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Forest inventory with sparse resolution Airborne Laser Scanning data – a literature review

Raita Säynäjoki, Matti Maltamo & Kari T. Korhonen

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Abstract
The article reviews 34 recent Nordic studies on the use of sparse resolution ALS data for forest inventory. Key parameters of the applied ALS data - study area, ground truth data, applied methods, resulting regression models and reliability of the models - are summarised for each study. The reviewed articles show that ALS techniques can be used for estimating forest resources at stand level and the accuracy is in most cases better than the accuracy of ocular field estimations. The review will be useful in selecting modeling approaches in further studies. The published models might be used also as a priori information when estimating new models, especially with sparse ground truth data or even without new data.

Keywords
lidar, forest inventory, regression models

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1 Introduction

Research on the use Airborne Laser Scanning (ALS) for forest inventory has rapidly increased in the past 10–15 years. In the Nordic countries, especially in Norway, ALS based inventory techniques are already used in practical forestry. With current instruments and data processing capacity, forest inventory techniques based on sparse resolution ALS data seem most promising approach for forest inventory in terms of cost-efficiency and reliability. In these ALS techniques, estimation of growing stock attributes at stand or plot level is based on modelling the relationship between ALS data features and growing stock data measured in the field.

This article reviews 34 recent Nordic studies on the use of sparse resolution ALS data for forest inventory. Key parameters of the applied ALS data - study area, ground truth data, applied methods, resulting regression models and reliability of the models - are summarised for each study. The literature review is limited to the Nordic countries, where the forest types are similar and variation in the relationship between ALS data and ground truth is expected to be lower than in regions with more diversity in forest types.

The reviewed articles show that ALS techniques can be used for estimating forest resources at stand level and the accuracy is in most cases better than the accuracy of ocular field estimations. The regression models used for estimating stem volume, mean or dominant height, basal area or stem number are very much alike in different studies. Typically, the mean or dominant height of growing stock is predicted with the 80–95% percentile or maximum value of the first pulse laser canopy heights. In some cases, variables describing the density of crown hits have been used as auxiliary regressors. Regression models for basal area, stem volume, mean diameter or stem number typically include both percentile values of laser canopy heights and variables related to canopy density. The canopy density is typically described by the proportions of laser hits above selected fractions. Regressors vary by studies, even if the independent variable is the same. This is partly due that different height percentiles, e.g., 20% and 30% percentiles, are strongly correlated and their choice in the different studies is just based on statistical significance.

We hope that this review will be useful in selecting modeling approaches in further studies. The published models might be used also as a priori information when estimating new models, especially with sparse ground truth data or even without new data, even though the variability of reviewed models indicates that regression between ALS features and growing stock variables is case specific.

2 Summaries of articles

Accuracy of forest inventory using airborne laser scanning: evaluating the first Nordic full-scale operational project

Næsset, E. 2004. Scandinavian Journal of Forest Research 19: 1-4.

1 Airborne laser scanner data

- Laser scanner: ALTM 1233 (Optech)
- Flying altitude: 800 m
- Pulse density: 0.7/m^2
- Recorded echoes: first and last

2 Study area and field data

The study area is located in Nordre Land, south-east Norway (60°50′N 10°05′E, 140-900 m a.s.l.). The total size of the area is 420 km^2 and the dominant tree species are spruce (*Picea abies*) and pine (*Pinus sylvestris*). The stands were divided in two strata: A) mature spruce-dominated stands on good sites and B) mature pine-dominated stands on good sites. There were 55 training plots in stratum A and 52 in stratum B. The size of a training plot was 250 m^2. On each plot, trees with diameter at breast height >10 cm were callipered. Tree heights were measured on sample trees.

The test data set consisted of circular plots (strata A 21 plots and strata B 18) with a size of 1000 m^2.

Another dataset was collected in Krødsherad (60°10′N 9°35′E, 130-660 m a.s.l.). The main tree species were spruce and pine, but the proportion of deciduous trees was high especially in young forests. The average size of test plots was 3685 m^2. Only plots corresponding to the definitions of the two strata were used.

3 Regression models

Separate models were estimated for the six variables for both strata. The variables were Lorey's mean height, (h_L), dominant height (h_{dom}), mean diameter by basal area (d_g), stem number (N), basal area (G) and volume (V).

4 Reliability of the models

Table 1. Differences (*D*) between predicted and ground reference values. NS = not statistically significant (p>0.05).

	D, local test data[a]			D, other test data[b]		
	Range	Mean	SD	Range	Mean	SD
Stratum A						
h_L	-2.17 - 0.95	-0.53	0.72	-0.42 - 0.83	0.12 NS	0.36
h_{dom}	-1.47 - 1.81	-0.21 NS	0.70	-1.72 - 0.15	-0.80	0.72
d_g	-2.08 - 2.67	0.21 NS	1.29	-3.13 - 1.25	-0.19 NS	1.29
N	-646 - 251	-33 NS	222	-267 - 261	-55 NS	155
G	-7.16 - 7.19	0.29 NS	4.88	-4.77 - 3.34	-0.23 NS	2.38
V	-79.0 - 75.2	2.9 NS	45.9	-31.3 - 36.4	11.6 NS	21.6
Stratum B						
h_L	-2.05 - 1.17	-0.45	0.82	-1.94 - 1.49	0.12 NS	1.37
h_{dom}	-1.57 - 1.51	-0.13 NS	0.78	-3.88 - 0.34	-1.31 NS	1.55
d_g	-4.32 - 5.09	0.12 NS	2.60	-2.72 - 2.19	-0.28 NS	2.22
N	-289 - 271	47 NS	164	-106 - 196	5 NS	120
G	-3.64 - 6.70	1.95	2.78	-3.43 - 4.18	0.31 NS	2.92
V	-15.9 - 61.2	13.3	17.9	-11.8 - 17.8	4.2 NS	13.9

[a] Test plots located in the inventory area.
[b] Test plots located in a different district.

Airborne laser scanning as a method in operational forest inventory: Status of accuracy assessments accomplished in Scandinavia

Næsset, E. 2007. Scandinavian Journal of Forest Research 22: 433-442.

1 Airborne laser scanner data

- Laser scanner: ALTM 1233 (Optech)
- Date of data acquisition: 16th September 2004
- Flying altitude: 1200 m
- Max. (half) scan angle, data collection: 17°
- Max. (half) scan angle, data proceccing: 15°
- Average point density: 0.7-0.8/m^2
- Recorded echoes: first and last

2 Study areas, photo interpretation and field data

Study area A and B are located in Hole (60°01′N 10°20′E, 240-540 m a.s.l.) and Fet (59°52′N 11°15′E, 100-320 m a.s.l.), respectively, both in south-eastern Norway. The distance between the areas is about 50 km. Both areas are dominated by conifer trees.

Forest stands were delineated using aerial photographs for digital stereo photogrammetry. Tree species, age and site productivity was also determined for each stand. The stands were divided into four strata, of which two were used in this study: I = mature stands with high or medium site productivity dominated by spruce, II = mature stands with poor site productivity dominated by spruce or pine.

Stratum I consisted of 50 and 21 plots collected in areas A and B, respectively. The plots were systematically distributed, and the plot size was 250 m^2. Correspondingly, stratum II consisted of 31 and 25 plots. Diameter at breast height was measured on trees with DBH >10 cm. Heights were measured on sample trees.

3 Regression models

- Lorey's mean height
- Dominant height
- Mean diameter by basal area
- Stem number
- Basal area
- Volume

Table 1. Explanations for variables.

Variable	Explanation
h_L	Lorey's mean height (m)
h_{dom}	Dominant height (m)
d_g	Mean diameter by basal area (cm)
N	Stem number (ha^{-1})
G	Basal area (m^2ha^{-1})
V	Volume (m^3ha^{-1})
h_{60f}, h_{90f}	Percentiles of the first pulse laser canopy heights for 60% and 90% (m)
h_{60l}, h_{70l}, h_{90l}	Percentiles of the last pulse laser canopy heights for 60%, 70% and 90% (m)
h_{meanf}	Arithmetic mean of first pulse laser canopy heights (m)
h_{cvf}	Coefficient of variation of first pulse laser canopy heights (%)
h_{meanf}, h_{meanl}	Maximum of first and last pulse laser canopy heights (m)
d_{0f}, d_{2f}, d_{6f} and d_{9f}	Canopy densities corresponding to the proportions of first pulse laser echoes above fractions no. 0, 2, 6 and 9 to total number of last pulses

Table 2. Regression models, R^2 values and dummy variables. Dummy variable in common model for areas A and B with value = 0 if area A and value = 1 if area B. Level of significance: **p<0.01, *p<0.05, ns=not significant (p>0.05).

Response variables	Explanatory variables	R^2	Dummy (p-value)
Stratum I (n=71)			
$\ln h_{dom}$	$\ln h_{maxl}$, $\ln d_{9f}$	0.88	0.01 **
$\ln d_g$	$\ln h_{60l}$, $\ln h_{cvf}$, $\ln d_{6f}$	0.65	0.30 ns
$\ln N$	$\ln h_{90l}$, $\ln d_{2f}$, $\ln d_{6f}$	0.61	0.13 ns
$\ln G$	$\ln h_{maxf}$, $\ln d_{4l}$	0.72	0.55 ns
$\ln V$	$\ln h_{meanf}$, $\ln d_{0l}$	0.84	0.96 ns
Stratum II (n=56)			
$\ln h_L$	$\ln h_{90f}$, $\ln d_{0f}$	0.87	0.93 ns
$\ln h_{dom}$	$\ln h_{70l}$, $\ln h_{maxl}$	0.84	0.02 *
$\ln d_g$	$\ln h_{60f}$, $\ln h_{90f}$, $\ln d_{9l}$	0.50	0.24 ns
$\ln N$	$\ln d_{0f}$	0.50	0.73 ns
$\ln G$	$\ln h_{90l}$, $\ln d_{3l}$	0.82	0.53 ns
$\ln V$	$\ln h_{maxl}$, $\ln d_{2f}$, $\ln d_{4l}$	0.89	0.92 ns

4 Reliability of the models

Table 3. Differences (*D*) between predicted and ground reference values, standard deviation (SD) for the differences, and root mean square error (RMSE) values.

		D			
Characteristics	Observed mean	Range	Mean	SD	RMSE
Stratum I (n=71)					
$\ln h_L$	18.41	-1.39 – 1.89	-0.01 ns	0.71	0.69
$\ln h_{dom}$	20.53	-1.98 – 1.30	0.14 ns	0.70	0.70
$\ln d_g$	23.38	-5.63 – 3.86	0.53 ns	2.43	2.44
$\ln N$	724	-317 – 249	-25 ns	155	154
$\ln G$	28.31	-2.80 – 8.31	0.88 ns	2.62	2.71
$\ln V$	233.3	-31.5 – 76.4	3.5 ns	25.1	24.8
Stratum II (n=56)					
$\ln h_L$	15.21	-2.34 – 0.86	-0.40 ns	0.85	0.92
$\ln h_{dom}$	16.89	-1.72 – 0.54	-0.28 ns	0.58	0.63
$\ln d_g$	22.13	-6.54 – 4.90	-0.03 ns	2.96	2.86
$\ln N$	531	-134 – 247	19 ns	103	101
$\ln G$	20.13	-3.54 – 5.46	0.82 ns	2.87	2.89
$\ln V$	146.0	-20.5 – 47.6	9.8 ns	18.7	20.5

Assessing effects of laser point density on biophysical stand properties derived from airborne laser scanner data in mature forest

Gobakken, T., Næsset, E. IAPRS Vol XXXVI, Part 3 / W52, 2007.

1 Airborne laser scanner data

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 8.-9.6.1999 (site A), 23.7.-1.8.2001 (site B),
- Recorded echoes: first echoes used in this study

The point clouds were thinned from about 1.2 m^{-2} and 0.9 m^{-2} for sites A and B, respectively, to 1 point per 4, 8, and 16 m^2 (0.25, 0.13 and 0.006 m^{-2}).

2 Study area and field data

Two forest areas were used in this study: site A in Våler and site B in Krødsherad, both in sout-eastern Norway. Dominant tree species in the two sites were Norway spruce (*Picea abies*) and Scots pine (*Pinus sylvestris*).

Two different datasets were collected on from sites A and B: sample plots and forest stands. Sample plots were used to assessing the effects of different point densities and to model developing. Forest stands were used to assessing the influence of point density on the stand level predictions.

82 and 70 sample plots were systematically distributed on sites A and B, respectively. Plot sizes were 200 m^2 in site A and 232.9 m^2 in site B. All trees with DBH >10 cm were callipered. The measurements were recorded in 2 cm classes. Tree heights were measured on sample trees. Basal area per hectare, Lorey's mean height and total plot volume were computed and prorated by means of growth functions to match with the laser scanning date. These values were used as ground-truth data. Plot centres were determined with GPS and GLONASS.

39 stands in site A were subjectively selected for the study. The average size was 1.7 ha. Stands were inventoried by plots within each stand. The average number of plots per stand was 20. Large test plots on subjectively selected stands were used in site B. The quadrate plot size was supposed to be 61×61 m, but the size varied somewhat. Large test plot data was denoted as stand data. Stand data was synchronized to the laser scanning date and these values were used as ground-truth.

3 Regression models and the reliability of the models

- Lorey's mean height (m)
- Basal area (m^2ha^{-1})
- Volume (m^3ha^{-1})

Table 1. Explanations for the variables.

Variable	Explanation
h_L	Lorey's mean height
G	Basal area
V	Volume
h_{10} and h_{90}	Percentiles of the laser canopy heights for 10% and 90%
h_{mean}	Arithmetic mean of first return laser heights (m)
d_1 and d_5	Canopy density corresponding to the proportion of laser hits above fraction 1 and 5

Table 2. Selected models, R^2 and RMSE values.

Response variable	Expl. variables	R^2	RMSE
Site A			
$\ln h_L$	$\ln h_{10}$, $\ln h_{90}$	0.87	0.07
$\ln G$	$\ln h_{90}$, $\ln d_5$	0.62	0.25
$\ln V$	$\ln h_{mean}$, $\ln d_1$	0.71	0.27
Site B			
$\ln h_L$	$\ln h90$	0.93	0.06
$\ln G$	$\ln h_{mean}$, $\ln d_1$	0.80	0.20
$\ln V$	$\ln h_{mean}$, $\ln d_1$, $\ln h_{90}$	0.90	0.20

Table 4. Mean differences (*D*) and standard deviations for the differences (S.D) between laser-derived and observed variables in sites A and B with different point densities.

Response variable	1.2 points m^{-2}		0.25 points m^{-2}		0.13 points m^{-2}		0.06 points m^{-2}	
	Mean *D*	S.D.	Mean *D*	S.D.	Mean *D*	S.D.	Mean *D*	S.D.
Site A								
h_L	-0.03	0.97	-0.01	0.96	-0.05	1.07	-0.06	1.15
G	-0.30	2.67	-0.08	2.73	0.01	3.37	-0.93	3.59
V	2.78	30.11	3.02	29.70	3.09	37.30	-6.01	39.10
Site B								
h_L	-0.35	0.55	-0.33	0.61	-0.06	0.85	-0.35	0.72
G	1.74	3.19	1.78	2.99	1.68	3.05	0.93	3.58
V	8.94	27.80	7.24	26.52	12.41	28.19	2.34	38.23

Comparing regression methods in estimation of biophysical properties of forest stands from two different inventories using laser scanner data

Næsset, E., Bollandsås, O.M. & Gobaggen, T. 2005. Remote Sensing of Environment 94: 541-553.

1 Airborne laser scanner data

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 8.-9.6.1999 (A), 23.8.-1.8.2001 (B)
- Flying altitude: 620-730 m (A), 430-980 m (B)
- Average speed: 71 m/s (A), 75 m/s (B)
- Pulse repetition frequency: 10 kHz
- Scan frequency: 21 Hz (A), 30 Hz (B)
- Max. scan angle, data collection: 17° (A), 16° (B)
- Max. scan angle, data processing: 14° (A), 15° (B)
- Footprint diameter: 21 cm (A), 23 cm (B)
- Average footprint spacing: 0.9 m (A), 1.0 m (B)
- Recorded echoes: first and last

2 Study area and field data

Data from two inventories are used. The first study area (A) is situated in Våler (59°30′N 10°55′E, 70-120 m a.s.l.) and the other in Krønsherad (60°10′N 9°35′E, 130-660 m a.s.l.) in south-east Norway. The sizes of the inventories are 1000 and 6400 hectares, respectively. Both areas consist of managed forest.

133 and 100 circular training plots were located systematically throughout the two study areas A and B, respectively. The plot size was 200 m^2 in inventory A and B 232.9 m^2 in inventory B. The plots were divided in three strata: young forest (stratum I), mature forest with poor site quality (stratum II) and mature forest with good site quality (stratum III). On young forest plots (total number of 80) all trees with diameter at breast height more than 4 cm were callipered. On mature forest plots (total number of 153) trees with DBH >10 cm were callipered. Tree heights were measured on sample trees.

61 test stands were selected for validation in inventory A. The size of the stands varied from 0.73 to 11.71 ha. In inventory B, 54 large test plots (with an average size of 0.37 ha) were selected. The test stands and plots were selected subjectively in order to represent different age, site quality and tree species combinations. The ground reference data was collected on systematically distributed sample plots. In young stand the plot size was 100 m^2 and in mature stands it was 200 m^2. The number of sample plots per stand varied from 14 to 30.

3 Regression models

- Lorey's mean height
- Dominant height
- Mean diameter by basal area
- Stem number
- Basal area
- Volume

The models were estimated using OLS (ordinary least squares), SUR (seemingly unrelated regression) and PLS (partial least squares) estimation. Separate models were estimated for each stratum.

Table 2. Relationships between logarithmic transformations of ground-based characteristics of the training plots (response variables) and laser-derived metrics from stepwise multiple regression analysis (OLS) for all training plots grouped together (common model) and estimated for each inventory separately. Dummy variable in common model with value=0 if inventory A and value=1 if inventory B. NS = Not statistically significant (p>0.05).

Response variable	Explanatory variables, common model	Dummy (p-value)	Explanatory variables, inventory A	Explanatory variables, inventory B
Young forest				
$\ln h_L$	$\ln h_{10l}$, $\ln h_{80l}$	0.046	$\ln h_{90l}$, $\ln d_{9l}$	$\ln h_{80l}$
$\ln h_{dom}$	$\ln h_{90l}$	0.173 NS	$\ln h_{90l}$, $\ln d_{2f}$, $\ln d_{5f}$	$\ln h_{90l}$, $\ln d0_f$, $\ln d_{6f}$, $\ln d_{9f}$
$\ln d_g$	$\ln h_{90l}$, $\ln d_{0f}$, $\ln d_{8f}$	0.986 NS	$\ln h_{10l}$, $\ln h_{80l}$, $\ln d_{0f}$	$\ln d_{8f}$, $\ln d_{2l}$
$\ln N$	$\ln h_{20f}$, $\ln h_{20l}$, $\ln h_{90l}$, $\ln d_{0f}$ $\ln h_{50f}$, $\ln f_{80l}$, $\ln h_{meanf}$,	0.929 NS	$\ln h_{80f}$, $\ln d_{0f}$	$\ln h_{0f}$, $\ln d_{0f}$, $\ln d_{8f}$
$\ln G$	$\ln d_{2f}$	0.749 NS	$\ln h_{10f}$, $\ln d_{1l}$	$\ln h_{90f}$, $\ln h_{cvf}$, $\ln d_{0l}$
$\ln V$	$\ln h_{10l}$, $\ln h_{meanf}$, $\ln d_{0f}$	0.28 NS	$\ln h_{meanl}$, $\ln d_{1f}$	$\ln h_{90l}$, $\ln h_{cvf}$, $\ln d_{0l}$

Table 2 continued

Mature forest, poor site quality					
$\ln h_{L}$	$\ln h_{20f}$, $\ln h_{90f}$, $\ln h_{90l}$	0.026		$\ln h_{90f}$	$\ln h_{90f}$, $\ln h_{90l}$
$\ln h_{dom}$	$\ln h_{90f}$	0.527	NS	$\ln h_{90f}$, $\ln h_{0l}$, $\ln h_{80l}$	$\ln h_{80l}$, $\ln h_{90l}$, $\ln d_{0l}$
$\ln d_{g}$	$\ln h_{60f}$, $\ln d_{0f}$, $\ln d_{5l}$, $\ln d_{9l}$	0.818	NS	$\ln h_{90l}$, $\ln d_{6f}$, $\ln d_{2l}$	$\ln h_{60f}$, $\ln h_{60l}$, $\ln d_{0f}$, $\ln d_{9l}$
$\ln N$	$\ln h_{80l}$, $\ln d_{0f}$, $\ln d_{9f}$, $\ln d_{4l}$	0.058	NS	$\ln h_{0f}$, $\ln d_{40l}$, $\ln h_{cvf}$, $\ln d_{4l}$	$\ln h_{50f}$, $\ln h_{60f}$, $\ln d_{0f}$, $\ln d_{4l}$
$\ln G$	$\ln h_{90f}$, $\ln d_{3f}$	0.706	NS	$\ln d_{4f}$	$\ln h_{80l}$, $\ln d_{0f}$
$\ln V$	$\ln h_{30f}$, $\ln h_{70f}$, $\ln h_{meanf}$, $\ln d_{0f}$	0.658	NS	$\ln h_{20f}$, $\ln h_{30f}$, $\ln h_{50f}$, $\ln d_{0f}$	$\ln h_{90l}$, $\ln d_{0f}$
Mature forest, good site quality					
$\ln h_{L}$	$\ln h_{90f}$	0.005		$\ln h_{90f}$, $\ln h_{9f}$	$\ln h_{90f}$
$\ln h_{dom}$	$\ln h_{30f}$, $\ln h_{90f}$, $\ln h_{50l}$, $\ln d_{4l}$	0.898	NS	$\ln h_{30f}$, $\ln h_{90f}$, $\ln d_{9f}$, $\ln d_{3l}$	$\ln h_{80f}$, $\ln h_{90f}$, $\ln d_{9l}$
$\ln d_{g}$	$\ln h_{90l}$, $\ln d_{1l}$	0.247	NS	$\ln h_{90l}$, $\ln d_{1l}$	$\ln h_{90f}$, $\ln d_{1f}$
$\ln N$	$\ln h_{90l}$, $\ln d_{2f}$, $\ln d_{7f}$, $\ln d_{1l}$	0.162	NS	$\ln h_{80f}$, $\ln d_{7f}$, $\ln d_{2l}$	$\ln h_{70l}$, $\ln d_{4f}$
$\ln G$	$\ln h_{90f}$, $\ln h_{cvf}$, $\ln d_{5f}$	0.27	NS	$\ln d_{5f}$, $\ln d_{1l}$, $\ln d_{8l}$	$\ln h_{70l}$, $\ln d_{4l}$
$\ln V$	$\ln h_{80l}$, $\ln d_{0f}$, $\ln d_{7f}$	0.646	NS	$\ln h_{90l}$, $\ln h_{5f}$	$\ln h_{30f}$, $\ln h_{70l}$, $\ln d_{1f}$

Table 3. Initial models used in SUR estimation and variables excluded from the initial SUR models when the final models were estimated.

Response variable	Explanatory variables, initial SUR model	Excluded variables
Young forest		
$\ln h_{L}$	$\ln h_{10l}$, $\ln h_{80l}$	
$\ln h_{dom}$	$\ln h_{90l}$	
$\ln d_{g}$	$\ln h_{90l}$, $\ln d_{0f}$, $\ln d_{8f}$	
$\ln N$	$\ln h_{20f}$, $\ln h_{20l}$, $\ln h_{90l}$, $\ln d_{0f}$	$\ln h_{20l}$
$\ln G$	$\ln h_{50f}$, $\ln f_{80l}$, $\ln h_{meanf}$, $\ln d_{2f}$	
$\ln V$	$\ln h_{10l}$, $\ln h_{meanf}$, $\ln d_{0f}$	
Mature forest, poor site quality		
$\ln h_{L}$	$\ln h_{20f}$, $\ln h_{90f}$, $\ln h_{90l}$	$\ln h_{20f}$, $\ln h_{90l}$
$\ln h_{dom}$	$\ln h_{90f}$	
$\ln d_{g}$	$\ln h_{60f}$, $\ln d_{0f}$, $\ln d_{5l}$, $\ln d_{9l}$	
$\ln N$	$\ln h_{80l}$, $\ln d_{0f}$, $\ln d_{9f}$, $\ln d_{4l}$	$\ln d_{9f}$
$\ln G$	$\ln h_{90f}$, $\ln d_{3f}$	
$\ln V$	$\ln h_{30f}$, $\ln h_{70f}$, $\ln h_{meanf}$, $\ln d_{0f}$	
Mature forest, good site quality		
$\ln h_{L}$	$\ln h_{90f}$	
$\ln h_{dom}$	$\ln h_{30f}$, $\ln h_{90f}$, $\ln h_{50l}$, $\ln d_{4l}$	$\ln h_{50l}$
$\ln d_{g}$	$\ln h_{90l}$, $\ln d_{1l}$	
$\ln N$	$\ln h_{90l}$, $\ln d_{2f}$, $\ln d_{7f}$, $\ln d_{1l}$	
$\ln G$	$\ln h_{90f}$, $\ln h_{cvf}$, $\ln d_{5f}$	$\ln h_{cvf}$
$\ln V$	$\ln h_{80l}$, $\ln d_{0f}$, $\ln d_{7f}$	

In PLS estimation, all six biophysical properties were estimated simultaneously. The numbers of explanatory variables included in the models were 41 for strata I, 33 for strata II and 31 for strata III. The numbers of latent variables were 7, 5 and 3, respectively.

4 Reliability of the models

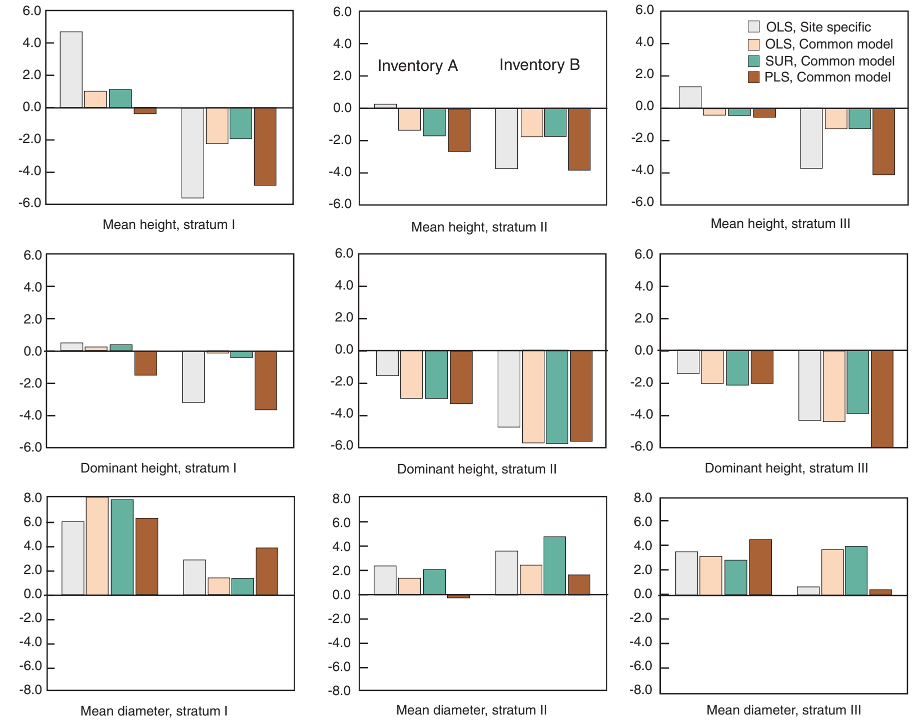

Figure 1 (Næsset et al. 2005). Standard deviation for the differences between predicted and ground-truth values using separate and common OLS models for inventories A and B, and SUR and PLS models common for both inventories.

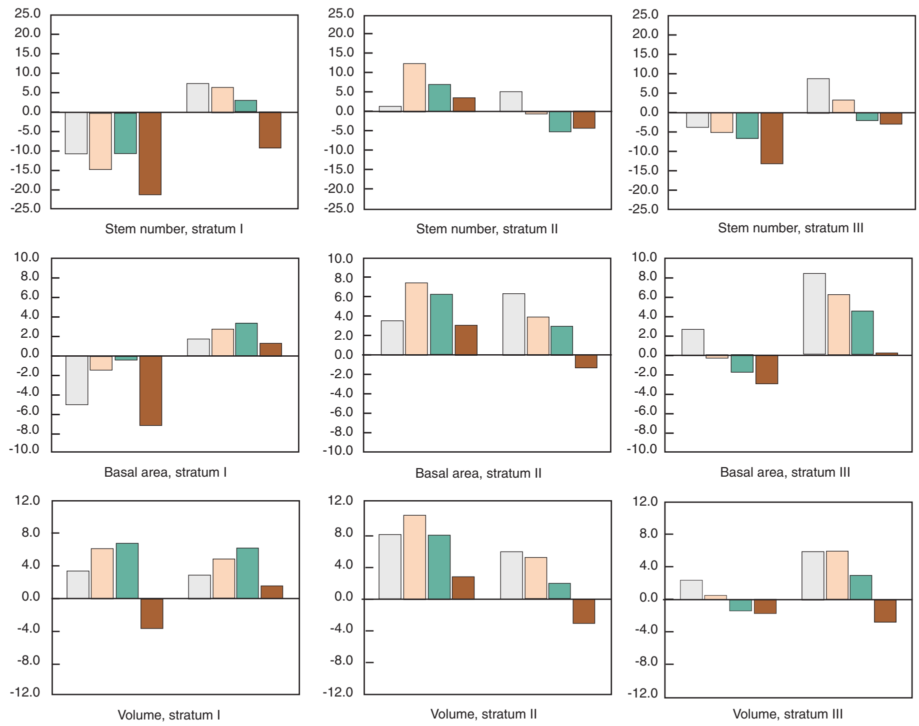

Figure 1 continued (Næsset et al. 2005). Standard deviation for the differences between predicted and ground-truth values.

Comparing stand inventories for large areas based on photo-interpretation and laser scanning by means of cost-plus-loss analyses

Eid, T., Gobaggen, T. & Næsset, E. 2004. Scandinavian Journal of Forest Research 19: 512-523.

1 Airborne laser scanner data

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 8.-9.6.1999
- Flying altitude: 700 m
- Average speed: 71 m/s
- Number of flight lines: 24+19
- Pulse repetition frequency: 10 kHz
- Scan frequency: 21 Hz
- Max. scan angle, data collection: 17°
- Max. scan angle, data processing: 14°
- Swath width: 420 m
- Footprint diameter: 21 cm
- Average footprint spacing: 0.92-0.94 m
- Recorded echoes: first and last

2 Study area and field data

Study areas are located in south-east Norway, in Våler (59°30′N 10°55′E, 70-120 m a.s.l.) and in Krønsherad (60°10′N 9°35′E, 130-660 m a.s.l.). The dominant tree species are spruce (*Picea abies*) and pine (*Pinus sylvestris*). Study area A (Våler) is the size of 1000 hectares and area B (Krønsherad) 6500 ha. 39 stands from area A and 38 from area B were selected for this study. All selected stands are mature forest.

In area A the average stand size was 1.7 ha, and the average number of plots located per stand was 19. The plot size was 200 m^2. All trees with breast height diameter >10 cm were callipered. Tree heights were measured on sample trees, the number of which per stand varied from 26 to 84 (with an average of 44).

In area B, the average stand size was 3740 m^2. Trees with DBH >10 cm were callipered. The number of sample trees varied from 49 to 77 (with an average of 61).

3 Method

Inventory methods were compared with cost-plus-loss-analysis, where the total costs are counted as a sum of net present values and inventory costs. In this study, inventories of basal area, dominant height and stem number based on airborne laser scanning data and aerial images were compared. The accuracies and biases of the variables were estimated, and the expenses of laser scanning data and aerial images data interpretation were assessed with cost-plus-loss analysis.

4 Reliability of the models

Table 1. Mean differences ($\overline{D}$) and standard deviation for the differences (SD) between estimated data and ground truth reference data in percentage of reference mean values for basal area (BA), dominant height (H_{dom}) and number of trees (N). NS = not statistically significant (p>0.05).

		BA			H_{dom}			N		
Inventory method	Site	Ref. (m^2ha^{-1})	$\overline{D}$ (%)	SD (%)	Ref. (m^2ha^{-1})	$\overline{D}$ (%)	SD (%)	Ref. (m^2ha^{-1})	$\overline{D}$ (%)	SD (%)
Photo-interpretation	Våler	24.9	-7.1	11.8	20.3	-5.6	8.9	720	-4.4 NS	30.3
Photo-interpretation	Krønsh.	25.4	-5.4 NS	20.0	20.3	-5.5	8.6	730	-0.6 NS	36.0
Laser scanning	Våler	24.9	0.1 NS	10.1	20.3	-1.8	5.2	720	-6.4 NS	20.6
Laser scanning	Krønsh.	25.4	7.5	12.2	20.3	-4.5	4.0	730	7.0	15.7

Comparison of basal area and stem frequency diameter distribution modelling using airborne laser scanner data and calibration estimation

Maltamo, M., Suvanto, A., Packalén, P. 2007. Forest Ecology and Management 247: 26-34.

1 Airborne laser scanner data

- Laser scanner: ALTM 2033 (Optech)
- Date of data acquisition: 4th August 2004
- Flying altitude: 1500 m
- Field of view: 30°
- Average point density: 0.7/m^2
- Recorded echoes: first and last

2 Study area and field data

Study area in Matalansalo, eastern Finland has a size of 1200 ha. The area is managed forest dominated by coniferous trees.

472 sample plots with a radius of 9 m were collected in the summer of 2004. 67 stands were selected randomly. 5-9 plots were systematically placed on each stand. Diameter at breast height, tree and storey class, and tree species were recorded for trees with DBH >5 cm. Height was measured on one sample tree for each species in each storey class on each plot. Heights for the rest of the trees were calculated with Veltheim's (1987) models. Tree volumes were calculated with Laasasenaho's (1982) models. The number of energy-wood stems (DBH 5-10 cm) and the volume of saw-wood-sized stems (DBH>17 cm) was calculated.

3 Regression models

- Weibull parameter β of the stem frequency distribution
- Weibull parameter γ of the stem frequency distribution
- Weibull parameter β basal area diameter distribution
- Weibull parameter γ basal area diameter distribution
- Stem number (N)
- Basal area (G)
- Volume (V)

Table 1. Seemingly unrelated regression (SUR) models for the Weibull parameters β and γ of the stem frequency distribution (fdd) and basal area diameter distribution (Gdd). Models for stem number (N), basal area (G) and volume (V) are also included.

Independent variable	$\ln(N)$	$\sqrt{G}$	$\ln(V)$	β_{fdd}	$\ln\gamma_{fdd}$	β_{Gdd}	$\sqrt{\gamma_{Gdd}}$
Intercept	7.019 (0.058)	-1.335 (0.191)	0.694 (0.208)	-25.902 (9024)	2.193 (0.079)	11.738 (0.544)	10.889 (0.783)
$fveg^2$	1.205 (0.090)			-6.275 (0.623)		-2.872 (0.644)	
lh_{mean}^2	-0.003 (0.0001)			0.042 (0.002)			
$lveg^2$	0.217 (0.07)						
$1/fh_{05}$	-0.566 (0.116)						
$\ln fh_{mean}$		1.582 (0.062)					
$fveg$		1.765 (0.259)					-1.072 (0.098)
$\sqrt{lveg}$		1.359 (0.280)			-0.356 (0.098)		
$\ln fh_{50}$			0.864 (0.042)				
$\sqrt{fveg}$			1.201 (0.206)				
$\ln lh_{mean}$			0.541 (0.042)				
$\ln lveg$			0.176 (0.048)				
fh_{20}^2				-0.004 (0.001)			
$1/lveg$				-0.546 (0.119)			
$1/fp_{95}$				38.896 (8.843)			
$\sqrt{fh_{std}}$					-0.744 (0.063)		
lh_{05}^2					0.001 (0.0002)		
fp_{10}^2					3.623 (1.085)		
fh_{20}					0.017 (0.006)		
fh_{90}^2						0.032 (0.001)	
fh_{40}^2						0.022 (0.006)	
lh_{20}						0.169 (0.040)	
fh_{30}^2						-0.027 (0.006)	
lh_{05}							0.022 (0.004)
fp_{80}							-9.489 (0.886)
s_i	0.026	0.033	0.008	1.624	0.008	1.543	0.010
ε_{ij}	0.049	0.133	0.029	3.637	0.043	4.462	0.063

4 Reliability of the models

Table 2. RMSE and bias of plot-level basal area and stem frequency diameter distributions.

Distribution type	Variable of interest	RMSE (%)	Bias
Stem frequency distribution	Volume	22.20	-3.60
	Basal area	18.35	-1.01
	Number of stems	27.87	-12.34
	Volume of saw-wood-sized trees	33.52	15.09
	Number of energy wood stems	70.28	298.77
Basal area diameter distribution	Volume	22.47	-1.92
	Basal area	16.77	0.06
	Number of stems	36.64	174.33
	Volume of saw-wood-sized trees	31.92	0.61
	Number of energy wood stems	83.39	421.88
Stem frequency distribution calibrated by *V*, *G* and *N*	Volume	20.56	-1.06
	Basal area	17.14	-0.18
	Number of stems	27.88	-12.04
	Volume of saw-wood-sized trees	36.1	21.28
	Number of energy wood stems	80.09	356.36
Basal area diameter distribution calibrated by *V*, *G* and *N*	Volume	21.32	-0.33
	Basal area	17.80	-0.11
	Number of stems	27.90	-11.89
	Volume of saw-wood-sized trees	36.46	18.55
	Number of energy wood stems	83.97	393.85

Table 3. RMSE and bias of stand-level basal area and stem frequency diameter distributions.

Distribution type	Variable of interest	RMSE (%)	Bias
Stem frequency distribution	Volume	12.34	-3.72
	Basal area	11.38	-0.99
	Number of stems	17.75	5.81
	Volume of saw-wood-sized trees	19.90	14.49
	Number of energy wood stems	55.75	300.95
Basal area diameter distribution	Volume	13.10	-2.21
	Basal area	8.80	0.06
	Number of stems	24.86	179.76
	Volume of saw-wood-sized trees	17.20	-0.11
	Number of energy wood stems	70.63	423.74
Stem frequency distribution calibrated by *V*, *G* and *N*	Volume	10.19	-1.30
	Basal area	9.06	-0.19
	Number of stems	17.81	-4.68
	Volume of saw-wood-sized trees	22.96	20.68
	Number of energy wood stems	65.6	360.32
Basal area diameter distribution calibrated by *V*, *G* and *N*	Volume	10.39	-0.63
	Basal area	9.23	-0.12
	Number of stems	17.81	-4.69
	Volume of saw-wood-sized trees	22.50	17.93
	Number of energy wood stems	70.22	398.92

Determination of mean tree height of forest stands using airborne laser scanner data

Næsset, E. 1997. ISPRS Journal of Photogrammetry & Remote Sensing 52: 49-56.

1 Airborne laser scanner data

- Laser scanner: ALTM 1020 (Optech)
- Date of data acquisition: 20.10.1995.
- Flying altitude: 640-825 m
- Average speed: 80 m/s
- Pulse repetition frequency: 2 kHz
- Scan frequency: 7 Hz
- Max. scan angle, data collection: 20°
- Max. scan angle, data processing: 19.2°
- Beam divergence: 0.25 mrad
- Footprint diameter: 13-16 cm
- Average footprint spacing: 2.8-3.3 m
- Recorded echoes: last

2 Study area and field data

Two study areas in Elverum (60°46′N 11°45′E, 170-200 m a.s.l.) and Grue (60°24′N 11°58′E, 200-360 m a.s.l.), south-east Norway were used in this study.

36 stands with a dominant tree species as spruce (*Picea abies*), pine (*Pinus sylvestris*) or mixture of the two were selected. In Elverum (1) the stands were dominated by pine (97%) and in Grue they were dominated by spruce (69 %). The age of the stands varied from 31 to 145 years. The sample plots were distributed systematically within each stand. In site 1 the number of sample plots per stand was in average 14 and in site 2 in average 15. The size of the sample plots was 100 m^2 and 200 m^2 in young and mature forests, respectively. In site 1, the average number of sample trees per stand was 18. In site 2, the corresponding number was 26. The height was measured for all sample trees. Lorey's mean height in site 1 was 17.9 m and in site 2 14.9 m.

3 Regression models

- Mean height of a stand (1-2)

Table 1. Explanations for the variables.

Variable	Explanation
h_{h1}	Arithmetic mean of the laser heights
$h_{15\times15}$	Laser height, computed by the grid approach using a grid cell size of 15×15 m
SITEDUMMY	A dummy variable. (Site 1 was pine-dominated and site 2 spruce-dominated forest)
G	Basal area
N	Stem number / ha
OFFNADIR	Off-nadir scan angle

Table 2. Ground truth stand mean height regressed against laser mean height. NS = not statistically significant (p>0.05).

Variable	Coefficient	
	laser height expressed by h_{h1}	laser height expressed by $h_{15\times15}$
(Intercept)	5.82	3.98
h_{h1}	0.96	
$h_{15\times15}$		0.75
SITEDUMMY	-1.61	0.65, NS
G	0.078	0.061, NS
N	-0.0018	-0.0019
OFFNADIR	0.039, NS	0.046, NS

4 Reliability of the models

Table 3. R-squared values for the two models.

	R^2
Model 1 (h_{h1})	0.94
Model 2 ($h_{15\times15}$)	0.94

Table 4. Mean difference ($\overline{D}$) between laser stand mean height (h_{h1}, h_{h2} and h_{h3}) and reference stand mean height (h_L), and standard deviation (SD) for the differences. h_{h2} and h_{h3} are weighted values. NS = not statistically significant (p>0.05).

Test site	Comparison	Mean h_L (m)	$\overline{D}$ (m)	SD (m)
1	h_{h1}-h_L	17.5	-4.1	1.6
1	h_{h2}-h_L	17.5	-2.9	1.4
1	h_{h3}-h_L	17.5	-2.1	1.4
1	$h_{15\times15}$-h_L	17.5	-0.4 NS	1.3
1	$h_{20\times20}$-h_L	17.5	0.3 NS	1.3
1	$h_{30\times30}$-h_L	17.5	1.1	1.3
2	h_{h1}-h_L	14.9	-5.5	1.3
2	h_{h2}-h_L	14.9	-3.6	1.1
2	h_{h3}-h_L	14.9	-2.4	1.1
2	$h_{15\times15}$-h_L	14.9	0.1 NS	1.2
2	$h_{20\times20}$-h_L	14.9	0.9	1.2
2	$h_{30\times30}$-h_L	14.9	1.9	1.3

Determination of tree size distribution models in mature forest from laser scanner data

Gobakken, T., Næsset, E., 2003. ScandLaser, Umeå, Sweden.
and Gobaggen, T. & Næsset, E. 2003. In: Hyyppä, J., Næsset, E., Olsson, H., Granqvist Pahlen, T., Reese, H. (Eds.), Proceedings of the Workshop Scandlaser Scientific Workshop on Airborne Laser Scanning of Forests. September 3–4, 2003, Umeå, Sweden. Working paper 112, 2003. Swedish University of Agricultural Sciences, Department of Forest Resource Management and Geomatics, pp. 71–77.

1 Airborne laser scanner data

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 23.7.-1.8.2001 (site 1) ja 8.-9.6.1999 (site 2)
- Recorded echoes: first and last

2 Study area and field data

Study area 1 is located in Krødsherad (60°10′N 9°35′E, 130-660 m a.s.l.) and area 2 in Våler (59°30′N 10°55′E, 70-120 m a.s.l.).

70 and 83 sample plots were measured on sites 1 and 2, respectively. All plots were located in mature forests. Plot size in site 1 was 233 m^2. In site 2, plot size was 200 m^2. Sample plots were divided in poor sites (71 plots) and good sites (83 plots). Site index in 40 years old forest on poor site was less than 11 meters. Diameter at breast height was measured on trees with DBH >10 cm. DBH was recorded in 2 cm classes. Heights were measured on sample trees. Basal area weighted mean height was 15.7 m in class 1 and 20.2 m in class 2.

Diameter distribution was produced by summing the measured trees in each diameter class. Basal area distribution was calculated by multiplying the stem number in each diameter class with the corresponding basal area.

Test data consisted of plots with a size of 0.4 ha. 38 of plots located on area 1 (Krødsherad) and 39 of them on area 2 (Våler). Stands and plots were selected subjectively. On the area 1, all trees were callipered. Circular sample plots with a size of 100-200 m^2 were systematically placed on the area 2. On these circular plots, all trees were callipered. Heights were measured on sample trees.

3 Regression models

– -Basal area distribution (1)

Table 1. Explanations for the variables.

Variable	Explanation
Y	G, (basal area, m^2/ha) and values of the percentiles d_{10}, d_{20}, …, d_{100} derived from field values of the basal area distributions for 10%, 20%, …, 100% (cm)
h_{0f}, h_{10f},…, h_{90f}	Percentiles of the first pulse laser canopy heights for 0%, 10%, …, 90% (m)
h_{0l}, h_{10l},…, h_{90l}	Percentiles of the last pulse laser canopy heights for 0%, 10%, …, 90% (m)
h_{meanf}, h_{meanl}	Mean of the first and last pulse laser canopy heights (m)
h_{cvh}, h_{cvl}	Coefficient of variation of the first and last pulse laser canopy heights (%)
d_{0f}, d_{1f}, …, d_{9f}	Canopy densities corresponding to the proportions of laser hits above fraction 0, 1, …, 9 to total number of pulses (first pulse data).
d_{0l}, d_{1l}, …, d_{9l}	Canopy densities corresponding to the proportions of laser hits above fraction 0, 1, …, 9 to total number of pulses (last pulse data).

Regression models for both sites were produced.

$$\begin{aligned}\ln Y &= \ln \beta_0 + \beta_1 \ln h_{of} + \beta_2 \ln h_{10f} + \ldots + \beta_{10} \ln h_{90f} \\ &+ \beta_{11} \ln h_{0l} + \beta_{12} \ln h_{10l} + \ldots + \beta_{20} \ln h_{90l} \\ &+ \beta_{21} \ln h_{meanf} + \beta_{22} \ln h_{meanl} + \beta_{23} \ln h_{cvf} + \beta_{24} \ln h_{cvl} \\ &+ \beta_{25} \ln d_{0f} + \beta_{26} \ln d_{1f} + \ldots + \beta_{34} \ln d_{9f} \\ &+ \beta_{35} \ln d_{0l} + \beta_{36} \ln d_{1l} + \ldots + \beta_{44} \ln d_{9l}\end{aligned} \quad (1)$$

Preliminary models were estimated by using stepwise method of least squares. A group of models for percentiles was estimated with seemingly unrelated regression (SUR).

4 Reliability of the models

Models were tested by producing mean height, stem number, basal area and volume from the predicted distribution.

Table 1. Bias between the predicted and measured values. h_L= basal area weighted mean height, N = stem number, G = basal area, V = volume. NS = not statistically significant (p>0.05).

Variable	Class (site)	Observed mean	Bias		
			Range	Mean	Standard deviation
h_L	1	15.8	-0.7 - 4.5	1.5	1.1
	2	19.8	-4.8 - 2.7	-0.4 NS	1.7
N	1	629	-274 - 234	-9 NS	117
	2	818	-454 - 216	-31 NS	145
G	1	20.33	-2.12 - 5.17	1.56	1.83
	2	29.4	-7.66 - 8.08	1.94	3.32
V	1	156.2	-13.9 - 50.4	26.0	16.3
	2	278.1	-61.0 - 89.5	11.8	33.5

Effects of different flying altitudes on biophysical stand properties estimated from canopy height and density measured with a small footprint airborne scanning laser

Næsset, E. 2004. Remote Sensing of Environment 91: 234-255.

1 Airborne laser scanner data

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 16.-17. 7. 2001
- Flying altitude: 530-540 m (low flight), 840-850 (high flight)
- Number of flight lines: 33 in both flights
- Pulse repetition frequency: 10 kHz
- Scan frequency: 30 Hz
- Max. scan angle, data collection: 16°
- Max. scan angle, data processing: 15°
- Footprint diameter: 16 cm (low flight), 26 cm (high flight)
- Pulse density: 0.6-1.3/m^2 (average 0.84-0.89/m^2)
- Recorded echoes: first and last

2 Study area and field data

Study area with a size of 1000 hectares is located in Våler, south-east Norway (59°30′N 10°55′E, 70-120 m a.s.l.). Main tree species are spruce (*Picea abies*) and pine (*Pinus sylvestris*).

133 circular sample plots were systematically distributed in a regular grid. Sample plots were divided in young forest (stratum I), mature forest with poor site quality (stratum II) and mature forest with good site quality (stratum III). The plot size was 300 m^2 and 400 m^2 in young and mature forests, respectively. In young forests, trees with diameter at breast height >4 cm were callipered. In mature forest, trees with DBH >10 cm were callipered. Heights were measured on sample trees, the amount of which per plot ranged from 2-17 (with an average of 9). Lorey's mean height in sample plots in stratum I was 14.5 m, in stratum II 16.4 m and in stratum III 20.5 m. Volume / ha in corresponding strata was 207.2 m^3, 157.0 m^3 and 292.1 m^3.

56 stands representing different combinations of age classes, site quality classes and tree species mixtures were subjectively selected for the study. The average number of plots per stand was 20. The plot size was 100 m^2 and 200 m^2 in young and mature forests, respectively. In young forests, trees with diameter at breast height >4 cm were callipered. In mature forest, trees with DBH >10 cm were callipered. The number of sample trees per plot varied from 24 to 87. Tree height was measured on sample trees. Loreys's mean height in stands in stratum I was 14.4 m, in stratum II 16.2 m and in stratum III 19.6 m. Volume / ha in corresponding strata was 184.7 m^3, 152.8 m^3, 280.8 m^3.

3 Regression models

There are separate models for each stratum in both flying heights.

- Lorey's mean height (h_L)
- Basal area (G)
- Volume (V)

Table 1. Explanations for the variables.

Variable	Explanation
h_L	Lorey's mean height (m)
G	Basal area (m²/ha)
V	Volume (m³/ha)
h_{10f}, h_{50f}, h_{90f}	10%, 50% and 90% percentiles of the first pulse laser canopy heights (m)
h_{10l}, h_{50l}, h_{90l}	10%, 50% and 90% percentiles of the last pulse laser canopy heights (m)
h_{maxl}	Maximum laser canopy height (m)
h_{cvf}	Coefficient of variation of laser canopy heights (%)
h_{meanf}	Arithmetic mean laser canopy height for first pulse data(m)
h_{meanl}	Arithmetic mean laser canopy height for last pulse data (m)
d_{1f}, d_{5f}, d_{9f}	Canopy densities corresponding to the proportions of laser hits above fraction 1, 5 and 9 respectively to total number of pulses (first pulse data).
d_{1l}, d_{5l}	Canopy densities corresponding to the proportions of laser hits above fraction 1 and 5 respectively to total number of pulses (last pulse data).

Models for young forest, low flight altitude:

$$\ln h_L = \beta_0 + \beta_1 \ln h_{90f} + \beta_2 \ln h_{meanl} \quad (1)$$

$$\ln G = \beta_0 + \beta_1 \ln h_{50f} + \beta_2 \ln d_{1f} \quad (2)$$

$$\ln V = \beta_0 + \beta_1 \ln h_{50f} + \beta_2 \ln d_{1f} + \beta_3 \ln d_{9f} \quad (3)$$

Models for young forest, high flight altitude:

$$\ln h_L = \beta_0 + \beta_1 \ln h_{90f} + \beta_2 \ln h_{meanl} \quad (4)$$

$$\ln G = \beta_0 + \beta_1 \ln h_{meanf} + \beta_2 \ln d_{1f} \quad (5)$$

$$\ln V = \beta_0 + \beta_1 \ln h_{meanf} + \beta_2 \ln d_{5l} \quad (6)$$

Models for mature forest, poor site quality, low flight altitude:

$$\ln h_L = \beta_0 + \beta_1 \ln h_{90l} \quad (7)$$

$$\ln G = \beta_0 + \beta_1 \ln h_{50f} + \beta_2 \ln h_{10l} + \beta_3 \ln d_{1f} \quad (8)$$

$$\ln V = \beta_0 + \beta_1 \ln h_{10l} + \beta_2 \ln h_{meanf} + \beta_3 \ln d_{1f} \quad (9)$$

Models for mature forest, poor site quality, high flight altitude:

$$\ln h_L = \beta_0 + \beta_1 \ln h_{90f} \quad (10)$$

$$\ln G = \beta_0 + \beta_1 \ln h_{10f} + \beta_2 \ln h_{50f} + \beta_3 \ln d_{1f} \quad (11)$$

$$\ln V = \beta_0 + \beta_1 \ln h_{10l} + \beta_2 \ln h_{meanf} + \beta_3 \ln d_{1l} \quad (12)$$

Models for mature forest, good site quality, low flight altitude:

$$\ln h_L = \beta_0 + \beta_1 \ln h_{90f} + \beta_2 \ln h_{cvf} + \beta_3 \ln d_{5l} \quad (13)$$

$$\ln G = \beta_0 + \beta_1 \ln h_{\max l} + \beta_2 \ln d_{5f} \quad (14)$$

$$\ln V = \beta_0 + \beta_1 \ln h_{meanf} + \beta_2 \ln d_{1f} + \beta_3 \ln d_{5l} \quad (15)$$

Models for mature forest, good site quality, high flight altitude:

$$\ln h_L = \beta_0 + \beta_1 \ln h_{90f} + \beta_2 \ln h_{10l} \quad (16)$$

$$\ln G = \beta_0 + \beta_1 \ln h_{90l} + \beta_2 \ln d_{1f} + \beta_3 \ln d_{5f} \quad (17)$$

$$\ln V = \beta_0 + \beta_1 \ln h_{meanf} + \beta_2 \ln d_{1f} + \beta_3 \ln d_{5l} \quad (18)$$

4 Reliability of the models

Table 2. Root mean square errors and R-squared values for the models.

Model	Response variable	RMSE	R^2
Young forest, low flight altitude			
1	h_L	0.08	0.90
2	G	0.11	0.90
3	V	0.14	0.94
Young forest, high flight altitude			
4	h_L	0.08	0.89
5	G	0.12	0.90
6	V	0.13	0.94
Mature forest, poor site quality, low flight altitude			
7	h_L	0.08	0.76
8	G	0.15	0.75
9	V	0.16	0.82
Mature forest, poor site quality, high flight altitude			
10	h_L	0.08	0.75
11	G	0.15	0.75
12	V	0.15	0.83
Mature forest, good site quality, low flight altitude			
13	h_L	0.06	0.86
14	G	0.12	0.84
15	V	0.13	0.91
Mature forest, good site quality, high flight altitude			
16	h_L	0.07	0.82
17	G	0.12	0.85
18	V	0.13	0.91

Table 3. Differences (*D*) between high- and low-flying altitudes for laser-derived metrics of small sample plots and standard deviation (SD) for the differences for first and last pulse data, respectively. NS = not statistically significant.

	D, first pulse		*D*, last pulse	
Metrics	Mean	SD	Mean	SD
Young forest				
h_{10} (m)	-0.01 NS	0.50	0.31 NS	1.09
h_{50} (m)	0.01 NS	0.23	0.16 NS	0.40
h_{90} (m)	0.05 NS	0.31	0.09 NS	0.40
h_{max} (m)	-0.01 NS	0.76	-0.08 NS	0.86
h_{mean} (m)	0.02 NS	0.17	0.20	0.32
h_{cv} (%)	0.17 NS	1.77	-0.61	3.55
d_1 (%)	0.15 NS	2.57	1.41 NS	3.80
d_5 (%)	0.85 NS	5.61	2.02	3.90
d_9 (%)	0.37 NS	1.86	0.54 NS	1.68
Mature forest, poor site quality				
h_{10} (m)	0.06 NS	0.75	0.14 NS	0.85
h_{50} (m)	-0.02 NS	0.43	0.40 NS	1.11
h_{90} (m)	0.01 NS	0.35	0.20 NS	0.44
h_{max} (m)	-0.20 NS	0.60	-0.23 NS	0.67
h_{mean} (m)	-0.01 NS	0.31	0.33	0.57
h_{cv} (%)	-0.05 NS	2.46	-2.19 NS	4.68
d_1 (%)	-0.47 NS	3.31	0.63 NS	2.98
d_5 (%)	-0.08 NS	2.86	1.43	2.47
d_9 (%)	0.74	1.26	0.75	1.32
Mature forest, good site quality				
h_{10} (m)	0.13 NS	0.59	0.52	1.20
h_{50} (m)	0.11 NS	0.39	0.23 NS	0.69
h_{90} (m)	0.10 NS	0.31	0.04 NS	0.39
h_{max} (m)	-0.01 NS	0.75	-0.03 NS	0.73
h_{mean} (m)	0.08 NS	0.23	0.32	0.47
h_{cv} (%)	-0.19 NS	1.75	-1.92	3.38
d_1 (%)	0.30 NS	2.17	2.10	3.69
d_5 (%)	0.88 NS	3.81	2.55	3.62
d_9 (%)	0.01 NS	2.13	0.05 NS	1.59

Table 4. Differences (D) between high- and low-flying altitudes for laser-derived metrics of forest stands and standard deviation (SD) for the differences for first and last pulse data, respectively. NS = not statistically significant.

	D, first pulse		D, last pulse	
Metrics	Mean	SD	Metrics	Mean
Young forest				
h_{10} (m)	0.06 NS	0.13	0.18 NS	0.30
h_{50} (m)	0.05 NS	0.17	0.29	0.24
h_{90} (m)	-0.01 NS	0.18	0.03 NS	0.16
h_{max} (m)	-0.12 NS	0.56	-0.03 NS	0.63
h_{mean} (m)	0.03 NS	0.14	0.21	0.16
h_{cv} (%)	-0.21 NS	0.49	-1.74	0.79
d_1 (%)	0.43 NS	1.04	1.84	1.43
d_5 (%)	0.96 NS	3.01	1.35 NS	2.34
d_9 (%)	0.08 NS	0.18	0.06 NS	0.13
Mature forest, poor site quality				
h_{10} (m)	0.01 NS	0.13	0.07	0.07
h_{50} (m)	-0.01 NS	0.13	0.47	0.19
h_{90} (m)	-0.02 NS	0.09	0.08 NS	0.13
h_{max} (m)	-0.04 NS	0.33	-0.08 NS	0.33
h_{mean} (m)	-0.02 NS	0.09	0.25	0.13
h_{cv} (%)	-0.01 NS	0.47	-1.59	0.72
d_1 (%)	0.13 NS	0.64	1.36	0.73
d_5 (%)	0.14 NS	1.05	1.20	0.84
d_9 (%)	-0.02 NS	0.13	-0.01 NS	0.11
Mature forest, good site quality				
h_{10} (m)	0.07 NS	0.16	0.29	0.23
h_{50} (m)	0.05 NS	0.15	0.50	0.23
h_{90} (m)	0.05 NS	0.16	0.10 NS	0.19
h_{max} (m)	-0.01 NS	0.80	-0.02 NS	0.84
h_{mean} (m)	0.06 NS	0.13	0.37	0.14
h_{cv} (%)	-0.24 NS	0.42	-2.16	0.65
d_1 (%)	0.19 NS	0.94	1.75	1.26
d_5 (%)	0.52 NS	2.44	1.90	1.81
d_9 (%)	0.08 NS	0.36	0.06 NS	0.28

Table 5. Testing of the selected regression models for biophysical properties derived from high- and low-altitude laser data against ground-truth data from the test stands, and test results from the simulation study of estimating regression equations and predicting biophysical properties of the test stands by mixing laser data from high and low altitude by a random procedure using 10 000 iterations.

	Low altitude		High altitude		Simulation results					
	$\overline{D}$	SD	$\overline{D}$	SD	$\lvert\overline{D}\rvert_{p10}$	$\lvert\overline{D}\rvert_{p90}$	Mean $\overline{D}$	SD_{p10}	SD_{p90}	Mean SD
Young forest										
h_L	0.49	0.86	0.48	0.79	0.42	1.09	0.75	0.69	1.17	0.83
G	-2.86	2.30	-2.66	2.49	2.48	5.16	-3.34	2.36	3.3	2.73
V	26.5	30.8	-32.4	29.0	4.6	34.4	1.50	25.5	37.5	30.7
Mature forest, poor site quality										
h_L	0.49	0.41	0.59	0.35	0.4	0.64	0.58	0.33	0.42	0.40
G	-0.18 NS	2.25	-0.01 NS	2.32	0.05	3.16	-0.71	2.24	2.72	2.40
V	3.7 NS	20.2	4.0 NS	20.3	2.5	6.5	1.90	19.2	21.5	20.2
Mature forest, good site quality										
h_L	0.41 NS	1.11	0.80	0.87	0.32	1.14	0.83	0.88	1.14	0.99
G	-4.68	3.52	-5.16	3.01	2.57	5.19	-4.34	2.82	3.52	3.26
V	-31.1	32.8	-14.2 NS	30.0	11.0	29.8	-18.8	29.9	33.7	31.7

Effects on estimation accuracy of forest variables using different pulse density of laser data

Magnusson, M., Fransson, J.E.S. & Holmgren, J. 2007. Forest Science 53: 619-626.

1. Airborne laser scanner and satellite data and preprocessing of ALS data

- Laser scanner: TopEye
- Date of data acquisition: 9.8.2003
- Flying altitude: 430 m
- Average speed: 25 m/s
- Pulse repetition frequency: 7 kHz
- Scan frequency: 17 Hz
- Max. scan angle, data collection: 20°
- Beam divergence: 1.0 mrad
- Pulse density: 2.5 / m^2
- Recorded echoes: first and last

The effect of pulse density was tested by thinning the pulse density of the first pulses using the TerraScan software. The thinning was carried out by allowing a minimum horizontal distance of 1, 2, …, 15 m between adjacent laser returns. The ground returns for each thinning level were then connected in a TIN and converted to a 1×1 DEM.

The accuracy of estimation was considered with grid and stand approaches. In the grid approach, squares with a size of 10×10, 20×20, …, 150×150 m were used to obtain about 100 pulses per square using the thinned point clouds. Laser variables were calculated for each square and averaged to the stand level using area-weighted averaging. Ground truth variables were linked with the averaged stand level laser variables. In stand approach, ground truth variables were linked with stand level laser variables.

2. Study area and field data

Study area with a size of 1200 ha is located in Remningstorp, southern Sweden (58°30′N 13°40′E). Dominant tree species are spruce (*Picea abies*), pine (*Pinus sylvestris*) and birch (*Betula spp*.).

Data for the study consists of a digital stand boundary map in vector format and a forest management plan. Stand boundaries were checked up using aerial orthoimages. 340 stands were divided into the stem volume ranges: 0-100, 100-200, …, 600-700 m^3ha^{-1}. 125 of these were randomly selected for field inventory. Circular sample plots with a radius of 10 m were randomly positioned in a stand-unique grid. On each plot, all trees were callipered and the heights were measured on randomly selected sample trees.

Standwise volume and tree heights from the field measurements collected in 1999-2002 were adjusted to year 2003. Conifer-dominated stands (total number of 70) with a soil type till (were selected for this study. Stand volume ranged from 30 to 620 m^3ha^{-1} with an average of 286. On average 10 sample plots were measured in each stand.

3. Regression models

Table 1. Explanations for the variables.

Variable	Explanation
*h*i	Tree height for stand i (m)
*h*laser,i	Laser height in stand i
εi	Random error
δi	Sampling error i
vi	Volume in stand i (m3ha-1)
*d*laser,i	Laser-derived canopy density in stand i
B1i-B4i	Average DN-values in stand i

Height:

$$h_i = \alpha_0 + \alpha_1 h_{laser,i} + \varepsilon_i + \delta_i \quad (1)$$

Volume:

$$\ln(v_i) = \alpha_0 + \alpha_1 \ln(h_{laser,i}) + \alpha_2 \ln d_{laser,i} + \varepsilon_i + \delta_i \quad (2)$$

4. Reliability of the models

Table 2. Root mean square error (RMSE), adjusted coefficient of determination (R^2_{adj}), regression coefficients (α_0-α_1) and corresponding *q*-value for tree height regression function derived from model 1 using high pulse density laser data, based on 70 stands. NS = not statistically significant.

Laser derived height	RMSE (m)	R2adj (%)	α0	α1	q
90th	0.819	97.5	0.414 NS	1.03	1.02
mean	0.946	96.8	0.937	1.32	1.02

Estimating forest growth using canopy metrics derived from airborne laser scanner data

Næsset, E. & Gobakken, T. 2005. Remote Sensing of Environment 96: 453-465.

1 Airborne laser scanner data

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 8.-9.6.1999 (A) and 16.-17.7.2001 (B)
- Flying altitude: 700 m (A) and 850 m (B)
- Number of flight lines: 43 (A) and 33 (B)
- Pulse repetition frequency: 10 kHz
- Scan frequency: 21 Hz (A) and 30 Hz (B)
- Max. scan angle, data collection: 17° (A) and 16° (B)
- Max. scan angle, data processing: 14° (A) and 15° (B)
- Footprint diameter: 21 cm (A) and 26 cm (B)
- Pulse density: 1.18/m^2 (A) and 0.87 /m^2 (B)
- Recorded echoes: first and last

2 Study area and field data

Study area with a size of 1000 hectares is located in Våler, south-east Norway (59°30′N 10°55′E, 70-120 m a.s.l.). The main tree species are spruce (*Picea abies*) and pine (*Pinus sylvestris*). Ground reference data consists of sample plots and forest stands.

133 sample plots were systematically distributed in the study area. The plots were divided in three strata: young forest (stratum I), mature forest with poor site quality (stratum II) and mature forest with good site quality (stratum III). The plot size was 300 m^2 in stratum I and 400 m^2 in strata II and III. In stratum I, trees with diameter at breast height >4 cm were callipered. In mature forests, trees with DBH >10 cm were callipered. Heights were measured on sample trees. Lorey's mean height in sample plots in young forests was 14.5, in mature forests with poor site quality 16.6 m and in mature forests with good site quality 20.5 m. Volumes per hectare in sample plots were 207.2 m^3, 157.0 m^3 and 292.1 m^3 in stratum I, II and III, respectively.

56 stands were selected to be used in this study. The average size of a stand was 1.7 ha and the amount of sample plots per stand was in average 20. In young forests, the plot size was 100 m^2 and in mature forests that was 200 m^2. Trees with DBH > 4 cm and >10 cm were callipered in young and mature forests, respectively. Heights were measured on sample trees. The amount of sample trees per stand ranged from 24 to 87 (with an average of 44). Lorey's mean height in young forest stands was 14.4 m, in mature forests with poor site quality 16.2 m and in mature forests with good site quality 19.6 m. Volumes per hectare in corresponding stands were 184.7 m^3, 152.8 m^3 and 280.8 m^3.

3 Regression models

Separate models were estimated for each stratum.

- Lorey's mean height (h_L)
- Basal area (G)
- Volume (V)

Table 1. Explanations for the variables.

Variable	Explanation
h_L	Lorey's mean height (m)
G	Basal area (m^2/ha)
V	Volume (m^3/ha)
h_{10f}, h_{50f}, h_{90f}	Percentiles of the laser canopy heights for 10%, 50% and 90% for first pulses (m)
h_{90l}	Percentile of the laser canopy heights for 90% for last pulses (m)
h_{cvl}	Coefficient of variation of the last pulse laser canopy heights (%)
h_{meanf}	Arithmetic mean laser canopy height of first pulses (m)
h_{meanl}	Arithmetic mean laser canopy height of first pulses (m)
d_{1f}	Canopy density corresponding to the proportion of laser hits above fraction 1 to total number of pulses (first pulses)
d_{1l}, d_{5l}	Canopy densities corresponding to the proportions of laser hits above fraction 1 and 5 to total number of pulses (last pulses)

Young forest:

$$\ln h_L = \beta_0 + \beta_1 \ln h_{10f} + \beta_2 \ln h_{meanl} \quad (1)$$

$$\ln G = \beta_0 + \beta_1 \ln h_{10f} + \beta_2 \ln h_{50f} + \beta_3 \ln d_{1f} \quad (2)$$

$$\ln V = \beta_0 + \beta_1 \ln h_{meanl} + \beta_2 \ln d_{1f} \quad (3)$$

Mature forest with poor site quality:

$$\ln h_L = \beta_0 + \beta_1 \ln h_{90l} \quad (5)$$

$$\ln G = \beta_0 + \beta_1 \ln h_{90f} + \beta_2 \ln h_{cvl} + \beta_3 \ln d_{5l} \quad (6)$$

$$\ln V = \beta_0 + \beta_1 \ln h_{50f} + \beta_2 \ln d_{1l} \quad (7)$$

Mature forest with good site quality:

$$\ln h_L = \beta_0 + \beta_1 \ln h_{10f} + \beta_2 \ln h_{90l} + \beta_3 \ln d_{5l} \quad (8)$$

$$\ln G = \beta_0 + \beta_1 \ln h_{meanf} + \beta_2 \ln d_{1f} + \beta_3 \ln d_{5l} \quad (9)$$

$$\ln V = \beta_0 + \beta_1 \ln h_{meanl} + \beta_2 \ln d_{1l} \quad (10)$$

4 Reliability of the models

Table 2. Root mean square values and R-squared values for the models.

Model	Response variable	RMSE	R^2
Young forest			
1	h_L	0.08	0.91
2	G	0.11	0.91
3	V	0.13	0.95
Mature forest with poor site quality			
4	h_L	0.09	0.71
5	G	0.14	0.78
6	V	0.15	0.85
Mature forest with good site quality			
7	h_L	0.07	0.85
8	G	0.12	0.86
9	V	0.13	0.91

Table 3. Growth (1999-2001) of the forest stands estimated from field measurements, growth predicted from laser data (1999-2001) using the regression equations and mean difference and standard deviation for the differences (SD). NS = not statistically significant (p>0.05).

Response variable	Mean field estimated growth	Mean laser predicted growth	Difference	
			Mean	SD
Young forest				
h_L (m)	0.70	0.33	-0.37	0.34
G (m^2 ha^{-1})	2.20	3.70	1.50	0.66
V (m^3 ha^{-1})	19.8	29.2	9.4	6.0
Mature forest, poor site quality				
h_L (m)	0.21	0.42	0.21	0.18
G (m^2 ha^{-1})	0.81	-0.37 NS	-1.18	1.15
V (m^3 ha^{-1})	7.7	1.2 NS	-6.5	10.2
Mature forest, good site quality				
h_L (m)	0.44	0.19 NS	-0.25	0.47
G (m^2 ha^{-1})	1.65	3.53	1.88	0.67
V (m^3 ha^{-1})	19.2	-1.0 NS	-20.2	9.9

Table 4. Growth (1999-2001) of the sample plots estimated from field measurements, growth predicted from laser data (1999-2001) using the regression equations and mean difference and standard deviation for the differences (SD). NS = not statistically significant (p>0.05).

Response variable	Mean field estimated growth	Mean laser predicted growth	Difference Mean	Difference SD
Young forest				
h_L (m)	0.95	0.80	-0.15 NS	0.94
G (m^2 ha^{-1})	2.92	4.23	1.31	1.49
V (m^3 ha^{-1})	28.6	38.4	9.8	17.7
Mature forest, poor site quality				
h_L (m)	0.23	0.50	0.27	0.47
G (m^2 ha^{-1})	0.84	-1.28	-2.12	2.37
V (m^3 ha^{-1})	8.4	-1.5 NS	-9.8	24.7
Mature forest, good site quality				
h_L (m)	0.48	0.25 NS	-0.23 NS	1.28
G (m^2 ha^{-1})	1.66	3.46	1.80	1.55
V (m^3 ha^{-1})	21.0	1.4 NS	-19.6	25.8

Estimating percentile-based diameter distributions in uneven-sized Norway spruce stands using airborne laser scanner data

Bollandsås, O.M., Næsset, E. 2007. Scandinavian Journal of Forest Research 22: 33-47.

1 Airborne laser scanner data

Inventory A

- Laser scanner: ALTM 1233 (Optech)
- Date of data acquisition: 9th October 2003
- Flying altitude: 600 m
- Average speed: 35 m/s
- Number of flight lines: 21
- Pulse repetition frequency: 33 kHz
- Scan frequency: 50 Hz
- Max. scan angle, data collection: 11°
- Max. scan angle, data proceccing: 10.5°
- Swath width: 230 m
- Footprint diameter: 18 cm
- Average point density: 5.0/m^2
- Recorded echoes: first and last

Inventory B

- Laser scanner: ALTM 1233 (Optech)
- Date of data acquisition: 9th October 2003
- Flying altitude: 800 m
- Average point density: 0.7/m^2
- Recorded echoes: first and last

2 Study area and field data

Inventory A

Field data used in model calibration consisted of 20 circular sample plots (0.1 ha). Study area is located in south-eastern Norway (59°50′N 11°02′E, 190-370 m a.s.l.). Dominant tree species were Norway spruce (*Picea abies*) and Scots pine (*Pinus sylvestris*). The area had a great within-stand variation of tree ages and tree sizes. The plots were selected subjectively. They should be spruce-dominated and have a multi-layered canopy structure. DBH was measured on trees with DBH≥3 cm. Tree heights were measured on sample trees. Gini coefficient (GC) was computed for each plot. It is an objective measure of the size distribution. GC is 0 when there is no variation in tree size, and when the size variation increases, the value approaches 1.

Inventory B

Validation data was collected in 2003 in Nordre Land in south-eastern Norway (60°50′N 10°05′E, 140-900 m a.s.l.). 0.1 ha circular plots were distributed systematically on the study area of 25 000 ha. Each plot had an inner circle with a radius of 8.92 m. All trees with DBH≥4 cm were callipered on a plot inner circle. On the rest of the plot area, trees with DBH≥10 cm were callipered. The number of trees 4≤DBH<10 cm for the entire 0.1 ha plot was extrapolated assuming, that the trees were evenly spatially distributed. Validation data was divided into three strata deriving from the GC values. Stratum 1 had GC values up to 0.30 (normal distributions). Stratum 2 included plots with values between 0.31 and 0.46 (approximately uniform distributions). Stratum 3 had the same GC range as the model calibration data (GC>0.47).

3 Regression model

– Percentiles 10%, 20%, …, 90% and 100% of the basal area distribution, and basal area.

The percentiles and basal area were modelled simultaneously by means of partial least squares regression.

4 Reliability of the model

Table 1. Estimation results from the partial least squares modelling: R^2 and RMSE for the 10 percentiles of the basal area distribution and basal area.

Dependent variable	R^2	RMSE (cm)
d_{10}	0.57	2.39
d_{20}	0.69	2.58
d_{30}	0.62	2.90
d_{40}	0.70	2.82
d_{50}	0.73	2.71
d_{60}	0.80	2.34
d_{70}	0.67	3.20
d_{80}	0.75	3.20
d_{90}	0.75	3.21
d_{100}	0.44	6.01
BA	0.67	3.09

Table 2. Mean difference between predicted volume according to the predicted distributions and observed plot volume (V), and standard deviation for the dirrerences (SD); from cross-validation (CV) and independent validation (IV). *p<0.05, ns= not significant (p>0.05).

Validation method	No. of observations	Observed mean *V*	Mean difference	SD (%)	Error index	
					Range	Mean
CV	20	360.7	-3.3 ns	11.0	41.5-126.8	78.1
$IV_{Stratum\ 1}$	3	281.0	0.6 ns	13.4	95.9-131.8	111.3
$IV_{Stratum\ 2}$	9	397.1	-12.6 *	13.7	49.4-106.2	77.0
$IV_{Stratum\ 3}$	6	338.9	-4.4 ns	13.1	26.5-97.6	59.7
$IV_{Full\ range}$	18	358.3	-8.3 *	14.2	26.5-131.8	77.0

Estimating timber volume of forest stands using airborne laser scanner data

Næsset, E. 1997. Remote Sensing of Environment 61: 246-253.

1 Airborne laser scanner data

- Laser scanner: ALTM 1020 (Optech)
- Date of data acquisition: 20.10.1995.
- Flying altitude: 640-825 m
- Average speed: 80 m/s
- Pulse repetition frequency: 2 kHz
- Scan frequency: 7 Hz
- Max. scan angle, data collection: 20°
- Max. scan angle, data processing: 19.2°

- Beam divergence: 0.25 mrad
- Footprint diameter: 13-16 cm
- Distance between footprints on the ground: 2.8-3.3 m
- Recorded echoes: last

2 Study area and field data

Two study areas in Elverum (60°46′N 11°45′E, 170-200 m a.s.l.) and Grue (60°24′N 11°58′E, 200-360 m a.s.l.), south-east Norway were selected for the study.

36 stands with a main tree species of spruce (*Picea abies*), pine (*Pinus sylvestris*) or mixture of both were used. In Elverum the stands were pine-dominated (97 %) and in Grue spruce-dominated (69 %). The age of the forest varied from 31 to 145. Sample plots were distributed systematically on the stands. The average number of sample plots per stand was 14 and 15 in Elverum and Grue, respectively. In young forests the plot size was 100 m^2 and in mature forests 200 m^2. In every sample plot, at least one sample tree was selected. In Elverum the number of sample trees per plot was in average 18. The corresponding number in Grue was 26. Tree height was measured on sample trees. Trees with diameter at breast height was >4 cm and >10 cm were callipered in young and mature forests, respectively.

3 Regression models

- Volume/ha inclusive bark (1-2)

Data for model 1 was pine-dominated and data for model 2 spruce-dominated.

Table 1. Explanations for the variables.

Variable	Explanation
V_f	Volume/ha inclusive bark (m^3/ha)
h_{15}	Laser stand mean height (m)
D	Mean laser canopy cover density (%)
h_a	Mean height of all laser pulses within a stand (m)

Total volume/ha inclusive bark:

$$V_f = \beta_0 h_{15}^{\beta_1} D^{\beta_2} \tag{1}$$

$$V_f = \beta_0 h_{15}^{\beta_1} h_a^{\beta_2} \tag{2}$$

Table 2. Regression coefficients for stand volume regressions. NS = not statistically significant.

Variable	Model 1, Elverum	Model 1, Grue	Model 2, Elverum	Model 2, Grue
(intercept)	2.595 NS	2.15 NS	1.22 NS	4.211 NS
h_{15}	1.775	1.706	0.886 NS	0.816 NS
D	0.809	0.667		
h_a			0.787 NS	0.823

4 Reliability of the models

Table 3. Root mean square error (RMSE) and coefficient of determination (R^2) for the models.

	Logarithmic scale		Original scale	
	RMSE	R^2	RMSE (m^3/ha)	R^2
Model 1, Elverum	0.306	0.744	81.8	0.472
Model 1, Grue	0.243	0.796	31.3	0.838
Model 2, Elverum	0.317	0.726	82.8	0.456
Model 2, Grue	0.218	0.836	26.1	0.887

Estimating tree height and tree crown properties using airborne scanning laser in a boreal nature reserve

Næsset, E. & Økland, T. 2002. Remote Sensing of Environment 79: 105-115.

1 Airborne laser scanner data

Østmarka:

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 9.6.1999
- Flying altitude: 590 m
- Average speed: 71 m/s
- Number of flight lines: 3+3
- Pulse repetition frequency: 10 kHz
- Scan frequency: 23 Hz
- Max. scan angle, data collection: 15°
- Max. scan angle, data processing: 13°
- Swath width: 320 m
- Beam divergence: 0.30 mrad
- Footprint diameter: 18 cm
- Average distance between footprints: 0.91 m
- Recorded echoes: first and last

Våler:

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 8.-9.6.1999
- Flying altitude: 690 m
- Average speed: 71 m/s
- Number of flight lines: 19+24
- Pulse repetition frequency: 10 kHz
- Scan frequency: 21 Hz
- Max. scan angle, data collection: 17°
- Max. scan angle, data processing: 14°
- Swath width: 420 m
- Beam divergence: 0.30 mrad
- Footprint diameter: 21 cm
- Average distance between footprints: 0.94 m
- Recorded echoes: first and last

2 Study area and field data

The first study area is situated in Østmarka (59°50′N 10°03′E, 240-290 m a.s.l.) south-east Norway. The area consists of natural boreal forest with a main tree species of spruce (*Picea abies*).

Size of the area was 2.5 hectares. 10 rectangular plots of 5×10 m were selected in order to represent different combinations of aspect, topographic conditions, and other ecological properties. Field data was collected in 1988 and 1998. In 1999 the data was modified: dead trees and trees of other species than spruce were discarded. Lorey's mean height (mean height weighted by basal area) for each plot was computed. This data was used for estimation of height and crown properties with cross-validation.

The other study are with a size of 1000 located in Våler (59°30′N 10°55′E, 70-120 m a.s.l.), south-east Norway. The dominant tree species were spruce and pine (*Pinus sylvestris*).

174 circular sample plots were distributed systematically in the study area. 27 of these plots were used in this study, and they were situated in spruce-dominated mature forests. The plot size was 200 m^2. Field data was collected in 1999. Trees with diameter at breast height >4 cm were callipered. On spruces with DBH >15 cm, height to the crown was also measured. This data was used in estimatin crown properties with cross-validation.

3 Regression models

Østmarka:

- Height (h)
- Height to the crown (h_c)
- Relative crown lenght (R_c)
- Lorey's mean height (h_L)
- Average height to the crown ($\bar{h}_c$)
- Average relative crown lenght $\bar{R}_c$

Våler:

- Avereage height to the crown ($\bar{h}_c$)
- Average relative crown lenght $\bar{R}_c$

Table 1. Explanations for the variables.

Variable	Explanation
h	Height
h_c	Height to the crown
R_c	Relative crown length
h_L	Lorey's mean height (mean height weighted by basal area)
h_{25f}	The quantile corresponding to the 25 percentiles of the first pulse laser heights (m)
$h_{25l}…h_{90l}$	The quantiles corresponding to the 25…95 percentiles of the last pulse laser heights (m)
h_{maxf}	Maxima of first pulse laser heights (m)
h_{maxl}	Maxima of last pulse laser heights (m)
h_{cvf}	Coefficient of variation of first pulse laser heights (%)

Østmarka:

$$\ln h = -0{,}0298 + 0{,}998 \ln h_{\max f} \quad (1)$$

$$\ln h_c = -0{,}904 + 0{,}561 \ln h_{25f} + 0{,}441 \ln h_{25l} \quad (2)$$

$$\ln R_c = 3{,}696 + 0{,}189 \ln h_{cvf} \quad (3)$$

$$\ln h_L = -1{,}079 + 1{,}298 \ln h_{\max l} \quad (4)$$

$$\ln \bar{h}_c = -4{,}561 + 2{,}116 \ln h_{75l} \quad (5)$$

$$\ln \bar{R}_c = 6{,}420 - 0{,}712 \ln h_{90l} \quad (6)$$

Våler:

$$\ln \bar{h}_c = -2{,}528 + 1{,}581 \ln h_{75l} \quad (7)$$

$$\ln \bar{R}_c = 4{,}972 - 0{,}877 \ln h_{75l} + 0{,}533 \ln h_{\max f} \quad (8)$$

4 Reliability of the models

Table 2. Root mean square error (RMSE) and coefficient of determination (R^2) for the models.

Model	Response variable	RMSE	R^2
1	h (m)	0.23	0.75
2	h_c (m)	0.37	0.53
3	R_c (%)	0.16	0.51
4	h_L (m)	0.06	0.91
5	$\bar{h}_c$ (m)	0.25	0.71
6	$\bar{R}_c$ (%)	0.08	0.60
7	$\bar{h}_c$ (Våler)	0.21	0.61
8	$\bar{R}_c$ (Våler)	0.11	0.47

Table 3. Differences between predicted and ground-truth values and standard deviation (SD) for the differences in cross-validation of the selected regressions.

		Differences		
Variable	Observed mean	Range	Mean*	SD
h (m)	17.85	-3.83 - 14.03	0.18	3.15
h_c (m)	5.6	-5.08 - 2.97	0.03	2.19
R_c (%)	70.44	-21.51 - 18.31	0.16	10.48
h_L (m)	19.51	-1.39 - 3.59	0.04	1.49
$\bar{h}_c$ (m)	5.31	-1.95 - 2.15	0.01	1.24
$\bar{R}_c$ (%)	7.51	-12.95 - 12.09	0.17	6.32
$\bar{h}_c$ (Våler)	7.28	-3.82 - 2.90	0.05	1.52
$\bar{R}_c$ (Våler)	65.25	-10.50 - 20.14	0.05	7.11

* Mean was not statistically significant (p>0.05) on any of the variables.

Estimating tree heights and number of stems in young forest stands using airborne laser scanner data

Næsset, E. & Bjerknes, K-O. 2001. Remote Sensing of Environment 78: 328-340.

1 Airborne laser scanner data

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 8.-9.6.1999
- Flying altitude: 690 m
- Average speed: 71 m/s
- Number of flight lines: 19+24
- Pulse repetition frequency: 10 kHz
- Scan frequency: 21 Hz
- Max. scan angle, data collection: 17°
- Max. scan angle, data processing: 14°
- Swath width: 420 m
- Beam divergence: 0.30 mrad
- Footprint diameter: 21 cm
- The average distance between footprints: 0.9 m
- Recorded echoes: first and last

2 Study area and field data

Study area with of 1000 hectares is situated in Våler, south-east Norway (59°30′N 10°55′E, 70-120 m a.s.l.). The main tree species in the area are spruce (*Picea abies*) and pine(*Pinus sylvestris*).

39 circular experimental plots of 200 m^2 were subjectively distributed in varying kinds of young forests. Each plot was divided in quarters (50 m^2). The number of trees with height >1.5 meters were counted in each quarters. Stem number per hectare ranged from 1650 to 7100. In every quarter, a tree height was measured on about five trees. The average height was 3.80 m. This data was used for assessing the accuracy of laser dominant height and laser stem number.

174 sample plots were distributed systematically as a regular grid. 29 of them were classified as young forest using photo interpretation, and ground-truth data of these plots was collected and used in this study. Plot size was 7.21×7.21 m. The corners of the plots were used as central points for subplots (with size of 40 m^2), which were divided in four quadrants (each size of 10 m^2). Two tallest trees in the first quadrant of a subplot were identified (species, assessed age). Height of the first tree was measured. Dominant height per hectar was computed.

12 test stands were subjectively selected to represent different height classes in young stands. Test stands were selected independently of sample plots. On each test stand, 16-26 circular plots (40 m^2) were distributed systematically. Plots were distributed in quadrants, each a size of 10 m^2. Two tallest trees of the first quadrant were indentified and height of the first tree was measured.

Sample plots and test stands were used in testing two-stage procedure for estimating dominant height of entire stands.

3 Regression models

– Dominant height (ha^{-1})
– Stem number (ha^{-1})

Table 1. Explanations for the variables.

Variable	Explanation
h	Ground-truth mean height of dominant trees
N	Number of stems (ha^{-1})
h_{90}	Quantile corresponding the laser height of 90 percentile of the laser canopy height
D_1	Laser canopy density

Dominant height (ha^{-1}):

$$\ln h = 1{,}406 + 0{,}269 \ln h_{90} + 0{,}406 \ln D_1 \qquad (1)$$

Stem number (ha^{-1}):

$$\ln N = 8{,}800 + 0{,}505 \ln D_1 \qquad (2)$$

4 Reliability of the models

Table 2. Root mean square errors (RMSE) and coefficients of determination (R^2) of the models.

Model	Variable	RMSE	R^2
1	$\ln h$	0.131	0.830
2	$\ln N$	0.279	0.421

Table 3. Differences between laser-derived and ground-truth values of mean height of dominant trees, and standard deviation for the differences in the test stands. NS = Not statistically significant (p>0.05).

		Difference		
Comparison	Mean height (m)	Range (m)	Mean (m)	Standard deviation (m)
$\hat{h}$-h	6.64	-0.67 - 1.1	0.23 NS	0.56

Estimation of above- and below-ground biomass in boreal forest ecosystems

Næsset, E. 2004. ISPRS Working Group VIII/2, Laser-Scanners for Forest and Landscape Assessments, Freiburg, Germany, 3–6 October 2004. International Society of Photogrammetry and Remote Sensing.

1 Airborne laser scanner data

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 8.-9.6.1999
- Flying altitude: 700 m
- Pulse repetition frequency: 10 kHz
- Max. scan angle, data processing: 14°
- Pulse density: 1.1 m^{-2}
- Recorded echoes: first and last

2 Study area and field data

Study area with a size of about 1000 ha is located in Våler, south-east Norway (59°30′N 10°55′E, 70-120 m a.s.l.). The main tree species are spruce (*Picea abies*) and pine (*Pinus sylvestris*).

143 circular sample plots were distributed systematically throughout the study area. The sample plots were divided in young forest, mature forest on poor sites and mature forest on good sites. In young forest, diameter at breast height was callipered on trees with DBH>4 cm. In mature forest, trees with DBH>10 cm were measured. Tree heights were measured on sample trees. Above- and below-ground biomasses were calculated separately for each tree species (pine, spruce and birch).

3 Regression models

- Above-ground biomass
- Above-ground biomass with a dummy variable
- Below-gound biomass
- Below-gound biomass with a dummy variable

Table 1. Explanations for variables.

Variable	Explanation
$\ln h_{60l}$	Percentile of the last pulse canopy height for 60% (m)
$\ln h_{meanf}$	Mean of the first laser canopy heights (m)
$\ln d_{1f}$	Canopy density corresponding to the proportion of first pulse laser hits above fraction 1 to total number of first pulses.
$\ln d_{1l}$	Canopy density corresponding to the proportion of last pulse laser hits above fraction 1 to total number of last pulses.

Table 2. Regression models for above-ground biomass (B_a) and below-ground biomass (B_b). NS = not statistically significant (p>0.05).

	Model			
Independent variable	$\ln B_a$	$\ln B_a$+dummy	$\ln B_b$	$\ln B_b$+dummy
(intercept)	1.94	1.84	0.55	0.55
$\ln h_{60l}$			1.12	1.13
$\ln h_{meanf}$	1.32	1.36		
$\ln d_{1f}$	0.31	0.28 NS		
$\ln d_{1l}$	0.48	0.48	0.59	0.62
dummy1		-0.03 NS		0.03 NS
dummy2		-0.04 NS		-0.02 NS

4 Reliability of the models

Table 3. Root mean square errors (RMSE) and coefficients of determination (R^2) of the models.

Model	RMSE	R^2
$\ln B_a$	0.14	0.92
$\ln B_a$+dummy	0.14	0.92
$\ln B_b$	0.17	0.86
$\ln B_b$+dummy	0.17	0.86

Estimation of diameter and basal area distributions in coniferous forest by means of airborne laser scanner data

Gobakken, T. & Næsset, E. 2004. Scandinavian Journal of Forest Research. 19: 529-542.

1 Airborne laser scanner data

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 23.7.-1.8.2001
- Flying altitude: 650 m
- Average speed: 75 m/s
- Number of flight lines: 129, sidelap 50 %
- Pulse repetition frequency: 10 kHz
- Scan frequency: 30 Hz
- Max. scan angle, data collection: 16°
- Max. scan angle, data processing: 15°
- Footprint diameter: 13-29 cm (average 23 cm)
- The average distance between footprints: 0.7-1.4 m (average 1.0 m)
- Recorded echoes: first and last

2 Study area and field data

Study area is located in Krødsherad, south-east Norway (60°10′N 9°35′E, 130-660 m a.s.l.). Dominant tree species are pine (*Picea abies*) and spruce (*Pinus sylvestris*).

54 field plots located on subjectively selected stands. Stands were selected to represent different combinations of age classes, site quality classes and tree species mixtures. The stands were divided in three strata: 1) young forest, 2) mature forest with poor site quality, 3) mature forest with good site quality. Site index was equal to or less than 11 for stands age of 40 years on poor sites, and more than 11 for stands age of 40 years on good sites. Reference data consisted of plots with an approximate size of 61×61 m. Trees with DBH more than 4 cm were callipered within young forest plots. Within mature forest plots, trees with DBH >10 cm were callipered.

Cross-validation was used to assess the accuracy of regression models.

3 Regression models

– Diameter distribution, young forest(1-4)
– Diameter distribution, mature forest, poor site quality (2-8)
– Diameter distribution, mature forest, good site quality (9-12)
– Basal area distribution, young forest (13-16)
– Basal area distribution, mature forest, poor site quality (17-20)
– Basal area distribution, mature forest, good site quality (21-24)
– Stem number, young forest (25)
– Stem number, mature forest, poor site quality (26)
– Stem number, mature forest, good site quality (27)
– Basal area, young forest (28)
– Basal area, mature forest, poor site quality (29)
– Basal area, mature forest, good site quality (30)

Table 1. Explanations for variables.

Variable	Explanation
b, c	Weibull parameters b and c
d_{24}, d_{93}	Weibull percentiles 24 and 93
$h_{0f}\ldots h_{90f}$	Percentiles of the first pulse laser canopy heights for 0-90 % (m)
h_{0l}-h_{90l}	Percentiles of the last pulse laser canopy heights for 0-90 % (m)
h_{meanl}	Mean of the first pulse laser canopy heights (m)
h_{cvf}	Coefficient of variation of the first pulse laser canopy heights (%)
h_{cvl}	Coefficient of variation of the last pulse laser canopy heights (%)
$d_{0f}\ldots d_{9f}$	Canopy densities corresponding to the proportions of the first pulse laser hits above fraction nos 0, 1, …, 9 to the total number of first pulses
$d_{0l}\ldots d_{9l}$	Canopy densities corresponding to the proportions of the last pulse laser hits above fraction nos 0, 1, …, 9 to the total number of last pulses

Diameter distribution (1-12)

Young forest:

$$\ln b = 0{,}237 + 1{,}005 \ln h_{401} - 1{,}067 \ln d_{1f} \quad (1)$$

$$\ln c = -0{,}3712 + 0{,}331 \ln h_{10l} + 1{,}634 \ln h_{meanl} - 1{,}874 \ln d_{0f} \quad (2)$$

$$\ln d_{24} = -3{,}206 + 2{,}157 \ln h_{meanl} - 1{,}888 \ln d_{0f} \quad (3)$$

$$\ln d_{93} = 6{,}492 - 6{,}283 \ln h_{0f} + 0{,}384 \ln h_{90l} \quad (4)$$

Mature forest, poor sites:

$$\ln b = 1{,}078 + 0{,}626 \ln h_{90l} - 0{,}283 \ln d_{0l} \quad (5)$$

$$\ln c = 4{,}782 - 1{,}035 \ln h_{cvf} \quad (6)$$

$$\ln d_{24} = 1{,}364 + 0{,}833 \ln h_{30l} - 0{,}721 \ln d_{0l} \quad (7)$$

$$\ln d_{93} = -0{,}060 - 2{,}396 \ln h_{70f} - 0{,}224 \ln h_{40l} + 3{,}360 \ln h_{90l} \quad (8)$$

Mature forest, good sites:

$$\ln b = -0{,}224 + 1{,}060 \ln h_{90l} - 0{,}201 \ln d_{4l} \quad (9)$$

$$\ln c = 4{,}863 - 1{,}073 \ln h_{cvf} \quad (10)$$

$$\ln d_{24} = -0{,}890 + 1{,}232 \ln h_{meanl} - 0{,}080 \ln d_{9f} \quad (11)$$

$$\ln d_{93} = 0{,}523 + 0{,}936 \ln h_{90f} - 0{,}352 \ln d_{4f} \quad (12)$$

Basal area distribution (13-24)

Young forest:

$$\ln b = 7{,}803 - 6{,}769 \ln h_{0l} \quad (13)$$

$$\ln c = -1{,}072 + 1{,}203 \ln h_{30l} - 1{,}275 \ln d_{0f} - 0{,}460 \ln d_{7f} + 0{,}404 \ln d_{8l} \quad (14)$$

$$\ln d_{24} = -0{,}196 + 1{,}058 \ln h_{70l} - 0{,}827 \ln d_{0f} \quad (15)$$

$$\ln d_{93} = 1{,}386 + 0{,}544 \ln h_{cvf} \quad (16)$$

Mature forest, poor sites:

$$\ln b = 3{,}142 - 0{,}164 \ln d_{0l} \tag{17}$$

$$\ln c = 4{,}674 - 1{,}021 \ln h_{cvf} - 0{,}213 \ln d_{0l} \tag{18}$$

$$\ln d_{24} = 0{,}823 + 0{,}660 \ln h_{80l} - 0{,}347 \ln d_{0l} \tag{19}$$

$$\ln d_{93} = 1{,}647 + 0{,}547 \ln h_{cvf} \tag{20}$$

Mature forest, good sites:

$$\ln b = -4{,}773 + 0{,}965 \ln h_{90f} + 7{,}186 \ln h_{0l} + 0{,}396 \ln d_{0f} - 0{,}154 \ln d_{7f} \tag{21}$$

$$\ln c = -2{,}035 + 1{,}273 \ln h_{30l} - 0{,}512 \ln d_{5f} \tag{22}$$

$$\ln d_{24} = -1{,}291 + 1{,}397 \ln h_{90l} - 0{,}632 \ln d_{1f} \tag{23}$$

$$\ln d_{93} = -6{,}887 + 0{,}523 \ln h_{10f} + 10{,}041 \ln h_{0l} - 0{,}379 \ln h_{70l} + 0{,}464 \ln h_{cvl} + 0{,}988 \ln d_{0f} \tag{24}$$

Stem number, young forest:

$$\ln N = 7{,}923 + 0{,}954 \ln d_{0l} \tag{25}$$

Stem number, mature forest, poor sites:

$$\ln N = 7{,}688 + 1{,}064 \ln d_{1l} \tag{26}$$

Stem number, mature forest, good sites:

$$\ln N = 7{,}051 + 2{,}554 \ln d_{0f} \tag{27}$$

Basal area, young forest:

$$\ln G = 3{,}867 + 0{,}789 \ln d_{0f} + 0{,}368 \ln d_{4l} \tag{28}$$

Basal area, mature forest, poor sites:

$$\ln G = 2{,}046 + 0{,}978 \ln h_{70f} - 0{,}408 \ln h_{50l} + 1{,}138 \ln d_{2f} \tag{29}$$

Basal area, mature forest, good sites:

$$\ln G = 4{,}662 + 0{,}853 \ln h_{90f} - 4{,}924 \ln h_{01} + 0{,}278 \ln d_{5l} \tag{30}$$

4 Reliability of the models

Table 2. Root mean square errors (RMSE) and coefficients of determination (R^2) of the models.

Model		Dependent variable	RMSE	R^2
1	Young forest	ln*b*	0.10	0.78
2		ln*c*	0.10	0.90
3		$\ln d_{24}$	0.12	0.86
4		$\ln d_{93}$	0.07	0.60
5	Mature forest,	ln*b*	0.05	0.87
6	poor sites	ln*c*	0.12	0.60
7		$\ln d_{24}$	0.07	0.83
8		$\ln d_{93}$	0.08	0.75
9	Mature forest,	ln*b*	0.06	0.85
10	good sites	ln*c*	0.15	0.68

Table 2 continued

11		lnd_{24}	0.10	0.81
12		lnd_{93}	0.09	0.64
13	Young forest	ln$b$	0.07	0.47
14		ln$c$	0.10	0.93
15		lnd_{24}	0.08	0.84
16		lnd_{93}	0.11	0.63
17	Mature forest,	ln$b$	0.13	0.20
18	poor sites	ln$c$	0.14	0.63
19		lnd_{24}	0.08	0.80
20		lnd_{93}	0.14	0.22
21	Mature forest,	ln$b$	0.05	0.90
22	good sites	ln$c$	0.11	0.88
23		lnd_{24}	0.09	0.84
24		lnd_{93}	0.06	0.90
25	Young forest	ln$N$	0.31	0.37
26	Mature *f.*, poor sites	ln$N$	0.16	0.87
27	Mature *f.*, good sites	ln$N$	0.16	0.72
28	Young forest	ln$G$	0.08	0.95
29	Mature *f.*, poor sites	ln$G$	0.07	0.96
30	Mature *f.*, good sites	ln$G$	0.11	0.58

Table 3. Mean difference between ground reference volume (V) and V predicted according to the distribution functions, standard deviation for the differences (SD) as percentages of the reference V using ground-truth stem number and basal area as scaling variables. NS = not statistically significant (p>0.05).

Method	Reference mean V (m^3ha^{-1})	Difference (%)	SD (%)
Young forest	222.7		
Diameter (parameter)		0.5 NS	29.1
Diameter (percentile)		2.7 NS	27.5
Basal area (parameter)		-3.1 NS	10.7
Basal area (percentile)		-3.0 NS	9.5
Mature forest, poor sites	161.4		
Diameter (parameter)		-1.6 NS	14.5
Diameter (percentile)		1.8 NS	14.2
Basal area (parameter)		-4.1 NS	13.3
Basal area (percentile)		-4.8	9.9
Mature forest, good sites	283.8		
Diameter (parameter)		0.9 NS	13.2
Diameter (percentile)		0.7 NS	14.1
Basal area (parameter)		-1.1 NS	7.2
Basal area (percentile)		-2.0 NS	5.6

Table 4. Mean difference between ground reference volume (V) and V predicted according to the distribution functions, standard deviation for the differences (SD) as percentages of the reference V using stem number and basal area predicted from laser data as scaling variables. NS = not statistically significant (p>0.05).

Method	Reference mean V (m^3ha^{-1})	Difference (%)	SD (%)
Young forest	222.7		
Diameter (parameter)		3.4 NS	21.9
Diameter (percentile)		6.6 NS	24.2
Basal area (parameter)		-2.7 NS	13.2
Basal area (percentile)		-2.6 NS	11.4
Mature forest, poor sites	161.4		
Diameter (parameter)		-0.1 NS	13.6
Diameter (percentile)		4.5 NS	22.7
Basal area (parameter)		-4.0 NS	15.5
Basal area (percentile)		-4.7 NS	13.1
Mature forest, good sites	283.8		
Diameter (parameter)		3.0 NS	13.6
Diameter (percentile)		2.8 NS	15.9
Basal area (parameter)		0.7 NS	13.0
Basal area (percentile)		-0.3 NS	12.2

Estimation of stem volume using laser scanning-based canopy height metrics

Maltamo, M., Eerikäinen, K., Packalén, P. & Hyyppä, J. 2006. Forestry 79: 217-229.

1 Airborne laser scanner data

- Laser scanner: Toposys Falcon
- Date of data acquisition: 4.8.2004
- Flying altitude: 400 m
- Pulse repetition frequency: 83 kHz
- Scan frequency: 653 Hz
- Max. scan angle: ±7.1°
- Pulse density: $10/m^2$
- Beam divergence: 1.0 mrad
- Footprint diameter: 40 cm
- Swath width: 100 m
- Recorded echoes: first and last (only first pulses were used in this study)

2 Study area and field data

Study area with a size of 50 ha is located in Kalkkinen, southern Finland. About 50 % of stem volume is spruce (*Picea abies*), 35 % pine (*Pinus sylvestris*) and 15 % birch (*Betula pendula* and *Betula pubescens*). The tree stock of the study area is naturally regenerated. The most of the area has not been managed for decades.

32 sample plots were distributed systematically on the study area. Plot size of 30×30 m was most commonly used. Also plot sizes 25×25 m and 30×40 m were used to have the amount of trees per sample plot about 100. Total 2612 trees were measured.

3 Regression models

- Plot level stem volume using different point densities (1-5) (V1)
- SUR estimates for stem volume (6-19) (V2)
- Stem volume based on Weibull distributions (V3)
- Stem volume based on Weibull distributions with field-measured stand characteristics (V4)

Table 1. Explanations for variables.

Variable	Explanation
G	Basal area
h_{GM}	Height of basal area median tree
$d_1 \ldots d_{100}$	modelled diameter 1…100 percentile
$h_1 \ldots h_{100}$	Percentiles for the canopy height for 1…100%
$p_1 \ldots p_{100}$	Relative proportion of the laser pulses in percentiles
h_{max}	The maximum of laser heights
h_{dev}	Standard deviation of laser heights

Point densities 6.3, 1.3, 0.6 and 0.13 were obtained by systematic data reduction. After the reduction, there were 50%, 10%, 5% and 1% left from the original data.

Volume using a point density of 12.7:

$$\ln(V) = -0{,}655 + 1{,}999 * \ln h_{80} + 0{,}049 * h_1 \quad (1)$$

Volume using a point density of 6.3:

$$\ln(V) = -0{,}771 + 2{,}054 * \ln h_{80} - 0{,}034 * h_5 + 0{,}106 * h_1 \quad (2)$$

Volume using a point density of 1.3:

$$\ln(V) = -0{,}527 + 2{,}013 * \ln h_{70} - 0{,}040 * h_5 + 0{,}116 * h_1 \quad (3)$$

Volume using a point density of 0.6:

$$\ln(V) = -0{,}415 + 1{,}980 * \ln h_{70} \quad (4)$$

Volume using a point density of 0.13:

$$\ln(V) = -0{,}947 + 6{,}506 * \ln h_{65} - 4{,}352 * \ln h_{60} - 2{,}355 * p_{95} \quad (5)$$

SUR estimates for volume prediction (basal area, height of basal area median tree and modelled diameter percentiles):

$$\ln G = -0{,}1970 + 0{,}1824 \ln h_1 + 1{,}1153 \ln h_{80} \quad (6)$$

$$\ln h_{GM} = -0{,}5817 + 1{,}1146 \ln h_{\max} \quad (7)$$

$$\ln d_1 = 1{,}2487 - 10{,}3601 \ln p_5 + 0{,}2825 \ln h_{10} \quad (8)$$

$$\ln d_5 = 2{,}1379 + 3{,}6963 \ln p_{40} + 4{,}4283 \ln p_{40}^2 - 0{,}1963 \ln h_1 0{,}1504 h_{dev} \quad (9)$$

$$\ln d_{10} = 1{,}8722 + 1{,}9647 \ln p_{30} - 0{,}1270 \ln h_1 + 0{,}1813 h_{dev} \quad (10)$$

$$\ln d_{20} = 0{,}6431 - 0{,}1507 \ln h_2 0{,}8342 \ln h_{40} \quad (11)$$

$$\ln d_{30} = -0{,}2660 - 0{,}1155 \ln h_1 + 1{,}0572 \ln h_{95} \quad (12)$$

$$\ln d_{40} = -0{,}1144 - 0{,}0767 \ln h_2 + 1{,}0443 \ln h_{95} \quad (13)$$

$$\ln d_{50} = -0{,}4424 + 1{,}1145 \ln h_{\max} \quad (14)$$

$$\ln d_{60} = -0{,}2721 + 1{,}0864 \ln h_{\max} \quad (15)$$

$$\ln d_{70} = -0{,}1845 + 1{,}0800 \ln h_{\max} \quad (16)$$

$$\ln d_{80} = -0{,}0782 + 1{,}0728 \ln h_{\max} \quad (17)$$

$$\ln d_{90} = 0{,}1514 + 1{,}0375 \ln h_{\max} \quad (18)$$

$$\ln d_{100} = 0{,}3529 + 1{,}0108 \ln h_{\max} \quad (19)$$

4 Reliability of the models

Table 2. Cross-equation variance-covariance matrix of the residual for the SUR-equation system.

	ln$G$	lnh_{GM}	lnd_1	lnd_5	lnd_{10}	lnd_{20}	lnd_{30}	lnd_{40}	lnd_{50}	lnd_{60}	lnd_{70}	lnd_{80}	lnd_{90}	lnd_{100}
lnG	0.0166	0.0024	0.0047	-0.0090	-0.0080	0.0032	0.0065	0.0057	0.0061	0.0044	0.0044	0.0028	0.0019	0.0041
lnh_{GM}		0.0130	0.0010	0.0012	-0.0011	-0.0026	0.0040	0.0001	0.0016	0.0005	0.0010	0.0002	-0.0003	0.0002
lnd_1			0.0162	0.0087	0.0033	0.0038	0.0055	0.0035	0.0035	0.0031	0.0037	0.0034	-0.0044	-0.0029
lnd_5				0.0276	0.0257	0.0180	0.0151	0.0130	0.0096	0.0088	0.0073	0.0048	-0.0010	-0.0017
lnd_{10}					0.0388	0.0254	0.0201	0.0175	0.0119	0.0110	0.0078	0.0045	0.0041	0.0018
lnd_{20}						0.0257	0.0171	0.0161	0.0122	0.0114	0.0100	0.0074	0.0067	0.0071
lnd_{30}							0.0240	0.0211	0.0155	0.0141	0.0123	0.0092	0.0099	0.0121
lnd_{40}								0.0200	0.0154	0.0143	0.0126	0.0098	0.0118	0.0141
lnd_{50}									0.0161	0.0151	0.0134	0.0101	0.0116	0.0125
lnd_{60}										0.0153	0.0136	0.0103	0.0125	0.0133
lnd_{70}											0.0133	0.0105	0.0122	0.0135
lnd_{80}												0.0095	0.0117	0.0128
lnd_{90}													0.0282	0.0280
lnd_{100}														0.0322

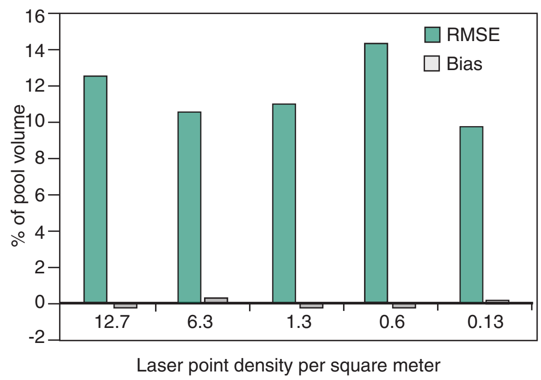

Figure 1 (Maltamo ym. 2006). Effect of data reduction on the accuracy of the constructed models for the plot-level stem volume (V1).

Table 3. RMSE(%) and bias(%) for the methods V2-V4.

Method	RMSE (%)	Bias (%)
V2	12.5	-1.4
V3	14.1	0.9
V4	27.5	7.0

Inventory of seedling stands of Norway spruce and assessment of the need for their management with help of laser scanning

Närhi, M. 2007. Master's thesis. University of Joensuu, Faculty of Forest Sciences.

1 Airborne laser scanner data

- Laser scanner: ALTM 3100 (Optech)
- Date of aquisition: 27.7.2006
- Average height: 2300 m
- Average speed: 75 m/s
- Max. scan angle: 34°
- Distance of the flight lines: 1070 m
- Side lap: 24%
- Pulse density: 0.5/m^2
- Recorded pulses: first and last

2 Study area and field data

Study area is located in Sonkajärvi, Eastern Finland.

Only spruce dominated sample plots with a dominant height less than 2 meters were included to the study. Site class should be Myrtillus type or better. 212 sample plots in clusters were located in 25 stands. One cluster consisted of four sample plots with a radius of 4 meters (50 m^2). 4 plots were measured on stands with a size less than one hectare. Stands of 1-3 hectares had 8 plots measured and larger stands had 12. All trees on plots were recorded by species. Heights were measured and recorded in 0.5 meter classes. Average values of the measurements in each cluster were used.

Information from a forest plan was also available.

3 Regression models

- Plot density, laser variables (1)
- Plot density, laser and forest plan variables (2)
- Average height (3)

$$\ln(tiheys) = 10.183 + 0.018 \times lveg + 0.332 \times \ln l\_h90 - 0.046 \times f\_p30 - 0.391 \times f\_h20 + \delta^2/2 \quad (1)$$

where $\delta^2 = 0.05951$

$$\ln(tiheys) = 10.389 + 0.021 \times lveg + 0.604 \times \ln ikä - 0.216 \times f\_h60 - 1.089 \times \ln f\_p20 + \delta^2 / 2 \quad (2)$$

where $\delta^2 = 0.056045$

$$\ln(pituus) = 0.996 + 0.112 \times f\_h40 + 0.159 \times \ln l\_h20 + \delta^2 / 2 \quad (3)$$

where $\delta^2 = 0.0105$

4 Reliability of the models

Table 1. Results of the best plot level regression models compared to field measurements.

Variable	Observed mean	Predicted mean	RMSE	RMSE (%)	Bias	Bias (%)
Height (m)	4.02	3.98	0.63	15.93	0.04	1.03
Density (trees/ha)	3982	3941	1782	45.20	40.28	1.01
Density (trees/ha), laser+fp	3982	4038	1608	39.83	-56.45	-1.42

The prediction of stand characteristics using airborne laser scanning

Suvanto, A., Maltamo, M., Packalén, P. & Kangas, J. 2005. Metsätieteen aikakauskirja 4/2005: 413-428.

1 Airborne laser scanner data

- Laser scanner: ALTM 2033 (Optech)
- Date of data acquisition: 3.8.2004
- Flying altitude: 1500 m
- Average speed: 75 m/s
- Number of flight lines: 7, sidelap 35 %
- Pulse repetition frequency: 33 kHz
- Max. scan angle: 15°
- Pulse density: 0.7/m^2
- Beam divergence: 0.2 mrad or 0.1 mrad
- Recorded echoes: first and last

2 Study area and field data

Matalansalo study area is located in Varkaus, eastern Finland. The area is owned by UPM-Kymmene Ltd.

Field data consisted of young, middle-aged and mature stands selected by sampling. Main tree species was pine or spruce on the most of the stands. Regular grid of circular plots (total 472) was established on the stands. Plot radius was 9 meters. Trees with diameter at breast height >5 cm were callipered. Proportion of pine, spruce and deciduous dominated plots were 57.1%, 34.1% and 8.1%, respectively. 26.9% of the plots were young forest, 42.2% were middle-aged and 30.9% were mature forest. Nearly half of the plots were established on stands with good site quality. Many other fertility classes were also represented.

3 Regression models

- Volume (with a dummy variable), 1
- Basal area (with a dummy variable), 2
- Stem number (with a dummy variable), 3
- Mean height (with a dummy variable), 4
- Mean diameter (with a dummy variable), 5
- Volume (percentile), 6
- Basal (percentile), 7
- Stem number (percentile), 8
- Mean height (percentile), 9
- Mean diameter (percentile), 10

Table 1. Explanations for variables

Variable	Explanation
F	First pulse
L	Last pulse
vege	Proportion of vegetation hits
hmea	Mean height of the laser pulses (m)
hajo	Standard deviation of the laser heights (m)
vari	Coefficient of variation of the laser heights
hmax	Maximum value of the laser heights (m)
p05-p95	Height of the 5, 10, 20, …, 90, 95 % percentile (m)
su05-su95	Relative proportion of laser pulses in percentiles
2	Dummy variable, young forest
4	Dummy v., mature forest
ku	Dummy v., main tree species spruce
ko	Dummy v., main tree species birch
ks	Dummy v., mineral soil, spruce dominated
kv	Dummy v., mineral soil, birch dominated
su	Dummy v., spruce swamp or pine bog
st	Dummy v., spruce swamp or pine bog, pine dominated
sv	Dummy v., spruce swamp or pine bog, birch dominated
om	Dummy v., site class OMT
vt	Dummy v., site class VT

Volume (with a dummy variable):

$$\ln(V) = 1{,}273 + 1{,}395 * \sqrt{Fvege} - 9{,}814 * \frac{1}{koFp90} + 0{,}238 * \sqrt{Lhmea} + 1{,}036 * \sqrt{Lvege} \quad (1)$$

$$- 2{,}511 * \frac{1}{Fp10} - 0{,}322 * \ln(kuFvege) - 0{,}575 * \frac{1}{04Lp05} + 0{,}372 * \sqrt{Lp70} - 0{,}002$$

$$* (koLp20)^2 - 8{,}769 * \ln(koFsu70) + 4{,}358 * \ln(koLsu60) + 4{,}249 * \frac{1}{ksFp10} - 6{,}907$$

$$* \frac{1}{ksFhmea} + \delta^2 / 2$$

where δ2= 0,0246.

Basal area (with a dummy variable):

$$\sqrt{PPA} = -2{,}135 + 2{,}064 * \ln(Fhmea) + 1{,}627 * Fvege - 0{,}078 * \frac{1}{koFsu05} + 1{,}596 * Lvege - 1{,}135 * (04L\,vari)^2 + 0{,}001 * (suLp10)^2 - 0{,}001 * (Lp10)^2 - 6{,}113 * (stF\,vari)^2 + 0{,}365 * \ln(02Fp10) + \delta^2 \quad (2)$$

Stem number (with a dummy variable):

$$\sqrt{N} = 35{,}386 + 11{,}775 * (Fvege)^2 - 0{,}041 * (Lhmea)^2 + 18{,}081 * (Lvege)^2 + 2{,}260 * \frac{1}{suF\,vari} - 0{,}077 * (omLhajo)^2 - 0{,}056 * (kuFp40)^2 + 0{,}032 * (kuFp80)^2 - 49{,}980 * \frac{1}{Fp10} - 43{,}191 * \frac{1}{svLhajo} + 51{,}533 * svF\,vari - 0{,}019 * (suFp80)^2 + 0{,}288 * (vtFhajo)^2 - 58{,}382 * \frac{1}{stFp40} + 0{,}150 * (stLp95)^2 - 0{,}138 * (stFp95)^2 + 78{,}037 * \frac{1}{02Fp70} - 0{,}158 * (vtLhajo)^2 + \delta^2 \quad (3)$$

Mean height (with a dummy variable):

$$HGM = 5{,}910 + 0{,}499 * Fp80 - 0{,}003 * (02Fp90)^2 + 0{,}016 * (Fp60)^2 - 14{,}210 * \frac{1}{04Lh\max} + 0{,}029 * (kuFhajo)^2 - 8{,}469 * \frac{1}{02Lp40} + 0{,}338 * (omLvege)^2 \quad (4)$$

Mean diameter (with a dummy variable)

$$\sqrt{DGM} = 3{,}401 + 0{,}070 * Fp80 - 0{,}001 * (02Fp90)^2 - 0{,}037 * koFp10 + 0{,}040 * omFhajo - 0{,}159 * Fp20 + 0{,}094 * Lp40 - 6{,}138 * \frac{1}{04Lp20} - 0{,}694 * \frac{1}{04Fvege} - 1{,}461 * \frac{1}{02Fp05} + 0{,}002 * (Fp50)^2\, 0{,}010 * (suFhajo)^2 + 3{,}834 * \ln(suFsu95) + 1{,}416 * (04Lvege)^2 - 2{,}138 * (04Fvege)^2 + 1{,}681 * \frac{1}{04Fsu70} + \delta^2 \quad (5)$$

Volume (percentile):

$$\ln(V) = 0{,}788 + 0{,}704 * \ln(Fp50) + 1{,}090 * \sqrt{Fvege} + 0{,}735 * \ln(Lhmea) + 0{,}267 * \ln(Lvege) + \delta^2/2 \quad (6)$$

Basal area (percentile):

$$\sqrt{PPA} = -0{,}795 + 1{,}329 * \ln(Fhmea) + 1{,}342 * Fvege + 2{,}270 * \sqrt{Lvege} - 1{,}773 * \frac{1}{Fp05} + \delta^2 \quad (7)$$

Stem number (percentile):

$$N = 11376{,}750 + 1556{,}143 * (Lvege)^2 + 1036{,}063 * (Fvege)^2 - 4{,}214 * (Lhmea)^2 - 5777{,}637 * (Fsu50)^2 - 5755{,}064 * \frac{1}{Lsu70} - 1471{,}856 * \sqrt{L\,vari} \quad (8)$$

Mean height (percentile):

$$\sqrt{HGM} = 0{,}950 + 0{,}594 * \sqrt{Fp80} + 0{,}055 * Fp60 + \delta^2 \tag{9}$$

Mean diameter (percentile):

$$\ln(DGM) = 2{,}970 + 0{,}440 * \sqrt{Lp70} - 0{,}383 * \sqrt{Fp20} - 0{,}337 * (Fvege)^2 + 0{,}409 * \sqrt{Fp50} - 1{,}900 * \sqrt{Lsu70} - 0{,}052 * \frac{1}{Lvege} + \delta^2 / 2 \tag{10}$$

4 Reliability of the models

Table 2. Reliability of the dummy variable models on plot level. N = 472. Models 1-5.

	Observed mean	Standard deviation	RMSE	RMSE (%)	Bias	Bias (%)
V (m^3/ha)	203.4	103.5	35.1	17.3	-1.3	-0.6
PPA (m^2/ha)	24.7	8.0	3.6	14.6	-0.3	-1.2
N (number/ha)	1506.9	692.3	317.2	21.1	5.4	0.4
HGM (m)	17.0	5.1	1.3	7.9	-0.1	-0.6
DGM (cm)	19.8	6.5	4.7	23.8	0.1	0.4

Table 3. Reliability of the percentile models on plot level. N = 472. Models 6-10.

	Observed mean	Standard deviation	RMSE	RMSE (%)	Bias	Bias (%)
V (m^3/ha)	203.4	103.5	40.4	19.9	0.2	-0.1
PPA (m^2/ha)	24.7	8.0	4.1	16.5	0.0	-0.1
N (number/ha)	1506.9	692.3	407.6	27.0	-6.4	-0.4
HGM (m)	17.0	5.1	1.4	8.4	0.0	-0.4
DGM (cm)	19.8	6.5	2.7	13.6	0.1	0.6

Table 4. Reliability of the dummy variable models of stand level. N = 67. Models 1-5.

	Observed mean	Standard deviation	RMSE	RMSE (%)	Bias	Bias (%)
V (m^3/ha)	202.5	89.2	19.9	9.8	1.8	0.9
PPA (m^2/ha)	24.6	6.3	2.0	8.3	-0.1	-0.4
N (number/ha)	1512.5	516.5	273.8	18.1	36.9	2.4
HGM (m)	17.0	4.7	0.9	5.4	-0.4	-2.4
DGM (cm)	19.7	5.6	2.0	9.9	-0.8	-4.0

Table 5. Reliability of the percentile models on stand level. N = 67. Models 6-10.

	Observed mean	Standard deviation	RMSE	RMSE (%)	Bias	Bias (%)
V (m^3/ha)	202.5	89.2	24.1	11.9	4.7	2.3
PPA (m^2/ha)	24.6	6.3	2.6	10.5	0.2	0.9
N (number/ha)	1512.5	516.5	312.0	20.6	53.0	3.5
HGM (m)	17.0	4.7	0.9	5.3	-0.3	-1.8
DGM (cm)	19.7	5.6	1.9	9.5	-0.6	-2.9

Mapping defoliation during a severe insect attack on Scots pine using airborne laser scanning

Solberg, S., Næsset, E., Hanssen, K.H. & Christiansen, E. 2006. Remote Sensing of Environment. 102: 364-376.

1 Airborne laser scanner data

- Laser scanner: ALTM 3100C (Optech)
- Date of data acquisition: 13.5, 26.6. and 1.9.2005
- Flying altitude: 650 m
- Average speed: 75 m/s
- Number of flight lines: 17
- Swath width: 325 m
- Pulse repetition frequency: 100 kHz
- Max. scan angle, data collection: 14°
- Max. scan angle, data processing: 12°
- Footprint diameter: 17 cm
- Pulse density: 3.1-9.8 /m^2
- Recorded echoes: first and last

2 Data

Study area of varying aged Scots pine (*Pinus silvestris* L.) forest is located in Åsnes, south-east Norway (60°41′N 12°18′E, 200-260 m a.s.l.).

Data consisted of leaf area index (LAI) ground truth data measured with LAI-2000 and Hemiview instruments, and laser data.

LAI-2000 instrument measures the amount of light above and below canopy. Below canopy measurements are taken in a forest. Above canopy measurements are taken on a nearby open location, e.g. a clear-cut, a field, or a bog.

Hemispherical images (Hemiview data) provide an upward-looking hemispherical view. In these images, the sky areas are darker near the horizon than those close to the zenith. Therefore the images were treated in Adobe Photoshop to increase the brightness in darker sky areas. The images were then transformed to binary images with a subjectively defined threshold value.

Gap fraction was derived as the ratio below canopy pulses to the total number of pulses (model 1). A threshold for canopy hits was set one meter above ground.

$$LAI = 1/k \ln(N_a / N_b) \quad (1)$$

where

Na = total number of reflected pulses

Nb = number of pulses reflected below canopy

k = regression of laser data against ground measurements of LAI

3 Regression models

- LAI for LAI-2000 data (first pulses)
- LAI for Hemiview data (first pulses)
- LAI for LAI-2000 data (last pulses)
- LAI for Hemiview data (last pulses)

Models for first pulses:

LAI-2000: $LAI = 0{,}08 - 1{,}38\ln(N_a / N_b)$ (2)

Hemiview: $LAI = 0{,}22 + 0{,}91\ln(N_a / N_b)$ (3)

Models for last pulses:

LAI-2000: $LAI = 0{,}24 - 3{,}75\ln(N_a / N_b)$ (4)

Hemiview: $LAI = 0{,}31 + 2{,}61\ln(N_a / N_b)$ (5)

4 Reliability of the models

Table 1. Coefficients of determination (R^2) of the models

Model		R^2
2	LAI-2000 first	0.93
3	Hemiview first	0.87
4	LAI-2000 last	0.83
5	Hemiview last	0.90

Mapping defoliation with LIDAR

Solberg, S., Næsset, E. IAPRS Volume XXXVI, Part 3 / W52, 2007.

1 Laser scanner data and field data

In this study, several LIDAR data sets for Norwegian forests were used. Field plot measurements from these study areas were available.

2 Models

- Leaf area index (LAI, this formula also in Solberg et al. 2006)
- Normal LAI
- Health indicator value *c*

$LAI = 1/k \ln(N_a / N_b)$ (1)

where
N_a = total number of reflected pulses
N_b = number of pulses reflected below canopy
k = regression of laser data against ground measurements of LAI

Normal leaf area index values were modelled as
SD = *h/dist* (2)
where *h* is the mean tree height, and *dist* is the mean distance between the trees.

Forest health indicator value, *c*, was defined as
c = LAI / SD (3)

Regression models with input variables such as crown size and tree height were used for estimating the woody area fraction of LAI. The woody area fraction values were used for assessing the defoliation incurred by insects.

Measures of spatial forest structure derived from airborne laser data are associated with natural regeneration patterns in an uneven-aged spruce forest

Bollandsås, O.M., Hanssen, K.H., Marthiniussen, S., Næsset, E. 2007. Forest Ecology and Management 255: 953-961.

1 Airborne laser scanner data

- Laser scanner: ALTM 1233 (Optech)
- Date of data acquisition: October 2003
- Flying altitude: 600 m
- Average speed: 35 m/s
- Number of flight lines: 21
- Pulse repetition frequency: 33 kHz
- Scan frequency: 50 Hz
- Max. scan angle, data collection: 11°
- Max. scan angle, data processing: 10.5°
- Swath width: 230 m
- Footprint diameter: 18 cm
- Average point density: 5.0/m^2
- Recorded echoes: first and last

2 Study area and field data

The study area is located in south-eastern Norway (59°50′N 11°02′E, 190-370 m a.s.l.) and the size of it is about 1400 ha. The area is considered as primeval forest. Some selective cuttings have been carried out before 1940.

72 circular plots with a size of 25 m^2 were established. The plots were located in 18 clusters. Each cluster had 4 plots each. Cluster locations were subjectively determined according to dominant tree species and age distribution. Plots should be spruce-dominated with a multi-layered canopy. Each plot was split into four quadrants. In each quadrant, the number of seedlings between 0.1 and 3 meters were recorded. Apical dominance ratio (ADR), total height (TH), absolute leader length (ALL) and leader length (LL) were measured on the tallest seedling on each quadrant. Also regeneration success rate (RSR) was computed for the quadrants.

3 Regression models

The models were ranked by Akaike information criterion (AIC), and according to the significance level of the explanatory variable. AIC ranking yielded a rank of laser variables according to the goodness of fit of the model for each response variable. The two rankings showed if the top ranked variables were robust when the ranking criterion changed. Laser variables were attributed to four categories: scale, return, fraction and type.

4 Reliability of the models

Table 1. The most frequent group of variables (modus group) of the best 5, 10 and 15 category assigned, AIC-ranked, laser variables after modelling regeneration success rate (RSR), apical dominance ratio (ADR), relative leader length (RLL), and leader length (LL).

Response	*n*	Scale[a] Modus group	Freq.(%)	Return[b] Modus group	Freq.(%)	Fraction[c] Modus group	Freq.(%)	Type[d] Modus group	Freq.(%)
RSR	5	Large	80	Last	80	Lower	80	Density	100
RSR	10	Large	50	Last	70	Middle	60	Density	100
RSR	15	Large	53	Last	60	Lower	40	Density	100
ADR	5	Small	40	Last	80	Middle	60	Density	100
ADR	10	Large	40	Last	50	Lower	50	Density	90
ADR	15	Medium	40	Last	53	Middle	53	Density	93
RLL	5	Small	80	Last	60	Middle	60	Density	100
RLL	10	Small	60	Last	60	Middle	70	Density	100
RLL	15	Small	47	Last	67	Middle	60	Density	100
LL	5	Medium	80	Last	80	Upper	40	Density	100
LL	10	Medium	70	Last	80	Middle	50	Density	100
LL	15	Medium	60	Last	60	Middle	47	Density	100

[a] Area from which laser variables originate. Small: 25 m2 circle, medium: 100 m^2 circle, large: 225 m^2 circle.
[b] First of last laser echo.
[c] Fraction (upper, middle, lower or full range) of the range of laser heights from which laser variables originate.
[d] Type of laser variable (height, density, or topographic variable).

Table 2. The 5 highest ranked laser variables according to AIC value after modelling regeneration success rate (RSR), apical dominance ratio (ADR), relative leader length (RLL), and leader length (LL).

Response	AIC rank	Laser variable[a]	Relationship	Scale[b]	Effect of cluster[c]	p-value
RSR	1	STDd02	Positive	Large	Yes	0.011
RSR	2	d02	Negative	Large	Yes	0.001
RSR	3	d02	Negative	Medium	Yes	0.004
RSR	4	CVd02	Positive	Large	Yes	0.007
RSR	5	STDd01	Positive	Large	Yes	0.167
ADR	1	STDd02	Negative	Small	Yes	0.020
ADR	2	d02	Negative	Large	Yes	0.191
ADR	3	d02	Negative	Medium	Yes	0.218
ADR	4	d12	Negative	Large	Yes	0.255
ADR	5	d11	Positive	Small	Yes	0.302
RLL	1	d32	Positive	Small	Yes	0.011
RLL	2	d31	Positive	Small	Yes	0.070
RLL	3	d22	Positive	Small	Yes	0.064
RLL	4	STDd01	Negative	Large	No	0.359
RLL	5	STDd02	Negative	Small	No	0.348

Table 2 continued

LL	1	d32	Negative	Medium	Yes	0.685
LL	2	d02	Positive	Medium	Yes	0.456
LL	3	d31	Negative	Medium	Yes	0.768
LL	4	d02	Positive	Large	Yes	0.484
LL	5	d22	Negative	Medium	Yes	0.743

[a] Variable name convention: STD: standard deviation; DV: coefficient of variation; lower case d: density metrics; first number: fraction number across the range of laser heights (0: lower, 1 and 2: middle, 3: upper, 4: full range); last number: first (1) or last (2) laser return echo.
[b] Area from which laser variables originate. Small = 25 m^2, medium = 100 m^2, large = 225 m^2.
[c] Indicates if the variable is correlated within clusters.

Table 3. The most frequent group of variables (modus group) of the best 5, 10 and 15 category assigned, significance-ranked, laser variables after modelling regeneration success rate (RSR), apical dominance ratio (ADR), relative laser length (RLL), and leader length (LL).

Response	*n*	Scale[a]		Return[b]		Fraction[c]		Type[d]	
		Modus group	Freq.(%)	Modus group	Freq.(%)	Modus group	Freq.(%)	Modus group	Freq.(%)
RSR	5	Large	80	Last	80	Lower	60	Density	60
RSR	10	Large	70	Last	70	Upper	60	Height	60
RSR	15	Large	73	Last	67	Upper	40	Height	60
ADR	5	Small	80	First	80	Middle	40	Density	60
ADR	10	Small	50	Last	50	Middle	30	Density	60
ADR	15	Small	40	Last	60	Middle	40	Density	67
RLL	5	Small	100	Last	80	Upper	40	Density	80
RLL	10	Small	90	Last	80	Middle	30	Density	70
RLL	15	Small	87	Last	67	Lower	33	Density	60
LL	5	Medium	60	Last	60	Middle	80	Density	80
LL	10	Small	60	First	60	Middle	60	Density	90
LL	15	Small	47	First	60	Middle	40	Density	80

[a] Area from which laser variables originate. Small: 25 m^2 circle, medium: 100 m^2 circle, large: 225 m^2 circle.
[b] First of last laser echo.
[c] Fraction (upper, middle, lower or full range) of the range of laser heights from which laser variables originate.
[d] Type of laser variable (height, density, or topographic variable).

Table 4. The 5 highest ranked laser variables according to p- value after modelling regeneration success rate (RSR), apical dominance ratio (ADR), relative leader length (RLL), and leader length (LL).

Response	p-value rank	Laser variable[a]	Relationship	Scale[b]	Effect of cluster[c]	p-value
RSR	1	d02	Negative	Large	Yes	0.001
RSR	2	d02	Negative	Medium	Yes	0.004
RSR	3	h32	Positive	Large	Yes	0.004
RSR	4	h31	Positive	Large	Yes	0.006
RSR	5	CVd02	Positive	Large	Yes	0.007

Table 2 continued

ADR	1	STDd02	Negative	Small	Yes	0.020
ADR	2	d31	Positive	Small	No	0.045
ADR	3	CVh41	Negative	Small	Yes	0.082
ADR	4	STDd01	Negative	Small	Yes	0.124
ADR	5	CVh41	Negative	Medium	Yes	0.148
RLL	1	d32	Positive	Small	Yes	0.011
RLL	2	CVh42	Negative	Small	No	0.034
RLL	3	d22	Positive	Small	Yes	0.064
RLL	4	d31	Positive	Small	Yes	0.070
RLL	5	d12	Positive	Small	Yes	0.082
LL	1	STDd02	Negative	Medium	No	0.147
LL	2	CVd01	Negative	Small	No	0.196
LL	3	STDd01	Negative	Small	Yes	0.237
LL	4	CVh42	Positive	Medium	Yes	0.306
LL	5	CVd02	Positive	Medium	No	0.317

[a] Variable name convention: STD: standard deviation; DV: coefficient of variation; lower case d: density metrics; lower case h: height metrics; first number: fraction number across the range of laser heights (0: lower, 1 and 2: middle, 3: upper, 4: full range); last number: first (1) or last (2) laser return echo.
[b] Area from which laser variables originate. Small = 25 m^2, medium = 100 m^2, large = 225 m^2.
[c] Indicates if the variable is correlated within clusters.

Nonparametric estimation of stem volume using airborne laser scanning, aerial photography, and stand-register data

Maltamo, M., Malinen, J., Packalén, P., Suvanto, A. & Kangas, J. 2006. Canadian Journal of Forest Research 36: 426-436.

1 Airborne laser scanner data

- Laser scanner: ALTM 2033 (Optech)
- Date of data acquisition: 4.8.2004
- Flying altitude: 1500 m
- Swath width: 800 m
- Max. scan angle: 15°
- Pulse density: 0.7/m^2
- Recorded echoes: first and last

2 Aerial image data

- Camera: Leica RC30
- Date of data acquisition: 22.7.2004
- Scale: 1:30 000
- Lens: UAGA-F 13158
- Focal length: 163.18 mm
- Number of images: 3
- Pixel size of the orthorectified images: 0.5 m

3 Study area and field data

Study area is located in Matalansalo, eastern Finland. The size of the area was 1200 ha and it was managed forest owned by UPM-Kymmene. The main tree species were pine (*Pinus sylvestris*) on 59 % of the sample plots and spruce (*Picea abies*) on 34 % of the plots. The area consisted of young, middle-aged and matureforest.

463 circular sample plots were established on 67 stands. Radius of the plots was 9 m. The location of the stands was chosen randomly. The number of sample plots on a stand varied from 5 to 9. The plots were placed systematically on a stand. Trees with DBH >5 cm were measured. On each sample plot, height was measured from one sample tree of each tree species and each storey class.

Stand level inventory data from 1990s was also available.

4 Method

k most similar neighbor (*k*-MSN) method was used to predict plot and stand volume. *k*-MSN is a non-parametric method that uses canonic correlation analysis to produce a weighting matrix. With the produced matrix, *k*-most similar neighbors are selected from reference data.

Cross validation was used for calculating of results.

5 Reliability of the models

Table 1. The accuracy of *k*-most similar neighbour based stand volume estimates using different data sources (independent variables). Class variables include main tree species, stand development class, and site fertility class.

Independent variables	RMSE (%)	Bias (%)
aerial photograph	29.04	-0.47
laser scanner	5.89	0.44
aerial photograph and laser scanner	5.88	0.28
aerial photograph and class variables	20.85	-0.32
laser scanner and class variables	5.75	0.13
class variables and updated old stand volume	19.15	-1.18
aerial photograph, laser scanner, and class variables	5.53	0.32
aerial photograph, class variables and updated old stand volume	17.03	-0.51
laser scanner, class variables, and updated old stand volume	6.22	-0.03
aerial photograph, laser scanner, class variables, and updated old stand volume	5.78	0.64

Table 2. The accuracy of k-most similar neighbor based plot volume estimates using different data sources. Class variables include main tree species, stand development class, and site fertility class.

Dependent variable(s)	Independent variable(s)	*k*	RMSE (%)	Harha (%)
V, V^2	aerial photograph	14	38.17	-0.40
V	laser scanner	11	15.58	0.28
V, V^2	aerial photograph and laser scanner	8	14.35	0.20
V	aerial photograph and class variables	11	31.43	-0.42
V, V^2	laser scanner and class variables	7	14.55	-0.01
V	class variables and updated old stand volume	18	31.25	-1.38
V, V^2	aerial photograph, laser scanner, and class variables	12	13.28	0.19
V	aerial photograph, class variables, and updated old stand volume	11	28.66	-1.29
V, V^2	laser scanner, class variables, and updated old stand volume	9	14.39	-0.08
V, V^2	aerial photograph, laser scanner, class variables, and updated old stand volume	10	13.70	-0.04

Practical large-scale forest stand inventory using a small-footprint airborne scanning laser

Næsset, E. 2004. Scandinavian Journal of Forest Research. 19: 164-179.

1 Airborne laser scanner data

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 23.7.-1.8.2001
- Flying altitude: 650 m
- Average speed: 75 m/s
- Number of flight lines: 129, sidelap 50 %
- Pulse repetition frequency: 10 kHz
- Scan frequency: 30 Hz
- Max. scan angle, data collection: 16°
- Max. scan angle, data processing: 15°
- Footprint diameter: 13-29 cm (average 23 cm)
- Average distance between footprints: 0.7-1.6 m (average 1.0 m)

2 Study area and field data

Study area in Krødsherad, south-east Norway (60°10′N 9°35′E, 130-660 m a.s.l.), with a size of 6500 ha was selected for this study. The main tree species were spruce (*Picea abies*) and pine (*Pinus sylvestris*), but especially in young stands the portion of deciduous trees was large.

116 circular training plots were used as a reference data. The plots were systematically distributed on the study area. Plot size was 232.9 m^2. Training plots were divided in young forests, mature forests on poor sites, and mature forests on good sites. Site index on poor sites was equal to or less than 11 meters at the age of 40 years. The number of young forest plots was 39, and the number of mature forest plots was 77. The number of sample trees per plot varied from 4 to 13 with an average of 10. Altogether 1118 sample trees were measured.

57 test plots with a size of 61×61 m^2 located in subjectively selected stands. Different combinations of age classes, site quality classes and tree species mixtures were represented. In young forest plots, trees with DBH >4 cm were callipered. In mature forest plots, trees with DBH >10 cm were callipered. The number of sample trees per plot varied from 36 to 77 (average 60, total 3429).

3 Regression models

Regression models are constructed for conifer dominated (>70 % of volume) forests. Young forest, mature forest on poor sites, and mature forest on good sites have their own models.

- Mean height weighted by basal area (Lorey's mean height) (1, 7 and 13)
- Dominant height (2, 8 and 14)
- Mean diameter by basal area (3, 9 and 15)
- Stem number (4, 10 and 16)
- Basal area (5, 11 and 17)
- Volume (6, 12 and 18)

Table1. Explanations for variables.

Variable	Explanation
h_L	Mean height weighted by basal area (Lorey's mean height)
h_{dom}	Dominant height (m)
d_g	Mean diameter by basal area (cm)
N	Stem number/ha
G	Basal area (m²/ha)
V	Volume (m³/ha)
h_{of}, h_{30f}, h_{50f}, h_{60f}, h_{80f}, h_{90f}	Percentiles of the first pulse laser canopy heights for 0%, 10%, …, 90%
h_{60l}, h_{70l}, h_{80l}, $h_{90}l$	Percentiles of the last pulse laser canopy heights for 60%, 70%, 80% and 90%
h_{cvf}	Coefficient of variation of the first and last pulse canopy heights (%)
d_{of}, d_{1f}, d_{4f}, d_{6f}, d_{8f}, d_{9f}	Canopy densities corresponding to the proportions of first pulse laser hits above fraction no. 0, 1, …, 9 to total number of first pulses
d_{0l}, d_{4l} d_{9l}	Canopy densities corresponding to the proportions of last pulse laser hits above fraction no. 0, 4 and 9 to total number of first pulses

Models for young forest:

$$\ln h_L = 0{,}691 + 0{,}758 * \ln h_{80l} \quad (1)$$

$$\ln h_{dom} = 0{,}214 + 0{,}995 * \ln h_{90l} + 0{,}296 * \ln d_{0f} - 0{,}232 * \ln d_{6f} + 0{,}065 * \ln d_{9f} \quad (2)$$

$$\ln d_g = 3{,}116 + 0{,}255 * \ln d_{8f} - 0{,}233 * \ln d_{2l} \quad (3)$$

$$\ln N = 6{,}278 + 0{,}55 * \ln h_{0f} + 1{,}881 * \ln d_{0f} - 0{,}374 * \ln d_{0l} \quad (4)$$

$$\ln G = 1{,}87 + 1{,}204 * \ln h_{90f} - 0{,}427 * \ln h_{cvf} + 0{,}628 * \ln d_{0l} \quad (5)$$

$$\ln V = 2{,}326 + 1{,}912 * \ln h_{90l} - 0{,}525 * \ln h_{cvf} + 0{,}679 * \ln d_{0l} \quad (6)$$

Models for mature forest, poor site quality:

$$\ln h_L = 0{,}52 + 0{,}473 * \ln h_{90f} + 0{,}337 * \ln h_{90l} \quad (7)$$

$$\ln h_{dom} = 0{,}714 - 0{,}459 * \ln h_{80l} + 1{,}249 * \ln h_{90l} + 0{,}082 * \ln d_{0l} \quad (8)$$

$$\ln d_g = 1{,}437 + 1{,}255 * \ln h_{60f} - 0{,}477 * \ln h_{60l} - 0{,}5 * \ln d_{0f} + 0{,}127 * \ln d_{9l} \quad (9)$$

$$\ln N = 10{,}55 + 2{,}602 * \ln h_{50f} - 3{,}719 * \ln h_{60f} + 1{,}185 * \ln d_{0f} + 0{,}472 * \ln d_{4l} \quad (10)$$

$$\ln G = 0{,}964 + 0{,}909 * \ln h_{80l} + 0{,}73 * \ln d_{0f} \quad (11)$$

$$\ln V = 1{,}0 + 1{,}609 * \ln h_{90l} + 0{,}774 * \ln d_{0f} \quad (12)$$

Models for mature forest, good site quality:

$$\ln h_L = 0{,}789 + 0{,}737 * \ln h_{90f} \quad (13)$$

$$\ln h_{dom} = 0{,}736 - 0{,}896 * \ln h_{80f} + 1{,}702 * \ln h_{90f} + 0{,}031 * \ln d_{9l} \quad (14)$$

$$\ln d_g = 0{,}406 + 0{,}892 * \ln h_{90f} - 0{,}374 * \ln d_{1f} \quad (15)$$

$$\ln N = 9{,}793 - 0{,}914 * \ln h_{70l} + 1{,}367 * \ln d_{4f} \quad (16)$$

$$\ln G = 0{,}883 + 1{,}081 * \ln h_{701} + 0{,}468 * \ln d_{41} \quad (17)$$

$$\ln V = 0{,}561 - 0{,}915 * \ln h_{30f} + 2{,}7 * \ln h_{701} + 1{,}078 * \ln d_{1f} \quad (18)$$

4 Reliability of the models

Table 2. Root mean square errors (RMSE) and coefficients of determination (R^2) of the models.

Model no.	Dependent variable	RMSE	R^2
Young forest			
1	$\ln h_L$	0.07	0.92
2	$\ln h_{dom}$	0.07	0.92
3	$\ln d_g$	0.17	0.55
4	$\ln N$	0.26	0.77
5	$\ln G$	0.13	0.94
6	$\ln V$	0.13	0.97
Mature forest, poor site quality			
7	$\ln h_L$	0.06	0.91
8	$\ln h_{dom}$	0.07	0.9
9	$\ln d_g$	0.13	0.69
10	$\ln N$	0.25	0.81
11	$\ln G$	0.22	0.77
12	$\ln V$	0.22	0.86
Mature forest, good site quality			
13	$\ln h_L$	0.07	0.77
14	$\ln h_{dom}$	0.06	0.85
15	$\ln d_g$	0.12	0.61
16	$\ln N$	0.25	0.6
17	$\ln G$	0.17	0.74
18	$\ln V$	0.18	0.83

Table 3. Differences between predicted and ground reference values in cross-validation. None of the mean differences was statistically significant (p>0.05).

Variable	Observed mean	Mean difference	Stand. dev. for the differences
Young forest			
h_L (m)	13.00	-0.01	0.93
h_{dom} (m)	15.79	-0.01	1.18
d_g (cm)	13.89	-0.04	2.79
N (ha^{-1})	1735	19	454
G (m^3 ha^{-1})	25.03	0.32	3.70
V (m^3 ha^{-1})	188.5	0.9	32.9
Mature forest, poor site quality			
h_L (m)	15.14	0.00	0.87
h_{dom} (m)	16.67	-0.03	1.10
d_g (cm)	22.32	0.03	3.27
N (ha^{-1})	628	6	161
G (m^3 ha^{-1})	22.59	0.08	5.26
V (m^3 ha^{-1})	175.1	0.7	39.4
Mature forest, good site quality			
h_L (m)	20.34	0.00	1.42
h_{dom} (m)	22.93	0.00	1.51
d_g (cm)	23.94	0.04	3.24
N (ha^{-1})	803	1	203
G (m^3 ha^{-1})	34.23	0.21	5.84
V (m^3 ha^{-1})	340.8	0.7	67.8

Table 4. Differences between predicted and ground reference values for the differences in predictions. NS=not statistically significant (p>0.05).

Variable	Observed mean	Mean difference	Stand. dev. for the differences
Young forest			
h_L (m)	15.79	-0.85	1.01
h_{dom} (m)	18.62	-0.61	0.67
d_g (cm)	15.36	0.45 NS	2.42
N (ha^{-1})	1592	108 NS	466
G (m^3 ha^{-1})	28.21	0.43 NS	3.84
V (m^3 ha^{-1})	225.2	5.9 NS	25.8
Mature forest, poor site quality			
h_L (m)	15.57	-0.58	0.64
h_{dom} (m)	17.65	-0.83	0.78
d_g (cm)	20.32	0.74 NS	1.63
N (ha^{-1})	680	34 NS	97
G (m^3 ha^{-1})	21.11	1.32	1.83
V (m^3 ha^{-1})	162.3	8.9	15.1
Mature forest, good site quality			
h_L (m)	20.27	-0.75	0.75
h_{dom} (m)	22.94	-0.99	0.84
d_g (cm)	22.58	0.15 NS	1.33
N (ha^{-1})	779	68	129
G (m^3 ha^{-1})	29.78	2.51	3.94
V (m^3 ha^{-1})	286.6	16.1 NS	35.1

Predicting forest stand characteristics with airborne scanning laser using a practical two-stage procedure and field data

Næsset, E. 2002. Remote Sensing of Environment. 80: 88-99.

1 Airborne laser scanner data

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 8.-9.6.1999
- Flying altitude: 700 m
- Average speed: 71 m/s
- Number of flight lines: 24+19
- Pulse repetition frequency: 10 kHz
- Scan frequency: 21 Hz
- Max. scan angle, data collection: 17°
- Max. scan angle, data processing: 14°
- Swath width: 420 m
- Footprint diameter: 21 cm
- Average distance between footprints: 0.92-0.94 m
- Recorded echoes: first and last

2 Study area and field data

A study area with a size of about 1000 ha is located in Våler, south-east Norway (59°30′N 10°55′E, 70-120 m a.s.l.)

144 circular plots (200 m^2) were systematically distributed throughout the study area. Sample plots were divided in young and mature forests. Mature forests were further divided in poor and good site quality. Site index value on poor sites at the age of 40 years was equal to or less than 11. On 81 of the plots, all tree heights were measured. On 63 remaining plots, tree heights were measured on sample trees. The number of sample trees per plot varied from 11 to 23 with an average of 16. The ground-truth mean height of each plot was computed as mean height weighted by basal area (h_L). In young forests, all trees were included in computations. In mature forests, trees with DBH >10 cm were used. Mean plot diameter (d_g) was computed as mean diameter by basal area.

61 stands were subjectively selected to represent different combinations of age class, fertility class and tree species combinations. Circular plots were systematically distributed on stands. The distances ranged from 20 to 60 meters. The number of plots per stand was 14-30. Plot size was 100 m^2 in young stands and 200 m^2 in mature stands. In young stands, trees with DBH >4 cm were callipered. In mature forests, DBH of callipered trees was >10 cm. At least one sample tree was selected on each plot. There were 24 to 84 sample trees per stand. Mean stand height was computed as the arithmetic mean of sample tree heights, which corresponds to h_L. Mean stand diameter was computed from callipered trees as mean diameter weighted by basal area (d_g).

3 Regression models

- Mean height weighted by basal area (Lorey's mean height)
- Dominant height
- Mean diameter weighted by basal area
- Stem number
- Basal area
- Volume

Table 1. Explanations for variables.

Variable	Explanation
h_L	Mean height weighted by basal area (m)
h_{dom}	Dominant height(m)
d_g	Mean diameter weighted by basal area (cm)
N	Stem number (/ha)
G	Basal area (m^2/ha)
V	Volume (m^3/ha)
$h_{0f}…h_{90f}$	Canopy densities corresponding to the proportions of first pulse laser hits above the 0, 10, …, 90 quantiles to the total number of first pulses
$h_{0l}…h_{90l}$	Canopy densities corresponding to the proportions of last pulse laser hits above the 0, 10, …, 90 quantiles to the total number of last pulses
h_{maxf}	Maximum first pulse laser canopy height
h_{maxl}	Maximum last pulse laser canopy height
h_{cvf}	Coefficient of variation of first pulse laser canopy height (%)
h_{meanl}	Mean of the last pulse laser canopy heights (m)
$d_{50f}…d_{90f}$	Canopy densities corresponding to the proportions of first pulse laser hits above the 0, 10, …, 90 quantiles to total number of first pulses
$d_{50l}…d_{90l}$	Canopy densities corresponding to the proportions of last pulse laser hits above the 0, 10, …, 90 quantiles to total number of last pulses

Young forest:

$$\ln h_L = 0{,}46 + 1{,}149 \ln h_{90l} - 0{,}28 \ln h_{\max l} \quad (1)$$

$$\ln h_{dom} = 0{,}568 + 1{,}169 \ln h_{90l} - 0{,}286 \ln h_{\max l} \quad (2)$$

$$\ln d_g = -0{,}867 + 0{,}217 \ln h_{10l} + 0{,}665 \ln h_{80l} - 0{,}805 \ln d_{80f} \quad (3)$$

$$\ln N = 15{,}99 - 1{,}182 \ln h_{80l} + 3{,}08 \ln d_{80f} \quad (4)$$

$$\ln G = 3{,}492 + 0{,}536 \ln h_{10f} + 1{,}388 \ln d_{50f} \quad (5)$$

$$\ln V = 3{,}473 + 1{,}336 \ln h_{meanl} + 1{,}477 \ln d_{50f} \quad (6)$$

Mature forest, poor site quality:

$$\ln h_L = 0{,}285 + 1{,}01 \ln h_{90f} - 0{,}107 \ln h_{50l} \quad (7)$$

$$\ln h_{dom} = -0{,}0187 + 1{,}002 \ln h_{\max f} \quad (8)$$

$$\ln d_g = 0{,}206 + 0{,}77 \ln h_{90l} 0{,}312 \ln d_{80l} \quad (9)$$

$$\ln N = 11{,}24 + 1{,}195 \ln h_{0f} - 1{,}662 \ln h_{\max f} + 1{,}156 \ln d_{20l} \quad (10)$$

$$\ln G = 4{,}253 + 4{,}304 \ln h_{50f} - 4{,}022 \ln h_{60f} + 0{,}584 \ln d_{90f} \quad (11)$$

$$\ln V = 4{,}951 - 1{,}278 \ln h_{30f} + 5{,}994 \ln h_{50f} - 3{,}8 \ln h_{60f} + 0{,}766 \ln d_{90f} \quad (12)$$

Mature forest, good site quality:

$$\ln h_L = 0{,}35 + 0{,}529 \ln h_{90f} + 0{,}355 \ln_{\max f} \quad (13)$$

$$\ln h_{dom} = 0{,}525 + 0{,}23 \ln h_{80f} + 0{,}637 \ln h_{\max f} + 0{,}084 \ln d_{10l} \quad (14)$$

$$\ln d_g = 0{,}441 + 0{,}64 \ln h_{90l} - 0{,}277 \ln d_{90l} \quad (15)$$

$$\ln N = 10{,}33 - 0{,}487 \ln h_{0l} - 0{,}667 \ln h_{cvf} + 1{,}187 \ln d_{50f} \quad (16)$$

$$\ln G = 3{,}608 + 2{,}629 \ln h_{80l} - 2{,}157 \ln h_{\max f} + 1{,}26 \ln d_{50f} \quad (17)$$

$$\ln V = 3{,}151 + 3{,}027 \ln h_{80l} - 1{,}66 \ln h_{\max f} + 1{,}223 \ln d_{50f} \quad (18)$$

4 Reliability of the models

Table 2. Root mean square errors (RMSE) and coefficients of determination (R^2) of the models.

Model		Dependent variable	RMSE	R^2
1	Models for young forest	$\ln h_L$	0.06	0.95
2		$\ln h_{dom}$	0.07	0.93
3		$\ln d_g$	0.12	0.78
4		$\ln N$	0.28	0.68
5		$\ln G$	0.14	0.89
6		$\ln V$	0.16	0.93
7	Models for mature forest, poor sites	$\ln h_L$	0.05	0.86
8		$\ln h_{dom}$	0.08	0.74
9		$\ln d_g$	0.12	0.54
10		$\ln N$	0.30	0.65
11		$\ln G$	0.21	0.69
12		$\ln V$	0.20	0.80
13	Models for mature forest, good sites	$\ln h_L$	0.01	0.82
14		$\ln h_{dom}$	0.07	0.85
15		$\ln d_g$	0.12	0.39
16		$\ln N$	0.35	0.50
17		$\ln G$	0.21	0.75
18		$\ln V$	0.22	0.8

Table 3. Differences between predicted and ground-truth values and standard deviation (SD) for the differences in cross-validation. NS=Not statistically significant (p>0.05).

Variable	Observed mean	Mean difference	SD
Young forest			
h_L (m)	14.09	-0.01 NS	0.87
h_{dom} (m)	16.26	0.02 NS	1.27
d_g (cm)	13.28	0.03 NS	1.66
N (ha^{-1})	2056	11 NS	649
G (m^3 ha^{-1})	25.83	0.07 NS	3.63
V (m^3 ha^{-1})	192.2	-0.1 NS	29.3
Mature forest, poor sites			
h_L (m)	16.45	-0.01 NS	1.00
h_{dom} (m)	17.69	-0.01 NS	1.50
d_g (cm)	22.84	0.02	3.20
N (ha^{-1})	522	6 NS	135
G (m^3 ha^{-1})	19.75	0.15 NS	4.54
V (m^3 ha^{-1})	155.0	1.1 NS	37.8
Mature forest, good sites			
h_L (m)	20.27	0.01 NS	0.36
h_{dom} (m)	22.50	0.03 NS	1.54
d_g (cm)	22.64	0.04 NS	2.84
N (ha^{-1})	722	38 NS	377
G (m^3 ha^{-1})	27.45	0.14 NS	5.64
V (m^3 ha^{-1})	269.2	1.1 NS	62.0

Table 4. Differences between predicted and ground-truth values in stand predictions. NS=Not statistically significant (p>0.05).

Variable	Observed mean	Mean difference	SD
Young forest			
h_L (m)	13.9	0.42	0.87
h_{dom} (m)	16.62	-0.08 NS	1.33
d_g (cm)	13.23	0.72	1.60
N (ha^{-1})	1844	-90 NS	400
G (m^3 ha^{-1})	23.79	-0.86 NS	2.48
V (m^3 ha^{-1})	168	6.2 NS	24.0
Mature forest, poor sites			
h_L (m)	16.37	-0.09 NS	0.61
h_{dom} (m)	18.07	-0.31 NS	0.70
d_g (cm)	21.17	0.78	1.61
N (ha^{-1})	577	15 NS	128
G (m^3 ha^{-1})	19.84	0.74 NS	2.33
V (m^3 ha^{-1})	154.8	8.2 NS	18.3
Mature forest, good sites			
h_L (m)	19.77	-0.01 NS	1.17
h_{dom} (m)	22.38	-0.43 NS	1.32
d_g (cm)	21.24	0.98	1.37
N (ha^{-1})	856	-103	145
G (m^3 ha^{-1})	29.66	-0.67 NS	2.54
V (m^3 ha^{-1})	280.5	0.3 NS	31.9

Predicting the plot volume by tree species using airborne laser scanning and aerial photographs

Packalén, P. & Maltamo, M. 2006. Forest Science 52: 611-622.

1 Airborne laser scanner data

- Laser scanner: ALTM 2033 (Optech)
- Date of data acquisition: 4.8.2004
- Flying altitude: 1500 m
- Average speed: 75 m/s
- Number of flight lines: 7, side lap 35%
- Max. scan angle:15°
- Swath width: 800 m
- Footprint diameter: 45 cm
- Pulse density: 0.7 / m^2
- Beam divergence: 0.3 mrad
- Recorded echoes: first and last

2 Aerial image data

- Camera: Leica RC30
- Date of data acquisition: 22.7.2004
- Scale: 1:30 000
- Lens: UAGA-F 13158
- Focal length: 163.18 mm
- Number of images: 3
- Pixel size of the orthorectified images: 0.5 m

3 Study area and field data

The study area is located in Varkaus, eastern Finland. The area is typical managed boreal forest and it is owned by UPM-Kymmene Oyj. The main tree species was pine (*Pinus sylvestris*) on 59% of the sample plots and spruce (*Picea abies*) on 34% of the plots.

463 circular sample plots with a radius of 9 meters were located on 67 stands. 27% of the sample plots were young forest, 42% were middle aged and 31% mature forest. DBH, tree and storey class and tree species were determined on trees with DBH >5 cm. Tree height was measured on one sample tree on each storey and tree species class on every sample plot. The heights for the rest of the trees were calculated with Veltheim's (1987) models. Height measurements were used for calibration of the height model. Tree volumes were calculated with Laasasenaho's (1982) models based on DBH and tree height.

265 sample plots were used for modeling and 198 for model testing.

4 Models

Fuzzy classification and *k*-MSN method were used to volume estimation. Maximum likelihood (ML), fuzzy classification based on the underlying logic of fuzzy sets (FZ) and linear mixture modelling (LMM) were studied in the fuzzy classification.

In fuzzy classification, total volume was first predicted using laser variables as predictor variables (Formula 1). Total volume was then divided into tree species using features from the aerial images and fuzzy classification.

$$\ln(V) = 2{,}100 - 0{,}0116 f\_pgh - 0{,}00995 l\_p30 + 1{,}452 \ln(f\_h60) + \frac{\delta^2}{2} \quad (1)$$

Classification variables in the second phase of fuzzy classification:

$mean_{\mathrm{NIR}}$, $mean_{\mathrm{RED}}$, var_{NDVI}, idm_{NIR}, $cont_{\mathrm{NIR}}$

The subscript denotes the band; mean = mean intensity; var = sum of squares: variance; idm = inverse difference moment; cont = contrast.

In *k*-MSN method, predictor variables were selected manually. The best combination of predictors were selected on the basis of RMSE and bias. Predictor variables in *k*-MSN-based volume model:

$\ln(mean_{\mathrm{NIR}})$, $\ln(mean_{\mathrm{RED}})$, $\ln(cont_{\mathrm{NIR}})$, $\ln(savg_{\mathrm{NIR}})$, ln(*f_h20*), ln(*f_h60*), ln(*f_h95*), ln(*l_h10*), ln(*l_p30*), ln(*l_p70*), ln(*f_pgh*)

The subscript denotes the band; *mean* = mean intensity; *cont* = contrast; *savg* = sum average; *f* tai *l* = laser pulse type, first or last pulse; p_{30}, p_{70} = 30% or 70% canopy density corresponding to the proportion of first or last pulse laser hits above the 30% or 70% quantile; h_{10}, h_{20}, h_{60}, h_{95} = the height at which 10%, 20%, 60% or 95% of the height distribution has accumulated; *pgh* = proportion of ground hits.

5 Reliability of the models

Table 1. Accuracy of the volume estimates by tree species and the total volume.

	RMSE (m^3)				RMSE (%)			
Method	Pine	Spruce	Deciduous	Total	Pine	Spruce	Deciduous	Total
ML	62.57	61.99	32.03	37.22	62.87	81.41	148.75	18.88
FZ	83.32	61.18	62.26	37.22	83.73	80.35	289.19	18.88
LMM	79.44	86.68	57.13	37.22	79.83	113.83	265.36	18.88
k-MSN	45.28	47.2	19.87	47.05	45.5	61.98	92.3	23.86
		Bias (m^3)				Bias (%)		
Method	Pine	Spruce	Deciduous	Total	Pine	Spruce	Deciduous	Total
ML	1.39	8.15	-12.46	-2.92	1.4	10.7	-57.87	-1.48
FZ	51.74	-2.17	-52.5	-2.92	52	-2.85	-243.85	-1.48
LMM	24.53	-16.18	-11.28	-2.92	24.65	-21.25	-52.39	-1.48
k-MSN	1.9	-7.55	0.76	-4.9	1.91	-9.92	3.51	-2.48

Prediction of tree height, basal area and stem volume in forest stands using airborne laser scanning

Holmgren, J. 2004. Scandinavian Journal of Forest Research. 19: 543-553.

1 Airborne laser scanner data

- Laser scanner: TopEye (from a helicopter)
- Date of data acquisition: 13.9.2000
- Flying altitude: 230 m
- Average speed: 16 m/s
- Pulse repetition frequency: 7 kHz
- Scan frequency: 16.67 Hz
- Max. scan angle: 20°
- Swath width: 167 m
- Footprint diameter: 1.8 m
- Beam divergence: 8 mrad
- Recorded echoes: first and last

2 Study area and field data

Test area is located in Remningstorp, south-western Sweden (58°30′N 13°40′E, 120-145 a.s.l.). Dominant tree species are spruce, (*Picea abies*), pine (*Pinus sylvestris*) and birch (*Betula* spp.).

Circular field plots with basal area weighted height (h_L) >5 m were used in this study. Field plots had a radius of 10 m. Trees with a stem diameter ≥5 cm were callipered.

Two separate field datasets was used. Dataset A consisted of circular plots, which were allocated on each stand using a randomly placed grid. Distance between the plots was determined by stand size and specification. 10 plots were allocated on each stand with a distance of 50, 75 or 100 meters. 664 field plots were inventoried in 1997-2000. Certain criteria were used for selecting a plot to be included in dataset A: 1) the plot must not be divided by a stand boundary, 2) the stand not been thinned or clear-felled during 1997-2000, 3) the plot must be totally covered by laser scanner data, and 4) there must be pulses reflected at least 3 meters above the ground level. 140 field plots within 22 stands fulfilled the criteria. Average tree height was 20 meters, average forest age 67 years, and average site index 27.

Dataset B consisted of 464 circular plots within 80×80 m^2 squares placed in the middle of forest stands. 70% of tree volume on stands must consist of conifer forest. Plots were located on a regular grid with a distance between the plots was 20 meters. Average tree height on measured trees was 19 meters. Average age was 52 years and average site index 32.

First, dataset A was used for parameter estimation. Parameters were used in the regression models for predictions at plot level within dataset B. Secondly, cross-validation was used for parameter estimation in dataset B.

3 Regression models

- Basal area weighted mean tree height (h_L)
- Basal area (G)
- Volume (V)

Table 1. Explanations for variables

Variable	Explanation
h_{90} and h_{95}	Plot level 90th and 95th percentiles
Dv	Vegetation ratio
Dp	Pulse type ratio $(n_1+n_3)/(n_1+n_2)$
	where n_1 = number of single returns
	n_2 = number of first returns of a double return
	n_3 = number of first returns of a double return that had their second return with a height value >3 m
relstd	Standard deviation divided by the 95th laser height percentile

Basal area weighted mean tree height:

$$h_L = \beta_0 + \beta_1 h_{95} + \varepsilon \quad (1)$$

Basal area:

$$\ln(G) = \beta_0 + \beta_1 \ln(h_{90}) + \beta_2 \ln(D_v) + \beta_3 D_p + \beta_4 relstd + \varepsilon \quad (2)$$

Volume:

$$\ln(V) = \beta_0 + \beta_1 \ln(h_{90}) + \beta_2 \ln(D_v) + \beta_3 D_p + \beta_4 relstd + \varepsilon \quad (3)$$

Table 2. Properties of the regression models. SE=standard error.

Model	Estimated variable		Dataset A value	SE	Dataset B value	SE
1	h_L	residual		1.42		1.05
		(intercept)	2.08		1.46	
		h_{95}	0.88		0.95	
2	$\ln(G)$	residual		0.24		0.15
		(intercept)			-2.28	
		$\ln(D_v)$	0.3		0.88	
		$\ln(h_{90})$	0.47		0.77	
		relstd	-2.73		-2.08	
		D_p	1.24			
3	$\ln(V)$	residual		0.29		0.18
		(intercept)			-2.5	
		$\ln(D_v)$	0.28		0.87	
		$\ln(h_{90})$	1.21		1.49	
		relstd	-3.09		-2.44	
		D_p	1.41		0.44	

4 Reliability of the models

Table 3. RMSE and RMSE (%) in datasets A and B at plot level and stand level.

Model	Estimated variable		Dataset A RMSE	Dataset A RMSE(%)	Dataset B RMSE	Dataset B RMSE(%)
1	h_L	plot level	0.99	5 %	1.07	6 %
		stand level			0.59	3 %
2	$\ln(G)$	plot level	4.2	15 %	4.8	17 %
		stand level			2.7	10 %
3	$\ln(V)$	plot level	50	19 %	55	20 %
		stand level			31	11 %

Accuracy of stand limits and estimating the stand characteristics

Havia, J. 2006. Master's thesis. University of Joensuu, Faculty of Forest Sciences.

1 Airborne laser scanner data

- Laser scanning operator: Blom Asa
- Date of data acquisition: 5.8.2004
- Flying altitude: 900 m
- Average speed: 75 m/s
- Pulse repetition frequency: 29 kHz
- Max. scan angle: 20°
- Recorded echoes: first and last

2 Study area and field data

Study area is located in Juupajoki, western Finland. Location in the Finnish coordinate system: I 3358500 P 6862000.

Modelling data consisted of 167 circular sample plots with a radius of 9 meters. Sample plots were placed on forest stands with a purpose to have enough plots on each age class. Sapling stands were not used in this study. Trees with DBH >5 cm were callipered.

Test data consisted of truncated angle sample plots on 22 stands. The number of plots per stand was depended on stand size and varied from 5-12.

3 Regression models

1. Volume / ha
2. Basal area / ha
3. Basal area weighted height / ha
4. Dominant height / ha
5. Basal area weighted median diameter / ha
6. Proportion of spruce in total volume

Table1: Variables in the models.

Variable	Explanation
$h_{5...95}$	Height of 5...95 % percentile (m)
$d_{5...95}$	Relative proportion of pulses in 5...95 % percentiles
h_{max}	Maximum value of laser heights
h_{med}	Median value of laser heights
veg	Proportion of vegetation hits
var	Coefficient of variation
lower *first*	First pulses
lower *last*	Last pulses

Volume / ha:

$$\ln V = 2{,}493 + 0{,}034 * h\max_{last} + 0{,}817 * \ln veg_{first} + 0{,}661 * \ln h_{40last} + 0{,}942 * veg_{last} \quad (1)$$

Basal area / ha:

$$\ln G = 0{,}386 * \ln veg_{last} + 0{,}248 * \ln h_{40\,first} + 1{,}272 * \ln veg + 0{,}019 * h_{\max last} + 2{,}535 \quad (2)$$

Basal area weighted height / ha:

$$\ln H_n = 0{,}892 + 1{,}772 * \ln h_{95\,first} + 0{,}018 * h_{50last} - 1{,}165 * \ln h_{90last} \quad (3)$$

Dominant height / ha:

$$\ln h_{dom} = 1{,}465 + 0{,}081 * \ln h_{95\,first} + 0{,}175 * \ln veg_{last} + 0{,}546 * \ln h_{70\,first} - 0{,}076 * h_{90\,first} \quad (4)$$

Basal area weighted median diameter / ha:

$$\ln d_{mean} = 1{,}974 + 0{,}059 * h_{95\,first} - 0{,}009 * h_{10last} - 0{,}361 * \ln veg_{first} - 0{,}944 * d_{95last} \quad (5)$$

Proportion of spruce in total volume:

$$\ln Kuusi\% = 3{,}841 + 10{,}147 * veg_{last} + 1{,}403 * \ln h_{10last} - 9{,}765 * veg_{first} + 1{,}262 * \ln \mathrm{var}_{first} \quad (6)$$

4 Reliability of the models

Table 2. RMSE and R^2 of the models.

Model	Independent variable	RMSE	RMSE (%)	R^2
1	$\ln V$	0.23	23.8	0.75
2	$\ln G$	0.19	18.9	0.6
3	$\ln h_n$	0.11	11	0.8
4	$\ln h_{dom}$	0.11	12	0.72
5	$\ln d_{mean}$	0.11	11.1	0.86
6	$\ln Kuusi\%$	0.94	27	0.63

Recovering plot-specific diameter distribution and height-diameter curve using ALS based stand characteristics

Mehtätalo, L., Maltamo, M., Packalén, P. ISPRS Workshop on Laser Scanning 2007 and SilviLaser 2007.

1 Airborne laser scanner data

- Laser scanner: ALTM 3100C (Optech)
- Date of data acquisition: 13.7.2005
- Flying altitude: 2000 m
- Max. scan angle, data collection: 15°
- Swath width: 1050 m
- Pulse density: $0.6/m^2$
- Recorded echoes: 4 range measurements for each pulse (only first and last echoes used)

2 Study area and field data

Study area is located in Juuka, eastern Finland.

506 circular sample plots with a 9 meters radius were placed rather systematically on young, middle-aged and mature forest stands. Sapling stands were left out. The centre of each plot was determined with the GPS. DBH, tree and storey class, and tree species was recorded for trees with DBH >5 cm. Height was measured on one sample tree of each species in each storey class on each plot. Heights for the rest of the trees were predicted with Näslund’s height model using a random constant for each plot. Tree volumes were calculated with Laasasenaho’s models.

3 Regression models

- Plot volume (1)
- Number of stems (2)
- Basal area median height (3)
- Basal area median diameter (4)

Table 1. Explanations for the variables.

Variable	Explanation
f or *l*	First or last pulse type
h_p	The height at which p% of the height distribution has accumulated
veg	Proportion of vegetation hits
i_{50}	50th percentile of intensity reflection
p_{20}	Proportion of laser hits which is accumulated at the height of 20%

$$\ln V = 0.134 + 1.202\ln(f\_h_{50}) + 0.198\sqrt{f\_veg} + 0.114\ln(l\_veg) \quad (1)$$

$$\ln N = 7.803 - 1.027\ln(f\_h_{95}) + 0.251\sqrt{f\_veg} + 7.988(\frac{1}{fi_{50}}) - 0.319\ln(l\_p_{20}) \quad (2)$$

$$\ln H = -26.075 + 5.747\ln(f\_h_{95}) + 3.581\ln(f\_h_{40}) + 38.371(\frac{1}{f\_h_{60}}) + 0.605\ln(l\_veg) + 4.907\ln(l\_h_{50}) \quad (3)$$

$$\ln D = 2.697 - 2.605(\frac{1}{f\_i_{50}}) - 37.812(\frac{1}{f\_p_{20}}) + 2.004\ln(f\_h_{50}) - 1.231\ln(f\_h_{40}) \quad (4)$$

4 Reliability of the models

Table 2. R-squared values, standard errors (s.e.) and relative standard errors for the models.

Model	R^2	s.e.	relative s.e.
1	0.924	0.155	16.3%
2	0.497	0.311	30.8%
3	0.783	0.119	12.9%
4	0.860	1.230	8.7%

Table 3. RMSE and bias of predicted stand characteristics in the data of feasible solutions.

	RMSE		Bias	
	Absolute	%	Absolute	%
H, m	1.22	8.70	0.00	-0.01
D, cm	2.35	12.96	-0.15	-0.80
N, ha^{-1}	279.8	31.00	-35.30	-3.91
V, m^3ha^{-1}	20.02	16.29	-1.64	-1.33

Simulating sampling efficiency in airborne laser scanning based forest inventory

Ene, L., Næsset, E., Gobaggen, T. ISPRS Workshop on Laser Scanning 2007 and SilviLaser 2007.

1 Airborne laser scanner data

- Laser scanner: ALTM 3100 (Optech)
- Date of data acquisition: June 2005 (leaf-off conditions)
- Flying altitude: 750 m
- Average speed: 75 m/s
- Pulse repetition frequency: 100 kHz
- Scan frequency: 70 Hz
- Max. (half) scan angle, data collection: 10°
- Swath width: 264 m
- Footprint diameter: 21 cm
- Average point density: 5.09/m^2
- Recorded echoes: first and last

2 Study area and field data

The first dataset consisted of 20 circular plots of 0.1 ha. Study area is located in south-eastern Norway. Norway spruce (*Picea abies*) and Scots pine (*Pinus sylvestris*) were the dominant species in this heterogeneous forest. The plots were established in subjectively selected stands with spruce as a dominant species. All trees with DBH >3 cm were callipered, and tree heights were measured on trees selected with probability proportional to stem basal area. GPS and GLONASS were used to determine the plot centre coordinates. Coordinates for single trees were computed.

The second dataset comprised 60 large plots. Study area with a size of 5000 ha is located in Krødsherad, south-eastern Norway. The area was managed forest and dominated by Norway spruce and Scots pine. Younger stands were dominated by deciduous trees. Plot size varied from 3121 m^2 to 4219 m^2. On each plot, all trees with DBH ≥4 cm and ≥10 cm were callipered in young and mature stands, respectively. The measurements were recorded on 2 cm classes. Tree heights were measured on trees selected with probability proportional to stem basal area at breast height.

3 Regression models

Models were created to estimate mean volume using laser scanning-based stripe sampling forest inventory. Stripe widths of 160, 180 and 200 m were used. Monte Carlo simulation was used to estimate the laser strip and ground-based volumes. 5 created models were 1) multiplicative, 2) log(y), 3) sqrt(y), 4) asin(sqrt(y)), 5) linear.

4 Reliability of the models

Table 1. Bias, standard error (S.D) and RMSE of mean volume estimates. *=p<0.05; ns, not significant=p>0.05.

		Plot area								
		200 m^2			400 m^2			600 m^2		
Strip width (m)	Model	bias	S.D.	RMSE	bias	S.D.	RMSE	bias	S.D.	RMSE
		Laser scanning based estimates (m^3ha^{-1})								
	1	-0.8^{ns}	5.5	5.6	-0.8^{ns}	5.7	5.8	-0.2^{ns}	6.4	6.4
	2	-2.2*	6.2	6.6	-1.3^{ns}	6.4	6.5	-0.8^{ns}	7.4	7.4
160	3	-1.5*	5.2	5.4	-1.6^{ns}	5.7	5.9	-0.9^{ns}	6.5	6.5
	4	-5.5*	5.6	7.8	-3.6*	6.1	7.1	-4.3*	7.2	8.4
	5	-0.6^{ns}	5.5	5.5	-1.0^{ns}	5.7	5.8	-0.5^{ns}	6.3	6.4
	1	-0.7^{ns}	5.5	5.6	-0.7^{ns}	5.0	5.1	-0.5^{ns}	5.6	5.7
	2	-3.0*	6.2	6.9	-0.4^{ns}	5.7	5.8	-0.8^{ns}	7.1	7.1
	3	-1.4^{ns}	5.3	5.4	-1.3^{ns}	5.1	5.3	-1.4^{ns}	6.0	6.2
180	4	-5.7*	5.6	8.0	-3.6*	5.7	6.7	-4.1*	7.0	8.1
	5	-0.5^{ns}	5.4	5.4	-0.8^{ns}	5.0	5.1	-0.9^{ns}	5.5	5.6
	1	-0.2^{ns}	5.4	5.4	-0.1^{ns}	5.1	5.1	-0.2^{ns}	6.0	6.0
	2	-2.5*	5.7	6.2	0.3^{ns}	5.9	5.9	-0.7^{ns}	7.2	7.2
	3	-0.9^{ns}	5.3	5.4	-0.7^{ns}	5.0	5.1	-1.2^{ns}	6.3	6.4
200	4	-5.2*	5.7	7.7	-3.2*	5.5	6.4	-3.1*	7.1	7.7
	5	0.0^{ns}	5.4	5.4	-0.2^{ns}	5.2	5.2	-1.0^{ns}	6.0	6.1
		Ground plot based inventory (m^3ha^{-1})								
160	-	-0.7^{ns}	13.7	13.7	3.9^{ns}	14.9	15.4	1.8^{ns}	18.4	18.4
180	-	-2.1^{ns}	14.8	15.0	2.2^{ns}	14.1	14.3	2.2^{ns}	17.5	17.6
200	-	-2.6^{ns}	14.5	14.8	2.2^{ns}	14.5	14.7	2.2^{ns}	18.2	18.4

Testing the usability of truncated angle count sample plots as ground truth in airborne laser scanning-based forest inventories

Maltamo, M., Korhonen, K.T., Packalén, P., Mehtätalo, L. & Suvanto, A. 2007. Forestry 80: 73-81.

1 Airborne laser scanner data

- Laser scanner: ALTM 2033 (Optech)
- Date of data acquisition: 4.8.2004
- Flying altitude: 1500 m
- Max. scan angle: 15°
- Swath width: 800 m
- Pulse density: 0.7/m^2
- Recorded echoes: first and last

2 Study area and field data

Test area in Matalansalo, eastern Finland is coniferous trees dominated managed forest. The size of the area is about 1200 ha. 27% of the area was young forest, 42% was middle-aged forest and 31% was mature forest.

472 circular plots were established in 67 stands with an average size of 2.8 ha. 5-9 plots were systematically placed on selected stands. Plot radius was 9 meters. DBH, tree class, tree storey and tree species were determined for trees with DBH >5 cm. Tree heights were predicted with Veltheim's (1987) species-specific models. The height of one sample tree of each species and storey class was measured on each plot. Measured heights were used to calibrate the estimated heights. Species-specific volumes were calculated with Laasasenaho's (1982) models. The plot characteristics were multiplied out at the hectare level.

A truncated angle count sample was generated on each plot. The locations of the trees were generated by assuming a random (Poisson) spatial pattern within.

3 Regression models

Models are for plot level variables.

- Volume
- Basal area
- Stem number
- Basal area weighted mean diameter
- Basal area weighted mean height

Table 1. Explanations for variables.

Variable	Explanation
V	Volume
G	Basal area
N	Stem number
d_{gM}	Basal area weighted mean diameter
h_{gM}	Basal area weighted mean height
f	First pulse
l	Last pulse
$h_{10...90}$	Height at which 10...90 per cent of the height distribution has accumulated
veg	Proportion of vegetation hits
$coeffva$	Coefficient of variation in laser canopy heights
h_{mean}	Mean of the laser canopy heights

Volume:

$$\ln(V) = 1{,}964 + 1{,}406\ln(fh_{60}) + 0{,}895\ln(fveg) - 0{,}949 lcoeffva^2 - \frac{0{,}390}{lstd} \tag{1}$$

Basal area:

$$\sqrt{G} = -2{,}113 + 2{,}080 fveg + 1{,}694\ln(fhmean) + 1{,}702\sqrt{fveg} \tag{2}$$

Stem number:

$$\ln(N) = 6{,}584 + 1{,}692 fveg^2 - 0{,}00448 lhmean^2 + 0{,}0010252 fh_{10}^2 \tag{3}$$

Basal area weighted mean diameter:

$$\sqrt{d_{gM}} = 3{,}161 + 0{,}220lh_{70} - 0{,}134fh_{20} - 0{,}727veg^2 - 0{,}355lh_{80} + 0{,}148fh_{80} + 0{,}757\sqrt{lh_{40}} + 0{,}00338lh_{80}^2 - \frac{0{,}125}{lveg} - \frac{3{,}549}{lhmean} \quad (4)$$

Basal area weighted mean height:

$$h_{gM} = -4{,}551 + 0{,}220fh_{80}^2 + 5{,}369\sqrt{fh_{60}} - 0{,}0115lh_{90}^2 - 1{,}067fveg^2 - 0{,}173fh_{20} \quad (5)$$

4 Reliability of the models

Table 2. Reliability of angle count plot-based estimates of plot-level stand attributes. The reference results obtained from models constructed using fixed-area plots are from the work of Suvanto et al. (2005).

Model		V	G	N	d_{gM}	h_{gM}
Fixed area	RMSE (%)	19.89	16.47	27.05	13.61	8.42
	Bias (%)	-0.07	-0.06	-0.42	0.57	-0.12
Angle count	RMSE (%)	20.83	16.57	30.83	13.82	14.44
	Bias (%)	-2.15	0.89	12.58	1.96	-11.22

Table 3. Reliability of angle count plot-based estimates of stand-level stand attributes. The reference results obtained from models constructed using fixed-area plots are from the work of Suvanto et al. (2005).

Model		V	G	N	d_{gM}	h_{gM}
Fixed area	RMSE (%)	8.97	8.42	16.02	7.25	3.52
	Bias (%)	-0.04	-0.02	0.69	-0.87	-0.12
Angle count	RMSE (%)	10.09	8.56	21.81	7.32	3.53
	Bias (%)	-2.17	0.92	12.94	0.71	0.03

The *k*-MSN method for the prediction of species-specific stand attributes using airborne laser scanning and aerial photographs

Packalén, P. & Maltamo, M. 2007. Remote Sensing of Environment 109: 328-341.

1 Airborne laser scanner data

- Laser scanner: ALTM 2033 (Optech)
- Date of data acquisition: 4.8.2004
- Flying altitude: 1500 m
- Average speed: 75 m/s
- Number of flight lines: 7, sidelap 35%
- Max. scan angle: 15°
- Swath width: 800 m
- Footprint diameter: 45 cm
- Pulse density: 0.7 / m^2
- Beam divergence: 0.3 mrad
- Recorded echoes: first and last

2 Aerial image data

- Camera: Leica RC30
- Date of data acquisition: 22.7.2004
- Scale: 1:30 000
- Lens: UAGA-F 13158
- Focal length: 163.18 mm
- Film: colour-infrared
- Number of images: 3
- Pixel size of the orthorectified images: 0.5 m

3 Study area and field data

Matalansalo study area is located in Varkaus, eastern Finland. It is owned by UPM Kymmene Ltd. On 59% of the sample plots the main tree species was pine (*Pinus sylvestris*) and on 34% it was spruce (*Picea abies*).

463 circular sample plots (radius 9 meters) were placed on 67 stands (average size 3.5 ha). Plots were placed systematically. The number of plots per stand varied from five to nine. Development class was young forest on 27% of the plots, middle aged on 42% of the plots and mature on 31% of the plots. DBH, tree class, tree storey and tree species was recorded for trees with DBH >5 cm. Tree height was measured from the basal tree of each species and storey class by plots. Heights for the rest of the trees were calculated with Veltheim's (1987) models. Volumes were calclulated with Laasasenaho's (1982) models.

4 Models

- Volume (V)
- Basal area (G)
- Stem number (N)
- Diameter of the basal area median tree (dgm)
- Height of the basal area median tree (hgm)

Estimation was made separately for different tree species (pine, spruce, deciduous trees).

Stand characteristics were estimated simultaneously using the non-parametric *k*-MSN method.

Predictor variables:
Aerial photo:

Mean, NIR, Median, Red; Inv(Median, Green); Sqrt(Contrast, NIR); Inv(IDM, NIR); Sqrt(DV, NIR); X2(ASM, NDVI).

Laser scanner data:

Ln(f_veg); Sqrt(f_mean); f_h20; X2(f_h40); X2(f_h60); Ln(f_h80); Inv(f_h95); X2(f_p60); X2(l_veg); Sqrt(l_havg); Sqrt(l_h20); Inv(l_h40); Sqrt(l_h60); l_h80; Inv(l_p20); l_p40; Ln(l_p60); Inv(l_p95).

Ln = natural logarithm, Sqrt = square root, X2 = power of two, Inv = 1/X, f or l = laser pulse type: first or last, hX = height at which X percent of the height distribution has accumulated, pX = canopy density corresponding to the proportion of laser hits above the X quantile, hmean = mean height of above-ground hits, veg = proportion of above-ground hits.

5 Reliability of the models

Table 1. Accuracy of the estimated stand characteristics at plot level. * = bias was significant at 95 % confidence level.

	RMSE	RMSE (%)	Harha
Deciduous			
V (m^3ha^{-1})	22.36	102.84	2.16*
G (m^2ha^{-1})	2.58	87.76	0.23
N (ha^{-1})	281	89.69	19.85
dgm (cm)	5.30	45.93	0.20
hgm (m)	4.14	32.15	0.10
Pine			
V (m^3ha^{-1})	50.25	51.55	-1.11
G (m^2ha^{-1})	2.56	45.56	-0.22
N (ha^{-1})	336	60.53	-13.50
dgm (cm)	4.55	23.08	0.72*
hgm (m)	2.60	16.04	0.37*
Spruce			
V (m^3ha^{-1})	47.47	55.72	-0.61
G (m^2ha^{-1})	5.09	51.19	-0.09
N (ha^{-1})	410	63.61	-2.34
dgm (cm)	4.35	33.06	-0.12
hgm (m)	3.48	29.97	-0.14
Total			
V (m^3ha^{-1})	41.92	20.51	0.44
G (m^2ha^{-1})	4.25	17.15	-0.08
N (ha^{-1})	453	29.95	4.01

Table 2. Accuracy of the estimated stand characteristics at stand level.

	RMSE	RMSE (%)	Harha
Deciduous			
V (m^3ha^{-1})	13.70	62.33	2.46
G (m^2ha^{-1})	1.56	52.53	0.29
N (ha^{-1})	151.00	47.55	27.92
dgm (cm)	2.87	25.34	-0.17
hgm (m)	2.32	18.40	-0.30
Pine			
V (m^3ha^{-1})	27.71	28.08	-2.07
G (m^2ha^{-1})	3.27	27.05	-0.34
N (ha^{-1})	232.00	40.81	-13.10
dgm (cm)	3.43	16.91	0.11
hgm (m)	1.40	8.46	0.09
Spruce			
V (m^3ha^{-1})	26.99	32.64	-0.41
G (m^2ha^{-1})	3.03	32.30	-0.08
N (ha^{-1})	240.00	38.07	-11.15
dgm (cm)	2.55	20.15	0.04
hgm (m)	1.96	17.63	-0.02
Total			
V (m^3ha^{-1})	21.07	10.36	-0.02
G (m^2ha^{-1})	2.14	8.63	-0.14
N (ha^{-1})	241.00	15.88	3.66

Usability of Truncated Angle Count Sample Plots in Laser Scanning

Kainulainen, A. 2007. Master's Thesis. University of Joensuu, Faculty of Forest Sciences.

1 Airborne laser scanning data

- Laser scanner: ALTM 3100C (Optech)
- Date of aquisition: 28.7.2006
- Average height: 2000 m
- Average speed: 75 m/s
- Scan angle (half): 15°
- Swath width: 857 m
- Beam divergence: 0.3 mrad
- Side lap: 20%
- Number of flight lines: 21+3
- Average pulse density: 0.64/m^2
- Recorded pulses: first and last

2 Study area and field data

Study area is located in Kuortane, western Finland.

Sample plots were placed in clusters like in National Forest Inventory VMI10. Clusters had a form of a square, and each had 18 plots. Data consisted of truncated angle count sample plots (with a maximum radius of 12.52 m) and circular plots (radius 9 m). These plots had the same centre point. Trees with DBH ≥50 mm were callipered on circular plots. Stem number and height on each species were recorded on trees with DBH less than 50 mm. All sampling trees within a circle determined by a 12.52 m radius were measured on truncated angle count sample plot. Basal area median tree was selected subjectively, and height, height of the lower crown, and age was measured on that tree. Total 427 sample plots were established on forest. 49 of them were used in this study.

3 Regression models

- Plot volume, circular plots (1 and 2)
- Plot volume, truncated angle count sample plots (3 and 4)
- Stem number, circular plots (5 and 6)
- Stem number, truncated angle count sample plots (7 and 8)

Table 1. Explanations for variables

Variable	Explanation
lh_p	p% percentile of the last pulse canopy heights
lh_{std}	Standard deviation of the last pulse canopy heights
fveg	Proportion of vegetation hits in first pulse data
fh_p	p% percentile of the first pulse canopy heights
lp_{70}	Proportion of last pulse laser hits which is accumulated at the height of 70%
fh_{avg}	Average of the first pulse canopy heights

$$V_{9m} = -38.785 - 0.766(lh_{10})^2 - 3.564(lh_{std})^2 + 0.016(fveg)^2 + 1.712(fh_{40})^2 \quad (1)$$

$$\ln V_{9m} = -0.764 - 0.0003(lp_{70})^2 + 0.853\ln(fveg) + 1.553\ln(fh_{40}) \quad (2)$$

$$V_{relaskooppi} = -39.515 + 1.093(fh_{avg})^2 + 0.014(fveg)^2 \quad (3)$$

$$\ln V_{relaskooppi} = 2.327 + 0.172(fh_{avg}) + 0.009(fveg) \quad (4)$$

$$RL_{9m} = 853.554 + 0.172(fveg)^2 - 2.343(fh_{90})^2 \quad (5)$$

$$\ln RL_{9m} = 6.605 + 0.0002(fveg)^2 - 0.002(fh_{90})^2 \quad (6)$$

$$RL_{relaskooppi} = 92.362 - \frac{25231.004}{lh_{20}} + \frac{128637.256}{lh_{70}} - \frac{77930.538}{fh_{80}} \quad (7)$$

$$\ln RL_{relaskooppi} = 6.158 - \frac{16.314}{lh_{20}} + \frac{80.940}{lh_{70}} - \frac{47.537}{fh_{80}} \quad (8)$$

4 Reliability of the models

Table 2. Reliability of the regression models

	Volume			Stem number		
Model	RSME (m^3)	RMSE%	Bias	RMSE (kpl/ha)	RMSE%	Bias
9m	50.93	27.84	1.9E-11	367.64	32.88	-1.2E-10
ln(9m)	55.36	30.26	0.438	395.68	35.39	3.1E-13
Relaskooppi	56.85	31.86	3.5E-11	489.64	40.70	-1.9E-09
ln(Relaskooppi)	64.52	36.16	9.9E-15	464.95	38.65	6.0E-14

Weibull and percentile models for lidar-based estimation of basal area distribution

Gobakken, T. & Næsset, E. 2005. Scandinavian Journal of Forest Research 20: 490-502.

1 Airborne laser scanner data

- Laser scanner: ALTM 1210 (Optech)
- Date of data acquisition: 8.-9.6.1999 (second flight 6.6.2000 to collect last return data for the surface model)
- Flying altitude: 700 m
- Average speed: 71 m/s
- Number of flight lines: 43
- Pulse repetition frequency: 10 kHz
- Scan frequency: 21 Hz

- Max. scan angle, data acquisition: 17°
- Max. scan angle, data processing: 14°
- Swath width: 420 m
- Footprint diameter: 21 cm
- Average distance between footprints: 0.94 m
- Recorded echoes: first and last

2 Study area and field data

Study area is located in Våler, south-east Norway (59°30′N 10°55′E, 70-120 m a.s.l.). The main tree species in the area of 1000 hectares were pine (*Picea abies*) and spruce (*Pinus sylvestris*).

141 circular sample plots were systematically placed on the area according to a regular grid. The plots were divided into three strata: young forest, mature forest on poor sites, and mature forest on good sites. Site index on poor sites was 11 or less at the age of 40 years. 56 plots (300 m^2) were classified as young forest, 36 plots (400 m^2) were classified as mature forest on poor sites, and 49 (400 m^2) plots were classified as mature forest on good sites. In young forests, diameter at breast height was measured for trees with DBH >4 cm. In mature forest, callipered trees had DBH >10 cm. Diameters at breast heights were recorded in 2 cm classes. Heights were measured on sample trees. Total 1316 sample trees were measured (2-17 per plot).

3 Regression models

- Weibull 24 and 93 percentiles of Weibull basal area distributions and basal area for the sample plots (1-9)
- Ground-based percentiles of the basal area distribution and basal area for the sample plots (10-42)

There are separate models for young forest, mature forest on poor sites and mature forest on good sites.

Table 1. Explanations for variables.

Variable	Explanation
d_{24}, d_{93}	Weibull 24 and 93 percentiles
G	Basal area
$d_{10}...d_{100}$	10%...100% percentiles
$h_{0f}...h_{90f}$	Percentiles of the first pulse laser canopy heights for 0%...90%
$h_{0l}...h_{90l}$	Percentiles of the last pulse laser canopy heights for 0%...90%
h_{meanf}	Mean of the first pulse canopy heights (m)
h_{cvf}	Coefficient of variation of the first pulse canopy heights (%)
$d_{0f}...d_{6f}$	Canopy densities corresponding to the proportions of first pulse laser hits above fraction numbers 1…6 to total number of first pulses
$d_{0l}...d_{7l}$	Canopy densities corresponding to the proportions of last pulse laser hits above fraction numbers 1…7 to total number of last pulses

Weibull 24 and 93 percentiles and basal area for young forest:

$$\ln d_{24} = 0{,}009 + 0{,}963 \ln h_{70l} - 0{,}651 \ln d_{1f} \quad (1)$$

$$\ln d_{93} = 0{,}253 + 1{,}022 \ln h_{90f} - 0{,}228 \ln d_{6l} \quad (2)$$

$$\ln G = 2{,}110 + 0{,}643 \ln h_{20f} + 1{,}075 \ln d_{0f} \quad (3)$$

Weibull 24 and 93 percentiles and basal area for mature forest, poor sites:

$$\ln d_{24} = -0{,}192 - 0{,}442 \ln h_{0f} + 1{,}212 \ln h_{90l} - 0{,}618 \ln d_{2l} + 0{,}296 \ln d_{6l} \quad (4)$$

$$\ln d_{93} = 1{,}879 - 2{,}164 \ln h_{90f} + 2{,}751 \ln h_{90l} \quad (5)$$

$$\ln G = 2{,}455 + 1{,}043 \ln h_{50f} - 0{,}506 \ln h_{40l} + 0{,}712 \ln d_{4l} \quad (6)$$

Weibull 24 and 93 percentiles and basal area for mature forest, good sites:

$$\ln d_{24} = 0{,}974 + 0{,}658 \ln h_{80l} - 0{,}281 \ln d_{0l} \quad (7)$$

$$\ln d_{93} = 2{,}859 - 2{,}563 \ln h_{70l} + 2{,}777 \ln h_{80l} + 0{,}059 \ln d_{5l} \quad (8)$$

$$\ln G = 2{,}737 - 1{,}262 \ln h_{70f} + 1{,}666 \ln h_{80l} + 0{,}705 \ln d_{4f} + 2{,}094 \ln d_{2l} - 1{,}947 \ln d_{3l} + 0{,}219 \ln d_{7l} \quad (9)$$

Ground-based percentiles and basal area for young forest:

$$\ln d_{10} = -0{,}628 + 0{,}685 \ln h_{20l} + 0{,}419 h_{cvf} - 0{,}560 \ln d_{1l} + 0{,}184 \ln d_{4l} \quad (10)$$

$$\ln d_{20} = 0{,}035 - 0{,}794 \ln h_{20f} + 0{,}484 \ln h_{20l} + 1{,}189 \ln h_{70l} - 0{,}394 \ln d_{2l} \quad (11)$$

$$\ln d_{30} = 0{,}363 - 0{,}436 \ln h_{20f} + 0{,}198 \ln h_{10l} + 1{,}082 \ln h_{70l} - 0{,}270 \ln d_{0l} \quad (12)$$

$$\ln d_{40} = 0{,}383 + 0{,}878 \ln h_{80l} - 0{,}442 \ln d_{1f} \quad (13)$$

$$\ln d_{50} = 0{,}339 + 0{,}910 \ln h_{90l} - 0{,}289 \ln d_{1f} \quad (14)$$

$$\ln d_{60} = 0{,}611 + 0{,}834 \ln h_{90l} - 0{,}138 \ln d_{3f} \quad (15)$$

$$\ln d_{70} = 0{,}592 - 0{,}309 \ln h_{20f} + 1{,}138 \ln h_{90l} \quad (16)$$

$$\ln d_{80} = 0{,}596 - 1{,}394 \ln h_{60f} + 2{,}215 \ln h_{90f} \quad (17)$$

$$\ln d_{90} = 0{,}479 - 0{,}709 \ln h_{60f} + 1{,}596 \ln h_{90f} - 0{,}154 \ln d_{6l} \quad (18)$$

$$\ln d_{100} = 0{,}685 + 0{,}899 \ln h_{90f} - 0{,}171 \ln d_{6l} \quad (19)$$

$$\ln G = 2{,}116 + 0{,}642 \ln h_{20f} + 1{,}102 \ln d_{0f} \quad (20)$$

Ground-based percentiles and basal area for mature forest, poor sites:

$$\ln d_{10} = -0{,}217 + 0{,}964 \ln h_{70f} - 0{,}489 \ln d_{1l} \quad (21)$$

$$\ln d_{20} = 0{,}144 - 0{,}265 \ln h_{0f} - 0{,}524 \ln h_{0l} + 1{,}329 \ln h_{50l} - 0{,}391 \ln d_{4l} \quad (22)$$

$$\ln d_{30} = 0{,}185 + 0{,}922 \ln h_{90l} - 0{,}330 \ln d_{2f} \quad (23)$$

$$\ln d_{40} = 0{,}698 + 0{,}773 \ln h_{90l} - 0{,}322 \ln d_{2l} \quad (24)$$

$$\ln d_{50} = 0{,}966 - 0{,}618 \ln h_{0f} + 0{,}889 \ln h_{90l} + 0{,}220 \ln d_{2f} - 0{,}420 \ln d_{2l} \quad (25)$$

$$\ln d_{60} = 1{,}284 - 0{,}317 \ln h_{0f} + 0{,}760 \ln h_{90f} - 0{,}780 \ln d_{2l} + 0{,}561 \ln d_{3l} \quad (26)$$

$$\ln d_{70} = 1{,}381 - 0{,}462 \ln h_{0f} + 0{,}750 \ln h_{90l} + 0{,}240 \ln d_{2f} - 0{,}395 \ln d_{2l} \quad (27)$$

$$\ln d_{80} = 0{,}957 + 0{,}850 \ln h_{90l} - 0{,}140 \ln d_{2l} \quad (28)$$

$$\ln d_{90} = 1{,}817 - 2{,}344 \ln h_{90f} + 2{,}952 \ln h_{90l} \quad (29)$$

$$\ln d_{100} = 1{,}988 - 2{,}268 \ln h_{90f} + 2{,}816 \ln h_{90l} \quad (30)$$

$$\ln G = 2{,}407 + 1{,}081 \ln h_{50f} - 0{,}519 \ln h_{40l} + 0{,}722 \ln d_{4l} \quad (31)$$

Ground-based percentiles and basal area for mature forest, good sites:

$$\ln d_{10} = 0{,}982 + 0{,}558 \ln h_{80l} - 0{,}340 \ln d_{0l} \quad (32)$$

$$\ln d_{20} = 1{,}241 + 0{,}557 \ln h_{80l} - 0{,}218 \ln d_{1l} \quad (33)$$

$$\ln d_{30} = 0{,}805 + 0{,}724 \ln h_{90l} - 0{,}169 \ln d_{2l} \quad (34)$$

$$\ln d_{40} = 0{,}792 - 0{,}097 \ln h_{10f} + 0{,}820 \ln h_{90l} - 0{,}182 \ln d_{3l} \quad (35)$$

$$\ln d_{50} = 1{,}710 - 0{,}112 \ln h_{10f} + 0{,}575 \ln h_{90l} - 0{,}120 \ln d_{0l} \quad (36)$$

$$\ln d_{60} = 2{,}064 - 0{,}776 \ln h_{90l} - 0{,}388 \ln h_{meanf} \quad (37)$$

$$\ln d_{70} = 2{,}848 - 0{,}737 \ln h_{70l} + 0{,}892 \ln h_{90l} \quad (38)$$

$$\ln d_{80} = 2{,}790 - 0{,}566 \ln h_{70l} + 0{,}772 \ln h_{90l} \quad (39)$$

$$\ln d_{90} = 3{,}523 - 0{,}063 \ln d_{6f} \quad (40)$$

$$\ln d_{100} = 3{,}322 - 1{,}085 \ln h_{70l} + 1{,}178 \ln h_{80l} + 0{,}047 \ln d_{5l} \quad (41)$$

$$\ln G = 2{,}491 - 1{,}319 \ln h_{70f} + 1{,}798 \ln h_{80l} + 0{,}723 \ln d_{4f} + 2{,}222 \ln d_{2l} - 2{,}156 \ln d_{3l} + 0{,}240 \ln d_{7l} \quad (42)$$

4 Reliability of the models

Table 2. Root mean square errors (RMSE) and coefficients of determination (R^2) of the models.

Dependent variable	RMSE	R^2
Young forest		
$\ln d_{24}$	0.18	0.62
$\ln d_{93}$	0.17	0.53
$\ln G$	0.11	0.91
Mature forest, poor sites		
$\ln d_{24}$	0.11	0.75
$\ln d_{93}$	0.09	0.45
$\ln G$	0.13	0.81
Mature forest, good sites		
$\ln d_{24}$	0.16	0.30
$\ln d_{93}$	0.11	0.41
$\ln G$	0.10	0.90
Young forest		
$\ln d_{10}$	0.16	0.73
$\ln d_{20}$	0.15	0.74
$\ln d_{30}$	0.15	0.73
$\ln d_{40}$	0.15	0.68
$\ln d_{50}$	0.17	0.61
$\ln d_{60}$	0.16	0.59
$\ln d_{70}$	0.14	0.67
$\ln d_{80}$	0.15	0.63
$\ln d_{90}$	0.16	0.58
$\ln d_{100}$	0.14	0.59
$\ln G$	0.11	0.91

Table 2 continue. Root mean square errors (RMSE) and coefficients of determination (R^2) of the models.

Dependent variable	RMSE	R2
Mature forest, poor sites		
$\ln d_{10}$	0.16	0.53
$\ln d_{20}$	0.13	0.64
$\ln d_{30}$	0.13	0.55
$\ln d_{40}$	0.12	0.53
$\ln d_{50}$	0.1	0.69
$\ln d_{60}$	0.1	0.61
$\ln d_{70}$	0.11	0.56
$\ln d_{80}$	0.11	0.5
$\ln d_{90}$	0.1	0.44
$\ln d_{100}$	0.1	0.45
$\ln G$	0.13	0.81
Mature forest, good sites		
$\ln d_{10}$	0.15	0.35
$\ln d_{20}$	0.16	0.31
$\ln d_{30}$	0.15	0.33
$\ln d_{40}$	0.13	0.5
$\ln d_{50}$	0.14	0.36
$\ln d_{60}$	0.13	0.3
$\ln d_{70}$	0.13	0.24
$\ln d_{80}$	0.15	0.21
$\ln d_{90}$	0.14	0.07
$\ln d_{100}$	0.11	0.33
$\ln G$	0.1	0.9

Table 3. Mean difference and standard deviation of the models. NS = not statistically significant (p>0.05).

Method	Observed mean V (m^3ha^{-1})	Mean difference (%)	Standard deviation (%)
Young forest	170.6		
Parameter recovery		0.1 NS	16.2
Discrete, 10 percentiles		-0.4 NS	16.4
Mature forest, poor sites	148.6		
Parameter recovery		-0.2 NS	15.5
Discrete, 10 percentiles		2.1 NS	15.8
Mature forest, good sites	265.6		
Parameter recovery		-1.2 NS	15.1
Discrete, 10 percentiles		-0.3 NS	15.7