying ecological model components that are instrumental in determining the economic surplus of forestry (cf. Battaglia and Sands 1998) and may need additional research inputs. This study shows, for example, that optimal solutions are highly sensitive to tree mortality and relative growth at different stocking levels. However, process-based descriptions of tree mortality (see e.g. Mäkelä et al. 2000a, Bigler and Bugman 2003) and growth of open-grown trees (Mäkelä and Sievänen 1992) are difficult tasks and remain as challenges to ecological research.



Notes: Average annual growth is calculated from 15-yr period after thinning a 38-yr-old stand. Thinning removes equal proportion of trees from each class.

Figure 10. Predicted growth of tree basal area after thinning according to process-based (solid curves) and empirical model (dotted curves).

Process-based growth models are new tools in economic research of forestry, and may open a new era particularly in stand-level optimization. They enable highly detailed analysis of several topical issues including joint production of several forest benefits and optimal management under changing climatic or athmospheric conditions. Detailed descriptions of tree structure over time and in response to different forest management activities enable more detailed specification of timber quality than is possible with the most advanced empirical models. Even though our specifications of timber quality and roundwood categories are more detailed than in earlier stand-level economic studies, we have used only a part of the crown morphology data available from the applied process model (Mäkelä and Mäkinen 2003). For example, accounting for the three-dimensional distribution of branches inside the tree boles would enable reliable predictions of sawn product mixture.

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APPENDIX

Stem taper over bark

The process model describes stem diameters under bark as functions of sapwood and disused pipe areas within each whorl. In timber markets, however, commercial roundwood is expressed in terms of cubic volume measured over bark. Tree diameters under bark, d, are converted to diameters over bark, $\tilde{d}$, using the equation $\tilde{d}=d(0.833+0.712\omega-1.77\omega^2+2.07\omega^3-1.01\omega^4)^{-1}$, where $\omega \in [0,1]$ is the height of tree whorl relative to total tree length. This equation is based on field data of Laasasenaho (1982) and on an observation that the ratio of bark to tree diameter varies considerably along the tree bole, but changes less with stand age.

Dimension and quality requirements for roundwood categories

The length of superior-quality butt log may vary between 37 and 55 dm with minimum top diameter of 20 cm. The log must be free of living and dry branches. The length of ordinary butt log may vary between 43 and 55 dm, with minimum top diameter of 15 cm. The ordinary butt log and middle logs may contain dry branches thinner than 6 cm in diameter. Butt logs must be along the whole length below the crown base. The length of the top log may vary between 37 and 55 dm, with minimum diameter of 15 cm. Top log may contain living branches thinner than 6 cm. Dry branches thinner than 4 cm are allowed, but only along 15 dm length at the large end of the top log. Pulpwood from tree tops is attained from bole section with diameter between 6 and 15 cm. These requirements are adapted from specifications applied by the Metsäliitto group in central Finland in 2002.

Due to quality defects, a certain proportion of roundwood within each size class is unsuitable for manufacturing sawn products, but can be used for pulpwood. This proportion is computed by tree whorls as a function of average diameter of branches for a bole section that is thicker than 15 cm. Below the crown base the proportion of inferior-quality stems is given by $\varphi=\min(-0.0073z^4+0.0272z^3+0.0706z^2, 1)$, and above the crown base by $\varphi=\min(-0.00144z^4+0.00806z^3+0.0314z^2, 1)$, where z is the average diameter of dry or living branch in each whorl (in cm). Branchless section is formed at tree butt after all dry branches from the five lowest whorls have been shed. This specification is based on average size distribution of branches within whorls and is determined from the data sets of Mäkinen and Colin (1998).

Roadside prices

The roadside prices are: p_1=71.52, p_2=52.98, p_3=39.73, p_4=45.03, p_5=26.24, p_6= 26.24 € / m^3. The prices reflect the marginal value of roundwood categories in further processing. They are based on average levels for pulpwood and sawlog (Hyytiäinen et al. 2003), and ratios between sawlog categories applied in timber sales in Central Finland in summer 2002 (T. Ahonen, pers. comm.). The unit prices for the two pulpwood categories is currently the same in the timber markets even though they may vary with respect to fiber length, proportion of juvenile wood and other relevant qualities in the paper and pulp industry.

Stand establishment and harvesting costs

Stand establishment consists of land preparation and sowing at yr 1 (297 € ha^{-1}) and tending of seedling stand at yrs 6 and 11 (218 € ha^{-1}). The logging cost models are based on an empirical productivity study of mechanized cutting and forest haulage (Kuitto et al. 1994). Cutting is divided into two and forest haulage into five active phases. The unit cost of felling and forest haulage are 67.28 and 47.09 € hr^{-1}, respectively. The fixed harvesting cost is 100 €.

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