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Abstract

Factors affecting optimal stand management consist of tree growth, harvesting technology and economics. In the present study, the optimal harvesting for a set of even-aged Norway spruce (*Picea abies* [L.] Karst.) stands located in eastern Finland are studied using an individual-tree growth model.

The optimal solutions are presented separately for artificially and naturally regenerated stands. The results show that two thinnings, thinning from above, and later thinnings, is optimal for moderate density stands (with initial density about 1900 trees per hectare), at 3% rate of interest.

The optimization results show that high bare land values are connected to stands that have large basal areas, tree diameters and dominant heights. The sparser the initial stand, the smaller the optimal number of thinnings. Typically, the higher the MAI and the fertile the site quality, the higher the bare land value. The higher the number of thinnings, the lower the thinning intensity. The rotation length in plots giving the highest bare land value is somewhat shorter than in most of earlier studies. However, the results show that optimal rotation may vary considerably (60-99 yrs) at 3% rate of interest depending on the initial stand state.

In order to interpret the underlying reasons behind the optimal results, additional computations concerning optimal solutions at different rates of interest, flexibility of thinning type definition, and butt rot effects are studied. The optimal rotation length decreases with an increasing rate of interest. MSY rotations are about 15 years shorter than rotations based on Forest Rent. Thinning from below is more profitable when the interest rate is less than 2%. By contrast, thinning from above is superior with 3-5% rates of interest. Increasing the rate of interest leads to a lower final volume, a narrower and more highly peaked diameter distribution, and a lower mean diameter. Allowing flexible thinnings may increases maximized bare land values and rotation lengths. Increased flexibility enables thinnings that remove trees strictly from the above. Butt rot has no significant effect on optimal management at rotations shorter than 90 years. However, the butt rot effects shorten the rotation length over 9% at rotations longer than 90 years and with a lower rate of interest (e.g., 1%).

In addition, the comparison of old and new growth models in SMA and MOTTI programs is studied. The slight differences between SMA and MOTTI simulations are caused by different specification of mortality. There was also some difference in growth. The comparison of law, recommendations and optimal solutions is studied as well. The optimal solutions are legal at lower rates of interest (0-2%), but illegal at higher rates of interest (4-5%). In the present study, the optimal rotation of Norway spruce was 20-30 years shorter than the recommendations for practice.

Keywords Optimal harvesting, Norway spruce, Individual-tree growth model

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Preface

This study was carried out at the Finnish Forest Research Institute (METLA) in the project "Economic-ecological Interactions in Sustainable Use of Forest Resources", led by Professor Olli Tahvonen. I sincerely thank Professor Olli Tahvonen and Ph.D. student Kari Hyytiäinen for the necessary resources, and for a background in the economics of optimal stand management. I am greatly indebted to Professor Lauri Valsta for the biological data, and for support in my graduate studies at the University of Helsinki.

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Helsinki, January 2003

Tianjian Cao

Contents

1 Introduction

1.1 Optimal Forest Management at the Stand Level

Valsta (1993) divides the forest management decisions into five different levels by the scope of the decisions and the level of available information. These five levels are tree, stand, forest, enterprise and region or sector. According to Valsta (1993), most forest economics theory is based on stand level formulations, and the stand level offers the first meaningful level of decision making. The advantages of such an approach include its mathematical simplicity and the generality and applicability of results.

A stand is a geographically contiguous parcel of land considered homogeneous in terms of tree vegetation (Davis et al. 2001, p.65). An even-aged stand generally has been defined as being composed of trees of more or less the same age (Smith et al. 1997). In practice, most private, industrial, and public forests are even-aged, and the final clearcut harvests are followed by new plantings (Davis et al. 2001, p.93).

Factors affecting optimal stand management consist of tree growth, harvesting technology and economics. According to Davis et al. (2001, p.97), the main decisions in even-aged management concern regeneration method, rotation length and thinnings.

Economic research on stand management typically studies how the chain of management activities from planting and thinning to clearcut should be timed and scaled in order to maximize the long-term economic output of timber production. This also includes the analysis of alternative purposes of using the forest land (such as agriculture, conservation or wasteland), to determine whether it is economically reasonable to sustain timber production.

Stand level economic research produces results on economically optimal planting density, optimal timing, intensity and type of thinnings, and on the length of the optimal forest rotation period. This type of knowledge is needed directly in the decision making of the 800,000 Finnish private forest owners and the foresters managing industrially and publicly owned forests. Government officials responsible for forest policy, silvicultural instructions and legal restrictions on forestry practices also need it.

1.2 A Review of Previous Studies

The first studies capable of determining optimal timing, number and intensity of thinnings use variable-density whole-stand models (e.g. Amidon and Akin 1968, Kilkki and Väisänen 1969, Brodie et al. 1978, Brodie and Kao 1979, Kao and Brodie 1980). These models contain a few state variables (such as basal area, volume or number of trees) that evolve over time due to growth, mortality and harvesting.

Recent Finnish studies on optimal timber management use more advanced descriptions of growth (e.g. Valsta 1992b, Pukkala et al. 1998, Vettenranta & Miina 1999). These and earlier studies do not, however, explain exhaustively how the characteristics of optimal harvests (number, timing and intensity of thinnings, and timing of clearcut) depend on economic parameters, logging conditions or site quality (Hyytiäinen and Tahvonen 2002a).

Most recent studies reveal that thinning that removes trees from above or from both ends of the diameter distribution is typically superior to conventional thinning from below (Haight 1987, Haight and Monserud 1990b, Solberg and Haight 1991, Vuokila 1977, Valsta 1992a, b, Eriksson 1994, Pukkala et al. 1998, Pukkala and Miina 1998, Vettenranta and Miina 1999). Instead, thinning from below may be optimal in the case of the first commercial thinning (Haight et al. 1985), precommercial thinning (Roise 1986) or valuable broad-leaved species (Rautiainen et al. 2000).

Even-aged forests have conventionally been, and still are, thinned from below in many countries (Hyytiäinen et al. 2002), despite the high numerous of research results showing consistently the superiority of thinning from above. Hyytiäinen et al. (2002) presume that there may be several reasons for slow adoption of research results in forestry extension documents and in practical forest management. First, a recommendation to favor the largest trees in thinning may be based on maximizing volume of timber production rather than economic. Secondly, most present studies pay scant attention to the underlying reasons for optimal results and thus the results cannot be adopted more generally. Thirdly, it may be believed that harvest specifications and other assumptions in the present models are too limiting to give valid results applicable in practice.

1.3 Objectives

The general objective of the study is to analyze the optimal harvest regime for a set of even-aged Norway spruce stands based on an individual-tree growth model. The sample plots are mostly located in eastern Finland. Empirical data on two site types were studied: *Myrtillus* (MT) and *Oxalis-Myrtillus* (OMT). Management of 12 Norway spruce stands were optimized, and the optimal solutions were characterized by bare land value, rotation length and number, timing, type and intensity of thinnings.

The specific objectives of the study are: (1) to analyze optimal thinnings and rotation for a set of initial states; (2) to find out how the rate of interest affects optimal thinnings and rotation; (3) to interpret how the type of thinning affects bare land value and rotation length; (4) to discuss what kind of structure is optimal for Norway spruce stands; (5) to examine how the inclusion of quality factors (butt rot) affects the optimal solutions; (6) to study the differences in forest growth as generated by SMA versus MOTTI software using a new generation of growth models; (7) to analyze differences between optimal solutions and previous studies, Finnish forest law, and Finnish silvicultural recommendations.

This paper is organized as follows: Section 2, Models for Stand Development, describes growth and yield models. Section 3, The Optimization Model, presents the methodological basis of optimization. Section 4 explains the materials and computations. The section titled Results and Discussion gives the main results on optimal stand management, sensitivity analyses and comparisons. The recommendations and limitations are discussed further in the section titled Conclusions and Limitations.

2 Models for Stand Development

2.1 Classification of Growth and Yield Models

Stand management optimization requires a projection model to compute the effects of chosen treatments. Such models are customarily called (stand) simulators (Valsta 1993).

Table 1 gives the classification of growth and yield models based on Valsta (1993) and Davis et al. (2001, p.186-187).

Variable-density whole-stand models are still popular in forest economics computations, and form a basis for many economic extensions. Besides diameter distribution models, most whole-stand models apply only to forest stands where the economic value of timber or aggregate growth does not depend on stand structure (Hyytiäinen et al. 2002).

Numerical methods have been used to solve whole-stand models (e.g. Kilkki & Väisänen 1969, Brodie et al. 1978) and increasingly complex individual-tree (e.g. Roise 1986, Valsta 1992b), distance-dependent (e.g. Pukkala et al. 1998, Vettenranta & Miina 1999) and stage-structured growth models (e.g. Haight 1987, Solberg & Haight 1991). Linear optimal control models have been used to derive analytic solutions to optimal growing stock and rotation (Clark 1976, Cawrse et al. 1984). Increasing the complexity may improve the reliability of the results. However, it has forced researchers to concentrate heavily on numerical solution techniques (Hyytiäinen and Tahvonen 2002a).

2.2 Individual-tree Growth Models

Davis et al. (2001, p.210) claim that individual-tree models give us the best available tool for simulating the way tree communities grow under different management prescriptions. Individual-tree models work by simulating the growth of each individual-tree in diameter, height, and crown; deciding whether it lives or dies; calculating its growth and volume; and then adding the trees together to get per hectare characteristics, volumes, and growth rates. Whether or not an individual-tree lives or dies and the rate at which it grows depend on its competitive position in the stand as determined by such attributes as relative size or distance to neighboring trees. Both stand and individual-tree models can use sample survey plot data as input, but only individual-tree models can simulate the competitive environment of each tree (Davis et al. 2001, p.211).

Individual-tree growth models use detailed data on single trees gathered from inventory plots to forecast how that tree or class of trees will grow and change in the future under different management regimes. Tree growth is initiated with the tree list from the current inventory. Using a mathematical simulation model, the future projection implements the chosen prescription

Table 1. A classification of growth and yield models.

Models of stand development	Definition and description
Whole-stand models - Density-free whole-stand models 1. Normal yield tables 2. Empirical yield tables for average current stands	Density-free models assume a predetermined stand density development over the rotation. For a given species, site, and location, stand development is a function of time only, and follows a predefined trajectory.
- Variable-density whole-stand models 1. Predict current volumes a. Explicit models b. Implicit models (diameter distribution) 2. Predict future growth and volumes a. Explicit models i. Direct growth prediction ii. Stand density prediction b. Implicit models (diameter distribution)	For a considerable period of time, these models were the prevailing basis of stand projection and they still are used in practice. A relationship between variables is implicit when the variables in the equation are defined and the dependent variable(s) identified, but the relationship is not quantified. When this relationship is specified, it becomes explicit.
Diameter class models - Empirical stand table projections - Diameter class growth models	The diameter class models separately simulate the growth in each diameter class by calculating the characteristics, volume, and growth of the average tree in each class and multiplying this average tree by the inventoried number of stems in each class. The two diameter class methods are distinguished by whether actual radial increment data collected from the subject stand are used to model the trees or whether generalized growth functions based on research sample data are used.
Age/stage-structured models	These models are based on grouping individuals into cohorts, characterized by the age, size, or developmental state of an individual. Tree growth is described as a transition from one stage to another.
Individual-tree models - Distance-dependent - Distance-independent	Individual-tree models are usually grouped into two classes: distance-dependent and distance-independent, based on whether or not they utilize information about the locations of other trees close to the subject tree.
Process based models	Process based models operate on a representation of the physiological processes of the tree. The structure and resolution varies but typical processes included are photosynthesis, respiration, allocation, and decomposition.

Source: Valsta, L. 1993. Stand management optimization based on growth simulators. p.11-13. Davis et al. 2001. Forest Management. p.186-187.

for one growth cycle of 1, 5 or 10 years, or even longer. At the start of the cycle, based on prescription and mortality rules, each tree on the list is selected to be harvested, to die naturally, or to remain as a living tree at the end of rotation. Then all remaining live trees, are grown in height, diameter, and crown to the end of the growth cycle. Each simulation cycle may involve four tree lists: the initial list, a harvest list, a mortality list, and an ending list to start the next cycle (Davis et al. 2001, p.211).

Getz and Haight (1989, p.239-242) explicitly describe the individual-tree growth models using another name—single-tree simulators. All single-tree simulators include equations for diameter growth, and many include equations for height growth and tree crown development. The growth equations are functions of either the location of the tree relative to its neighbors (distance-dependent models) or aggregate stand density variables (distance-independent models). In distance-dependent models, each tree is described by spatial coordinates that change due to harvesting or mortality. In distance-independent models, each tree record includes a tree expansion factor that represents the number of trees of its kind in the stand. The tree factor is reduced by harvesting or mortality, and aggregate stand attributes are obtained by summing the products of tree attributes and tree factors over all tree records. In general single-tree simulators can allow for the projection of 500 tree records or more (Getz and Haight 1989, p.240).

2.3 Present Models for Stand Development

2.3.1 The Growth Models

Tree growth is predicted with models for tree basal area growth (measured at breast height), height growth and tree crown ratio. Models for individual-tree basal area growth and height growth, as well as models for height development of dominant trees, are based on data from the Finnish National Forest Inventory (Hynynen et al. 2002).

The basal area growth, ΔBA is a function of tree variables (tree basal area BA, number of trees n, crown ratio cr and crown competition factor ccf) and stand variable (stand age t). In addition, the parameters affecting growth are: regeneration method, age at breast height, mean height of the dominant trees H_{dom}, stand density, site quality, north code, east code, elevation, temperature, sea index, lake index, administration code, and so on:

$$\Delta BA = f\,(t, BA, n, cr, ccf, H_{dom}, THIN, RDFL, RDF, SI, SC, TS), \qquad (1)$$

where $THIN$ denotes the time period after the previous thinning; $RDFL$ and RDF denote within-stand competition; SI, SC and TS denote site properties (Hynynen et al. 2002).

The number of state variables is the number of stand level variables plus number of tree level variables multiplied by number of tree classes. The number of tree classes differs from 9 to 16 in the sample plots. The total number of state variables for the plots is varied from 37 to 65. Stand age, basal area, diameter at breast height *dbh*, dominant height, crown ratio, and expansion factor are initially given and updated at each time period.

Tree crown ratio is defined as the ratio between the length of the live crown to tree height. Live crown base is determined as the height of lowest living branch over which the number of death nodes is less than two (Hynynen et al. 2002). For predicting tree crown ratio, the following non-linear model form was applied:

$$\hat{cr}_t = f\,(\hat{cr}_{t-1},\ 1-e^{-f(x)}), \tag{2}$$

where $\hat{cr}_{t-1}$ denotes crown ratio at the end of the previous five-year growth period. The model formulation restricts the predicted values between 0 and 1. The effects of site, geographical location, and stand and tree characteristics are included in the function *f*. The height of the crown base is specified to increase over time.

2.3.2 Mortality Model

Individual-tree survival rate is obtained with models predicting the probability of a tree dying during the next five-year growth period P_{tot5}. This can be predicted with the following formula by Hynynen et al. (2002):

$$P_{tot5} = 1-(1-P_{comp5})(1-P_{old5}), \tag{3}$$

in which P_{comp5} is the probability of mortality due to within-stand competition and P_{old5} the life-span mortality. Within-stand mortality function is effective only for high levels of basal area and is due to self-thinning.

2.3.3 Auxiliary Models

Volumes of stems are predicted by applying the polynomial model for the stem curve of Laasasenaho (1982). The stem curve is predicted using information on tree diameter and height. Timber categories are determined as functions of stem dimensions. The minimum diameter for pulpwood is 6 cm and the minimum length is 2 m. Log volumes obtained from the timber assortment model are based on stem dimensions. In practice this overestimates log volumes, because it ignores all the various defects that generally appear in stems (Hynynen et al. 2002).

3 The Optimization Model

Over 150 years ago, Martin Faustmann (1849) wrote his world famous paper on the 'Berechnung des Wertes, welchen Waldboden, sowie noch nicht haubare Holzbestände in der Waldwirtschaft besitzen' (Calculation of the Value which Forest Land and Immature Stands Possess for Forestry). His formula evaluates forest land as a source of permanent periodic revenue from timber sales.

Assuming that even-aged management would be practiced indefinitely, Faustmann defined forest value as the sum of the present values of harvests from the ongoing rotation, and land expectation value (LEV), which is the present value of an infinite series of plantations (Getz and Haight 1989, p.266).

The optimization problem for even-aged Norway spruce stands presented here is based on Hyytiäinen et al. (2002). When stand development is projected with an individual-tree growth model, the objective function for maximizing the bare land value specified for even-aged Norway spruce stands is:

$$\max_{\{\mathbf{h}_u, t_u, u=1,\ldots,k; \mathbf{Z}_0\}} \quad V = \frac{\sum_{u=1}^{k} \left\{ \left[\sum_{j=1}^{2} \mathrm{p}_j \cdot \mathrm{g}_j(Z_{t_u}, \mathbf{h}_u) \right] - \mathrm{c}_u(Z_{t_u}, \mathbf{h}_u) \right\} (1+\mathrm{r})^{-t_u} - \mathrm{c}_0(Z_0)}{1 - (1+\mathrm{r})^{-t_k}}, \tag{4}$$

subject to

$$\mathbf{Z}_{t_{u+1}} = \mathrm{f}(\mathbf{Z}_{t_u}, t_{u+1} - t_u, \mathbf{h}_u), \qquad u = 0, \ldots, k-1, \tag{5}$$

$$t_u \leq t_{u+1}, \qquad u = 1, \ldots, k-1, \tag{6}$$

$$\mathbf{h}_u \in \sigma_{t_u}, \qquad u = 1, \ldots, k, \tag{7}$$

where

V = bare land value,
u = 1,..., k harvests,
i = 1,..., n trees,
j = timber categories (sawlog, $j = 1$; pulpwood, $j = 2$),
k = the final harvest,
$\mathbf{Z}_0$ = matrix describing the given state of the stand at the beginning of the rotation,
t_u = stand age at the u^{th} harvest time,
$\mathbf{Z}_{t_u}$ = matrix describing the stand state before the u^{th} harvest at stand age t_u,
$\mathbf{h}_u$ = n-dimensional vector as the ratio of trees removed in the u^{th} harvest,
p_j = road-side prices of timber categories (€/m^3),
g_j = harvested volumes of timber categories (m^3 ha^{-1}),
c_u = harvest cost of the u^{th} harvest,

c_0 = establishment cost,
r = rate of interest (decimal),
σ_{t_u} = the set of admissible thinnings at stand age t_u.

The bare land value in equation (4) is maximized by changing the harvesting time and harvesting ratio in the condition of the initial stand state matrix. Stand structure is described for *n* trees. Each tree is characterized by variables reflecting its current dimensions (age, diameter, height etc.) and an expansion factor representing the number of trees of its kind per hectare or other area unit (Getz and Haight 1989).

The objective function is maximized subject to the stand dynamics equation (5), which defines the stand state before harvest at age t_{u+1} as a function of stand state before the previous harvest $\boldsymbol{Z}_{t_u}$, time difference between two harvests, and intensity of the previous harvest $\mathbf{h}_u$.

The thinning rate equation (7) defines thinning rates for each tree diameter classes. In the present study, the thinning rate in relation to tree diameter is defined by a piecewise linear function (for details, see Valsta & Linkosalo 1996). The thinning rates are linearly restricted depending on the specified number of thinning points.

Figure 1 illustrates the thinning type definition . Increasing the number of thinning points (type variables) enables a more detailed specification of the type of thinnings (thinning from below/above) in the simulation. Each number of thinnings forms a different optimization problem with different variables to be optimized. The flexibility of the thinning type increases with the number of parameters (Valsta 1992b, Valsta & Linkosalo 1996).

The optimization algorithm is based on the direct search method of Hooke and Jeeves (1961), which has proved successful in solving complex non-differentiable forestry problems (see e.g., Haight and Monserud 1990a). The

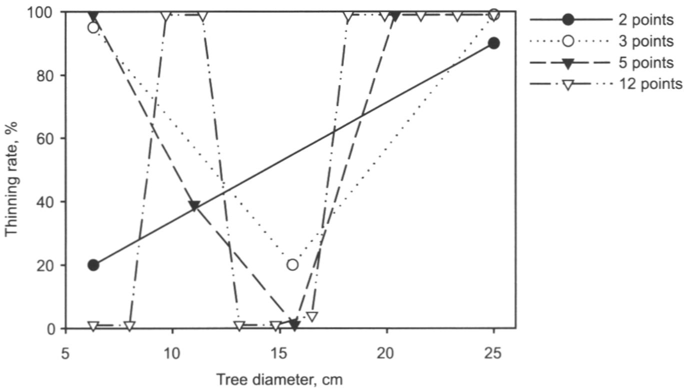


Figure 1. Thinning rates in relation to tree diameter in optimal solutions in Valsta (1992b) with four different thinning specifications.

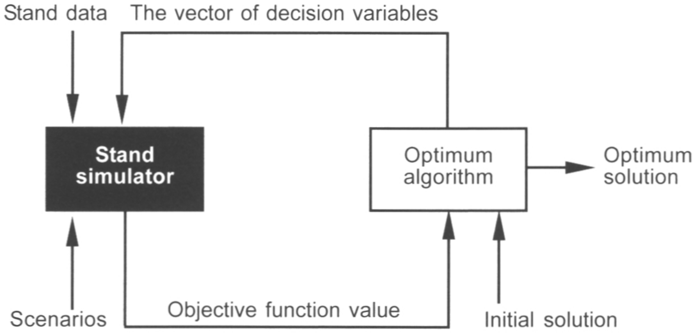


Figure 2. Structure of simulation-optimization system (Valsta 1992b & 1993).

modifications to the original algorithm are based on Osyczka (1984), and are explained in detail by Valsta (1992b). The number of variables N in the problem of optimizing thinnings and rotation is given by $N = k(s + 1) + 1$, where k denotes the number of thinnings and, s the number of thinning type variables (Valsta 1992b).

Figure 2 shows the overall structure of the simulation-optimization model. The applied solution procedure does not utilize the dynamic structure of the problem explicitly. This appears potentially inefficient (solving a dynamical problem without recognizing dynamics), but because of the complex nature of individual-tree based stand dynamics it has been a successful approach, as compared to dynamic programming (Valsta 1993). It also allows complete flexibility of the structure of the stand simulator, which is inside the "black box" (Roise 1986). The optimization problem appears to the algorithm as a standard nonlinear programming problem (Valsta 1993).

4 Materials and Computations

4.1 Biological and Economic Data

The biological data are collected from 12 sample Norway spruce stands. Besides the artificial regeneration method, four plots were established by the natural method. Figure 3 shows the locations of the 12 sample stands. Most plots are located on favorable sites for Norway spruce in eastern Finland, except plots 43 and 45. These two plots are located in the Kainuu region, where the climate is more like that of northern Finland. Among the sample stands, MT plots (59, 74, 51, 55, 70 and 25) are located more in the south, and OMT plots (100, 83, 78, 61, 43 and 45) are located more in the north.

The economic data consist of regeneration cost and roadside prices of Norway spruce sawlog and pulpwood. A 3% rate of interest is employed to calculate the optimal solutions. Sensitivity analysis is carried out in section 5.2.

It is obvious that the regeneration cost of the natural establishment method is less than the cost of artificial establishment. The regeneration cost for the artificial method is assumed to be €672.8 per hectare (including land preparation cost, planting and seedling cost and silvicultural operations), while

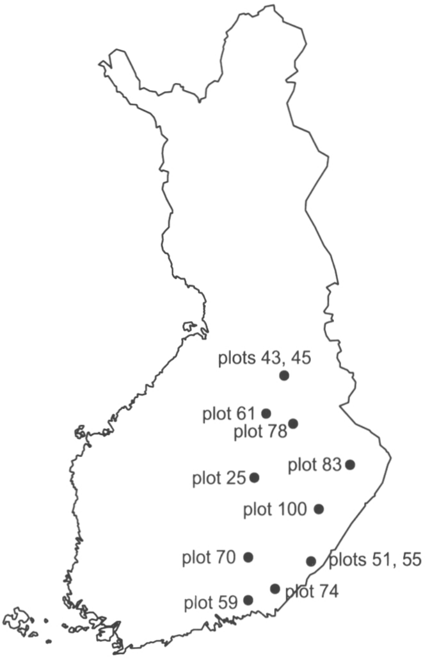


Figure 3. Locations of the 12 sample stands included in the biological data.

the regeneration cost for the natural method is €328 per hectare (including land preparation cost and tending cost). In order to better compare with previous studies, the roadside prices of Norway spruce employed here are based on Hyytiäinen and Tahvonen (2002a), and are 43.2 €/m^3 and 29.4 €/m^3 for Norway spruce sawlog and pulpwood, respectively.

4.2 Logging Cost Model

The logging cost model by Kuitto et al. (1994) is based on mechanized cutting and forest haulage specification. They defined the logging cost per hectare C_u as a function of the volume of harvested timber category, g_j (where $j = 1$ denotes sawlog and $j = 2$ pulpwood) and harvest method, u (thinnings, $u = 1$; or clearcutting, $u = 2$), total number of removed stems n, transport distance d and terrain class w:

$$C_u = f(g,u,n,d,w) \tag{8}$$

From forest harvesting point of view, logging cost consists of felling cost, transport cost and fixed cost. The felling cost equals total volume times felling cost per hour divided by felling productivity α. The transport cost equals total volumes by categories times transport cost per hour divided by transport productivity β. The fixed cost of logging is €100 per hectare. The costs per for felling phase $C_{fell} = 47.1$ (€/h) and for haulage phase $C_{trans} = 61.3$ (€/h). The logging cost model is explicitly described in equations (9)-(11):

$$C_u = C_{fell} \cdot \frac{\sum_{i=1}^{n}\sum_{j=1}^{2} g_{uij}}{\alpha} + C_{trans} \cdot \left[\frac{\sum_{i=1}^{n} g_{ui1}}{\beta_1} + \frac{\sum_{i=1}^{n} g_{ui2}}{\beta_2} \right] + C_{fixed}, \tag{9}$$

$$\alpha = \frac{\sum_{i=1}^{n}\sum_{j=1}^{2} g_{uij} \cdot 60}{n \cdot \left[\frac{z_1}{w} + z_2 \right] \cdot 1.197 \cdot 1.276}, \tag{10}$$

$$\beta_j = \frac{60}{(y_1 + y_2 + y_3 + y_4 + y_5) \cdot 1.084 \cdot 1.224}, \tag{11}$$

where z_1 is time expenditure on shifting (min/stem), z_2 is time expenditure on handling (min/stem); y_1 is time expenditure on loading (min/m^3), y_2 is time expenditure on delay (min/m^3), y_3 is time expenditure on hauling load (min/m^3), y_4 is time expenditure on driving without load (min/m^3), y_5 is time expenditure on unloading (min/m^3). The details are given in Kuitto et al. (1994).

4.3 Computations

Stand Management Assistant (SMA) is a program for analyzing silvicultural and economic options for stand management. Based on deterministic or stochastic optimization, the program allows the user to determine optimal solutions for the specified conditions and view the results of optimization as well as the sensitivity to changes in the prescription (Valsta and Linkosalo 1996). Here the deterministic version is used. The SMA software is used to solve the harvesting problem for the sample plots, various thinning type definitions, and all rates of interest that yield a positive bare land value. The computer framework combines stand-level simulation and numerical optimization.

The computations consist of three parts: optimal solutions, sensitivity analyses and comparisons. The sample plots are divided into natural and artificial groups. Maximum bare land value, optimal rotation, optimal thinning frequency, optimal thinning type, optimal thinning intensity, and timing of thinning and clearcutting are calculated using a 3% rate of interest and three thinning points.

The butt rot effects are computed by the equation for butt rot rate:

$$R_{rot} = \frac{1}{1 + 50 \cdot \exp(-0.023 \cdot t)}, \tag{12}$$

where R_{rot} denotes rate of butt rot.

5 Results and Discussion

5.1 Optimal Solutions at 3% Rate of Interest

Tables 2 shows the optimal solutions at 3% rate of interest for plots. The symbols are as follows: Dg denotes mean diameter weighted by tree basal area in cm; S-trees (%) denotes the portion of small size tree removed; M-trees (%) denotes the portion of medium size tree removed; L-trees (%) denotes the portion of large size trees removed; Ro.dg denotes mean diameter weighted by tree basal area at the end of rotation, in cm; M.A.I. denotes mean annual increment in cubic meter per year; H_{100} denotes the dominant height in meters at the age of 100 years. H_{100} corresponds to dominant height site classification and illustrates the growth potential of the stand. In this study, H_{100} was predicted without thinning. The advantage of the H_{100} classification is that it is more precise and detailed than classification based on ground vegetation (OMT, MT sites classification).

The analyzed sample plots were different in initial stand age, site fertility, regeneration methods, initial density and geographic location. In addition, the success of prior management (silvicultural activities) may vary. However, the bare land values were calculated assuming no prior harvest revenues and constant establishment costs.

The optimization results indicated that the bare land values were associated with stands with high basal area, tree diameters and dominant height. This affirmed that development of basal area, diameter and dominant height were the key factors affecting bare land value. In general, fertile site plots had higher bare land value than infertile sites. Plots 51 ($H_{100} = 29.2$) and 59 ($H_{100} = 30.7$) produced the highest bare land values among all the plots. Note that for plot 51 the initial dominant height was low, yet the plot ranked second in dominant height at the age of 100 years.

Stands that had the highest mean annual increment (9.7 m^3/yr/ha in plots 51 and 59) gave the highest bare land value. Plot 51, at early ages, was characterized by a low initial density (1400 trees ha^{-1}) but high growth of tree diameters, dominant height, basal area and commercial volume. These lead to a short optimal rotation and a high level of timber production. There was only one thinning with 38.3% thinning intensity at the stand age of 47.3 years, and the thinning type was from above. Low initial density was the reason why the one-thinning regime was optimal. The dense stands (2250-2300 trees ha^{-1}) involved more thinnings (4-5). Thus the sparser the initial stand, the smaller the optimal number of thinnings (see Table 2).

Naturally regenerated plots (74, 55, 25) were characterized by a long optimal rotation, with the exception of plot 61 (see Table 2). The two highest bare land values at the OMT site were given by plot 61 (natural regeneration)

Table 2. Optimal solutions at 3% rate of interest with 3 thinning type variables.

Initial states	Plot 59	Plot 51	Plot 74	Plot 83	Plot 70	Plot 61	Plot 78	Plot 100	Plot 55	Plot 45	Plot 43	Plot 25
H_{100} (m)	30.7	29.2	28.0	27.5	26.9	26.7	25.8	25.7	25.7	24.6	24.3	22.3
Biological age (yrs)	34	25	42	33	36	36	36	38	37	30	45	48
Age at breast height (yrs)	23	14	29	22	23	25	20	31	27	13	28	34
Regeneration code	Artificial	Artificial	Natural	Artificial	Artificial	Natural	Artificial	Artificial	Natural	Artificial	Artificial	Natural
Site type	MT	MT	MT	OMT	MT	OMT	OMT	OMT	MT	OMT	OMT	MT
Number of trees	1700	1400	2300	1825	2250	1825	2300	1475	1375	1700	1475	1875
H_{dom} (m)	15.9	11.7	15.9	13.8	14.1	14.5	13.1	12.6	13.0	10.9	14.5	13.9
Basal area ($m^2 ha^{-1}$)	26.5	20.5	31.0	26.6	29.9	29.3	33.6	17.0	12.0	26.3	31.6	23.3
Dg (cm)	15.3	14.5	14.1	14.8	13.8	15.8	15.0	13.2	11.6	15.0	17.9	13.3
1st thinning (yrs)	51.3	47.3	44.5	46.5	43.9	45.2	37.1	64.8	63.5	40.8	57.1	66.2
- Intensity (%)	42.3	38.3	21.0	45.1	16.0	42.8	23.3	40.2	31.0	37.2	39.6	39.2
- S-trees (%)	16.0	16.0	25.2	16.0	16.1	16.0	31.4	16.0	16.0	21.3	16.0	16.0
- M-trees (%)	16.0	16.0	16.6	16.0	16.0	16.0	16.0	16.0	16.0	17.8	16.0	16.0
- L-trees (%)	99.0	99.0	32.9	99.0	16.0	99.0	36.8	99.0	99.0	99.0	99.0	99.0
2nd thinning (yrs)	57.5		56.1	57.7	54.0	61.0	49.3	75.5	76.6		63.6	77.8
- Intensity (%)	18.2		20.4	18.8	30.0	14.8	24.5	31.9	66.8		18.7	13.9
- S-trees (%)	1.0		1.3	1.0	1.0	1.0	1.1	1.0	1.0		1.0	1.0
- M-trees (%)	1.0		1.2	1.0	1.0	1.0	1.0	24.0	99.0		1.0	1.0
- L-trees (%)	99.0		99.0	99.0	99.0	99.0	99.0	99.0	99.0		99.0	99.0
3rd thinning (yrs)			62.4		58.5		56.5					83.5
- Intensity (%)			12.7		16.3		14.6					36.4
- S-trees (%)			1.7		1.0		1.0					1.0
- M-trees (%)			1.9		1.0		1.5					41.0
- L-trees (%)			99.0		99.0		99.0					99.0
4th thinning (yrs)			67.7		70.4		62.4					
- Intensity (%)			11.4		43.8		10.3					
- S-trees (%)			1.9		1.0		1.0					
- M-trees (%)			3.8		44.6		1.0					
- L-trees (%)			99.0		99.0		99.0					
5th thinning (yrs)			76.1				63.9					
- Intensity (%)			45.9				13.9					
- S-trees (%)			1.0				1.0					
- M-trees (%)			48.0				1.0					
- L-trees (%)			99.0				99.0					
6th thinning (yrs)												
- Intensity (%)												
- S-trees (%)												
- M-trees (%)												
- L-trees (%)												
Rotation (yrs)	73.8	60.4	90.6	71.0	86.1	69.2	77.6	89.3	97.9	61.6	76.3	99.2
Ro.dg (cm)	24.5	25.5	24.8	23.6	24.3	23.0	24.3	26.8	28.0	23.7	24.2	23.9
M.A.I. (m^3/year)	9.7	9.7	8.8	8.7	9.0	8.3	8.3	8.0	6.3	7.8	6.7	5.9
Bare land value (€)	3081.2	3644.5	2335.8	2759.7	2379.6	2893.4	2147.7	1744.0	1603.3	2462.6	1589.7	1146.0

and plot 83 (artificial regeneration) with the same planting density (1825 trees ha^{-1}). H_{100} was higher for plot 83, but low stand establishment cost resulted in a high bare land value for plot 61. Recall the assumption that a new stand can be established naturally without shelter tree cuttings before the end of rotation. It is also assumed that there is no regeneration delay.

Figures 4a-h show the stand development in plots. In Figures 4a-b, when all plots reach the age of 40 years, the highest average diameter was twice as high as the lowest. For dominant height (see Figures 4c-d), the difference is similar. This implies large variation in optimal rotation length between plots. Figures 4e-f show that the sample plots yielding the higher bare land values were characterized by high basal area. Before the first thinning, plots 51 and 59 reached the highest basal area. This made fast growth of basal area possible after thinning. Then, the optimal rotation length was shorter than that for plots which still needed to achieve the diameter goal.

Many factors can affect rotation age, such as planting density, regeneration method, and thinning frequency. Table 3 compares the optimal solutions with previous studies. The optimal rotation length in plots giving the highest bare land values for this study was a little shorter compared to most of the previous studies. Möykkynen et al. (2000) and Möykkynen & Miina (2002) report shorter rotation length in Norway spruce and Scots pine mixture (when initial butt rot level of the stand was 0% at first thinning, in winter). Zero to two thinnings and a small difference between sawlog and pulpwood prices (sawlog 35.3 €/m^3, pulpwood 30.3 €/m^3) might be the key reasons for short rotation in their studies.

With the thinning type from below, Hyytiäinen and Tahvonen (2001) show that a 5-7 cm larger diameter at breast height was optimal at the end of rotation, compared to most of the previous studies. In the study of Hyytiäinen and Tahvonen (2002a), the optimal rotation periods were about 5-20 years longer than those of other Finnish optimization studies.

Thinning removes some trees to give the remaining trees more resources: space, moisture, nutrients, and sunlight for growth. Thinning can increase the merchantable volume growth and can provide a number of other advantages. For instance, thinning can increase the quality of the remaining stock, improve the value growth of the remaining trees and, also yield income before rotation-end (Klemperer 1996, p.242-243). However, higher fixed harvesting costs decrease the optimal number of thinnings (Eriksson 1999, Hyytiäinen and Tahvonen 2002a) and increase the losses from a nonoptimally high number of thinnings in distant and small stands that are costly to access (Hyytiäinen and Tahvonen 2002b).

Figure 5 shows the relative bare land value for plots at 3% rate of interest as a function of the number of thinnings, respectively. The optimal number of thinnings was typically two. In a study by Valsta (1992b), the optimal number of commercial thinnings was two for 1100 trees per hectare. Three thinnings was optimal for 2200 trees per hectare, and three for 4400 trees per hectare

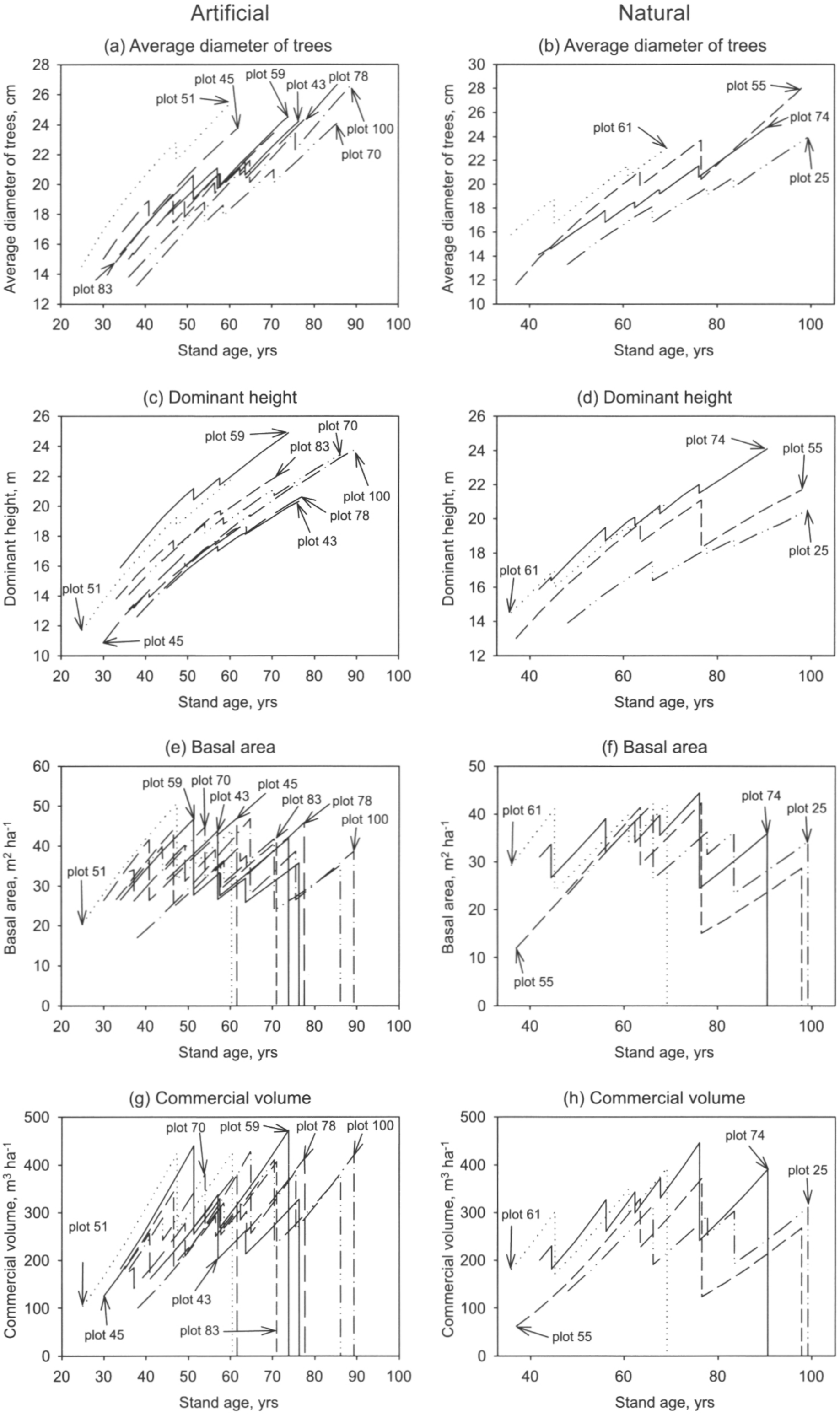


Figure 4. Optimal stand development for plots.

Table 3. Comparison of published results on optimal rotation periods (age in years/average dbh at the end of rotation period in cm) for Norway spruce stands with 3% rate of interest.

Studies	OMT	MT
Nyyssönen 1958	70/-	85/-
Pesonen & Hirvelä 1992	75-80/-	80-90/-
Salminen 1993a	70/-	80/-
Valsta 1992b	77/20-22	-
Vettenranta & Miina 1999	-	70-78/-
Möykkynen et al. 2000	61/-	-
Hyytiäinen & Tahvonen 2001	75/30	80/29
Möykkynen and Miina 2002	61/-	-
Hyytiäinen & Tahvonen 2002a	60-80/-	80-115/-
Best solutions in the present study	69/23	60/25.5
All plots in the present study	62-89/23-27	60-99/24-28

Note: All the studies are based on different optimization techniques, and varying economic data on price and cost parameters. Extended from Hyytiäinen and Tahvonen (2001).

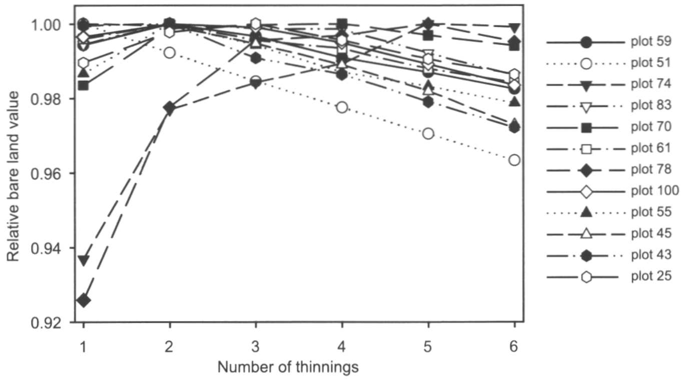


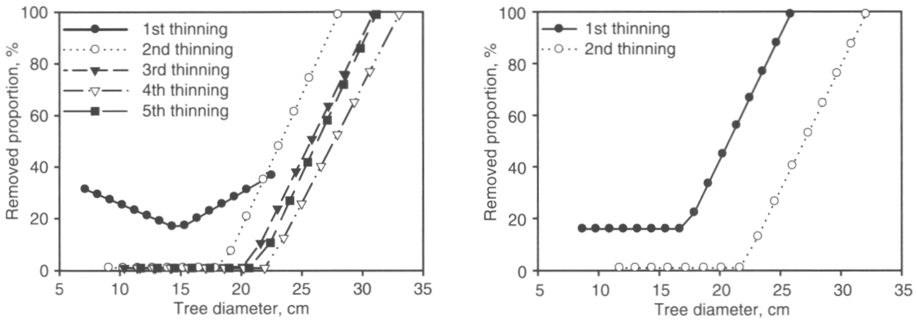
Figure 5. Bare land values with 1-6 thinnings relative to the maximum at 3% rate of interest.

(the first thinning was precommercial). In Hyytiäinen and Tahvonen (2002a), the two-thinning regime was optimal for most Norway spruce site indices and for most of the rates of interest studied, and one thinning for poor sites and high (r > 4%) rates of interest, as well as for the most fertile site at medium rates.

The exceptions were plots 78 and 74. For these, five thinnings was optimal because of high initial densities of 2300 trees ha^{-1}. It seems that if two light thinnings are combined, then the economic loss due to mortality is greater than the losses due to too frequent thinning. The effects of different numbers of thinning points on results for plot 74 are discussed in section 5.3.

Figures 6a-d illustrate the proportion of trees removed in optimal thinnings for four plots at 3% rate of interest. The thinning type is defined with three

(a) Plot 78 (OMT, artificial, 2300 trees ha^{-1}) (b) Plot 61 (OMT, natural, 1825 trees ha^{-1})



(c) Plot 51 (MT, artificial, 1400 trees ha^{-1}) (d) Plot 74 (MT, natural, 2300 trees ha^{-1})

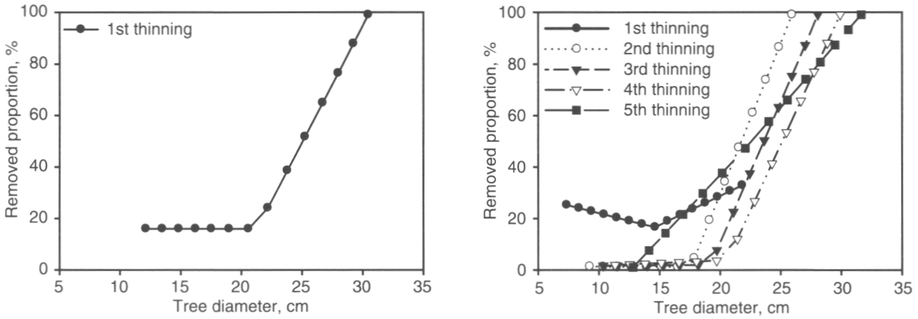


Figure 6. Removal rates in optimal thinnings (1st, 2nd,...) at 3% rate of interest with 3 thinning type

optimized variables that define thinning rate for minimum, average and maximum diameters. Thinning rates for other diameters are interpolated. Sparse stands were typically thinned from above and dense stands from the both ends of the diameter distribution in the first thinning. In addition, for all the sample plots, the first thinning removed intermediate trees with 16% or higher thinning rate (see Tables 2).

The results of this study are in line with those of Valsta (1992b) which suggested that thinning from above was optimal in most cases for pure Norway spruce stands. However, according to current silvicultural suggestions, for Norway spruce stands in Finland, one should carry out two thinnings, first from below and then from above (Mielikäinen & Riikilä 1997, p.75). This procedure was also optimal in Möykkynen et al. (2000) and Möykkynen and Miina (2002). In addition, Pukkala et al. (1998) report that it is optimal to apply first thinning from below and then thinning from above for Scots pine and Norway spruce mixtures.

The first thinning typically removed pulpwood, with the exception of plot 51 where the development of diameter has been so fast that the largest trees have already reached sawlog dimensions before the first thinning. The intensity of the first thinning was 40-45% in the majority of the plots (Figure 7). All

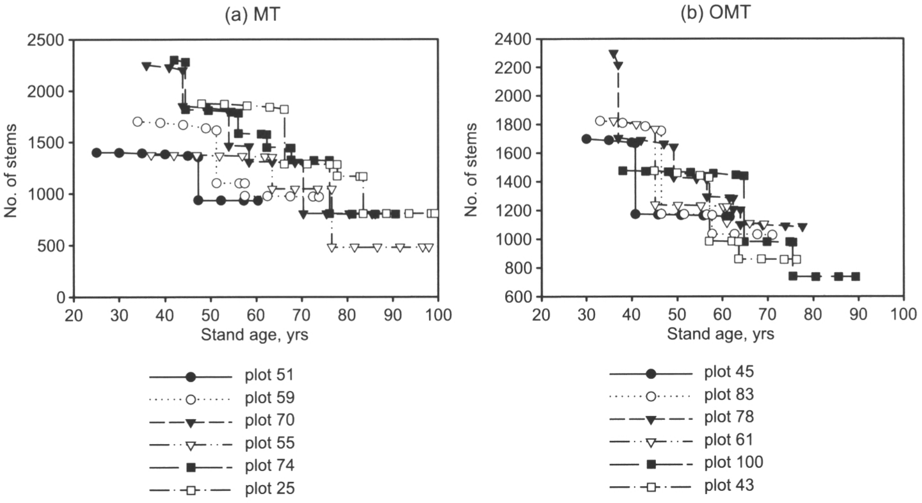


Figure 7. Mortality at MT and OMT sites.

subsequent thinnings removed mainly sawlogs. However, in Hyytiäinen and Tahvonen (2002a), for a typical Norway spruce site at 3% rate of interest, the intensity of the first thinning was 26% and the second thinning 74%.

Self-thinning due to high density and competition between individual-trees is one reason for the first thinning. Small trees were removed, because otherwise they would die before reaching sawlog dimension. For example there was high mortality before the first thinning for plot 78 (see Figure 7b). In three dense stands (plots 70, 74 and 78), where initial density was around 2250-2300 trees ha^{-1}, the first thinnings were early and removed trees only from the hauling roads. These removed larger proportions of small and large compared to medium-sized trees (see Table 2 and Figures 6a,d). The optimal numbers of thinnings of these three stands were four or five, and the thinning rates were mostly low, i.e. 10-30%. Overall, the greater the number of thinnings, the lower the thinning intensity.

After the first thinning, the subsequent thinnings removed trees mainly from above with typically quite light or medium thinning intensities (see Table 2 and Figures 6a-d). The purpose of subsequent thinnings might be to keep the level of basal area high, and to reduce mortality.

There is no price premium for Norway spruce sawlog. Despite this, in pure Norway spruce stands, the dominant trees were not removed soon after reaching the minimum sawlog dimensions. A study of Scots pine by Hyytiäinen et al. (2002) reported the contrasting result that the dominant trees are removed soon after they reach the minimum sawlog dimensions for Scots pine. One obvious reason is that the ratio of pulpwood to sawlog price is higher for Scots pine than for Norway spruce. Figures 8a-b show how merchantable stand volume was distributed over tree diameter at the end of the rotation period at

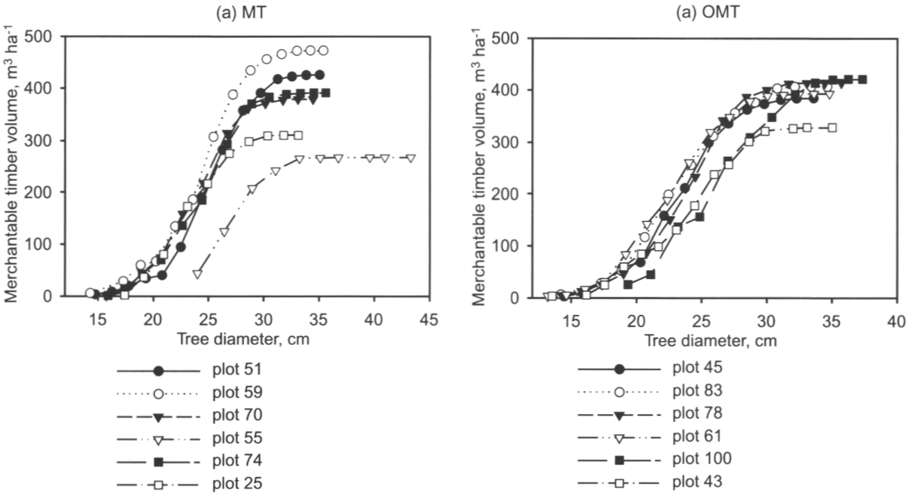


Figure 8. Cumulative distribution of merchantable stand volume at 3% rate of interest at final harvest at MT and OMT sites.

3% rate of interest. Vuokila (1985) and Pukkala et al. (1998) describe the sawtimber requirements for Norway spruce: minimum dbh 17 cm, minimum top diameter of logs 16 cm, mean log length 49 dm, and minimum top outside bark diameter 17.5 cm. The proportion of trees that fulfill the dimension requirements for sawlog trees (dbh > 18.1 cm) was high for all plots analyzed in this study.

Figure 9 illustrates the development of stand structure at 3% rate of interest for plot 61. To better illustrate the differences, the actual distributions were fit to a 4-parameter Weibull distribution. In the first thinning, large trees were removed, as well as small and medium trees, but the removed proportions were smaller for small than for large trees. In the second thinning, only trees

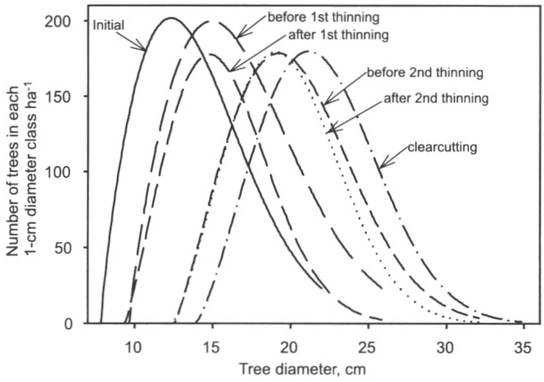


Figure 9. Stand structure development of optimal solutions of plot 61 at 3% rate of interest, fit to 4-parameter Weibull distributions.

that fulfilled the dimension requirements for sawlog trees, over 18.1 cm, were lightly thinned. In clearcutting, there were still a few trees under sawlog dimension requirements. After each thinning, the diameter frequency distribution was narrower than before thinning, and then it wided up to the next thinning and clearcutting.

5.2 Optimal Solutions at Different Rates of Interest

Table 4 shows the maximized bare land values computed for three stands and all rates of interest that yield positive bare land values. Three plots (59, 61 and 83) were selected because they had an initial density close to the optimal planting density. According to Solberg and Haight (1991) and Valsta (1992b), the optimal planting density for Norway spruce (*Picea abies* [L.] Karst.) is about 1900 trees ha^{-1} at 3% rate of interest. The initial states of the three stands were also similar in age, dominant height, basal area and average diameter (see Tables 2 and 3).

Plot 59 (MT, artificial) gave the highest bare land values for 0% to 3% rates of interest, plot 61 (OMT, natural) gave the highest bare land values for 4% to 5% rates of interest. Compared to plot 83 (OMT, artificial), this implies that artificially established stands obtain better bare land value for 0-2% rates of interest, and naturally regenerated stands obtain better bare land value for 4-5% rates of interest.

The optimal number of thinnings decreased with the rate of interest (Table 5). It was sensitive to change between 1-2%. In the naturally established stand (plot 61), the number of thinnings was sensitively decreased. But it was not sensitive in artificially regenerated stands (plots 59 and 83).

Valsta (1992b) alleged that the higher the interest rate, the shorter the rotation. In addition, a high interest rate leads to earlier, fewer and heavier the thinnings. Figure 10 shows how the optimal rotation length decreased with the rate of interest. The rotation length was more sensitive in the range of 0-2%

Table 4. Maximized value of bare land at 0-5% rates of interest (at 0% rate of interest, the values represent maximized annual net revenues).

	Maximized value of bare land (€/ha)		
Rate of interest (%)	Plot 59	Plot 61	Plot 83
0	431	356	379
1	24813	21030	22108
2	7598	6718	6829
3	3081	2893	2768
4	1310	1375	1149
5	470	641	369

Table 5. Optimal number of thinnings at 0-5% rates of interest.

	Optimal number of thinnings		
Rate of interest (%)	Plot 59	Plot 61	Plot 83
0	4	5	3
1	3	4	4
2	2	3	3
3	2	2	2
4	1	1	2
5	2	1	2

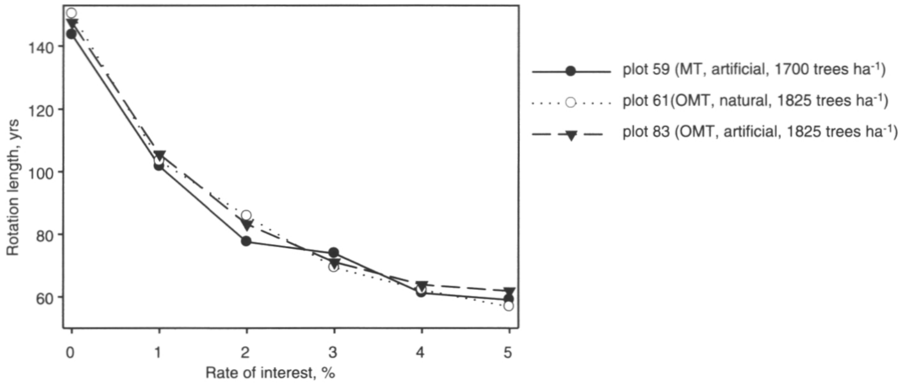


Figure 10. Optimal rotation length with 0-5% rate of interest.

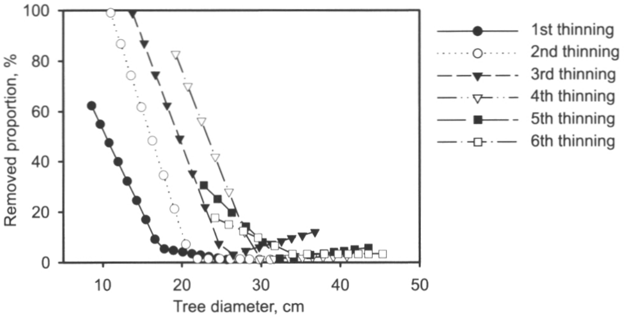
rates of interest, and from 4% to 5%, the slopes were almost flat. Rotation length was almost insensitive to the rate of interest for r > 4%.

The MSY rotation was about 15 years shorter than Forest Rent rotation, when the maximum number of thinnings was six (see Appendix). In a comparison of MSY and Forest Rent by Hyytiäinen and Tahvonen (2002b), neglecting all prices and costs, MSY deviated more from the economic criterion than did the Forest Rent criterion.

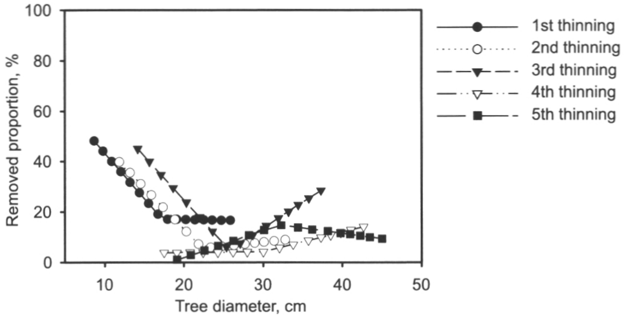
Valsta (1992b) shows that the optimal thinning type was affected by the objective of stand management and by stand density. In the present study, different objectives clearly changed the type of thinning in plot 61 with three thinning points. For MSY and Forest Rent, the thinning types were clearly from below (Figures 11a-b). Thinning from below was more profitable with interest rates less than 2%, whereas thinning from above was superior with interest rates of 3-5% (Figure 11c). This result is similar to that of Hämäläinen (1978) using Vuokila (1967) growth functions for Scots pine stands.

Figure 12a shows the basal area development at 1, 3 and 5% rates of interest for plot 61. The rotation length and the number of thinnings decreased

(a) MSY solution



(b) Forest rent solution



(c) MSY, forest rent and 1-5% rates of interest (1st thinning)

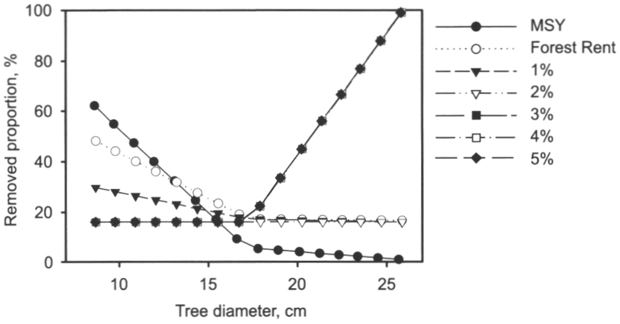


Figure 11. Thinning types in plot 61 with MSY, Forest Rent and 1-5% rates of interest.

with the rate of interest. The first thinnings occurred at the same time with all rates of interest (see Appendix). At 0-2% rate of interest, the first thinnings were light (16-20.6%). At 3-5% rate of interest, the first thinnings were heavier with the same thinning intensity, 42.8%. Similar results can be seen for plots 59 and 83 in the Appendix.

Figure 12b shows the diameter distributions at the end of rotation for plot 61 at 1, 3 and 5% rates of interest. To better illustrate the differences, the actual distributions were fit to the 4-parameter Weibull distribution. Increasing

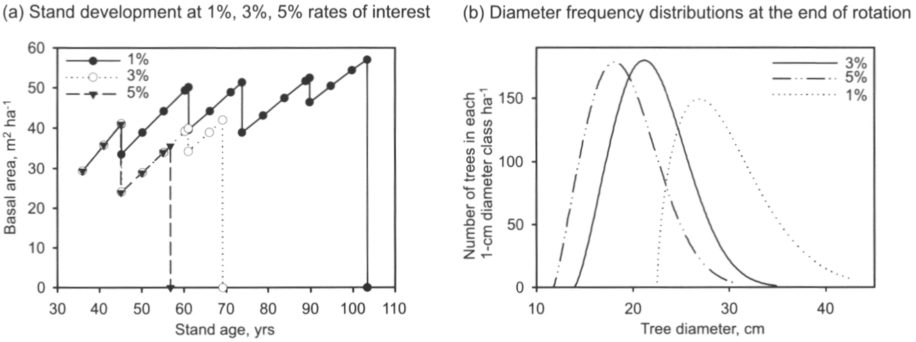


Figure 12. Basal area development and diameter distributions at 1, 3 and 5% rates of interest for plot 61 (OMT, natural).
Note: In Figure 12b, the actual distributions are fit to the 4-parameter Weibull distribution

Table 6. Proportion of sawlog and pulpwood volume at the end of rotation with 1, 3 and 5% rates of interest for plot 61.

Rate of interest	Vtot (m^3ha^{-1})	Vlogs(m^3ha^{-1})	Vpulpw(m^3ha^{-1})	Vlogs/Vtot
1%	654.9	564.3	87.8	0.86
3%	393.1	255.3	133.7	0.65
5%	297.9	135.7	157.1	0.46

Note: Vtot = total volume, Vlogs = sawlog volume, Vpulpw = pulpwood volume

the rate of interest resulted in a lower final volume, a narrower and more highly peaked diameter distribution, and a lower mean diameter.

Table 6 shows that the proportion of sawlog volume of total volume in clearcutting decreases significantly with the rate of interest. By contrast, the pulpwood volume increased slightly while sawlog volume decreased rapidly with the rate of interest.

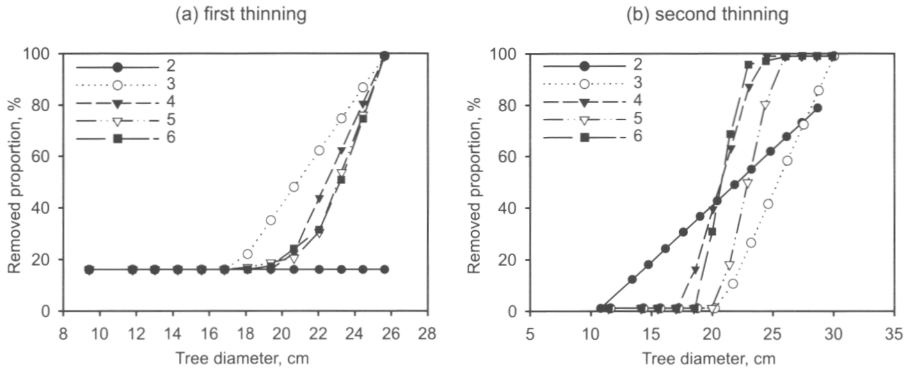
5.3 Flexibility of Thinning Type Definition

Thinning type is characterized by how many trees in tree classes should be removed in a thinning. Table 7 shows that flexible thinnings may increase the maximum bare land value and the rotation length. Increased flexibility enables thinnings that remove trees strictly below or above certain diameter limits. It should be noted that the alternative of one thinning point was programmed to produce thinning from below.

Figures 13a-b show how increasing thinning type flexibility changed the type of thinnings in optimal solutions for plot 83. The intensity of the first thinning was light with two thinning points but heavier (about 45%) with a higher number of thinning points. Figure 13a shows that more large trees were

Table 7. Rotation length and bare land value affected by thinning type variables for plot 83 with 1-3 thinnings.

No. of thinnings	Optimal solutions	Thinning type variables 1	2	3	4	5	6
1 thinning	Bare land value (€)	2609.4	2640.5	2744.5	2741.6	2779.3	2787.5
	Rotation (yrs)	61.8	66.9	70.8	66.8	75.1	70.9
2 thinnings	Bare land value (€)	2585.7	2654.6	2759.7	2817.8	2817.1	2834.9
	Rotation (yrs)	67.6	71.7	71.0	76.0	76.9	76.5
3 thinnings	Bare land value (€)	2559.9	2641.8	2756.2	2801.3	2816.1	2826.8
	Rotation (yrs)	67.6	76.9	75.8	76.0	76.6	76.2


(a) first thinning

(b) second thinning

Figure 13. The effects of optimized thinning type variables (2-6) on thinning rates in plot 83.

thinned with thinning type variables. When applying the most flexible thinning type definition (with five endogenous thinning type variables), the second thinning removed all trees above certain tree diameters, and retained all small trees (Figure 13b). However, a different result is reported in a study of Scots pine by Hyytiäinen et al. (2002), i.e. the second thinning removed all trees above and below certain tree diameters, and retained all the intermediate trees when applying the most flexible thinning type definition.

Table 8 shows that a two-thinnings regime was optimal for plot 74 with one thinning point, i.e. when thinning from below, and a three-thinnings regime was optimal with two thinning points. Increasing thinning points beyond three with more than five thinnings became infeasible because of computational constraints.

5.4 Butt Rot Effects

Figure 14a shows the optimal basal area development in plot 51 in optimal solutions with three thinnings at 3% rate of interest, with and without butt rot effects. Before the age of 60 years, the development of basal area was the

Table 8. Optimal number of thinnings and thinning points for plot 74 at 3% rate of interest.

Thinning points	1	2	3
Optimal No. of thinnings	2	3	5
1st thinning (yrs)	44.5	44.5	44.5
- intensity (%)	24.0	24.8	21.0
2nd thinning (yrs)	60.7	58.3	56.1
- intensity (%)	25.7	18.6	20.4
3rd thinning (yrs)		69.7	62.4
- intensity (%)		53.1	12.7
4th thinning (yrs)			67.7
- intensity (%)			11.4
5th thinning (yrs)			76.1
- intensity (%)			45.9
Rotation (yrs)	78.4	89.8	90.6
Bare land value (€)	2136.7	2227.3	2335.8

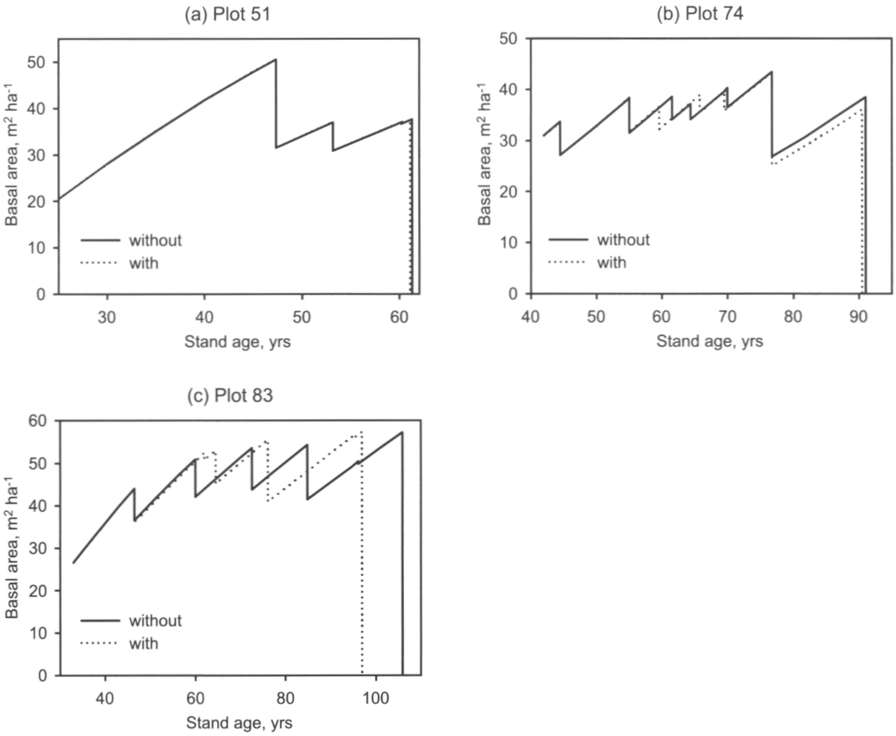


Figure 14. Basal area development in optimal solutions, with or without butt rot effects, 3% rate of interest.

same in both cases. With butt rot, the bare land value decreased by 2.6% (with €3459.9, without €3587.3), and the rotation length shortened from 61.3 to 61.1 years. This implies that butt rot affects only mature Norway spruce stands.

According to Piri et al. (1990), mycelium originating from old-growth stumps may be viable for up to 60-120 yrs, and spread after felling of butt-rotted trees (Stenlid & Redfern 1998). Frequent summer thinnings without stump treatment are the most important operation increasing the proportion of trees with butt rot at the end of rotation (Swedjemark & Stenlid 1993, Venn & Solheim 1994, Vollbrecht & Agestam 1995).

In the study of Möykkynen et al. (2000), at 3% rate of interest, the rotation was shortened by six years as compared to the optimal winter-thinning schedule, if any stump infection occurred (from 61 to 55 years). The thinning rate under infection risk was higher than in the first winter thinning.

In the present study, the butt rot effects started from a stand age of about 60 years. Thus, the rot had no significant effect on the optimal management at rotations shorter than 90 years (Figures 14a-b). In addition, there were no big differences for thinning intensity. Consequently, the butt rot effects shortened the rotation length in plot 83 by 8.9 years (with, 97 yrs; without, 105.9 yrs) at 1% rate of interest (Figure 14c). This implies that the butt rot becomes serious when rotation ages exceed 100 years, which become optimal at a very low rate of interest.

5.5 Comparison of Stand Development when Using SMA and MOTTI

MOTTI is a computer program for tree growth simulation that is designed and maintained by the Finnish Forest Research Institute. In MOTTI, the latest updated growth model is used.

Generally, the tree diameter growth was not significantly different as between SMA and MOTTI. For plot 51, MOTTI showed slightly faster growth than SMA, without thinning (Figure 15a). In a two-thinning case (Figure 15b), SMA and MOTTI tended to the same growth rate after the first thinning

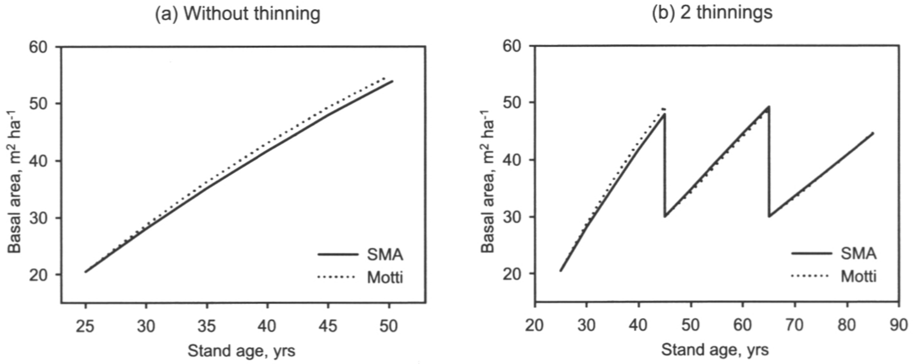


Figure 15. Comparison of SMA and MOTTI for plot 51.

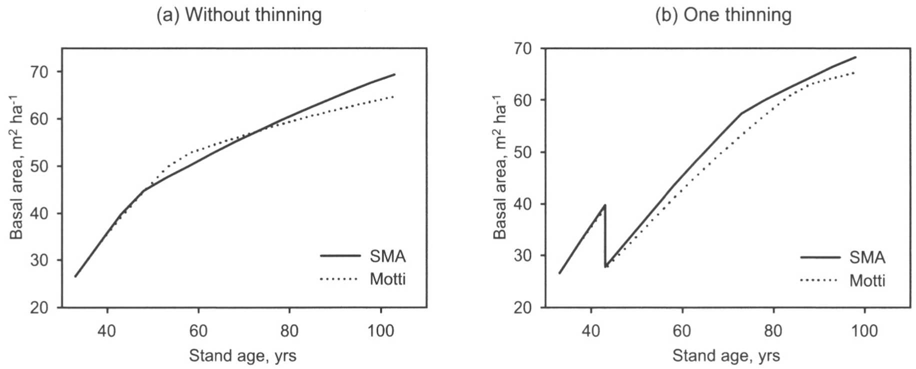


Figure 16. Comparison of SMA and MOTTI for plot 83.

(thinnings controlled by the same thinning time and the same basal area after thinning).

When extending the rotation length, the growth curves definitely differed (see Figure 16). The growth curves of MOTTI were smooth, both with and without thinning. However, the growth curves of SMA had small kinks. These were caused by within-stand mortality (Figure 16a-b). When rotation length increased to 100 years, there were two intersections, at around 50 and 75 years. This implies that the mortality functions differ as between SMA and MOTTI (Figure 16a). In contrast to plot 51, plot 83 grew faster in SMA than in MOTTI, after thinning (Figure 16b).

5.6 Comparison of Finnish Forest Law, Recommendations and the Optimal Solutions.

According to requirements set out in statutory provisions (Ministry of Agriculture and Forestry 1997), a stand must reach either a certain minimum average dbh, expressed in cm, or a certain minimum age before it can be clearcut. For Norway spruce, the requirements are: stand age ≥ 70 years or average dbh ≥ 25 cm for an OMT site; stand age ≥ 80 years or average dbh ≥ 24 cm for an MT site. The legal limits and recommendations (Tapio 2001 p.26, 2002 p.172) were initially designed for natural forests, but they have been identically applied to artificially regenerated forests.

Besides dbh and rotation, basal area after thinning is also a leagal constraint. In this study, the basal area constraint was not violated at 3% rate of interest for any plot. This constraint was also not violated for plots 59, 61 and 83 at 0-5% rate of interest.

Table 9 shows the length of the optimal rotation period for plot 59, plot 61 and plot 83 at 0-5% rates of interest. The optimal solutions were described by

Table 9. Optimal rotation length and average diameter at rotation age.

Rate of interest (%)	Plot 59 (MT) Rotation (yrs)	Plot 59 (MT) Dg (cm)	Plot 61 (OMT) Rotation (yrs)	Plot 61 (OMT) Dg (cm)	Plot 83 (OMT) Rotation (yrs)	Plot 83 (OMT) Dg (cm)
0	143.6	39.2	150.3	40.4	147.5	40.8
1	101.7	30.0	103.4	30.4	105.9	30.4
2	77.5	25.5	85.8	26.4	83.2	26.1
3	73.8	24.5	69.2	23.0	71.0	23.6
4	61.2	22.1	62.2	21.8	63.8	21.8
5	59.0	21.1	56.8	20.3	61.8	21.7

Note: Shaded solutions were illegal.

average tree diameter (weighted with tree basal area) and stand age at the end of the rotation period. For plot 59 and plot 83, the optimal solutions at 4% and 5% were illegal, and for plot 61, solutions at 3%-5% were illegal. In the study of Hyytiäinen and Tahvonen (2002a), similar results are obtained: for lower rates, the rotations were typically longer, and for high rates shorter than recommended.

Most of the early studies reported that the optimal rotation can be shorter than the recommendations (see Table 4). In the present study, the rotation of Norway spruce can be 20-30 years shorter than the recommendations for practice. A similar result was obtained in the growth and yield study of Vuokila (1985) as well.

Figure 17a illustrates that all the optimal solutions at 3% interest rate for an MT site were legal. However, Figure 17b shows that the optimal solutions of plots 45 and 61 were illegal for an OMT site. Note that plot 61 gave the highest bare land value of all OMT site plots.

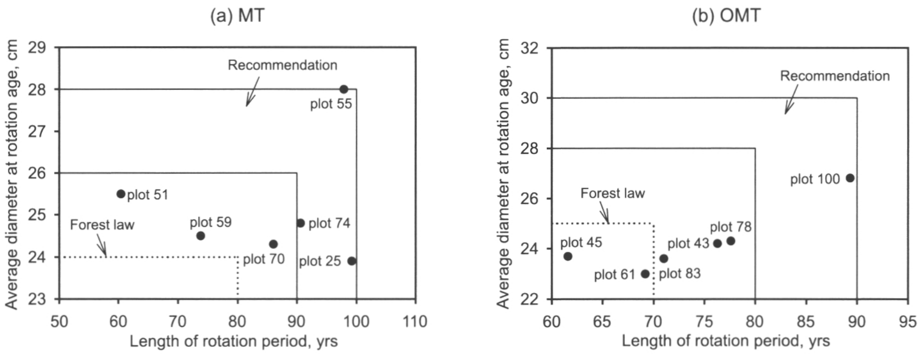


Figure 17. Optimal solution, law and recommendations for MT and OMT sites at 3% rate of interest.

6 Conclusions and Limitations

From this study lead some clear conclusions can be drawn about optimal stand management of Norway spruce in eastern Finland. It was found that two thinnings, thinnings from above, postponed thinnings, was optimal for moderate density stands (with initial density about 1900 trees per hectare), at 3% rate of interest.

With 3% rate of interest, the legal limits did not restrict the optimal management of most Norway spruce stands analyzed, but the legal limits restricted the average diameter for an OMT site in this study (see Table 9). In addition, the optimal rotation length for most plots in this study are shorter than in Valsta (1992b) and Hyytiäinen and Tahvonen (2001).

The optimal rotation length decreased with an increasing rate of interest. The MSY rotation was about 15 years shorter than the rotation based on Forest Rent. Thinning from below was more profitable when the interest rate was less than 2%. By contrast, thinning from above was superior with 3-5% rates of interest. Increasing the rate of interest led to a lower final volume, a narrower and more highly peaked diameter distribution, and a lower mean diameter.

This study was restricted by the availability of economic and biological data. First of all, the roadside prices of pulpwood and sawlog were fixed in this study. In practice, the market price fluctuates with demand and supply over time. Changes in demand and supply may also cause the changes in relative prices of timber products (sawlog, pulpwood). This suggests the need for modeling roadside timber prices. On the other hand, non-timber values also sometimes affect decision making. This may require further study as well. In this study, the analyzed plots were different in initial age, establishment methods and the other biological characteristics. The strong assumption was made that the naturally regenerated stand gets seeds from adjacent stands after clearcutting. If the shelter establishment method for natural regeneration is employed, the optimal solutions might be different. However, owing to the lack of good regeneration models, it is not possible to optimize the method of regeneration (Pukkala et al. 1998).

Valsta (1992b) assumes that thinnings that are late, heavy or from above may expose the stand to wind fall or snow break. Such natural hazards and logging damage, were ignored in the present study and other Finnish optimization studies (Pukkala et al. 1998, Hyytiäinen & Tahvonen 2002a). From this point of view, one would need an applicable model in which the probability of natural hazards or logging damage depends on logging.

The individual-tree growth models are based on statistical treatment of extensive field measurements. This means that the parameter determination in the statistical models is based on measurements from present forests and experiments and hence these values can be applied only within the domain of the data. Process-based models enable causal analysis of the effects of various silvicultural activities and harvesting on growth, since they often include a description of the mechanisms conveying the effects. It might be worthwhile to extend the existing economic research on stand-level forest management by applying a process-based model of forest growth with forest economic optimization.

References

Amidon, E. L., and Akin, G. S. 1968. Dynamic Programming to Determine Optimal Levels of Growing Stock. Forest Science 14: 287-291.

Brodie, J. D., Adams, D. M. and Kao, C. 1978. Analysis of Economic Impacts of Thinning and Rotation for Douglas-fir Using Dynamic Programming. Forest Science 24: 513-522.

Brodie, J. D. and Kao. C. 1979. Optimizing Thinning in Douglas-fir with Three Descriptor Dynamic Programming to Account for Accelerated Diameter Growth. Forest Science 25: 665-672.

Cawrse, D. C., Betters, D. R. & kent, B. M. 1984. A variational solution technique for determining optimal thinning and rotational schedules. Forest Science 30(3): 793-802.

Clark, C. M. 1976. Mathematical bioeconomics. Wiley, New York. 352 pp.

Davis, L. S., Johnson, K. N., Bettinger, P. S., Howard, T. E.. 2001. Forest Management: To Sustain Ecological, Economic, and Social Values. Fourth Edition. McGraw-Hill. 804 pp.

Eriksson, L. O. 1994. Two Methods for Solving Stand Management Problems Based on a Single Tree Model. Forest Science 40: 732-758.

Getz, W. M. and Haight, R. G. 1989. Population Harvesting: Demographic Models of Fish, Forest and Animal Resources. Princeton University Press. N. J. 225-319.

Faustmann, M. 1849. Reprinted in Translation as: Calculation of the Value which Forest Land and Immature Stands Possess for Forestry. Journal of Forest Economics. 1: 7-44. (The original article was published in Allgemeine Forst- und Jagd-Zeitung, vol.15, 1849.)

Haight, R. G. 1987. Evaluating the Efficiency of Even-aged and Uneven-aged Stand Management, Forest Science 33: 116-134.

Haight, R. G. and Monserud, R. A. 1990a. Optimizing Any-aged Management of Mixed-Species Stands: I. Performance of a Coordinate-search Process. Canadian Journal of Forest Research 20: 15-25.

Haight, R. G. and Monserud, R. A. 1990b. Optimizing Any-aged Management of Mixed-species Stands: II. Effects of Decision Criteria. Forest Science 36: 125-144.

Haight, R. G., Brodie, J. D. and Dahms, W. G. 1985. A Dynamic Programming Algorithm for Optimization of *Lodgepole* Pine Management. Forest Science 31: 321-330.

Hooke, R. and Jeeves, T. A. 1961. “Direct Search” Solution of Numerical and Statistical Problems. Journal of the Association for Computing Machinery 8: 212-229.

Hynynen, J., Ojansuu, R., Hökkä, H., Siipilehto, J., Salminen, H. and Haapala, P. 2002. Models for Predicting Stand Development in MELA System. Finnish Forest Research Institute, Research Papers 835. 116 pp.

Hyytiäinen, K. and Tahvonen, O. 2001. The Effects of Legal Limits and Recommendations in Timber Production: The Case of Finland. Forest Science 47 (4): 443-454.

Hyytiäinen, K. and Tahvonen, O. 2002a. Economics of Forest Thinnings and Rotation Periods for Finnish Conifer Cultures. Scandinavian Journal of Forest Research 17: 274-288.

Hyytiäinen, K. and Tahvonen, O. 2002b. MSY, Forest Rent or Faustmann – Does It Really Matter? (manuscript). 31 pp.

Hyytiäinen, K., Tahvonen, O. and Valsta, L. 2002. Optimal Juvenile Density, Harvesting and Stand Structure in Even-aged Scots Pine Stands. (manuscript). 35 pp.

Hämäläinen, J. 1978. Harvennustavan vaikutus metsikön hakkuutuloihin, puuston arvoon ja kasvatuksen edullisuustunnuksiin. Puuntuotannon nykyhetken ongelmia. Metsäntutkimuslaitoksen 60-vuotisjuhlaretkeily 12.-13.6.1978. Metsäntutkimuslaitos. 15-21.

Kao, C., and Brodie, J. D. 1980. Simultaneous Optimization of Thinnings and Rotation with Continuous Stocking and Entry Intervals. Forest Science 26: 338-346.

Kilkki, P. and Väisänen, U. 1969. Determination of the Optimal Cutting Policy for the Forest Stand by Means of Dynamic Programming. Acta Forestalia Fennica 102: 1-23.

Klemperer, W. D. Forest Resource Economics and Finance. 1996. McGraw-Hill. 551 pp.

Kuitto, J.-P., S. Keskinen, J. Lindroos, T. Ojala, J. Rajamäki, T. Räsänen, and Terävä. 1994. Mechanized Cutting and Forest Haulage. Metsäteho Report 410. Painovaliste KY, Helsinki. (In Finnish). 38 pp.

Laasasenaho, J. 1982. Taper Curve and Volume Functions for Pine, Spruce and Birch. Communicationes Instituti Forestalis Fenniae. 108: 74 pp.

Mielikäinen, K. & Riikilä, M. 1997. Kannattava puuntuotanto. Gummerus, Jyväskylä. 140 p. ISBN 952-5118-06-1. (In Finnish)

Möykkynen, T., Miina, J. & Pukkala, T. 2000. Opimizing the Management of a *Picea abies* Stand Under Risk of Butt Rot. Forest Pathology 30: 65-76.

Möykkynen, T. & Miina, J. 2002. Optimizing the Management of a Butt-rotted *Picea abies* Stand Infected by *Heterobasidion annosum* from the Previous Rotation. Scandinavian Journal of Forest Research 17: 47-52.

Niemelä, T. & Korhonen, K. 1998. Taxonomy of the Genus *Heterobasidion*. *In* Woodward, S., Stenlid, J., Karjalainen, R. & Hüttermann, A. (eds). *Heterobasidion annosum*. Biology, Ecology, Impact and Control, 27-33. CAB International, Oxon. ISBN 0-85199-275-7.

Nyyssönen, A. 1958. Rotation and Its Determination. Communicationes Instituti Forestalis Fenniae. 49(6).

Osyczka, A. 1984. Multicriterion Optimization in Engineering with FORTRAN Programs. Ellis Horwood, Chichester. 178 pp.

Pesonen, M., and Hirvelä, H. 1992. Thinning Models Based on Profitability Calculations for Southern Finland. Folia Forestalia. 800.

Piri, T., Korhonen, K. & Sairanen, A. 1990. Occurrence of *Heterobasidion annosum* in Pure and Mixed Spruce Stands in Southern Finland. Scandinavian Journal of Forest Research. 5: 113-125.

Pukkala, T. and Miina, J. 1998. Tree-selection Algorithms for Optimizing Thinning Using a Distance-dependent Growth Model. Canadian Journal of Forest Research 28: 693-702.

Pukkala, T., Miina, J., Kurttila, M., and Lolström, T. 1998. A Spatial Model for Optimizing the Thinning Regime for Mixed Stands of *Pinus sylvestris* and *Picea abies*. Scandinavian Journal of Forest Research 13: 31-42.

Rautiainen, O., Pukkala, T., and Miina, J. 2000. Optimizing the Management of Even-aged *Shorea robusta* Stands in Southern Nepal Using Individual Tree Growth Models. Forest Ecology and Management 126: 417-429.

Roise, J. P. 1986. An Approach for Optimizing Residual Diameter Class Distribution when Thinning Even-aged Stands. Forest Science 32: 871-881.

Salminen, O. 1993. Simultaneous Optimization of Thinnings and Rotation of Cultivated Stands Using Dynamic Programming. Finnish Forest Research Institute. Research Papers. 480. 48 pp.

Smith, D., Larson, B., Kelty, M., and Ashton, P.. 1997. The Practice of Silviculture: Applied Forest Ecology. (9th edition). New York: John Wiley & Sons. 537 pp.

Stenlid, J. & Redfern, D. B. 1998. Spread within the Tree and Stand. *In* Woodward, S., Stenlid, J., Karjalainen, R. & Hüttermann, A. (eds). *Heterobasidion annosum*. Biology, Ecology, Impact and Control, 125-141. CAB International, Oxon. ISBN 0-85199-275-7.

Solberg, B. and Haight, R. G. 1991. Analysis of Optimal Economic Management Regimes for *Picea abies* Stands Using a Stage-Structured Optimal-Control Model. Scandinavian Journal of Forest Research 6: 559-572.

Swedjemark, G. & Stenlid, J. 1993. Population Dynamics of the Root Rot Fungus *Heterobasidion annosum* Following Thinning of *Picea abies*. Oikos 66: 247-254.

Tapio. 2001. Recommendations for Good Silviculture. Metsätalouden Kehittämiskeskus Tapio, Libris Oy, Helsinki. (In Finnish). 95 pp.

Tapio. 2002. Tapio's Handbook (Tapion Taskukirja, in Finnish). Edition 24. Metsätalouden Kehittämiskeskus Tapio, Gummerus Kirjapaino Oy. 555 pp.

Valsta, L. 1992a. A Scenario Approach to Stochastic Anticipatory Optimization in Stand Management. Forest Science 38: 430-447.

Valsta, L. 1992b. An Optimization Model for Norway Spruce Management Based on Individual-tree Growth Models. Acta Forestalia Fennica 232. 20 pp.

Valsta, L. 1993. Stand Management Optimization Based on Growth Simulators. Finnish Forest Research Institute, Research Papers 453. 51 pp.

Valsta, L. and Linkosalo, T. 1996. Stand Management Assistant (SMA). Puuston Kehityksen Ennustaminen - MELA ja Vaihtoehtoja. Finnish Forest Research Institute. Research Papers 612: 85-106.

Venn, K. & Solheim, H. 1994. Root and Butt Rot in First Generation of Norway Spruce Affected by Spacing and Thinning. *In* Johansson, M. & Stenlid, J. (eds). Proceedings of the Eighth International Conference on Root and Butt Rots, 642-645. Swedish University of Agriculture Science, Uppsala. ISBN 91-576-4803-4.

Vettenranta, J. and Miina, J. 1999. Optimizing Thinnings and Rotation of Scots Pine and Norway Spruces Mixtures. Silva Fennican 33: 73-84.

Vollbrecht, G. & Stenlid, J. 1999. Transfer of the P-type of *Heterobasidion annosum* from Old-growth Stumps of *Picea abies* to *Picea abies* and *Larix X eurolepis*. European Journal of Forest Pathology. 29: 153-159.

Vuokila, Y. 1967. Growth and Yield Tables for Pine Stands Treated with Intermediate Cuttings of Varying Degree for Southern Central-Finland. Communicationes Instituti Forestalis Fenniae 63(2). 123 pp. (In Finnish, with English summary)

Vuokila, Y. 1977. Selective Thinning from Above as a Factor of Growth and Yield. Acta Forestalia Fennica 110. (In Finnish, with English summary)

Vuokila, Y. 1985. Puuston määrän vaikutus istutuskuusikon kehitykseen, kasvuun ja tuotokseen. Summary: The Effect of Growing Stock Level on the Development, Growth and Yield of Spruce Plantations in Finland. Folia Forestalla 636: 1-18. (In Finnish, with English summary)

Appendix 1. MSY, Forest Rent and optimal solutions for plots 59, 61 and 83 with different rates of interest.

MSY, Forest Rent and optimal solutions in plot 59 with different rates of interest

			Rate of interest (%)				
	MSY	Forest Rent	1	2	3	4	5
1st thinning (yrs)	51.6	52.0	52.0	52.0	51.3	46.9	44.4
- Intensity (%)	7.1	21.0	16.0	25.0	42.3	42.1	42.1
- S-trees (%)	73.7	72.4	16.0	16.0	16.0	16.0	16.0
- M-trees (%)	1.3	16.8	16.0	16.0	16.0	16.0	16.0
- L-trees (%)	1.2	16.0	16.0	44.4	99.0	99.0	99.0
2nd thinning (yrs)	64.1	81.4	67.3	61.4	57.5		46.6
- Intensity (%)	9.9	11.2	15.5	28.8	18.2		17.6
- S-trees (%)	96.9	69.4	1.0	1.0	1.0		1.0
- M-trees (%)	2.5	11.6	1.26	1.0	1.0		1.0
- L-trees (%)	10.2	2.3	45.1	99.0	99.0		99.0
3rd thinning (yrs)	84.7	107.3	80.0				
- Intensity (%)	6.9	6.7	28.7				
- S-trees (%)	81.3	55.5	1.0				
- M-trees (%)	4.7	3.1	1.0				
- L-trees (%)	4.8	8.3	98.9				
4th thinning (yrs)	97.6	123.3					
- Intensity (%)	10.5	10.7					
- S-trees (%)	1.0	1.0					
- M-trees (%)	7.5	1.4					
- L-trees (%)	15.7	24.4					
5th thinning (yrs)	112.7						
- Intensity (%)	6.6						
- S-trees (%)	1.1						
- M-trees (%)	5.2						
- L-trees (%)	10.2						
6th thinning (yrs)	123.0						
- Intensity (%)	1.6						
- S-trees (%)	1.2						
- M-trees (%)	2.0						
- L-trees (%)	1.2						
M.A.I. (m^3/year)	12.2	11.9	11.3	10.3	9.7	9.0	8.5
Rotation age (yrs)	127.8	143.6	101.7	77.5	73.8	61.2	59.0

MSY, Forest Rent and optimal solutions in plot 61 with different rates of interest

			Rate of interest (%)				
	MSY	Forest Rent	1	2	3	4	5
1st thinning (yrs)	44.7	45.2	45.2	45.2	45.2	45.2	45.1
- Intensity (%)	10.7	20.6	18.2	16.0	42.8	42.8	42.8
- S-trees (%)	62.2	48.2	29.7	16.0	16.0	16.0	16.0
- M-trees (%)	5.6	17.1	17.1	16.0	16.0	16.0	16.0
- L-trees (%)	1.0	16.6	16.0	16.0	99.0	99.0	99.0
2nd thinning (yrs)	60.8	65.6	61.1	56.8	61.0		
- Intensity (%)	9.3	9.9	21.5	33.1	14.8		
- S-trees (%)	99.0	39.9	7.0	1.0	1.0		
- M-trees (%)	1.4	5.5	1.1	1.6	1.0		
- L-trees (%)	1.0	8.9	60.6	99.0	99.0		
3rd thinning (yrs)	81.9	82.7	73.8	71.7			
- Intensity (%)	10.6	16.2	24.6	20.4			
- S-trees (%)	99.0	45.0	3.0	1.1			
- M-trees (%)	2.1	4.8	2.0	8.3			
- L-trees (%)	12.1	28.5	95.0	99.0			
4th thinning (yrs)	100.5	107.4	89.8				
- Intensity (%)	6.0	7.6	11.5				
- S-trees (%)	95.5	3.8	4.2				
- M-trees (%)	1.2	4.2	11.9				
- L-trees (%)	2.4	14.1	17.2				
5th thinning (yrs)	115.4	120.8					
- Intensity (%)	4.6	12.2					
- S-trees (%)	35.5	1.0					
- M-trees (%)	1.4	14.8					
- L-trees (%)	5.8	9.4					
6th thinning (yrs)	126.3						
- Intensity (%)	4.0						
- S-trees (%)	20.1						
- M-trees (%)	3.4						
- L-trees (%)	3.3						
M.A.I. (m^3/year)	10.3	9.9	9.6	9.1	8.3	7.9	7.5
Rotation age (yrs)	136.9	150.3	103.4	85.8	69.2	62.2	56.8

MSY, Forest Rent and optimal solutions in plot 83 with different rates of interest

			Rate of interest (%)				
	MSY	Forest Rent	1	2	3	4	5
1st thinning (yrs)	43.5	46.5	46.5	46.5	46.5	46.5	41.2
- Intensity (%)	9.8	26.9	16.2	16.0	45.1	45.1	44.9
- S-trees (%)	78.1	97.5	16.0	16.0	16.0	16.0	16.0
- M-trees (%)	1.6	19.9	16.1	16.0	16.0	16.0	16.0
- L-trees (%)	1.0	16.0	16.6	16.0	99.0	99.0	99.0
2nd thinning (yrs)	62.3	84.6	59.1	57.9	57.7	52.6	50.3
- Intensity (%)	9.0	14.7	9.5	35.6	18.8	18.4	19.8
- S-trees (%)	99.0	62.0	5.8	1.0	1.00	1.0	1.0
- M-trees (%)	2.5	18.5	2.8	1.0	1.00	1.0	3.0
- L-trees (%)	1.9	3.1	20.9	99.0	99.0	99.0	99.0
3rd thinning (yrs)	81.3	118.8	66.8	65.0			
- Intensity (%)	7.9	15.6	25.3	18.8			
- S-trees (%)	99.0	22.7	2.2	1.0			
- M-trees (%)	5.0	3.2	2.6	1.0			
- L-trees (%)	2.5	28.6	70.5	99.0			
4th thinning (yrs)	97.3		83.9				
- Intensity (%)	7.8		26.5				
- S-trees (%)	89.7		3.4				
- M-trees (%)	3.0		3.4				
- L-trees (%)	1.5		97.9				
5th thinning (yrs)	118.6						
- Intensity (%)	4.3						
- S-trees (%)	78.5						
- M-trees (%)	1.2						
- L-trees (%)	1.3						
6th thinning (yrs)	133.6						
- Intensity (%)	2.1						
- S-trees (%)	1.1						
- M-trees (%)	2.7						
- L-trees (%)	1.5						
M.A.I. (m^3/year)	11.0	10.5	10.2	9.5	8.7	8.3	7.9
Rotation age (yrs)	134.6	147.5	105.6	83.2	71.0	63.8	61.8

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