

Modelling tree growth for managed stands

Jari Hynynen



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ERRATA

Page 40: Error in table 5 concerning the parameters a_2 , a_3 and a_4 .
Corrected Table 5 is as follows:

Table 5. Parameter estimates of diameter growth model [17].

Parameter	Estimate	Asymptotic Std. Dev.
a_0	0.4338	0.0683
a_1	0.3761	0.0375
a_2	0.5846	0.0246
a_3	-0.0006	0.00007
a_4	-0.0012	0.00005
a_5	-0.4424	0.0408
a_6	1.1076	0.0430
a_7	-0.5575	0.0147
Fertilization with 150 kg N ha ⁻¹		
d_{11}	0.3475	0.0148
d_{12}	0.7531	0.0196
d_{13}	0.7576	0.0285
Fertilization with 300 kg N ha ⁻¹		
d_{21}	0.4314	0.0156
d_{22}	0.7804	0.0203
d_{23}	0.6600	0.0277
\hat{i}_{d5}	1.524	
RMSE	0.516	
Observations	9669	

Page 48: In the text, it is referred to Table 6. The correct reference is Table 9

Page 50: Error in the title of the table. The number should be Table 9.

Error concerning the parameters a_2 , a_3 and a_4 .

Corrected Table 9 is as follows:

Table 9. Parameter estimates of model for form factor [21].

Parameter	Estimate	Asymptotic Std. Dev.
a_0	0.5653	0.0179
a_1	-0.0231	0.0025
a_2	2.9502	0.1859
a_3	2.5657	0.2442
a_4	-0.9295	0.0658
Fertilization with 150 kg N ha ⁻¹		
d_{11}	0.0128	0.0017
d_{12}	0.0167	0.0017
d_{13}	0.0075	0.0021
Fertilization with 300 kg N ha ⁻¹		
d_{21}	0.0126	0.0018
d_{22}	0.0175	0.0018
d_{23}	0.0089	0.0022
$f_{1,3}$	0.534	
RMSE	0.0255	
Observations	12 959	

Study IV, page 15, Table 5.

In the table, $\overline{IH_{dom}^{\hat{}}}$ should be $\ln(\overline{IH_{dom}^{\hat{}}})$

Study IV, page 22, Table 9:

Corrected Table 9, in which the corrected numbers are marked in bold font, is as follows:

Table 9. Behaviour of the models against the independent test data.

	Diameter growth ¹⁾		Height growth ¹⁾	Tree volume (form factor)
	Model [1]	Model [3]	Model [5]	Model [8]
Observed, mean	1.302 cm	1.302 cm	1.949 m	150.09 dm ³
Predicted, mean	1.274 cm	1.221 cm	1.939 m	158.38 dm ³
Absolute bias	0.028 cm	0.081 cm	0.011 m	-8.296 dm ³
Relative bias	0.0387	0.088	0.0426	-0.0577
RMSE	0.639	0.647	0.383	13.039
RMSE _r	0.716	0.761	0.307	0.067
No. of obs.	3551	3551	3523	8583

¹⁾ Five-year growth period

Study IV, page 27, 2bd paragraph:

.....thinning response was at its maximum (Fig. 4).....

should bethinning response was at its maximum (Fig. 3).....

Modelling tree growth for managed stands

Jari Hynynen

Academic dissertation

*To be presented, with the permission of the Faculty of
Agriculture and Forestry of the University of Helsinki,
for public criticism in Auditorium M II, Metsätalo,
Unioninkatu 40 B, Helsinki, on 15 December 1995,
at 12 o'clock noon.*

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In this report, individual-tree distance-independent models for managed stands are presented. The purpose of the modelling work was to develop modelling methods in prediction of growth responses to silvicultural practices, and to construct models for forest management planning purposes based on the information that is available in the data from forest inventories.

The aim was to develop models that are capable for growth prediction in managed stands with varying intensities of thinning and fertilization treatments. Models for prediction of tree growth in fertilized stands include tree basal area growth models for Scots pine (*Pinus sylvestris* L.) stands, and tree diameter and height growth models for loblolly pine (*Pinus taeda* L.) stands. The effects of thinning from below in Scots pine stands were quantified by developing the models for tree crown ratio, for tree diameter and height growth, and for cylindrical stem form factor. The interaction of thinning and fertilization on the development of these tree characteristics was quantified and modelled for Scots pine stands, as well.

In the growth models for tree basal area, diameter and height, the magnitude of growth response to treatment (thinning or fertilization) was expressed as a function of intensity of the treatment. Weibull function was employed to describe the temporal distribution of the response.

Models were based on data from repeatedly measured permanent experimental stands established in even-aged stands for Scots pine located in southern and central Finland, and in midrotation plantations for loblolly pine located across southeastern United States.

The results confirmed that it is possible to successfully predict tree growth and growth response to thinning and fertilization based on taxatorical tree and stand characteristics, that are commonly available in the data from practical forest inventories.

Keywords: growth modelling, individual-tree, distance-independent, growth response, Weibull distribution, thinning, fertilization, crown ratio, stem form, *Pinus sylvestris* L., *Pinus taeda* L.

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Preface

This study was mostly carried out in the former Department of Forest Production, and completed in Vantaa Research Centre of the Finnish Forest Research Institute. I thank the Institute for including the study in the research program, and for providing me with excellent working facilities. Ministry of Agriculture and Forestry of Finland, The Academy of Finland, and Metsämiesten Säätiö are gratefully acknowledged for financing this study.

The idea of modelling the individual-tree growth response to silvicultural practices initiated when I worked on forest fertilization research with Mr. Mikko Kukkola (M. For.). I am very thankful for the advice and the comments that I have received from him during many phases of this work. I am indebted to Dr. Risto Ojansuu for many inspiring discussions concerning growth modelling, and for numerous excellent comments that have proved to be most valuable throughout the work.

The one year period I worked at Virginia Polytechnic Institute and State University in Blacksburg, USA, had a strong influence in my modelling approach. I am very grateful to Professor Harold E. Burkhart for the guidance during that year, and for teaching me growth and yield modelling. I am also grateful to Mr. Ralph Amateis (M. Sc.) for encouraging criticism and inspiring discussions. I gratefully acknowledge Professor Lee Allen from North Carolina State University, and North Carolina State Forest Nutrition Cooperative for providing data for this study.

I am indebted to Professor Simo Poso for the encouraging support in my graduate studies. I am grateful to Professor Kari Mielikäinen for the support and advice he has provided me in the research work. I thank Professor Thomas E. Burk and Dr. Annikki Mäkelä-Carter who reviewed the manuscript, for their valuable comments and clarifying suggestions. Thanks are also due to Dr. Helena Henttonen, Mr. Hannu Salminen (M. For.), and Mr. Sauli Valkonen (Lic. For.) for comments on my work.

I am indebted to Mr. Tapio Ylimartimo, who strongly contributed to the data collection, and to Ms. Marja-Liisa Herno for finishing the figures and the layout of the manuscript. I appreciate the revision of the English language by Mr. Erkki Pekkinen (M. For.).

Finally, I thank my wife Päivi, and my daughters Suvi-Kukka and Sini-Tuuli for the continuous support, patience and understanding.

Vantaa, November 1995

Jari Hynynen

The following studies are reviewed in the course of this dissertation. They are referred to in the text using Roman numerals:

- I **Hynynen, J. 1993.** Modelling tree basal area growth response after nitrogen fertilization. In: Burkhart, H.E., Gregoire, T.G. & Smith, J.L. (eds.). Modelling Stand Response to Silvicultural Practices. Proceedings of the IUFRO S4.01 Conference, Blacksburg, Virginia, USA, Sept. 27–Oct. 1, 1993. p. 61–72.
- II **Hynynen, J., Burkhart, H. E., and Allen, H. L. 1994.** Modelling tree growth in fertilized midrotation loblolly pine plantations. Manuscript. Submitted to be published in Forest Science. Revised as suggested by the reviewers.
- III **Hynynen, J. 1995.** Predicting tree crown ratio for unthinned and thinned Scots pine stands. Canadian Journal of Forest Research 25:57–62.
- IV **Hynynen, J. 1995.** Predicting the growth response to thinning for Scots pine stands using individual-tree growth models. Manuscript. Accepted to be published in Silva Fennica 9.11.1995.

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List of symbols

Stand variables

Age	= Stand age
H_{dom}	= Stand dominant height
H_{domt}	= Stand dominant height at time of thinning
D_{dom}	= Stand dominant diameter
$IH_{dom n}$	= Increment of stand dominant height during growth period of n years
H_{100}	= Site index for Scots pine
H_{25}	= Site index for loblolly pine
H_g	= Mean height, weighted with stand basal area
D_g	= Mean diameter (over bark), weighted with stand basal area
G	= Stand basal area, over bark
G_b	= Stand basal area (over bark) before thinning
G_a	= Stand basal area (over bark) after thinning
FN	= Nitrogen dose
FP	= Categorical variable referring to phosphorus fertilization; FP = 1, if fertilized with phosphorus, otherwise FP = 0
I	= Thinning intensity, defined as: $(G, \text{ before thinning} - G, \text{ after thinning})/G, \text{ after thinning}$
T	= Time elapsed from treatment (thinning or fertilization), years
$D_{502} \dots D_{558}$	= Categorical variable referring to experimental stands
$Y_1 \dots Y_n$	= Categorical variable referring to growth periods 1 to n
N_{ij}	= Categorical variable referring to fertilization treatment i during growth period j
H_{ij}	= Categorical variable referring to thinning treatment i during growth period j

Tree variables

d	= Diameter at breast height, over bark
i_{dn}	= Tree diameter growth during growth period of n years
i_g	= Annual tree basal area growth, over bark
$\Delta(p_g)$	= Relative annual tree basal area growth response
$d_{6,0}$	= Diameter at 6 m height, over bark
g	= Tree basal area at breast height, over bark
h	= Tree height
i_{hn}	= Tree height growth during growth period of n years
v	= Tree volume
GL	= Basal area of trees (over bark) larger than subject tree
cr	= Tree crown ratio, defined as: Length of live crown/total tree height
$f_{1.3} = v/gh$	= Cylindrical form factor

Other symbols

a_i, c_i, d_i, b, c	= Parameters
e	= Error term

1 Introduction

1.1 Growth prediction for forest management planning

Forest management planning requires information about existing forest resources and forecasts about their development. Growth and yield models are tools for obtaining information about the future development of forests. Growth predictions are needed for several purposes in the field of forest management planning. Depending on the objectives and the level of forest management planning, different requirements are imposed on models. In the management planning focusing on an individual stand, the main interest is usually in evaluating alternative management schedules and in assessing the effects of different silvicultural treatments. On the other hand, when forest management is planned on the country level, a basic requirement for growth prediction may be to obtain unbiased estimates of the development of the forest resources at the country level. Also, the time span of planning affects the requirements for growth and yield models. In updating forest inventory data, the time horizon in growth prediction may be only a few years. However, when strategies for forest policy are planned, future forecasts covering some decades are needed, together with the ability to evaluate the effects of various management schedules. Burkhart (1992) has classified the primary use of the information provided by growth and yield models into the following categories: (1) inventory updating, (2) evaluation of silvicultural alternatives, (3) management planning, and (4) harvest scheduling.

It is obvious that there is no one growth model that would fulfil all the requirements of the different management purposes. There are situations, however, when a single set of models is employed to provide information for many purposes covering the aforementioned categories. Growth and yield models developed for the Finnish MELA System (Siitonen 1983, Siitonen 1994, Hynynen et al. 1995) serve as a good example of a set of models applied in forest management planning at various levels with varying planning time spans. Although MELA was originally designed for the purpose of analysing long-term timber production potentials at regional and national levels, the models used in MELA are also widely applied in practical forestry in stand level applications, as well as in forest research (Siitonen 1983, 1994).

1.2 General model characteristics

Models developed to be directly applicable in forest management planning programs are often referred to as empirical models (Munro 1974). The name empirical indicates that the models are based on periodically collected tree measurement data, but that no attempt is made to measure every factor that may affect tree growth (Bruce and Wensel 1987). Despite their name, many empirical models for estimating tree size and stand development include mathematical functions appropriate for describing biological processes.

Growth models may be classified according to many different criteria based on some of the characteristics of models. The most widely applied classification is that provided by Munro (1974), who divided growth models into single-tree models and whole-stand models. Regardless of the categories into which models may be grouped, there are some general requirements that empirical growth and yield models should meet, but there are also some constraints needing to be taken into account in model development.

In countries like Finland, where intensive forest management of commercial forests has been practised for many decades, evaluation of alternative management schedules is an essential aspect of forest management planning. Therefore, the growth and yield models applied in the planning should be capable of reliably predicting the effects of different silvicultural treatments on the development of managed stands. For long-term planning, growth models are required for predicting both the effects of silvicultural practices currently applied in practical forestry as well as the effects of the more extreme levels of these practices that may not be applied at the present.

In forest management planning and in decision-making in forest policy, long-term forecasts of forest resource development are needed. At regional and country levels, information about forest resources is obtained from national forest inventories. Therefore, models used for these purposes need to be compatible with forest inventory data. They should be relatively simple and straightforward; after all, they are based only on a few measured stand and tree variables. On the other hand, such models should be well-designed in order to be reliable and behave in a logical way when applied in long-term forecasts. The relationships between the model variables should be described on a sound, biological and ecological basis by applying functions suitable for describing biological processes. Well-designed model structure ensures logical behaviour even when applied outside the limits of the data the models are based on. Sometimes, compromises must be made at the expense of the best possible statistical fit in order to achieve model structures that behave logically.

One of the most important constraints in developing models for forest management planning purposes arises from the data available for model development. Forest inventories are the only source of representative and reliable information about the existing forest resources. This information provides the basis and the starting point for future forecasts. A large amount of information is gathered in forest inventories covering large land areas. Thus, the information on a single stand and individual trees cannot be very detailed. There are many stand and tree characteristics that could provide valuable information for growth prediction, but recording all such interesting variables would be too time consuming, and therefore too expensive, to accomplish. In practice, the input variables of models need to be confined to those available in forest inventory data (Burkhart 1992). The driving variables in models should be available both in modelling data as well as in the data models are applied to. It is of little use to develop detailed models for forest management planning that include numerous variables, if these variables are not present in the database used as the basis of simulations.

1.3 The role of modelling data

Ideal modelling data would be a representative sample of the forests in the region in which the models will be applied, e.g. national forest inventory data. At the same time, the said data should include a wide range of silvicultural treatments (spacing, thinning, fertilization) as in purpose-designed experiment data. In addition, the data should include observations over extensive time periods. Obviously, these kind of data never exist in reality. Therefore, compromises have to be made in choosing the modelling data.

Data sources for model development may be seen to be divided into two main groups; forest inventory data and purpose-designed experimental data. Both types of the data are usable in model development. Forest inventory data meet the requirement according to which data should be a representative sample of forests. Inventory data are needed for growth models to be used in inventory updating and in growth predictions for large areas. However, problems arise when models are intended to be suitable for evaluating silvicultural practices. Assuming that the management practices in forestry do not change in the future, it would be adequate to use forest inventory data collected from commercial forests, and including all the treatments applied in practical forestry, as the modelling data. However, if the purpose is to develop models capable of also predicting the effects of more extreme silvicultural treatments, forest inventory data are no longer adequate

because extreme treatments may not be sufficiently represented in the data.

The most suitable sources of information for modelling the effects of silvicultural treatments are purpose-designed permanent experiments for growth and yield research. Such experiments enable one to obtain information about the responses to various silvicultural practices while minimising other disturbing variation. Information obtained from experimental stands helps growth modellers understand the interaction between the factors influencing tree and stand growth, and appreciate change in these relationships due to silvicultural treatments (e.g. thinning, fertilization), or genetic improvement. There are, however, some serious disadvantages in using these data for modelling. The experimental stands are usually subjectively chosen, stand structure is more homogeneous in them than it is in commercial stands, and the applied silvicultural treatments are more controlled than they are in commercial forests on average. When growth models based on this kind of non-representative data are applied to average commercial forests, the resultant growth predictions may be seriously biased.

One possibility for combining inventory data and data from purpose-designed experiments is to use data collected from designed experimental stands in developing the basic structure of growth and yield models. Thereafter, the final parameter estimates can be obtained by refitting the models to representative and more comprehensive data, or by calibrating the models in order to arrive at unbiased overall growth predictions in the forests the models are planned to be applied to.

1.4 Purpose of the study

The main purpose of this doctoral dissertation study was develop individual-tree, distance-independent models capable of predicting development of managed stands. In model development, the goal was to construct models appropriate for use in forest management by restricting the input variables of the models to those usually measured in forest inventories.

The models are based on data collected from experimental stands. Therefore, they are not directly applicable for growth prediction in typical commercial stands. Nevertheless, the goal was to elaborate model structures applicable in developing models based on more representative data. With each model, the aim was to develop a model structure resulting in logical behaviour even outside the range of the modelling data.

One of the main interests of this study was to examine modelling methods in predicting growth responses to thinning from below and responses to forest fertilization, the two most widely applied silvicultural practices in commercial forests of Finland. The purpose was to elaborate on flexible models capable of predicting tree growth in both untreated stands and in stands with different levels of treatment intensity.

The length of the live crown has been recently included among the tree characteristics measured in connection with many forest inventories. It is the only variable that directly describes one dimension of the tree crown available for modelling purposes. The suitability of the tree-crown ratio was studied by incorporating it in the growth models. A matter of special interest was to examine the importance of the tree crown ratio in predicting growth responses to thinning and fertilization. As a regressor variable in the growth models, the tree crown ratio also needs to be updated when simulating stand development. Therefore, a model for predicting the tree crown ratio was developed in this study as well. Again, attention was paid to modelling the effects of thinning and fertilization on the development of the tree crown ratio.

This dissertation reviews four studies with the following specific objectives:

- Study I: – to develop a model for predicting tree basal area growth response to nitrogen fertilization in Scots pine stands
- Study II: – to develop tree diameter and height growth models for fertilized, mid-rotation loblolly pine stands
- Study III: – to examine the effect of thinning on development of tree crown ratios in Scots pine stands, and to develop a model for tree crown ratio for thinned and unthinned Scots pine stands
- Study IV: – to examine the effect of thinning from below on tree diameter growth in Scots pine stands, and to develop a tree diameter growth model for thinned and unthinned Scots pine stands
 - to examine the effect of thinning on the increment in stand dominant height, and on individual-tree height growth in Scots pine stands, and to develop a tree height growth model for unthinned and thinned Scots pine stands
 - to examine the effect of thinning on the development of tree stem form in Scots pine stands, and develop a model for the cylindrical stem form factor for Scots pine stands.

Chapter 2 sets out in brief the study material used in this dissertation, including the reviewed studies. Chapter 3 is a brief review of the results of earlier fertilization studies, as well as of the main results of studies I and II. The main results and applicability of the models constructed in I and II are discussed. As regards the effect of fertilization on the development of the tree crown ratio and stem form, only earlier research results are reviewed. Models for these tree

characteristics could not be constructed in studies **I** or **II** due to the lack of suitable modelling data. Chapter 4 discusses growth prediction in unthinned and thinned stands. The main results of studies **III** and **IV** concerned with modelling the crown ratio, individual-tree growth and stem form for unthinned and thinned Scots pine stands are reviewed and discussed. The interaction between thinning and fertilization is examined and models are presented in Chapter 5. The purpose of the analysis was to examine whether such an interaction exists between thinning and fertilization that cannot be predicted by applying thinning and fertilization response models separately. The effect of fertilization was incorporated in the models for Scots pine stands presented in studies **III** and **IV**. Finally, the foremost conclusions based on the modelling efforts are discussed in Chapter 6.

2 Study material

The models presented in this dissertation are based on three different data sources. Individual-tree basal area growth models developed in study **I** are based on data collected from eight experimental Scots pine stands (*Pinus sylvestris* L.) located in southern Finland. The experiments were established in the early 1970s in naturally regenerated, middle-aged stands growing on mineral soil sites. The experiments were arranged using factorial design. Two types of nitrogen fertilizers (ammonium nitrate with lime and urea), and three levels of nitrogen doses were studied. The nitrogen doses varied between 80 kg N ha⁻¹ and 450 kg N ha⁻¹. The number of treatment replications within an experimental stand varied between 1 and 10. There was at least one untreated control plot in each stand. The size of the circular sample plots was 1 000 m².

The study period covered 10–15 years after fertilization. An average of 17 randomly located sample trees were selected from each sample plot. Increment cores taken from sample trees were examined to determine annual tree diameter growths for the preceding 15 years. The study material consisted of 2 158 sample trees from 117 sample plots. The number of annual growth observations was 37 800 (Table 1).

Individual-tree growth models for fertilized loblolly pine stands (*Pinus taeda* L.) were constructed in study **II**. Models were based on data from the North Carolina State Forest Nutrition Cooperative's (NCSFNC) Regionwide 13 Study. The study material included thirteen mid-rotation plantations of loblolly pine located across south-eastern United States. The experiments were established in 1984 and 1985. Four levels of nitrogen doses (0, 100, 200, and 300 lbs ac⁻¹), and three

levels of phosphorus doses (0, 25, 50 lbs ac⁻¹) were examined using a factorial experimental design. The number of treatment replicates within a stand varied between two and four. The size of the rectangular plots varied between 0.045 acres and 0.516 acres with the average size being 0.09 acres.

The study period covered eight years following fertilization treatment. The sample plots were measured at two-year intervals. All the trees on the sample plots were measured for their diameter and height, and these were included in the modelling data. Tree diameter and height growths were obtained as the difference between successive measurements. The study material consisted of 16 126 sample trees from 432 sample plots. The number of two-year growth observations was 57 900 (Table 2).

The modelling data for studies III and IV were obtained from permanent sample plots established in experimental Scots pine stands (*Pinus sylvestris* L.). The study material consisted of eleven even-aged stands located in southern and central Finland (Fig. 1). The experiments were established by the Finnish Forest Research Institute

Table 1. Study material from fertilized Scots pine stands(I).

Exp no	Age a	H ₁₀₀ m	H _g m	G m ² ha ⁻¹	Number of sample plots	Number of sample trees
224	85	24.1	20.0	19.7	52	866
301	75	24.6	18.7	16.3	21	440
312	60	26.9	19.5	25.1	7	173
315	80	21.5	17.2	16.9	7	127
319	120	20.0	21.2	16.4	7	116
320	120	19.6	21.4	17.9	6	101
332	85	22.9	19.0	17.3	7	141
336	65	23.1	16.5	18.5	10	194

Table 2. Study material from fertilized mid-rotation loblolly pine stands (II).

Exp no	Age a	H ₂₅ ft	H _g ft	G ft ² ac ⁻¹	Number of sample plots	Number of sample trees
130802	14	64	42.4	67.5	48	1597
130901	14	59	36.7	101.8	24	894
131101	12	74	42.7	103.3	24	948
132401	14	62	35.7	102.4	48	1900
132601	14	61	41.5	99.2	24	843
132602	12	58	34.2	75.9	24	904
132603	12	67	36.8	92.1	24	883
132701	12	53	31.3	93.3	48	2249
132801	11	70	32.4	71.0	48	1619
132804	11	58	25.8	57.2	24	651
132805	11	60	27.9	68.2	24	757
133001	14	56	35.7	97.5	48	2046
133002	14	59	39.4	94.9	24	835

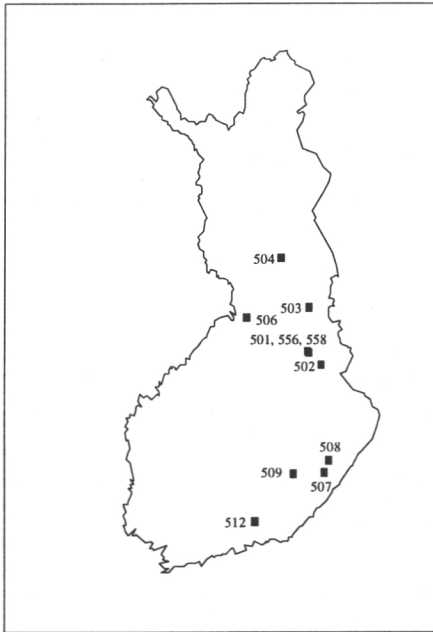


Figure 1. Location of the experimental stands for the thinning and fertilization study on Scots pine.

in the early 1970s. The purpose of the experiments was to study the effects of varying thinning intensities (thinning from below) and nitrogen fertilization on the growth and yield of Scots pine stands. The same experimental data were also used in analysing the combined effect of thinning and fertilization presented in this paper. While **III** and **IV** include only data from unfertilized sample plots in the modelling data, the analysis concerning the interaction of thinning and fertilization on tree growth was based on data in which all the sample plots were included.

The effects of three levels of thinning intensity and three levels of fertilization (NPK fertilizer) were studied using a factorial experimental design (Fig. 2). One-third of the sample plots were left unthinned, one-third were thinned moderately (30% of the stem number removed) and one-third were thinned heavily (60% of the stem number removed). In the moderately thinned plots, the second thinning was done ten years after the first thinning by again removing 30% of the stem number. Only the data from the measurement instances preceding the second thinning were included in the analyses involving these sample plots.

The fertilization treatments included the following levels: unfertilized, 150 kg N ha⁻¹, and 300 kg N ha⁻¹. NPK fertilizer with doses of 150 kg N ha⁻¹ or 300 kg N ha⁻¹ was repeatedly applied at 5-year intervals on the fertilized plots. Because of the repeated fertilizer applications, growth responses to individual applications could not be separated, and this was a constraint as regards model development. The size of the rectangular sample plots was 1 000 m².

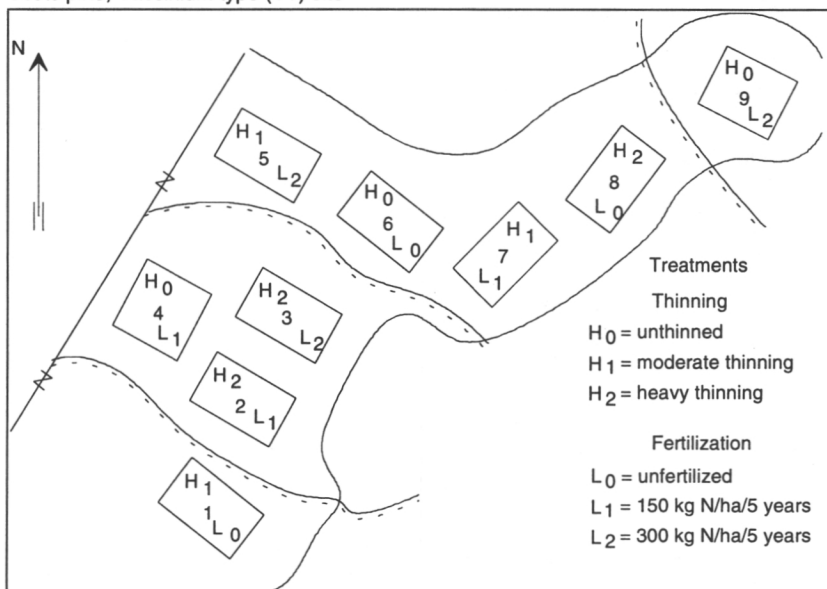


Figure 2. An example of the experimental design.

The stands were measured at five-year intervals during the 15-year study period. In the first measurement instance, 42 sample trees, on average, were selected from each sample plot and used throughout the study period. In the selection of sample trees, the probability of a tree to be selected was proportional to its diameter and independent of its location within the sample plot. Two-thirds of the sample trees were thicker than the stand's mean tree diameter. The height and crown height of every sample tree were measured. Crown height was defined as the height above ground of the lowest live contiguous branch whorl. In addition of breast height diameter, also diameter at six meters as well as diameters at the relative heights of 2.5%, 10%, 30% and 50% along the stem were measured. Sample tree volumes were calculated using simultaneous equations developed by Laasasenaho (1982). The increment of the trees and stand variables were calculated as the differences between the values of the variables at the end and at the beginning of five-year growth periods. Only the sample tree data were used in model development.

The study material included 12 999 measurements made of 3 940 sample trees located on 103 sample plots (Table 3). The number of tree diameter growth observations was 9 669, and the number of tree height growth observations was 9 569.

Only data from unfertilized sample plots were used in studies **III** and **IV**. The modelling data for study **III** consisted of 4 655 sample trees. The growth models for study **IV** were based on 3 479 tree diameter growth observations and 3 406 tree height growth observations.

Table 3. Study material from thinned and fertilized Scots pine stands.

Exp no	Age a	H ₁₀₀ m	H _g m	G m ² ha ⁻¹	Number of sample plots	Number of sample trees
501	40	23.4	10.0	18.2	9	336
502	55	20.5	10.9	17.9	9	439
503	41	24.2	11.8	23.9	9	375
504	45	21.5	11.9	21.7	9	315
506	56	22.1	14.7	21.7	13	507
507	38	25.6	10.7	22.8	9	339
508	39	24.9	10.7	22.9	9	351
509	29	28.8	10.7	25.6	9	376
512	48	21.0	10.6	20.1	9	310
556	40	23.8	10.3	18.7	9	338
558	44	26.5	15.2	25.9	9	254

3 Growth prediction for fertilized stands (I, II)

3.1 Development of tree crowns – a review of prior research

The allocation of foliage production within tree crowns following fertilization has been described for many conifers. Nitrogen fertilization increases the concentration of nitrogen in the needles resulting in an increased photosynthetic capacity (e.g. Viro 1965, Brix and Ebell 1969, Brix 1971, Kellomäki et al. 1982). Fertilization also increases the size and the number of needles, and the growth of lateral branches. As a result, the foliage biomass increases due to fertilization (Keay et al. 1968, Brix and Ebell 1969, Miller and Miller 1976, Valinger 1990, 1993). In fertilized stands, the increase in biomass is allocated to the upper parts of tree crowns (Saramäki and Silander 1982, Brix 1981, Valinger 1990). According to Brix (1971), nitrogen fertilization promotes the net photosynthetic rate of the foliage most of all under favourable light conditions, i.e. in the upper crown.

With increasing stand density and foliage biomass of tree crowns, the canopy density increases, leading to reduced light conditions in lower parts of the canopy. As shading increases within the stand, branch mortality in the lower canopy increases and it can be assumed to lead to enhanced crown recession, as observed by Brix (1982). On the other hand, Gillespie et al. (1994) reported that fertilization in young loblolly pine stands increased the amount of foliage throughout the crown. In their study, branch depth within the crown had a smaller effect on the amount of foliage on a branch of a given size in fertilized stands than it did in unfertilized stands. Accordingly, they suggest that lower branches may be nitrogen-limited rather than light-limited.

3.2 Tree diameter and basal area growth (I, II)

The magnitude and duration of tree diameter and basal area growth responses to fertilization are well documented in many earlier studies. Literature reviews concerning prior research are provided by, for instance, Kukkola and Saramäki (1983) and Hynynen (1993).

In quantifying the fertilizer-induced growth response and its change over time, the total response may be partitioned into direct and indirect effects (Miller and Tarrant 1983, Auchmoody 1985, Opalach and Heath 1988). The direct fertilization effect refers to the growth response directly due to improved nutrition. The indirect effect may be defined as the growth response due to the altered state of the stand brought about by fertilization. In analyses of fertilization data, usually obtained from purpose-designed fertilization experiments, various methods have been applied to determine these two effects or their combination. First, the indirect effect can be removed using covariance analysis by subtracting the mean growth on the control plots from that on the fertilized plots (Opalach and Heath 1988, Peterson and Hazard 1990). The second method is to develop a growth model for unfertilized plots, and use that model to obtain predicted growth for fertilized plots. The direct fertilization effect can then be calculated as the difference between the actual growth of fertilized plots and the predicted growth (e.g. Kukkola and Saramäki 1983, Peterson et al. 1984, Miller et al. 1988, Hynynen 1993). The third method is to develop a model for the total effect of fertilization including both direct and indirect components in its formulation (e.g. Bailey et al. 1989, Stegemoeller and Chappel 1990, Shafii et al. 1990).

In most fertilization studies, the effects of fertilization have been analysed on stand-level using analysis of variance and analysis of covariance (e.g. Paavilainen and Simpanen 1975, Miller and Tarrant 1983, Opalach and Heath 1988, Hynynen and Kukkola 1989), and

regression analysis (e.g. Gustavsen and Lipas 1975, Wells et al. 1976, Rosvall 1980, Kukkola and Saramäki 1983, Ballard 1984, Miller et al. 1988, Bailey et al. 1989).

Earlier research on modelling growth response to fertilization at the tree level is less frequent. Arney (1985) developed a regression model for predicting the relative diameter growth response to nitrogen fertilization. The growth response was predicted as a function of the nitrogen dose and site index, but the model did not include the temporal distribution of the growth response. Another individual-tree model was presented by Shafii et al. (1990). The effect of fertilization was predicted by adding categorical fertilization variables in the growth model. Prediction of the temporal distribution of the growth response was not presented in this model either; instead, separate models for 5-, 10- and 14-year growth periods following treatments were presented.

Despite the small number of individual-tree models for fertilized stands, the effect of fertilization has been incorporated into many growth simulators employing individual-tree models in growth prediction. In the MELA system, a stand-level fertilization response model is applied to predict the growth response of individual trees. The relative stand-level growth response predicted using the model developed by Kukkola and Saramäki (1983) is assumed to be evenly distributed among all the trees in the stand (Ojansuu et al. 1991). A similar approach is also employed in the Swedish HUGIN system (Hägglund 1981) in applying the stand-level models of Rosvall (1980) and Pettersson (1980). In the PTAEDA2 simulator for loblolly pine plantations (Burkhart et al. 1987), the effect of fertilization is incorporated into the models via a site adjustment factor, which is used as a modifier of site index for fertilized stands. The magnitude of the growth response can be incorporated into the model by means of user-defined parameters. In the PROGNOSIS model (Wykoff et al. 1982), the effect of fertilization can be predicted by applying the individual-tree models developed by Shafii et al. (1990).

When modelling the response to fertilization in studies **I** and **II**, the goal was to develop a flexible model structure capable of predicting the magnitude, as well as the temporal distribution, of the response to fertilization with varying doses of applied fertilizer. The model for tree basal area growth response for Scots pine stands was developed in study **I**. Study **II** presents the tree diameter growth model for fertilized, mid-rotation loblolly pine plantations.

The following multiplicative model form was employed in both studies as the basic model structure:

$$\hat{i}_{tree} = F_1(ref) \cdot F_2(fert) \quad [1]$$

where \hat{i}_{tree} refers to predicted tree growth, $F_1(ref)$ accounts for the effects of growth factors other than the direct fertilization effect, and $F_2(fert)$ refers to the direct relative growth response to fertilization. In order to avoid excessively complex model structure, a simplifying assumption was tested and applied, according to which the relative growth response is not affected by tree size (Moore et al. 1994). The results from both studies verified the assumption to be valid for managed stands of Scots pine and loblolly pine. There was no biased model behaviour with respect to either absolute or relative tree size.

In modelling the reference growth of trees, the goal was to include the effects of the relevant growth factors in the model, except for the direct effects of fertilization itself. The regressor variables in the reference growth models included tree and stand characteristics referring to absolute and relative tree size, stand density, stand age (in **II**), as well as to information about site quality. The tree crown ratio was not employed in the models because it was not measured for either of the modelling data sets. In order to minimise the disturbing effects of time-dependent factors, such as climatic growth variation, on the growth response to fertilization, categorical variables were employed in both studies.

The magnitude and the temporal distribution of the growth response to fertilization were predicted by applying identical modelling approaches in both studies **I** and **II**. The temporal distribution was predicted by employing the two-parameter Weibull function, which was then multiplied by the variable referring to the magnitude of the growth response. The basic structure of the growth response models is as follows:

$$F_2(fert) = k \cdot \frac{c}{b} \left(\frac{T}{b}\right)^{(c-1)} \exp\left[-\left(\frac{T}{b}\right)^c\right], \text{ where } (T > 0) \quad [2]$$

- where T = Time elapsed from fertilizer treatment
- b = Weibull scaling parameter (> 0)
- c = Weibull shape parameter (> 0)
- k = Multiplier referring to magnitude of growth response

In the tree basal area growth model for fertilized Scots pine stands (**I**), the reference growth was modelled based on the data from unfertilized control plots. The following model was fitted separately to each experimental stand:

$$\ln(i_g) = a_0 + a_1 \ln(d) + a_2 G + a_3 Y_1 + a_4 Y_2 + \dots + a_i Y_{n-1} + e \quad [3]$$

- where i_g = Annual tree basal area growth, cm^2
- d = Tree diameter at breast height, cm
- G = Stand basal area, m^2

Y_1, Y_2, \dots, Y_{n-1} = Categorical variables referring to growth periods 1 to $n-1$
 n = Number of years in study period
 a_0, a_1, \dots, a_i = Parameters
 e = Error term

The model was then applied in predicting the reference growth for trees growing on fertilized plots. The growth response to fertilization was calculated as the difference between the observed growth and the predicted reference growth. A separate model for the growth response was then developed, based on the data from fertilized sample plots. A similar approach in predicting the fertilization response has earlier been applied by Peterson et al. (1984), Arney (1985), and Heath and Chappel (1988). The model for the tree basal area growth response is as follows:

$$\Delta(p_g) = k \left\{ \frac{c}{b} \left(\frac{T}{b} \right)^{(c-1)} \exp \left[- \left(\frac{T}{b} \right)^c \right] \right\} + e, \text{ in which}$$

$$k = \left(\frac{a_2}{a_1} \right) \left(\frac{H_{100}}{a_1} \right)^{(a_2-1)} \exp \left[- \left(\frac{H_{100}}{a_1} \right)^{a_2} \right] \left(a_3 \left(\frac{FN}{10} \right) - a_4 \left(\frac{FN}{10} \right)^2 \right) \quad [4]$$

$$b = a_5 + a_6 k$$

$$c = a_7$$

where $\Delta(p_g)$ = Relative annual basal area growth response
 T = Time elapsed from fertilization treatment, years
 H_{100} = Site index, m
 FN = Nitrogen dose, kg N/ha
 a_1, \dots, a_7 = Parameters
 e = Error term

In the model for Scots pine stands (model [4]), the magnitude of the growth response was influenced by the fertilizer dose and site quality. Thus, multiplier k was expressed as a function of the fertilizer dose and site index. Due to the high correlation between scaling parameter b and parameter k , b was expressed as a function of k . Separate models for the two types of nitrogen fertilizer (ammonium nitrate with lime and urea) were developed.

According to the results obtained in study I, the total response in tree basal area growth increased with increasing nitrogen doses within the range of 0–450 kg N ha⁻¹. Moreover, the time of the maximum annual growth response varied according to the nitrogen dose. When the fertilizer dose increased from 50 to 400 kg N ha⁻¹, the maximum

relative growth response occurred 3 to 6 years after fertilization. The direct fertilizer effect levelled off 8 to 12 years after fertilization, depending on the dose of nitrogen applied. Fertilization with ammonium nitrate with lime resulted in a 30% greater growth response than the response induced by urea fertilization. According to model [4], the relative growth response in Scots pine stands reached its maximum level on sites with site index (H_{100}) equal to 21–22 m.

The models were tested against independent data collected from experimental stands of Scots pine located in southern Finland. The test data included 7 969 growth observation recordings made of 1 104 sample trees growing on 74 sample plots. The model obtained resulted in a mean relative bias of 1.01% in predicting annual stand basal area growth. Correspondingly, the relative bias in predicting mean diameter increment averaged at -0.58%. In general, model [4] performed satisfactory when tested against independent test data. However, a slight underprediction of the relative tree basal area growth response was observed, especially in young stands showing rapid basal area growth.

In the model for fertilized loblolly pine stands (II), both reference growth and growth response to fertilization were estimated simultaneously, based on the data that included observations made on both unfertilized and fertilized sample plots. The estimation method applied in II can be regarded to be statistically on a more sound basis compared to that applied in study I. By estimating all the model parameters simultaneously, all the information contained in the data could be utilized. The individual-tree diameter growth model developed in II is as follows:

$$i_{d2} = a_0 \cdot (d/G)^{a_1} \cdot \exp(a_2G + a_3 \ln(H_{dom})/Age^2) \cdot \exp(d_1Y_{85} + d_3Y_{87} + d_4Y_{88} + d_5Y_{89} + d_6Y_{90} + d_7Y_{91}) \cdot F_2(fert) + e \quad [5]$$

$$F_2(fert) = 1 + ((a_4 + a_5FP)FN^{a_6}) \left(\frac{c}{b} \right) \left(\frac{T}{b} \right)^{c-1} \exp\left(-\left(\frac{T}{b} \right)^c \right), \text{ where}$$

- i_{d2} = Two-year tree diameter growth, inches
- d = Tree diameter at beginning of growth period, inches
- G = Stand basal area at beginning of growth period, $\text{ft}^2\text{ac}^{-1}$
- H_{dom} = Stand dominant height at beginning of growth period, ft
- Age = Stand age at beginning of growth period, years
- T = Time elapsed from fertilization treatment, years
- FN = Nitrogen dose, lbs ac^{-1}
- FP = Categorical variable referring to phosphorus fertilization;
 $FP = 1$, if fertilized with phosphorus, otherwise $FP = 0$

Y_{85}, \dots, Y_{91} = Categorical variables referring to 2-year growth periods of 1985–1986, ..., 1991–1992, respectively; e.g. $Y_{85} = 1$, if growth period is 1985–1986, otherwise $Y_{85} = 0$
 $a_0, \dots, a_6, d_1, \dots, d_7, b, c$ = Parameters
 e = Error term

Varying doses of nitrogen and phosphorus fertilizers were applied in loblolly pine stands. Thus, parameter k of model [2] was expressed as a function of the fertilizer dose and the type of fertilizer applied (model [5]).

The magnitude of the direct, relative growth response to fertilization in mid-rotation loblolly pine plantations was strongly dependent on the nitrogen dose and the type of fertilizer applied. The response increased with increasing nitrogen doses, but not linearly, across the variation of nitrogen doses applied in the experimental stands (0–300 lbs N ac⁻¹). Adding phosphorus in addition to nitrogen resulted in an enhanced growth response. However, phosphorus did not increase growth when added alone without nitrogen. Increasing the phosphorus dose from 25 lbs P ac⁻¹ to 50 lbs P ac⁻¹ did not affect the growth response.

The results obtained in study II showed that temporal pattern of the growth response was not affected by neither site index nor fertilizer dose. The maximum relative growth response was reached within one to three years after the fertilization treatment. The direct fertilization effect levelled off by eight years after fertilization.

Growth models for the fertilized stands presented in I and II are simulation models. Tree growth was simulated in a stepwise manner. In model [4] for Scots pine, the time step (i.e. growth period) is one year, and in the model for loblolly pine [5] it is two years. The input variables of the models have to be updated in the beginning of each growth period during the simulation. Therefore, the increased tree size and stand density caused by fertilization until the beginning of the growth period in question, i.e. the indirect fertilization effect, is taken into account by the part of the model that predicts the reference growth ($F_1(ref)$). The additional growth response not explained by these growth factors is predicted by ($F_2(fert)$). Thus, $F_2(fert)$ refers to the relative growth response to fertilization during the growth period in question, i.e. the direct fertilization effect. The total, absolute growth response to fertilization can be obtained as the result of growth simulation, using a simulation period that is longer than the duration of the fertilization effect. According to the results of studies I and II, the duration of the total growth response after nitrogen fertilization is 6–12 years in Scots pine stands, and 6–8 years in mid-rotation loblolly pine stands, depending on the fertilization treatment.

Fertilization increases stand density (expressed in terms of stand basal area) which in turn tends to decrease the growth of individual trees. Although the absolute value of the relative growth response predicted using ($F_2(fert)$) is always positive, the opposite effect of increased stand density on tree growth is included in $F_1(ref)$, which also enables the negative effects of fertilization on the growth of individual trees. This kind of situation may occur in a fertilized stand, once the direct fertilization response has leveled off.

In study **II**, information about the average foliar nutrient concentration of the trees on the sample plots was available in the study material. Adding the nutrient concentrations into the tree diameter growth model as regressor variables did not result in significant model improvement.

3.3 Height growth (II)

Improved height increment following fertilization has been documented in many growth and yield studies, but models predicting individual tree height growth responses have not been developed so far, except for the growth response model of Arney (1985). In that model, height growth response to nitrogen fertilization was predicted as a function of site index and fertilizer dose.

In general, the relative height growth response has been found to be smaller compared to response in diameter (or basal area) growth (Brix and Ebell 1969, Saramäki 1980, Burgtorf 1981, Saramäki and Silander 1982, Arney 1985, Pettersson 1985, Hynynen and Kukkola 1989, Hirvelä and Hynynen 1990, Valinger 1990). In the study by Saramäki (1980), fertilization increased height growth significantly in dense and unthinned stands, but in stands of low stand density, height growth was not affected by fertilization. Results concerning the temporal distribution of the response are few in number. For Scots pine and Douglas-fir, the temporal distribution has been observed to follow closely that of basal area growth response (Brix and Ebell 1969, Saramäki 1980, Valinger 1990).

The individual-tree height growth model for fertilized stands of loblolly pine is presented in **II**. In the case of Scots pine stands in study **I**, suitable data for height growth modelling were not available. The height growth of individual trees in loblolly pine stands was modelled as the product of potential height growth and the modifier function. The analysis resulted in the following model:

$$i_{h2} = IH_{dom}(d/D_{dom})^{a_1}IH_{dom} e^{(-a_2G)} + e, \text{ where} \quad [6]$$

- i_{h2} = Two-year height growth of a tree, ft
 IH_{dom} = Two-year dominant height increment, ft
 d = Tree diameter, inches
 D_{dom} = Mean diameter of dominant and co-dominant trees, inches
 G = Stand basal area, $\text{ft}^2\text{ac}^{-1}$
 a_1, a_2 = Parameters
 e = Error term

Similar stratification of tree growth has been widely applied in the construction of height growth models (e.g. Ek and Monserud 1974, Leary 1979, Arney 1985, Burkhart et al. 1987). The increment of stand dominant height was assumed to refer to the potential height growth. The height growth of individual trees was assumed to be smaller or greater than the dominant height increment, depending on relative tree size and stand basal area.

Height growth model [6] of study II was fitted to the data containing height growth observations from both unfertilized and fertilized stands. The measured increments in dominant height were employed as the potential growth of the model. Analysis of the model residuals with respect to relative tree size confirmed the unbiased behaviour of the model. It was concluded that fertilization does not alter the distribution of height growth among trees of different sizes. Therefore, it was assumed to be sufficient to construct a model for dominant height increment including the fertilization effect, instead of elaborating the tree-level model.

The basic assumption underlying the model for dominant height increment for fertilized stands was similar to that applied in models for tree basal-area and diameter growth. Reference growth was predicted using site index and the categorical variables referring to time-dependent growth factors, such as climatic variation. The magnitude of the growth response to fertilization was expressed as a function of the fertilizer dose and the applied nutrient element. The temporal distribution was modelled using the Weibull function in a manner similar to that in modelling the diameter growth response. The model for dominant height increment can be expressed as follows:

$$IH_{dom2} = a_0 \cdot H_{25}^{a_1} \cdot \exp(a_2 \text{Age}^{-1} + d_1 Y_{85} + d_2 Y_{86} + d_3 Y_{89} + d_6 Y_{90} + d_7 Y_{91}) \cdot F_2(\text{fert}) + e \quad [7]$$

$$F_2(\text{fert}) = 1 + FP(a_3 FN) \left(\frac{c}{b} \right) \left(\frac{T}{b} \right)^{c-1} \exp \left(- \left(\frac{T}{b} \right)^c \right), \text{ where}$$

- IH_{dom2} = Two-year dominant height increment, ft
 H_{25} = Site index (base age 25 years), ft
 Age = Stand age at beginning of growth period, years
 T = Time elapsed from fertilization treatment, years

FN	= Nitrogen dose, lbs ac ⁻¹
FP	= Categorical variable referring to phosphorus fertilization; $FP = 1$, if fertilized with phosphorus, otherwise $FP = 0$
Y_{85}, \dots, Y_{91}	= Categorical variables referring to 2-year growth periods of 1985–1986, ..., 1991–1992, respectively; e.g. $Y_{85} = 1$, if growth period is 1985–1986, otherwise $Y_{85} = 0$
$a_0, \dots, a_3, d_1, \dots, d_7, b, c$	= Parameters
e	= Error term

All the parameters of model [7] were estimated simultaneously.

According to the models for loblolly pine presented in study II, the relative response to fertilization in tree height growth is smaller than the response in tree diameter growth. Both nitrogen and phosphorus fertilizer applications were needed to induce a height growth response. The direct, relative growth response increased linearly with increasing nitrogen dose within the range of 0 to 300 lbs N ac⁻¹. The increase in phosphorus dose from 25 to 50 lbs P ac⁻¹ did not have any effect on the magnitude of the growth response. The temporal pattern of the direct response was not affected by the type and dose of fertilizers, nor by the stand characteristics. The maximum response occurred two to four years after fertilization, i.e. somewhat later than the response in diameter growth.

3.4 Stem form development – a review of prior research

Earlier research results confirm that fertilization improves stem form in Scots pine stands. According to the results obtained by Saramäki (1980), fertilization slightly improves stem form development. The absolute increase in the form factor induced by fertilization was greatest in young stands, but the relative change was not affected by stand age or tree size. Fertilization increased the form factor during the two years following fertilizer application. Valinger (1990, 1992) examined stem form development in fertilized Scots pine stands with the help of the form quotient, i.e. the ratio between stem diameter at the relative height of 80% of the total tree height and diameter at breast height. He found that the form quotient increases due to fertilization, because the relative diameter growth was greater in the upper parts of the stem. Results for Douglas-fir are more conflicting. According to Brix and Ebell (1969), tree diameter and basal area growth response to fertilization decreases upwards along the stem. On the other hand, for the same species, Brix (1982) found the stem growth response to be evenly distributed along the stem.

4 Growth prediction for thinned and unthinned stands (III, IV)

4.1 Development of tree crown ratio (III)

The dimensions of tree crowns are known to be markedly affected by thinning. Thinning improves the light conditions within a stand, especially in the lower parts of tree crowns. While fertilization promotes the development of the upper parts of tree crowns, thinning has been found to promote the development of the lower parts of crowns (Saramäki and Silander 1982, Brix 1981, Valinger 1990). Improved crown development results in the longer retention of live branches in the lower crown, and slower crown recession.

In many individual-tree growth models, the tree crown ratio is used as a regressor variable. Growth simulation programs employing these kinds of growth models also include models for predicting the tree crown ratio. There are two different situations where crown ratio models are needed in simulating stand development. First, if not recorded in the data, models can be used to predict the tree crown ratio. Second, models are used in updating the crown ratio at the beginning of each growth period during the simulation. In the PROGNOSIS model (Wykoff et al. 1982), the crown ratio is predicted using a linear regression model developed by Hatch (1980). The non-linear model structure of the crown ratio is applied in STEMS (Belcher et al. 1982) as well as in PTAEDA2 (Burkhart et al. 1987). Tree size and stand density are usually employed as major driving variables in predicting the tree crown ratio. The effects of thinning on the development of the crown ratio in Scots pine trees were studied, and the model for the tree crown ratio was presented in study III.

The following non-linear model form was employed as a basic structure of the model for the tree crown ratio:

$$cr = 1 - e^{(-\Phi(x))}, \quad [8]$$

where cr = Tree crown ratio, defined as: length of live crown/total tree height

$\Phi(x)$ = Function of tree and stand characteristics (> 0).

Because of the model formulation, the predicted crown ratio is inherently restricted to value between 0 and 1. Thus, the model behaviour is logical regardless of the values of the input variables included in $\Phi(x)$.

A model for the tree crown ratio constructed in III is applicable for unthinned stands as well as for thinned stands. The stand basal area, stand dominant height and d/h ratio are the variables required for predicting the crown ratio for unthinned stands. To predict the effect of thinning, thinning intensity and stand dominant height at the time of thinning are needed in addition to the variables mentioned above. The thinning intensity was expressed as the difference between stand basal area before and after thinning. The effect of thinning was incorporated into the crown ratio model by modifying the effect of the stand basal area with the thinning variable. The model for the tree crown ratio is as follows:

$$cr = 1 - \exp\left\{-\left[\exp(-a_1(G + THIN)) + a_2 H_{dom}^{-1}\right]\left(\frac{d}{h}\right)^{a_3}\right\} + e, \text{ where} \quad [9]$$

$$THIN = (G_b - G_a) \exp\left[-\left(\frac{H_{dom} - H_{domt}}{a_4}\right)\right]$$

where G = Stand basal area, $m^2 \text{ ha}^{-1}$
 H_{dom} = Stand dominant height defined as:
average height of 100 thickest trees per hectare, m
 d = Tree diameter at breast height, cm
 h = Tree height, m
 G_b = Stand basal area before thinning, $m^2 \text{ ha}^{-1}$
 G_a = Stand basal area after thinning, $m^2 \text{ ha}^{-1}$
 H_{domt} = Stand dominant height at time of thinning, m
 a_0, \dots, a_4 = Parameters
 e = Error term

Instead of stand age, the stand dominant height was used to express the stage of stand development. No explicit variable referring to site quality was incorporated into the model. Instead, the effect of site quality is reflected through the stand dominant height. Therefore, the duration of the thinning effect on the crown ratio varies with varying site quality, i.e. with varying rate of dominant height increment.

The behaviour of the crown ratio model may be illustrated with the help of an example of the simulated development of the tree crown ratio and crown height (Fig. 3).

The crown ratio decreases with increasing stand dominant height, i.e. with increasing tree age. The rate of crown recession is faster in young stands than in mature stands. Within a stand, trees with more taper will have larger crown ratios. The increase in the stand basal area enhances crown recession. As a consequence of thinning, crown recession decreases. The impact of thinning on the tree crown ratio increases with increasing thinning intensity. The length of time that trees require for adapting their crowns to increased growing space after

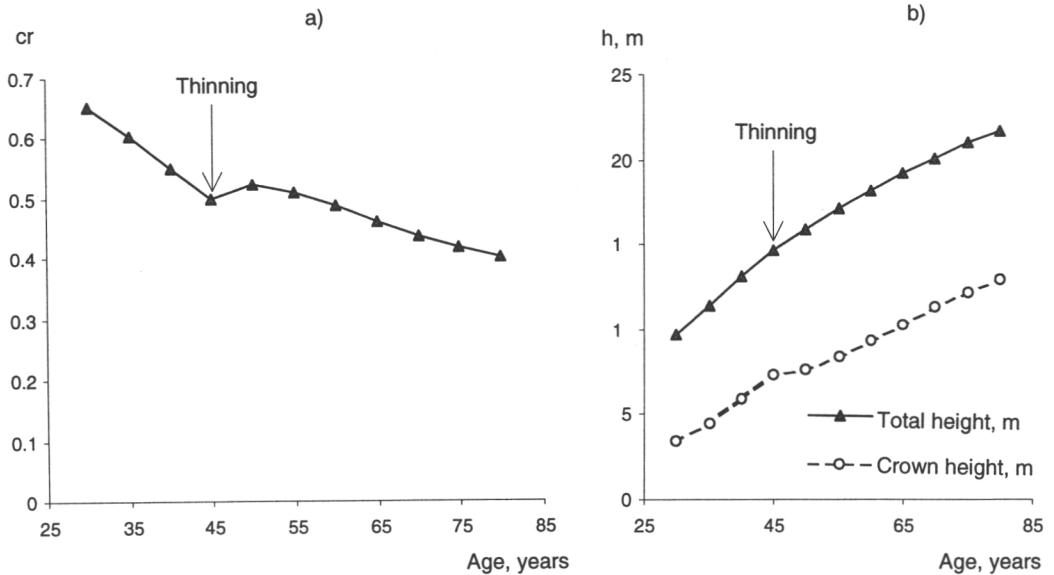


Figure 3. An example of simulated development of individual-tree crown ratio (a) and crown height (b). Thinning was carried out when stand dominant height was 14.5 m. In thinning, the stand basal area was dropped from $27.5 \text{ m}^2\text{ha}^{-1}$ to $17.5 \text{ m}^2\text{ha}^{-1}$, i.e. by 40%. For the simulation, the stand basal area and dominant height development were obtained from the growth and yield tables of Vuokila and Väliäho (1980). The simulated tree was a dominant tree in a stand growing on a site with site index (H_{100}) equal to 24 m. The stand basal area at age 30 years was $16 \text{ m}^2\text{ha}^{-1}$, tree diameter was 13 cm, and tree height was 9.7 m. At age 85 years, the basal area was $36 \text{ m}^2\text{ha}^{-1}$, tree diameter was 26.5 cm and tree height was 22.4 m.

thinning depends on the rate of dominant height increment. Trees in the stands with slow dominant height increment, i.e. trees growing on poor soils, or trees in older stands, react more slowly to thinning compared to the trees on fertile sites, or trees in young stands.

Because the model for the crown ratio is a static model, a change in crown ratio can be calculated as the difference between two predicted crown ratio values at the end and at the beginning of the growth period. There is one disadvantage in applying a static model in simulation; i.e. there is no constraint preventing a decrease in crown height (= the lower limit of the live crown) during the simulation period. To prevent the possibility of illogical behaviour, the simulation program must be capable of modifying the predicted tree crown ratio by restricting the predicted height of the live crown at the end of the growth period to be always greater or equal to the crown height at the beginning of the growth period.

An alternative approach to modelling crown development is to directly model the change in crown ratio or crown height. This has, however, proven to be difficult to model (Short and Burkhart 1992). In measuring crown height, there are often problems with subjectivity in

judging at which point the live crown actually begins. Therefore there is a high probability for variation in judging the height of the crown, as well as for measurement errors to occur in repeated crown height measurements made of a particular tree. Further, the change in crown height is not smooth; it is often stepwise. Therefore, it often happens that the crown height increment between two successive measurements is zero, and this is problem in parameter estimation.

4.2 Tree diameter growth (IV)

It is well documented in forestry literature that stand density and thinning of the growing stock effectively regulate tree diameter increment. The prior efforts made in modelling the thinning response, are discussed in detail in study IV.

Two alternative models for tree diameter growth of Scots pine were developed in study IV in order to examine the necessity of an explicit thinning variable in predicting the growth response to thinning. First, a diameter growth model was developed, in which the effect of thinning was implicitly taken into account. It represents the commonest modelling approach among growth and yield models applied in simulation programs. The regressor variables of the model referring to the measures of individual trees were tree diameter at breast height and the crown ratio. The relative size of a tree within the stand was expressed using the basal area of trees larger than the subject tree (GL). The stand dominant height was used to express the phase of stand development. The productive capacity of the site was described by means of the site index (H_{100}) calculated using the models presented by Vuokila and Väliäho (1980). The effect of stand density was described using the stand basal area. The growth response to thinning was assumed to be reflected through the change in the stand basal area. The tree diameter growth model without an explicit thinning variable is as follows:

$$i_{d5} = a_0 d^{a_1} cr^{a_2} \exp(a_3 d^2 + a_4 GL^2) H_{dom}^{a_5} H_{100}^{a_6} G^{a_7} + e \quad [10]$$

where	i_{d5}	= Five-year growth of tree diameter, cm
	d	= Tree diameter at breast height, cm
	cr	= Tree crown ratio
	GL	= Basal area of trees larger than subject tree, $m^2 ha^{-1}$
	H_{dom}	= Stand dominant height, m
	H_{100}	= Site index, m
	G	= Stand basal area, $m^2 ha^{-1}$
	$a_0, a_1 \dots a_7$	= Parameters
	e	= Error term

In the second diameter growth model, the effect of thinning was expressed using an explicit thinning variable. The modelling approach was identical to that applied in predicting the growth response to fertilization. The basic model structure was expressed as:

$$\hat{i}_{tree} = F_1(ref) \cdot F_2(thin) \quad [11]$$

In the reference growth part of the model ($F_1(ref)$), all the regressor variables were formulated in a manner similar to that in the first diameter growth model [10], except for the stand basal area that was excluded. $F_1(ref)$ refers to the growth of a tree in an unthinned stand. Although no variable referring directly to stand density is present in the model, the effect of stand density is reflected in the growth rate of individual trees through the tree crown ratio and the variable GL , which also depicts the stand basal area in addition to depicting the relative tree size. The effect of thinning was expressed by means of the thinning response variable, $F_2(thin)$, instead of the stand basal area. The magnitude and duration of the thinning response were modelled in a manner similar to that used with the models for the growth response to fertilization (studies I and II). The relative growth response was assumed to be independent of tree size. This assumption has been verified for Douglas-fir by Moore et al. (1994). The temporal distribution of the growth response was modelled using the two-parameter Weibull function. It was scaled with a multiplier formulated as a function of thinning intensity. Thinning intensity was expressed with the help of stand basal area before and after thinning. The time elapsed since thinning and the thinning intensity are required for predicting the relative growth response to thinning. The Weibull function has earlier been applied in modelling the growth response to thinning by Horne et al. (1986), who employed the Weibull function in a stand-level model. The tree diameter growth model with an explicit thinning variable can be expressed as:

$$id_5 = a_0 d^{a_1} cr^{a_2} \exp(a_3 d^2 + a_4 GL^2) H_{dom}^{a_5} H_{100}^{a_6} \cdot F_2(thin) + e, \text{ in which} \quad [12]$$

$$F_2(thin) = 1 + a_7 I \left(\frac{c}{b} \right) \left(\frac{T}{b} \right)^{(c-1)} \exp \left(- \left(\frac{T}{b} \right)^c \right)$$

where I = Thinning intensity, defined as: $(G, \text{ before thinning} - G, \text{ after thinning}) / G, \text{ after thinning}$

T = Time elapsed from thinning, years

$a_0, a_1, \dots, a_7, b, c$ = Parameters

other symbols as in model [10].

According to model [12], the magnitude of the relative diameter growth response to thinning is linearly dependent on the thinning

intensity. Thinning increases relative diameter growth without any time lag. The response reaches its maximum 5–10 years after thinning, and levels off by 30 years after thinning.

The diameter growth models presented in study IV were tested against independent data collected from six thinning trials. The test data included 3 551 trees growing on 24 sample plots. Both models, [10] and [12], resulted in a slight underprediction when applied with the test data. However, no bias was observed with respect to thinning intensity. In most of the stands of the test data, less than 35% of the stand basal area was removed in thinnings.

According to the results obtained in study IV, the diameter growth model [10], without an explicit thinning variable, performed satisfactory in unthinned stands and in stands subjected to moderate thinning intensities. However, growth predictions were biased in stands in which more than 50% of the stand basal area had been removed in thinning. Therefore, this kind of model can be applied quite safely in predicting growth for most of the commercial forests where thinning removals do not exceed 50% of the stand basal area. The risk of biased prediction is essentially greater if the model is applied in evaluating the growth effects induced by heavy thinnings.

The most important advantage of the model without an explicit thinning variable is that no information on thinning is required. For example, in forest inventory data there is seldom any information about the timing and intensities of thinnings carried out in the stands inventoried. Thus, when updating forest inventory data, usually the only applicable growth model is the one in which the thinning effect is implicitly expressed through variables referring to stand density.

The second diameter growth model [12] with an explicit thinning variable resulted in better overall performance, and showed no biased behaviour regardless of the thinning intensity. Because of the thinning response variable, model can provide fairly detailed information about the temporal variation and duration of the thinning response with varying thinning intensities. Therefore, it is preferable when evaluating the effects of alternative thinning schedules on stand development. The capacity of the model to produce detailed information about the thinning response has not been achieved without expenses. The model is rather demanding as regards the input information; in addition to the tree and stand characteristics of the growing stock, information is also required about thinning intensity and the time of thinning. Therefore, the model is not applicable when updating inventory data not containing thinning information.

In growth simulation, there are no major differences between the predictions provided by the models, despite differences in the basic model structure (Fig 4). Model [10], without an explicit thinning

variable, predicts smaller responses to thinning compared to model [12], which includes an explicit thinning variable.

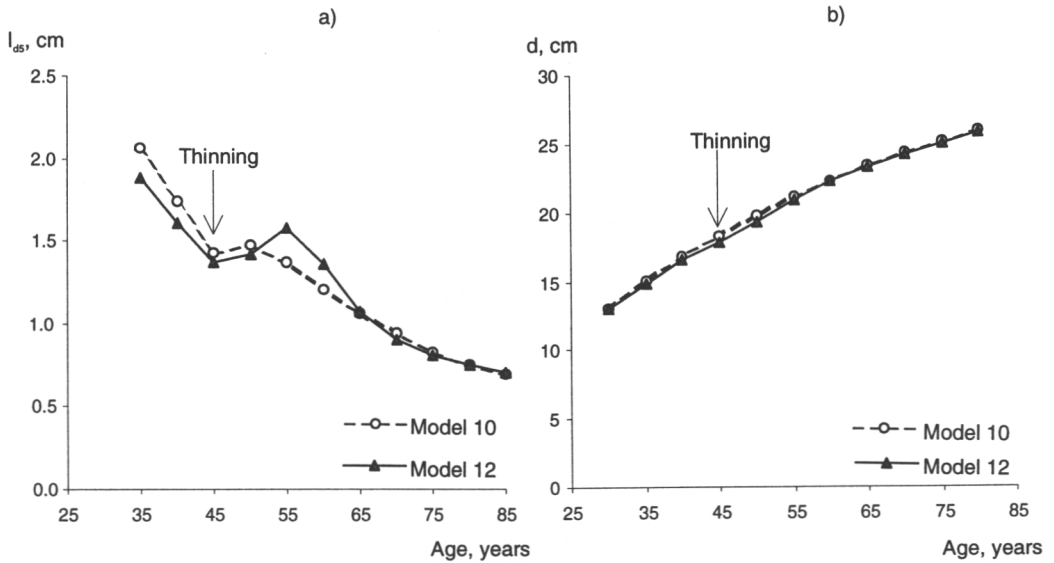


Figure 4. An example of the simulated individual-tree diameter growth (a) and development of diameter (b) with two alternative diameter growth models. In model [10], thinning is taken into account implicitly. In model [12], an explicit thinning variable is included. Thinning was carried out when the stand dominant height was 14.5 m. In thinning, the stand basal area was dropped from 27.5 m²ha⁻¹ to 17.5 m²ha⁻¹, i.e. by 40%. For the simulation, the stand basal area and dominant height development were obtained from the growth and yield tables presented by Vuokila and Väliäho (1980). The simulated tree was the dominant tree in a stand on a site with site index (H_{100}) equal to 24 m. The stand basal area at age 30 years was 16 m²ha⁻¹, tree diameter was 13 cm, and tree height was 9.7 m. At age 85 years, the stand basal area was 36 m²ha⁻¹, tree diameter was 26.5 cm and tree height was 22.4 m.

4.3 Height growth (IV)

Dominant height increment is regarded to be fairly insensitive to thinning from below (Assmann 1970, Clutter et al. 1983). Results from thinning studies focusing on Scots pine stands have confirmed that thinning does not have any significant effect on the increment of dominant height (Lampola 1991, Hynynen and Kukkola 1989, Hynynen and Saramäki 1995). However, during the first 3 to 5 years following thinning, Saramäki and Silander (1982) and Valinger (1990) have reported decreased height increment. In Douglas-fir stands, Brix (1981) found that thinning decreased height growth for the first 2 or 3 years, but tended then to increase height growth after five years.

The most commonly applied approach in height growth models for individual trees is to express tree growth as the product of potential height growth multiplied by the modifier function (e.g. Ek and Monserud 1974, Leary 1979, Arney 1985, Burkhart et al. 1987, Ojansuu et al. 1991). Increment of the stand dominant height is often used to express the height growth potential. The growth rate of an individual tree is regulated by a modifier, i.e. a function of variables referring to absolute and/or relative tree size, within-stand competition, and stand density. The thinning effect is usually implicitly included in the models; it is assumed to be reflected through measures describing stand density, e.g. stand basal area.

In study IV, the following simple model for stand dominant height increment was first constructed to see whether thinning from below affects dominant height increment:

$$\ln(IH_{dom}) = a_0 + a_1I + a_2D_{502} + a_3D_{503} + \dots + a_{11}D_{558} + e \quad [13]$$

where IH_{dom} = Five-year increment of dominant height, m
 I = Thinning intensity
 $D_{502} \dots D_{558}$ = Categorical variables referring to experimental stands
 $a_0, a_1 \dots a_{11}$ = Parameters
 e = Error term

The results showed that thinning had no significant effect on the dominant height increment during the 15-year study period. After that, the individual-tree height growth model was elaborated, with growth being stratified into potential height growth, which was multiplied by a modifier function. The actual, observed dominant height increment was used as the potential height growth. Relative tree size was described using the ratio between tree diameter at breast height and the mean diameter of the dominant trees (100 thickest trees ha⁻¹). No explicit function referring to thinning was incorporated in the model. As a result of model development, the following growth model was presented, with tree growth expressed as a function of the stand dominant height increment and relative tree size:

$$i_{h5} = IH_{dom}(d/D_{dom}) \left[a_1IH_{dom} + a_2(d/D_{dom})^{a_3} \right] + e \quad [14]$$

where i_{h5} = Five-year growth of tree height, m
 d = Tree diameter at breast height, cm
 D_{dom} = Stand dominant diameter defined as:
mean diameter of 100 thickest trees per hectare, cm
 a_1, a_2, a_3 = Parameters
 e = Error term

According to height growth model [14], a tree with diameter equal to the mean diameter of the dominant trees will have height growth equal to the dominant height increment. The relationship between the tree height growth and relative tree size is of curvilinear form. Starting from the most suppressed trees in a stand, height growth will increase with increasing relative tree size until it reaches its maximum. After that, growth begins to decrease with increasing relative size. The position of maximum height growth depends on the rate of the dominant height increment. The growth of a single tree is more affected by relative tree size within stands with fast dominant height increment than in stands with slow dominant height increment. Therefore, the differentiation in height growth among trees is greatest in stands with rapid height growth, i.e. in young stands and in stands growing on fertile sites.

The performance of the model proved to be unbiased within the variation of tree and stand variables, as well as within the variation of the thinning intensities applied in the modelling data. The validation of the height growth model against independent test data confirmed that the applied model structure is viable in predicting height growth in both thinned and unthinned stands. The analyses confirmed that no explicit thinning variable is required in predicting individual-tree growth in Scots pine stand thinned from below.

4.4 Stem form development (IV)

Thinning from below changes the stem form of trees. The increased growing space results in improved diameter growth at breast height while the increase in height growth remains smaller. In growth simulation, this kind of change in stem form can be taken into account by applying separate models for tree diameter and height growth that correctly reflect altered growth rates due to thinning.

In study IV, the aim was to examine whether there is an increase in diameter growth in the upper parts of the stem resulting in increased stem volume, but one that cannot be explained by the change in the ratio between breast height diameter and tree height (d/h). The study material contained diameter readings taken at five or six different heights along the stems of trees. These were used in stem volume calculations together with the measured tree heights. For each tree, a cylindrical form factor was calculated based on the calculated stem volumes, measured breast height diameters and tree heights. Two models for the cylindrical form factor were developed. In the first model, tree diameter and height were the only regressor variables:

$$f_{1.3} = 1 - \exp\left[-\left(a_0 + a_1(d/h)^{a_2} + a_3h^{a_4}\right)\right] + e \quad [15]$$

where $f_{1.3}$ = Cylindrical stem form factor
 d = Tree diameter at breast height, cm
 h = Tree height, m
 a_0, \dots, a_4 = Parameters
 e = Error term

In the second model, the crown ratio, stand dominant height, and the stand basal area were also employed as regressors in following manner:

$$f_{1.3} = 1 - \exp\left[-\left(a_0 + a_1(d/h)^{a_2} + a_3h^{a_4} + a_5cr + a_6H_{dom} + a_7G\right)\right] + e \quad [16]$$

where cr = Tree crown ratio
 H_{dom} = Stand dominant height, m
 G = Stand basal area, m^2ha^{-1}

Other symbols as in model [15].

Residual analysis was employed to examine whether the tree and stand characteristics are adequate for predicting the stem form factor for unthinned as well as for thinned stands, or whether variables referring to thinning treatment are required for unbiased prediction. According to the results, thinning does not cause such changes in stem form that cannot be predicted by the change in the d/h ratio. Adding crown ratio, stand dominant height and stand basal area into the model as regressors improved the model only slightly. Neither of the models showed biased behaviour with respect to thinning intensity in the modelling data nor the independent test data.

5 Interaction of thinning and fertilization

5.1 Development of tree crown ratio

A treatment interaction of thinning and fertilization on the development of tree crowns has been reported by Brix (1982) for Douglas-fir. He found that thinning and fertilization resulted in a considerable interaction on the net assimilation rates and on the production of foliage. Because nitrogen fertilization has a more stronger effect on the rate of photosynthesis at high than at low light intensities, the improvement of light conditions resulting from thinning will produce

the treatment interaction. Valinger (1990) has reported a similar synergistic effect for Scots pine associated with combined thinning and fertilization treatment.

In order to examine the effect of simultaneous fertilization and thinning on the development of the crown ratio, the crown ratio model [9] developed in study III was fitted to Scots pine data including all the treatment combinations (of thinning and fertilization) contained in the experimental stands. There were 12 999 tree crown ratio observations in the modelling data. The analysis was based on the assumption that fertilization affects the development of tree crown ratio by increasing the stand basal area, the effect of which is already incorporated in the model. A study was made of whether the crown ratio model would be valid in predicting the crown ratio for trees in fertilized stands without it being necessary to include any explicit variable referring to fertilization. The results of the analysis showed that there were no major differences in the parameter estimates (Table 4) compared to those of the model that was fitted to unfertilized sample plots only.

Table 4. Parameter estimates of the allometric model for the crown ratio fitted to the complete data and to the data obtained from unfertilized sample plots only (data of study III).

Parameter	Complete Data		Data from unfertilized plots	
	Estimate	Asymptotic Std. Dev.	Estimate	Asymptotic Std. Dev.
a_1	0.0353	0.0008	0.0314	0.0011
a_2	3.7198	0.1200	3.0994	0.1883
a_3	0.4329	0.0099	0.4496	0.0159
a_4	1.4930	0.1164	1.8491	0.2158
\hat{cr}	0.493		0.500	
RMSE	0.0729		0.0707	
Observations	12999		4655	

The crown ratio model proved to act satisfactory regardless of the fertilization treatment. The overall performance of the model in the various fertilization treatments was as follows:

Fertilization treatment	Mean of predicted crown ratio	Absolute bias	Relative bias, %
Unfertilized	0.501	-0.0004	-0.081
150 kg N ha ⁻¹ in 5 years	0.487	0.0055	1.139
300 kg N ha ⁻¹ in 5 years	0.490	-0.0057	-1.156

No serious biases were observed with respect to the regressor variables and with respect to the thinning and fertilization treatments. The residuals showed no major trends as regards thinning intensity within the scope of the fertilization treatments (Fig 5).

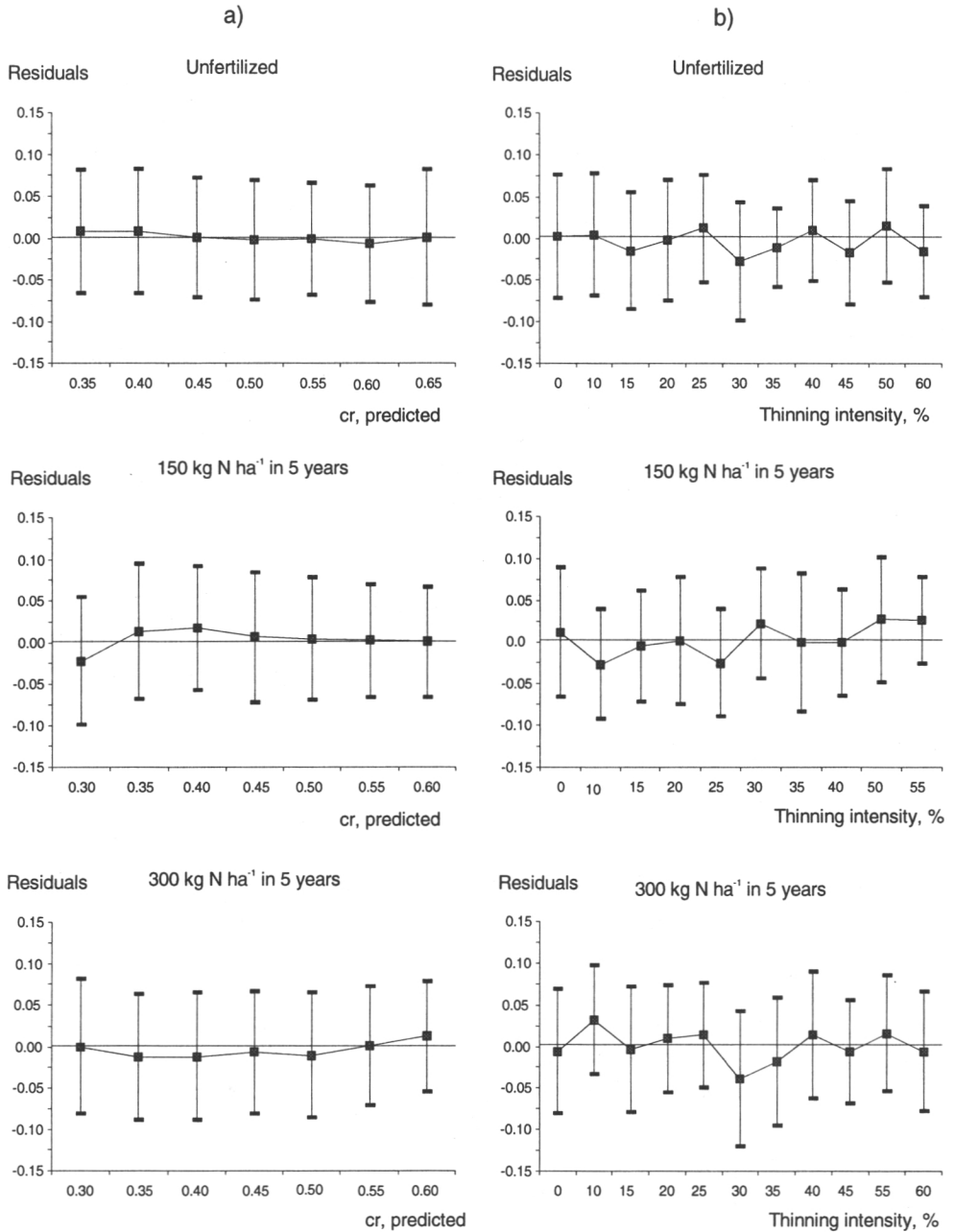


Figure 5. The residuals (\pm standard deviation) of the model for tree crown ratio plotted against the predicted crown ratio (a), and against thinning intensity (b) grouped by fertilization treatments

On the basis of the results obtained, it can be concluded that the crown ratio model is applicable in predicting the crown ratio for practical purposes with adequate accuracy regardless of the fertilization treatment applied. There seems to be no need to

incorporate any explicit variable referring to fertilization in model [9] developed in connection with study III. The analysis did not reveal any such fertilization effects on the development of the crown ratio that could not be explained by changes in the stand basal area (G). In the crown ratio model, the effect of fertilization was reflected through the stand basal area, which is strongly affected by fertilization. It is known that fertilization markedly promotes the growth of tree crowns, and thereby significantly increases their leaf area and photosynthetic capacity (see section 3.1). However, this growth increase takes place inside the crown, and it does not necessarily result in an increased crown ratio. Therefore, a tree crown ratio is a fairly ineffective variable in modelling the growth response of trees following fertilization. Instead of the crown ratio, other variables more closely correlated with leaf area should be used in models predicting the response to fertilization. Variables that have been found to closely correlate with leaf area and leaf biomass include tree diameter at the base of the living crown (e.g. Gillespie et al. 1994), the sapwood area at breast height (Grier and Waring 1974), and the area of sapwood at the base of the crown (e.g. Marchand 1984).

5.2 Tree diameter growth

Thinning and fertilization in combination are known to result in a greater growth response than when carried out alone (Jonsson and Möller 1977, Haapanen et al. 1979, Saramäki and Silander 1982, Brix 1982, Valinger 1990, 1992, Shafii et al. 1990). In their study focusing on four mature Scots pine stands, Jonsson and Möller (1977) found a significant interaction effect between thinning and fertilization on diameter growth in two trials with high stand densities, but the interaction was not significant in the other two stands of lower stand density. Saramäki and Silander (1982) reported a significant interaction of thinning and fertilization acting upon tree volume growth in a young Scots pine stand.

The aim in analysing the modelling of diameter growth for thinned and fertilized stands was to incorporate the fertilization effect into the diameter growth models developed in study IV (models [10] and [12]). The purpose was to see whether there is interaction between thinning and fertilization acting upon tree growth, in addition to the separate growth effects induced by these treatments.

The effect of fertilization was described using categorical variables. Since all the fertilized plots included in the data set were refertilized every five years, the magnitude and the temporal distribution of the response to single applications could not be separated. Thus, the model

structure described in studies **I** and **II** could not be applied with these data. The dummy variables referring to fertilization treatments were applied, based on the assumption that the relative growth response is not affected by tree size; this is similar to the assumption underlying the fertilization response models of studies **I** and **II**.

The effect of fertilization was incorporated in both diameter growth models presented in study **IV**. Diameter growth, which did not include an explicit thinning response function, is obtained as follows:

$$id_5 = a_0 d^{a_1} cr^{a_2} \exp(a_3 d^2 + a_4 GL^2) H_{dom}^{a_5} H_{100}^{a_6} \cdot G^{a_7} \cdot f(FERT) + e, \quad [17]$$

in which

$$f(FERT) = 1 + d_{11}N_{11} + d_{12}N_{12} + d_{13}N_{13} + d_{21}N_{21} + d_{22}N_{22} + d_{23}N_{23}$$

where

- N_{11} = Dummy variable, = 1, if 1st 5-year growth period and fertilized with 150 kg N ha⁻¹, otherwise 0.
- N_{12} = Dummy variable, = 1, if 2nd 5-year growth period and fertilized with 150 kg N ha⁻¹, otherwise 0.
- N_{13} = Dummy variable, = 1, if 3rd 5-year growth period and fertilized with 150 kg N ha⁻¹, otherwise 0.
- N_{21} = Dummy variable, = 1, if 1st 5-year growth period and fertilized with 300 kg N ha⁻¹, otherwise 0.
- N_{22} = Dummy variable, = 1, if 2nd 5-year growth period and fertilized with 300 kg N ha⁻¹, otherwise 0.
- N_{23} = Dummy variable, = 1, if 3rd 5-year growth period and fertilized with 300 kg N ha⁻¹, otherwise 0.
- d_{11}, \dots, d_{23} = Parameters,

Other symbols as in model [10].

All the parameters were estimated simultaneously by fitting the model [17] to the complete data set comprising 9 669 growth observations (Table 5). The relative standard error of the estimate of model [17] fitted to the complete data set was 33.7%, whereas for model [10] it was 35.2% when fitted to the data for unfertilized plots only.

The dummy variables in the models can be interpreted so that N_{11} and N_{21} refer to the relative growth response to 150 kg N ha⁻¹ and 300 kg N ha⁻¹, respectively, during the first five-year period following the first fertilization treatment. Parameters N_{12} and N_{22} depict the combined response to the second fertilization treatment plus the remaining effect of the first fertilization treatment during the second growth period. Consequently, variables N_{13} and N_{23} include the effect of all three repeated fertilization treatments during the third growth period.

The input variables of the model refer to the stand and tree characteristics at the beginning of the 5-year growth period in question.

Table 5. Parameter estimates of diameter growth model [17].

Parameter	Estimate	Asymptotic Std. Dev.
a_0	0.4338	0.0683
a_1	0.3761	0.0375
a_2	-0.0006	0.00007
a_3	-0.0012	0.00005
a_4	0.5846	0.0246
a_5	-0.4424	0.0408
a_6	1.1076	0.0430
a_7	-0.5575	0.0147
Fertilization with 150 kg N ha ⁻¹		
d_{11}	0.3475	0.0148
d_{12}	0.7531	0.0196
d_{13}	0.7576	0.0285
Fertilization with 300 kg N ha ⁻¹		
d_{21}	0.4314	0.0156
d_{22}	0.7804	0.0203
d_{23}	0.6600	0.0277
\widehat{i}_{d5}	1.524	
RMSE	0.516	
Observations	9669	

Table 6. Parameter estimates of diameter growth model [18].

Parameter	Estimate	Asymptotic Std. Dev.
a_0	0.4907	0.0758
a_1	0.2052	0.0366
a_2	0.5438	0.02446
a_3	-0.00050	0.00007
a_4	-0.0014	0.00005
a_5	-0.3239	0.0405
a_6	0.5444	0.0406
a_7	9.3937	1.0674
b	13.7594	1.4158
c	1.5796	0.1463
Fertilization with 150 kg N ha ⁻¹		
d_{11}	0.4209	0.0160
d_{12}	0.6009	0.0187
d_{13}	0.5147	0.0251
Fertilization with 300 kg N ha ⁻¹		
d_{21}	0.5165	0.0170
d_{22}	0.6300	0.0196
d_{23}	0.4721	0.0258
\widehat{i}_{d5}	1.525	
RMSE	0.507	
Observations	9669	

Therefore, when assessing the magnitude of the parameter values of the variables d_{12} , d_{13} , d_{22} , and d_{23} , it must be observed that the effect of the preceding fertilization(s) on tree size and stand density are already included in the values of these input variables.

The residuals of the model without an explicit thinning variable, when plotted against thinning intensity, showed biased behaviour on heavily thinned plots, as was observed already in connection with study IV. There were no differences in model behaviour among the fertilization treatments (Fig. 6a).

The response to fertilization was included in a similar manner in diameter growth model [12], in which an explicit thinning response variable was incorporated as well. The resulting model can be expressed as:

$$id5 = a_0 d^{a_1} c r^{a_2} \exp(a_3 d^2 + a_4 GL^2) H_{dom}^{a_5} H_{100}^{a_6} \cdot f(THIN) \cdot f(FERT) + e,$$

in which

$$f(THIN) = 1 + a_7 I \left(\frac{c}{b} \right) \left(\frac{T}{b} \right)^{(c-1)} \exp \left(- \left(\frac{T}{b} \right)^c \right) \quad [18]$$

$$f(FERT) = 1 + d_{11} N_{11} + d_{12} N_{12} + d_{13} N_{13} + d_{21} N_{21} + d_{22} N_{22} + d_{23} N_{23}$$

where all the symbols are as in models [12] and [17].

The relative standard error of the estimate of model [18], when fitted to the complete data set, was 33.2%, whereas for model [12] of study IV it was 33.5%, when fitted to the data for the unfertilized plots only (Table 6).

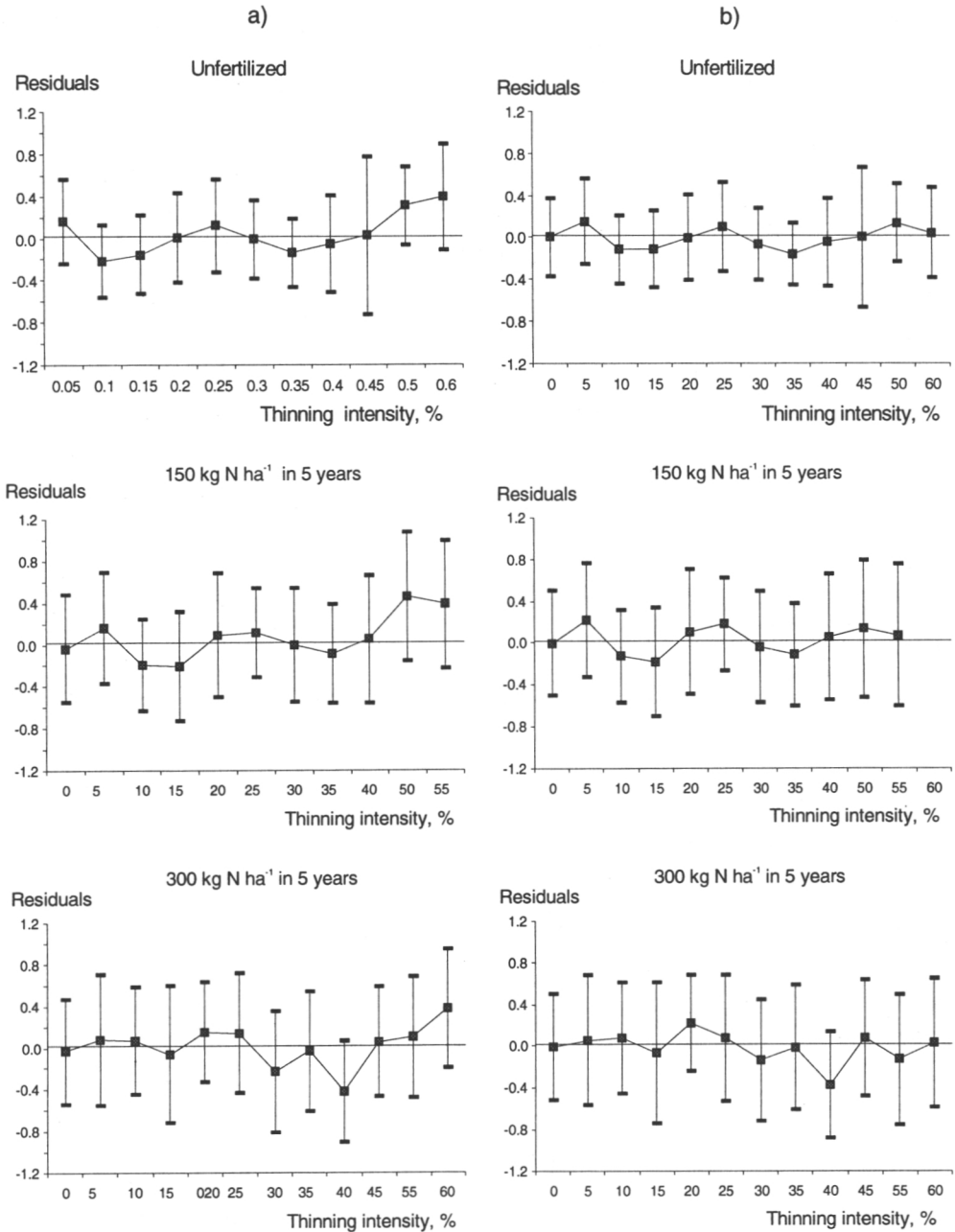


Figure 6. Model residuals (\pm standard deviation) plotted against thinning intensity. a) model [17], b) model [18].

There were no trends among the residuals as regards thinning intensity connected to any of the fertilization treatments (Fig. 6b).

To verify the assumption that the relative growth response is not affected by tree size regardless of the stand treatment, the model residuals with respect to relative tree size were studied by treatment. The models tended to slightly overpredict the diameter growth of all the smallest trees in the stand regardless of the thinning or fertilization treatment applied. However, the bias was relatively small, and there were no differences in model behaviour among the various stand treatments (Fig 7).

The magnitude of the predicted relative growth response to fertilization can be interpreted by examining the parameter values of the categorical variables N_{11} to N_{23} (Table 6). According to model [18], fertilization with 150 kg N ha⁻¹ increased growth by 42.1% during the first 5-year growth period. The application of 300 kg N ha⁻¹ resulted in a response of 51.6%. In the second growth period following the second

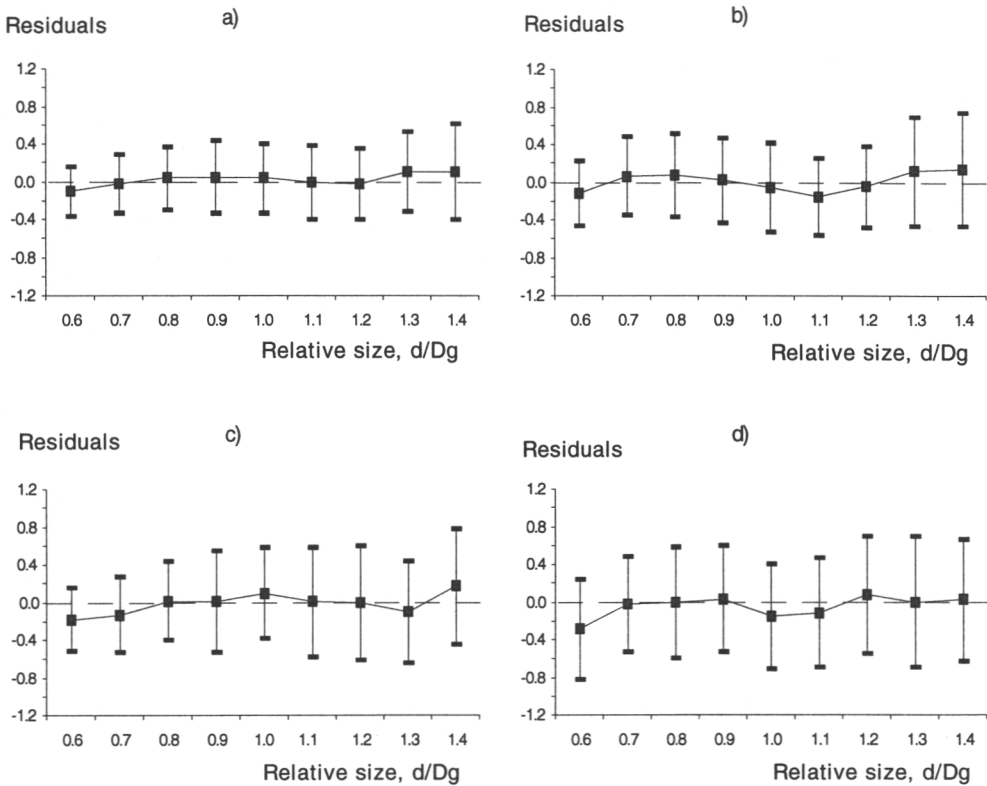


Figure 7. Residuals (\pm standard deviation) of diameter growth model [18] plotted against relative tree size: a) control plots, b) unfertilized and heavily thinned (60% of stem number removed) plots, c) fertilized (300 kg N ha⁻¹) and unthinned plots, and d) fertilized (300 kg N ha⁻¹) and heavily thinned plots.

fertilization treatment, the growth responses further increased, with the response to 300 kg N ha⁻¹ being greater than the response to 150 kg N ha⁻¹. However, in the last growth period, the fertilization responses were smaller compared to the responses of the second growth period. The results support the findings of earlier fertilization studies, in which diminishing growth responses to repeated fertilization were observed (e.g. Kukkola and Saramäki 1983). In the third growth period, the response to 300 kg N ha⁻¹ was smaller than the response to 150 kg N ha⁻¹; this was probably due to the massive dose of applied fertilizer (900 kg N ha⁻¹ in 15 years). According to results obtained in earlier fertilization studies, the growth response to single applications increases until the doses amount to 600 kg N/ha (e.g. Erken 1970, Malm and Möller 1975, Hynynen 1993).

5.3 Height growth

5.3.1 Stand dominant height increment

Simultaneous thinning and fertilization have been found to increase height growth (Saramäki and Silander 1982, Brix 1981, Valinger 1990, 1992). However, in the studies conducted by Saramäki and Silander (1982) and Valinger (1990), the growth response to thinning and fertilization of stands remained smaller than the response to fertilization alone.

The effect of fertilization on dominant height increment was examined in the same manner as is presented in study **IV**, in which the thinning effect was studied. First, a model for dominant height increment was developed, incorporating the responses to thinning and fertilization by using categorical variables. The following stand-level model was fitted to the data:

$$\ln(IH_{dom5}) = a_0 + a_2D_{502} + a_3D_{503} + \dots + a_{11}D_{558} + a_1 \ln(H_{dom}) + c_{11}H_{11} + c_{12}H_{12} + c_{21}H_{21} + c_{22}H_{22} + c_{23}H_{23} + d_{11}N_{11} + d_{12}N_{12} + d_{13}N_{13} + d_{21}N_{21} + d_{22}N_{22} + d_{23}N_{23} + e \quad [19]$$

where

H_{11} = Dummy variable, = 1, if 1st 5-year growth period and moderate thinning otherwise 0.

H_{12} = Dummy variable, = 1, if 2nd 5-year growth period and moderate thinning otherwise 0.

H_{21} = Dummy variable, = 1, if 1st 5-year growth period and heavy thinning otherwise 0.

H_{22} = Dummy variable, = 1, if 2nd 5-year growth period and heavy thinning otherwise 0.

H_{23} = Dummy variable, = 1, if 3rd 5-year growth period and heavy thinning otherwise 0.

c_{11}, \dots, c_{23} , = parameters,

Other symbols as in models [13] and [17].

Study **IV** looked at the effect of thinning on the dominant height growth increment over the entire 15-year study period. In model [19], the treatment effects during three successive 5-year growth periods were examined separately with the help of categorical variables. This analysis revealed temporal variation in the thinning response within the 15-year study period. During the first 5-year period after thinning, a decrease in dominant height increment was observed, especially after heavy thinning (Table 7). This is similar to the findings reported by Brix (1981), Saramäki and Silander (1982) and Valinger (1990). The decrease in height increment was then compensated for by improved height increment during the second and third 5-year growth periods; a similar pattern has been reported for Douglas-fir by Brix (1981). Thus, over the entire 15-year study period, the effect of thinning from below on dominant height increment proved not to be significant, which is in agreement with result obtained in study **IV**.

Table 7. Parameter estimates of the dominant height increment model [19].

Parameter	Estimate	Std. Error	t-value	Prob. > T
<i>Intercept</i>	0.0988			
a_1	-0.7621	0.2267	-3.361	0.0009
	Moderate thinning			
c_{11}	-0.1035	0.0600	-1.173	0.0855
c_{12}	0.1248	0.0587	2.127	0.0345
	Heavy thinning			
c_{21}	-0.1722	0.0584	-2.950	0.0035
c_{22}	0.1105	0.0576	1.918	0.0563
c_{23}	0.0402	0.0575	0.700	0.4846
	Fertilization with 150 kg N ha ⁻¹			
d_{11}	0.0709	0.0571	1.241	0.2160
d_{12}	0.2511	0.0581	4.320	0.0001
d_{13}	0.2848	0.0712	4.001	0.0001
	Fertilization with 300 kg N ha ⁻¹			
d_{21}	0.1077	0.0579	1.859	0.0642
d_{22}	0.2323	0.0590	3.933	0.0001
d_{23}	0.1135	0.0718	1.581	0.1152
$\ln(\hat{IH}_{dom})$	-1.1505			
R ²	0.574			
RMSE	0.254			
Observations	258			

Note: *Intercept* = Parameter a_0 + mean of the parameters a_{21}, \dots, a_{13} .

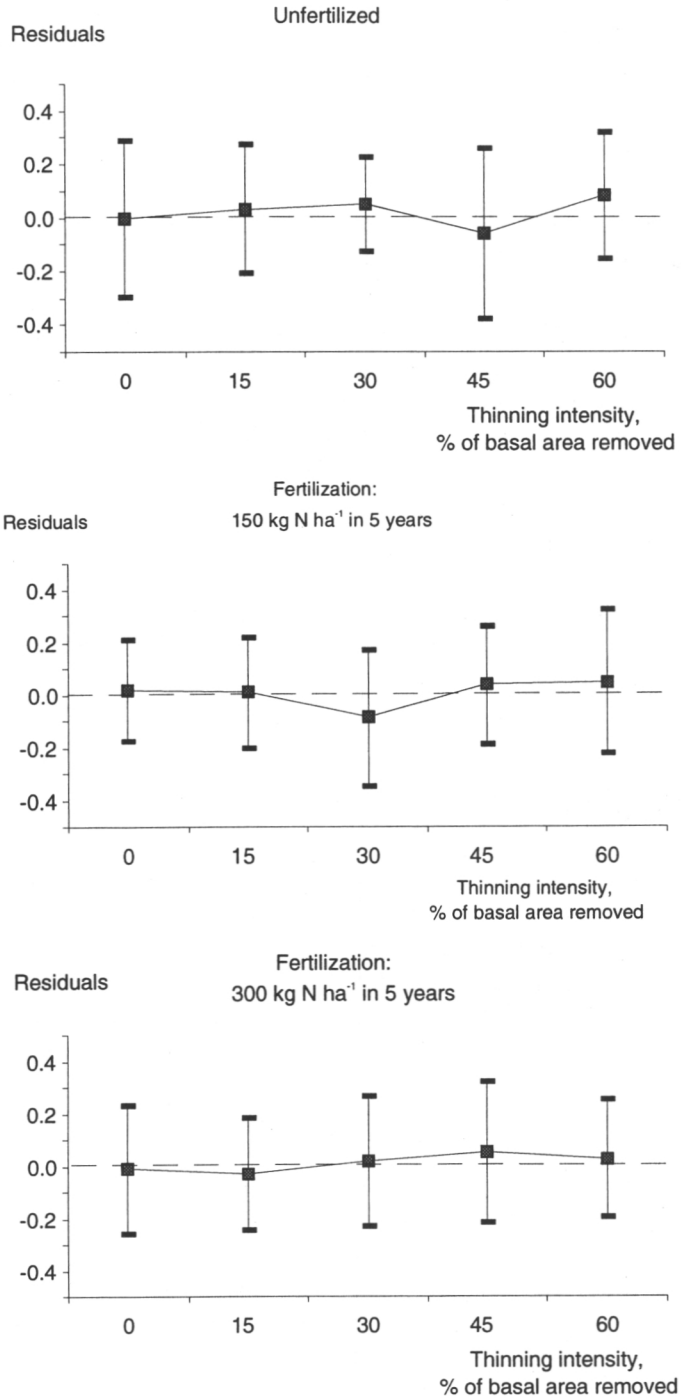


Figure 8. Residuals (\pm standard deviation) of model [19] for dominant height increment plotted against thinning intensity with varying fertilization treatment.

Repeated fertilization with doses of 150 kg N ha⁻¹ induced significant growth responses during the second and third 5-year growth periods. The growth response was not significant in the first growth period. The results suggest that the response in dominant height increment reaches its peak later than diameter growth response, although the effect of single applications could not be clearly separated due to the design of the experiments. Also, the response in terms of dominant height increment in loblolly pine stands reaches its maximum level somewhat later than the response in basal area growth (II). An increase in fertilizer dose from 150 to 300 kg N ha⁻¹ resulted in a decrease in growth response, which suggests that massive fertilization will result in decline in height growth. In the study material, the proportion of growth disturbances in tree crowns was the greater the more intensively the sample plots were fertilized (Hynynen and Saramäki 1995).

The behaviour of model [19] with respect to the fertilization treatment and thinning intensity was studied by examining the model residuals. There were no significant biases with any fertilization treatments as regards thinning intensity (Fig. 8). Therefore, there was no interaction between thinning and fertilization in addition the separate growth effects of these treatments.

5.3.2 Height growth of individual tree

In all the models for predicting thinning and fertilization responses presented earlier in the study, the basic assumption has been that the relative growth response to thinning or fertilization is not affected by tree size. To verify this assumption with regard to the combined effect of thinning and fertilization, a model for individual tree height growth, similar to model [14] developed in study IV, was fitted to the complete data set:

$$ih_5 = IH_{dom}(d/D_{dom}) \left[a_1 IH_{dom} + a_2 (d/D_{dom})^{a_3} \right] + e, \quad [20]$$

where all the symbols are as in model [14].

In model [20], the observed stand dominant height increment of the sample plots was employed as the potential height growth. Therefore, the effects of thinning and fertilization on dominant height increment were included in the model. The parameter estimates of model [20] fitted to the complete data set were quite similar to those of model [14] developed in study IV and based on unfertilized stands only (Table 8).

The model residuals were studied by treatment. The model behaved satisfactory with respect to relative tree size, regardless of the stand treatment (Fig. 9). There were small biases in the predicted growths among all the smallest trees. In unthinned and fertilized (300 kg N ha⁻¹)

Table 8. Parameter estimates of tree height growth model [20] fitted to the complete data and those of model [14] fitted to the data pertaining to the unfertilized plots only.

Parameter	Complete Data		Data for unfertilized plots	
	Estimate	Asymptotic Std. Dev.	Estimate	Asymptotic Std. Dev.
a_1	0.2166	0.0095	0.2445	0.0151
a_2	-0.5350	0.0329	-0.4710	0.0558
a_3	0.7936	0.0961	0.8045	0.1804
\widehat{i}_{h5}	1.621		1.479	
RMSE	0.483		0.449	
Observations	9596		3406	

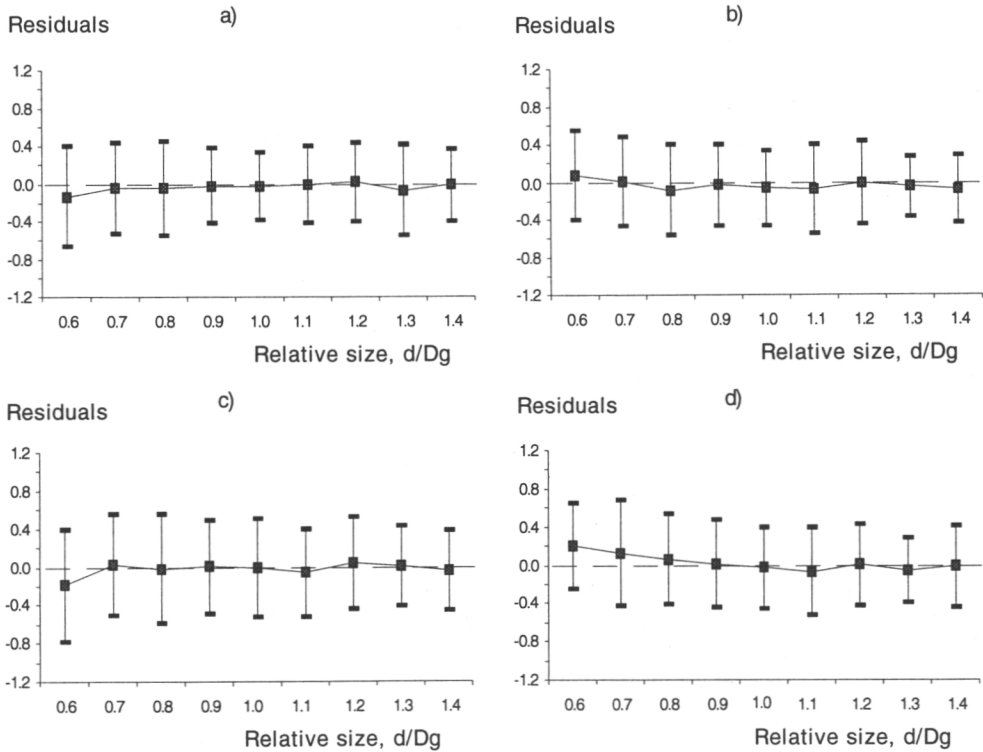


Figure 9. Residuals (\pm standard deviation) of height growth model [20] plotted against relative tree size: a) control plots, b) unfertilized and heavily thinned (60% of stem number removed) plots, c) fertilized (300 kg N ha⁻¹) and unthinned plots, and d) fertilized (300 kg N ha⁻¹) and heavily thinned plots.

stands, the growths of the smallest trees were overpredicted, whereas in heavily thinned and fertilized stands growth was underpredicted, respectively. These results suggest that the slightly biased model behaviour is mainly due to thinning, not to fertilization. The biases were not, however, statistically significant.

5.4 Stem form development

The opposite effects of thinning and fertilization on the development of stem form are well documented in research work delving into the effects of these treatments when applied simultaneously (Brix 1982, Valinger 1990, 1992). Brix (1982) reported that the thinning response in tree basal area growth in Douglas-fir stands was concentrated in the lower parts of the stem, whereas fertilization increased basal area growth evenly along the stem. Valinger (1990, 1992) has reported similar results for Scots pine.

The interaction of thinning and fertilization on stem form was studied by modifying the model for cylindrical stem form factor developed in connection with study IV. It was assumed, on the basis of earlier research, that fertilization improves stem form. The effect of fertilization was added into the nonlinear model for the stem form factor (model [15]). The effect of thinning was not incorporated in the model based on the results obtained in study IV. Again, categorical variables referring to fertilization treatment and the 5-year growth period in question, were included in the model. The model can be presented as follows:

$$f_{1.3} = 1 - \exp\left[-\left((a_0 + F(\text{fert})) + a_1(d/h)^{a_2} + a_3h^{a_4}\right)\right] + e, \text{ in which} \quad [21]$$

$$F(\text{fert}) = d_{11}N_{11} + d_{12}N_{12} + d_{13}N_{13} + d_{21}N_{21} + d_{22}N_{22} + d_{23}N_{23}$$

where all the symbols are as in models [15] and [17].

The effect of fertilization was significant with both fertilization treatments during all the growth periods (Table 6). The amount of applied fertilizer had hardly any effect on the magnitude of the change in the stem form factor. Stem form changed already after the first fertilization treatment during the first 5-year growth period. The change in the form factor after the third successive fertilization treatment was smaller than the change after the preceding fertilization treatments.

Residual analysis showed that there were no systematic trends in the residuals with varying thinning intensity in any of the fertilization treatments (Fig. 10). Therefore, no interaction on thinning and fertilization were observed.

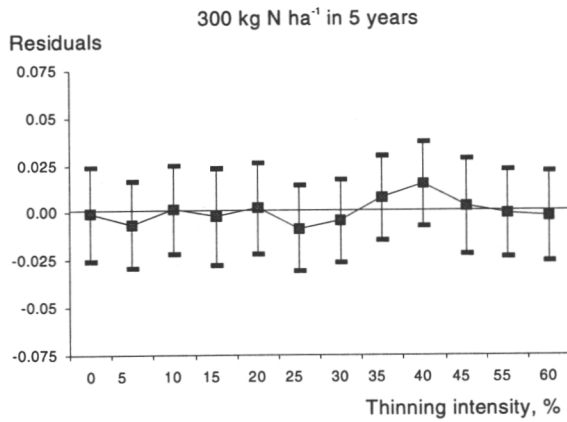
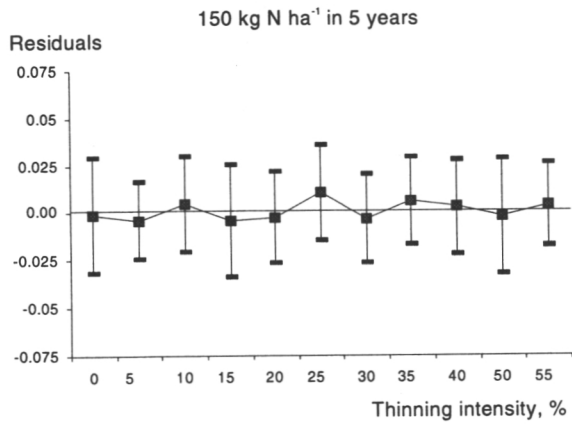
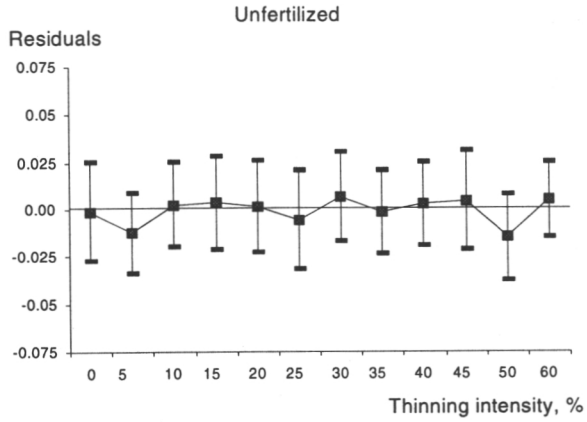


Figure 10. Residuals of the model for stem form factor [21] plotted against thinning intensity with varying fertilization treatments.

Table 6. Parameter estimates of model for form factor [21].

Parameter	Estimate	Asymptotic Std. Dev.
a_0	0.5653	0.0179
a_1	-0.0231	0.0025
a_2	2.5657	0.2442
a_3	-0.9295	0.0658
a_4	2.9502	0.1859
Fertilization with 150 kg N ha ⁻¹		
d_{11}	0.0128	0.0017
d_{12}	0.0167	0.0017
d_{13}	0.0075	0.0021
Fertilization with 300 kg N ha ⁻¹		
d_{21}	0.0126	0.0018
d_{22}	0.0175	0.0018
d_{23}	0.0089	0.0022
$f_{1,3}$	0.534	
RMSE	0.0255	
Observations	12 959	

Residuals grouped by treatments confirmed unbiased performance with varying relative tree size (Fig. 11) regardless of stand treatment.

The results verified that fertilization slightly improves stem form, i.e. that it increases the value of the cylindrical stem form factor. The cylindrical stem form factor in thinned and fertilized stands can be predicted by adding the effect of fertilization into the model. There seems to be no need for model modifications because of thinning. Based on the residual analysis of model [21], no interaction of thinning and fertilization exists

that is not taken into account in model [21].

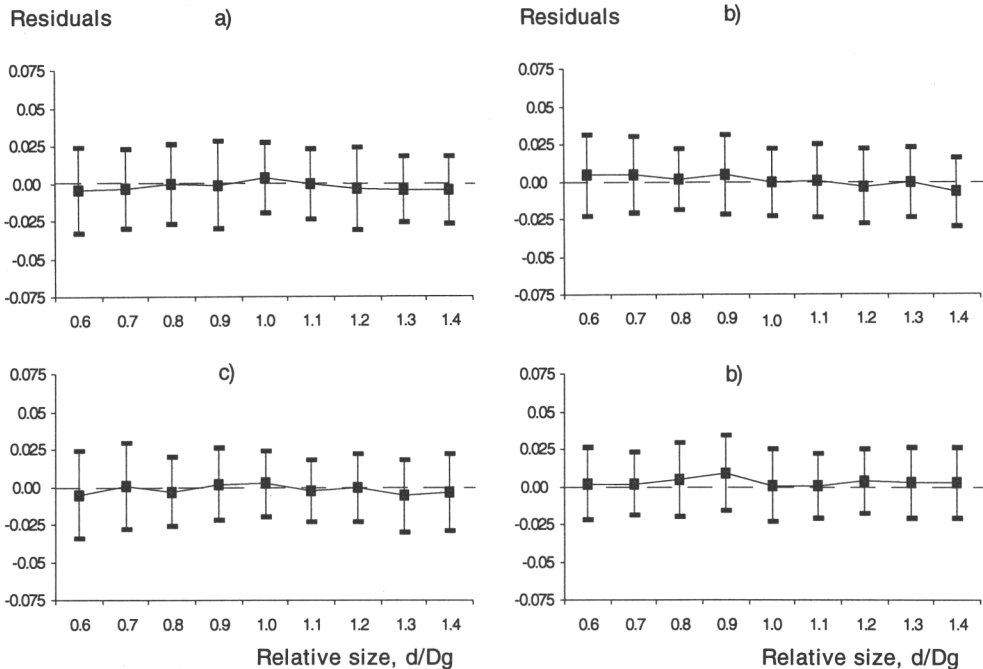


Figure 11. Residuals (\pm standard deviation) of stem form model [21] plotted against relative tree size: a) control plots, b) unfertilized and heavily thinned (60% of stem number removed) plots, c) fertilized (300 kg N ha⁻¹) and unthinned plots, and d) fertilized (300 kg N ha⁻¹) and heavily thinned plots.

6 Conclusions

6.1 Model characteristics

In the course of this dissertation, empirical models were developed, in which tree dimensions were predicted with the help of taxatorical tree and stand characteristics commonly available in the data from practical forest inventories. The results obtained confirm that it is possible to successfully predict tree growth and growth response to thinning and fertilization based on this kind of relatively limited information.

In the models for tree height growth, a widely applied model structure was applied, in which tree growth is stratified into potential growth and the modifier effects. The model structure, in which individual-tree growth is restricted to asymptotic potential growth, ensures a model that behaves logically regardless of the values of the input variables. This feature is advantageous especially in long-term growth predictions, when the values of input variables exceed the range of variation in the modelling data.

Increment of stand dominant height was employed to describe potential height growth. Earlier research results, as well as the results from study IV, have confirmed that dominant height increment is fairly insensitive to stand density and to thinning from below. In model development, the observed increment of stand dominant height was used as potential height growth. In applying the models, dominant height increment has to be obtained from site index equations. Thus, the prediction method sets high requirements on site index equations, because the quality of prediction is largely dependent on the behaviour of the equations for dominant height increment. Today, there are two applicable published site index equations in Finland for Scots pine stands; the models presented by Vuokila and Väliäho (1980) for seeded or planted stands of Scots pine, and the models presented by Gustavsen (1980) for naturally regenerated stands.

An analogical modelling method for predicting tree diameter growth would be to use the increment information from open-grown trees as potential growth. In this study, that kind of model structure was not applied in predicting diameter growth due to the lack of increment data from open-grown trees.

The idea of stratifying tree growth into separate growth factors was also applied in predicting the growth response to thinning and fertilization. In the first part of the models, the reference growth was predicted, and this was then modified with the second part of the model referring to the effects of the applied treatment. The model for reference growth ($F_1(ref)$ in models [1] and [11]), i.e. the growth of a tree in the

absence of treatment, was constructed in order to facilitate predicting all the other growth factors except those directly related to the treatment itself. In applying the models in the simulation process, all the input variables are updated at the beginning of each growth period. Therefore, the model for reference growth also accounts for the indirect treatment effects causing changes in stand density and tree size by the beginning of the growth period in question. The remaining part of tree growth not explained by the reference growth model ($F_1(ref)$), i.e. the direct treatment effect, is predicted with the equation for the relative growth response ($F_2(fert)$ in model [1], and $F_2(thin)$ in model [11]).

The magnitude of the absolute growth response to treatment cannot be directly obtained from the model parameters, because of the applied model structure. Instead, it can be obtained by simulating stand development with and without treatment, and by calculating the differences among alternative predictions. Although the total response cannot be obtained directly from the model parameters, the method applied guarantees a logical way to predict the dynamics of the growth response.

Incorporating the responses to various silvicultural treatments into a growth model easily leads to complicated models with large numbers of parameters, which can be difficult to interpret. In the research work reviewed in this dissertation, a simplified assumption has been employed, that the relative growth response to thinning or fertilization treatment is not affected by tree size. Therefore, it was possible to predict the growth response with a relatively simple model structure and a reasonable number of input variables and parameters needing to be estimated. Behavioural analyses of the models confirmed that the assumptions underlying the models were valid for even-aged, managed stands of Scots pine and loblolly pine. This assumption should not be directly and incautiously extended to stands with more heterogeneous stand structures, nor to other tree species. However, similar results on growth responses in thinned and fertilized Douglas-fir stands is provided by Moore et al. (1994).

A modified Weibull function was applied in predicting the growth response to thinning and fertilization. A modified cumulative Weibull function was applied in constructing a thinning variable in the model for tree crown ratio. The Weibull function proved to be a flexible, non-linear function in predicting responses to the applied treatments. All the parameters of the models, except for those in study I, were estimated simultaneously. Therefore, it was possible to use all the information from the experimental stands, regardless of the treatment, in model development. Despite the simplifying assumption regarding to the growth response, the total number of model parameters was fairly

large. However, it did not cause any major difficulties in the estimation of the parameters.

The importance of the tree crown ratio as a predictor in the growth model was assessed in study **IV**. The crown ratio proved to be a significant regressor in the diameter growth model resulting in improved accuracy in diameter growth prediction. Thus, the results confirm that the tree crown ratio provides valuable information for model development, and that it should be employed in modelling whenever available in the data.

6.2 Effect of thinning and fertilization on tree growth

The results on the magnitude and duration of the growth response following thinning and fertilization confirm findings from the earlier growth and yield studies referred to in this dissertation. Probably the most important new knowledge provided by the present study concerns the combined effect of thinning and fertilization. According to the results obtained, there is no synergistic effect of thinning and fertilization on tree growth that cannot be predicted using the models, in which the responses to these treatments are predicted separately.

The effect of fertilization on the development of tree crown ratio and stem form were examined with the help of experimental data in which the effects of both fertilization and thinning were studied. The experimental design applied in the trial for Scots pine prevented the separate modelling of the fertilization effects. However, the results of the analysis presented in Chapter 5 showed that the development of the tree crown ratio in fertilized stands can be predicted with adequate accuracy by applying a crown ratio model without any explicit variable referring to fertilization. The only significant effect of fertilization on the development of crown ratio was the accelerated development of stand density (G) leading to increased crown recession. Nevertheless, this feature is already taken into account by the model for the crown ratio presented in study **III**.

The changes in stem form, caused by thinning from below, can be explained by the altered growth rates in the height and breast height diameter of the trees. However, fertilization changes tree stem form in a way that cannot be explained by changes in the height and breast height diameter of the trees. In fertilized stands, the maximum diameter growth response occurs in the upper parts of the stem. This was confirmed by the models for cylindrical form factor presented in Chapter 5, in which the categorical variables referring to fertilization

proved to be statistically significant. Therefore, if the stem volume of fertilized stands is predicted with models based only on information about diameter at breast height and tree height, underprediction results.

The effects of repeated fertilization or thinning could not be modelled in this context because of lack of suitable data. As regards repeated thinning, the diameter growth model with an explicit thinning variable may be applicable because the effect of actual stand density is taken into account in predicting the reference growth by some of the regressor variables. However, further studies must be conducted to confirm its applicability. As regards fertilization, it is known that the growth response to repeated fertilization is not equal to the effect of the first fertilization treatment (e.g. Kukkola and Saramäki 1983). The magnitude of the growth response to repeated fertilization depends on the time interval between treatments, and on the fertilizer doses. Kukkola and Saramäki (1983) have constructed a model for predicting the growth response to repeated fertilization at the stand level. The effect of successive fertilization treatments is based on the "Law of Diminishing Growth Increases". It is likely that this model structure is applicable also in individual-tree growth models, although further studies are required to confirm the suitability of the said modelling approach at the tree level.

6.3 Application of models

The models are based on data collected from well-managed, pure, even-aged stands of Scots pine (**I**, **III** and **IV**) and loblolly pine (**II**). The results presented in studies **III**, **IV** and in Chapter 5, are based on data collected from young or middle-aged stands of Scots pine at the stage of first commercial thinning when the experiments were established. Therefore, the models, as well as the conclusions about the responses to the applied silvicultural practices, are directly applicable only in such stands. The models are applicable in evaluating the effects of alternative silvicultural practices on the development of even-aged, managed pine stands.

The modelling data represent experimental forests, where growth conditions are controlled. The stands are even-aged and even-sized in structure, and the silvicultural practices carried out in them are performed with greater care than in the average commercial stand. The growth and yield predictions obtained using these kinds of models are likely to be biased when applied directly to commercial stands in subjected to treatments typical of practical forestry. The absolute level of tree growth, as well as the magnitude of the growth responses to thinning and fertilization predicted using the models presented here, are

likely to be greater than in commercial forests on average. Nevertheless, the model formulation developed in studies I through IV are likely to be applicable in constructing individual-tree growth models for forest management planning, based on more comprehensive and representative forest inventory data; e.g. in the development of new growth and yield models for the MELA system.

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MODELLING TREE BASAL AREA GROWTH RESPONSE AFTER NITROGEN FERTILIZATION

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ABSTRACT

Individual-tree simulation models were developed for predicting tree basal area growth response following nitrogen fertilization. The models were based on data from permanent research plots located in middle-aged, managed Scots pine (*Pinus sylvestris* L.) stands in southern Finland. Using data from unfertilized control plots, a reference growth model for basal area growth was developed for calculation of the annual tree basal area growth response. The growth response of a fertilized tree was calculated as the difference between observed growth and predicted reference growth. The temporal distribution of tree basal area growth response was modelled using the Weibull function. The parameters of the Weibull function were expressed as a function of stand characteristics. Relative tree basal area growth response increased with increasing nitrogen doses between 80 - 450 kg N/ha. The growth response also varied according to site index; it reached its maximum level with the site index between 21 - 22 m. The relative response was not affected by neither stand density nor tree size.

INTRODUCTION

In Scots pine stands growing on mineral soil sites in Finland, the lack of available nitrogen is one of the most important factors limiting tree growth. Nitrogen fertilization has been used to increase timber production in commercial forests in Finland since the early 1970s.

In forest management planning, growth simulators are used for predicting the development of forest growth and yield. To forecast growth reliably for fertilized stands, growth simulators should include not only growth models, but also models that predict the growth response after nitrogen fertilization. At stand-level, growth response following nitrogen fertilization has been modelled by Rosvall (1980), Kukkola and Saramäki (1983) and Ballard (1984), among others. At present, growth in most simulators is predicted using individual-tree models. Therefore, growth response should also be modelled at the tree-level. Individual-tree growth models for fertilized stands have earlier been introduced by Burkhart et. al. (1987) and Shafii et. al. (1990), for example.

The aim of this study was to develop a model for predicting tree basal area growth response after nitrogen fertilization. A method was introduced to describe the temporal distribution of the growth response. The models were developed for application in growth simulators used for forest management planning purposes. Therefore, the independent variables of the models were restricted to those measured in practical forest inventories.

MATERIAL

The study material consisted of eight experimental Scots pine stands (*Pinus sylvestris* L.) established at the beginning of the 1970s for growth and yield studies. The experiments were set up in naturally regenerated, middle-aged stands growing on mineral soil sites in southern Finland. The experiments were arranged using factorial design. They consisted of sample plots representing two types of nitrogen fertilizers and three levels of nitrogen doses. There was at least one unfertilized control plot in every experimental stand. The nitrogen fertilizers used were ammonium nitrate with lime (Os) and urea (U). In one of the experiments, ammonium sulphate (As) was used instead of ammonium nitrate with lime. The nitrogen doses varied from 80 to 450 kg N/ha. Fertilizations were carried out in the spring of the same year that the experiments were set up. The number of replications varied between 1 and 10. Sample plot size was 1000 m² on average.

Study periods of the experimental stands varied between 10 - 15 years following fertilization. Although the stands were well managed, no thinnings were carried out during the study period, nor during the five-year period prior to the establishment of the experiments.

The sample plots were measured at intervals of five years. Each tree was measured for its breast height diameter over bark. An average of 17 sample trees were selected from each sample plots. Each sample tree was measured for its bark thickness and increment cores were examined to determine the annual radial growths for the preceding 15 years. The study material consisted of 2158 sample trees from 117 sample plots (Table 1). The number of annual radial growth observations was 37 800.

TABLE 1. Information about the experimental stands.

Exp. no.	Site index m	Age a	Stem number no./ha	Basal area m ² /ha	Mean height m	Nitrogen doses kg N/ha	No. of sample plots	Number of sample trees		
								Control	Os	Urea
224	24.1	85	410	19.7	20.0	0- 80-160-240	52	176	353	337
301	24.6	75	520	16.3	18.7	0-150-300-450	21	60	194	186
312	26.9	60	745	25.1	19.5	0-150-300-450	7	26	77	70
315	21.5	80	460	16.9	17.2	0-150-300-450	7	19	56	52
319	20.0	120	290	16.4	21.2	0-150-300-450	7	16	51	49
320	19.6	120	445	17.9	21.4	0-150-300	6	29	38	34
332	22.9	85	410	17.3	19.0	0-150-300-450	7	20	59	62
336	23.1	65	645	18.5	16.5	0-150-300-450	10	36	89	69

On the basis of the measurements, tree diameters (d), basal areas (g) and relative diameters (d_r) at breast height without bark of all trees were calculated for every year of the study period. Relative tree diameter (d_r) was defined as the ratio between tree diameter and the diameter of the thickest tree in the sample plot (d/D_{\max}). For sample trees, annual basal area growths (i_R) were calculated from increment core measurements. The largest experiment no. 224 (with an area of 11 ha), was divided into three blocks before analysis. Separate reference growth models were developed for each of the blocks.

The models were tested against independent data collected from six repeatedly-measured experimental Scots pine stands located in southern Finland. Two of the stands were middle-aged, while four of them were considerably younger and denser than those included in the study material. The test material consisted of 74 sample plots with 1104 sample trees and 7969 annual growth observations. Ammonium nitrate with lime (Os) was the only fertilizer used in the sample plots. The nitrogen doses varied between 55 and 750 kg N/ha.

MODEL DEVELOPMENT

Determination of growth response

The aim of the study was to develop a model for predicting annual tree basal area growth after nitrogen fertilization. To calculate the growth response of trees following nitrogen fertilization (ΔI_f), it was necessary first to estimate tree growth as if it had never been fertilized. This "hypothetical" growth was defined as reference growth (I_0).

The information collected from the control plots was used to develop reference growth model that was then applied to predict the annual reference growths of fertilized trees (\hat{I}_0). The annual growth response of a fertilized tree was defined as the difference of observed growth (I_f) and the estimated reference growth (\hat{I}_0):

$$\Delta \hat{I}_f = I_f - \hat{I}_0. \quad [1]$$

Reference growth model

Annual basal area growth measurements from sample trees of the unfertilized control plots were used in developing the reference growth model. Model parameters were estimated separately for each control plot. A logarithm of annual tree basal area growth, $\ln(i_k)$, was chosen as a dependent variable in the linear regression model [2]. The independent variables in the model were the logarithm of tree diameter without bark, $\ln(d)$, describing tree size, and stand basal area without bark (G), describing stand density. Neither site index nor relative tree size was included in the model, because the parameters were estimated separately for each experimental stand. The variation in tree growth between successive years was taken into account by adding fixed annual effects (v_1, v_2, \dots, v_n) into the model [2].

$$\ln(\hat{i}_k) = a_0 + a_1 \ln(d) + a_2 G + v_1 + v_2 + \dots + v_{n-1}. \quad [2]$$

where \hat{i}_k = predicted tree basal area growth, cm²
 d = tree diameter at breast height, cm
 G = stand basal area, m²
 v_1, v_2, \dots, v_{n-1} = fixed annual effects
 n = number of years in study period
 a_0, a_1, a_2 = parameters

A logarithm of tree diameter ($\ln(d)$) and stand basal area (G) proved to be significant independent variables in the model, except in experiments 224 and 301, where $\ln(d)$ was the only significant explanatory variable (Table 2).

TABLE 2. Annual tree basal area growth models for unfertilized sample plots (= reference growth models). Dependent variable $\ln(i_k)$.

Variable	Experiment									
	224(1)	224(2)	224(3)	301	312	315	319	320	332	336
	Coefficients									
<i>Constant</i>	-0.668	-4.015	-3.620	-1.616	-1.081	1.597	4.767	-0.079	1.414	0.367
$\ln(d)$	0.834	1.901	1.721	1.147	1.462	1.819	0.402	1.158	1.357	1.216
G					-0.062	-0.381	-0.317	-0.115	-0.305	-0.144
R^2	0.242	0.492	0.357	0.543	0.480	0.530	0.244	0.235	0.450	0.558
S_f	0.143	0.150	0.154	0.119	0.099	0.168	0.122	0.289	0.139	0.126
$S_e\%$	14.4	15.1	15.5	11.9	9.9	16.2	12.2	29.5	14.0	12.7

Constant = a_0 (in equation [2]) + average of fixed annual growth effects (v_1, v_2, \dots, v_n); R^2 = coefficient of determination; S_f = Residual standard deviation; $S_e\%$ = Relative standard error of the estimate.

The residuals were examined separately for each stand with regard to the independent variables of the models and relative tree size (dr). No trends in the residuals were observed with respect to the examined variables.

The basal area reference growths of fertilized trees were calculated using model [2]. When model [2] was applied to predict the annual reference growths for fertilized trees, the observed values of stand and tree characteristics at the beginning of the year in question were used as the independent variables. Thus, the annual reference growth of fertilized tree was estimated separately for each year regardless of the growth predictions of the previous years.

Growth response model

Relative annual basal area growth response, $\Delta(p_k)$, was chosen as the dependent variable of the response model. It was calculated on the basis of observed tree basal area growth (i_k) and the predicted reference growth (\hat{i}_k).

$$\Delta(p_k) = (i_k - \hat{i}_k) / \hat{i}_k, \quad [3]$$

The temporal distribution of response following nitrogen fertilization was modelled using the Weibull function. The three-parameter Weibull distribution can be expressed as

$$f(X) = \frac{\chi}{\beta} \left(\frac{X - \alpha}{\beta} \right)^{(\chi-1)} \exp \left[- \left(\frac{X - \alpha}{\beta} \right)^\chi \right], \text{ when } (\alpha \leq X < \infty) \quad [4]$$

$$= 0, \text{ otherwise}$$

where α = location parameter
 β = scaling parameter (> 0)
 χ = shape parameter (> 0)

When model [4] was applied to describe the distribution of growth response over time, it was multiplied by coefficient k referring to the total growth response caused by fertilization. Location parameter (α) was set to zero assuming that fertilization starts to increase tree growth already in the first growing season following nitrogen application. The modified Weibull model is given by equation [5]:

$$\Delta \hat{p}_k(t) = k \left\{ \frac{\chi}{\beta} \left(\frac{t}{\beta} \right)^{(\chi-1)} \exp \left[- \left(\frac{t}{\beta} \right)^\chi \right] \right\}, \text{ when } (0 \leq t < \infty) \quad [5]$$

where $\Delta \hat{p}_k(t)$ = relative basal area growth response in year t after fertilization
 t = year after fertilization
 β, χ, k = parameters

To examine the effects of stand-level characteristics on growth response, model [5] was fitted separately to each fertilized plot. Accordingly, the average growth response of trees was calculated for every fertilized sample plot. Next, the relationships between stand characteristics (the amount and the type of fertilizer, site type, stand density and stand age) and parameters of the Weibull model were examined. The relationships were formulated so that k , β and χ were expressed as a function of the nitrogen dose and stand characteristics.

Type of fertilizer, nitrogen dose and site index were the stand-level characteristics that correlated with coefficient k and parameter β . Further, parameter β and coefficient k were positively correlated. The shape parameter χ did not correlate with the studied stand-level variables, neither was it closely correlated with β or k .

The effects of tree characteristics on the growth response (coefficient k) were studied by calculating the total response of every sample tree. It was computed by summing up the annual responses of the study period. Annual responses were calculated according to equation [3]. The relationships between total response and tree diameter (d) as well as relative tree diameter (d_r) were studied. There was no significant interdependence between total relative growth response and studied tree-level variables.

The final, nonlinear growth response model can be expressed as follows:

$$\Delta(\hat{p}_8) = k \left\{ \frac{\chi}{\beta} \left(\frac{t}{\beta} \right)^{(\chi-1)} \exp \left[- \left(\frac{t}{\beta} \right)^\chi \right] \right\}, \text{ in which} \quad [6]$$

$$k = \left(\frac{a_2}{a_1} \right) \left(\frac{H_{100}}{a_1} \right)^{(a_2-1)} \exp \left(- \left(\frac{H_{100}}{a_1} \right)^{a_2} \right) \left(a_3 \left(\frac{FN}{10} \right) - a_4 \left(\frac{FN}{10} \right)^2 \right)$$

$$\beta = a_5 + a_6 k$$

$$\chi = a_7,$$

where $\Delta(\hat{p}_8)$ = relative annual basal area growth response
 t = year after fertilization
 H_{100} = site index, m
 FN = nitrogen dose applied, kg N/ha
 a_1, \dots, a_7 = parameters

In the final response model [6], coefficient k is depicted as the function of site index (H_{100}) and nitrogen dose (FN). Scaling parameter (β) in the Weibull model is stated as a linear function of coefficient k . Shape parameter χ is independent of fertilization factors and stand characteristics.

Model [6] was fitted separately for OS- and U-fertilizers (Table 3). The model was based on the data that included annual growth responses of all sample trees growing on the fertilized sample plots. The time period covered the studied years following the fertilizations. All the parameters were estimated simultaneously using an iterative nonlinear regression program (BMDP 3R) with the Gauss-Newton algorithm (Jennrich 1990).

The residuals were studied separately against the independent variables of model [6] and against stand density (G), tree diameter (d) and relative tree diameter (d_r). Residual variance was evenly distributed over time, except for the first two years after fertilization. The model resulted in slight overestimation in the first year, followed by the underestimation in the second year after fertilization. The Weibull model appeared to be too robust for describing the sudden increase in growth response during the first two years after fertilization. The residuals were evenly distributed with respect to other studied variables.

TABLE 3. Parameter estimates of the relative tree basal area growth response model [6].

Os fertilization			Urea fertilization		
parameter	estimate	asymptotic standard deviation	parameter	estimate	asymptotic standard deviation
a_1	22.225	0.107	a_1	23.301	0.201
a_2	4.561	0.134	a_2	3.868	0.163
a_3	3.390	0.139	a_3	2.574	0.144
a_4	0.016	0.003	a_4	0.005	0.002
a_5	2.976	0.110	a_5	3.230	0.135
a_6	0.569	0.022	a_6	0.649	0.035
a_7	2.466	0.036	a_7	2.611	0.048
Mean of the dependent variable: 0.317			Mean of the dependent variable: 0.245		
Residual mean square: 0.284			Residual mean square: 0.270		
Degrees of freedom: 11323			Degrees of freedom: 10952		

According to the model, relative basal area growth response varies with nitrogen dose and the type of fertilizer. Within the range of variation of nitrogen doses used in the study material (80 - 450 kg N/ha), the response increased with increasing dose. The year of maximum growth response also varied according to the dose of nitrogen. When dose increased from 50 to 400 kg N/ha the maximum response increased respectively from three to six years after fertilization. With Os fertilizer the response, on average, was 30 % greater than after application using the same amount of urea fertilizer.

An increase in the site index increases the response until it reaches the maximum level at the site index of 21 - 22 m, thereafter the response decreases as the site becomes more fertile.

APPLICATION OF GROWTH RESPONSE MODEL

When model [6] is applied to growth simulation of a fertilized tree, it can be used only in combination with the tree basal area growth model of unfertilized tree by which the reference growth for fertilized tree is predicted. The relative annual growth response can then be calculated using model [6] and added to the predicted reference growth. Simulation is carried out year by year. Stand basal area and tree diameters are updated every year after computing the annual growths and growth responses. Thus, both the reference growth model and the growth response model affect the final, absolute growth response caused by nitrogen fertilization (Fig 1.).

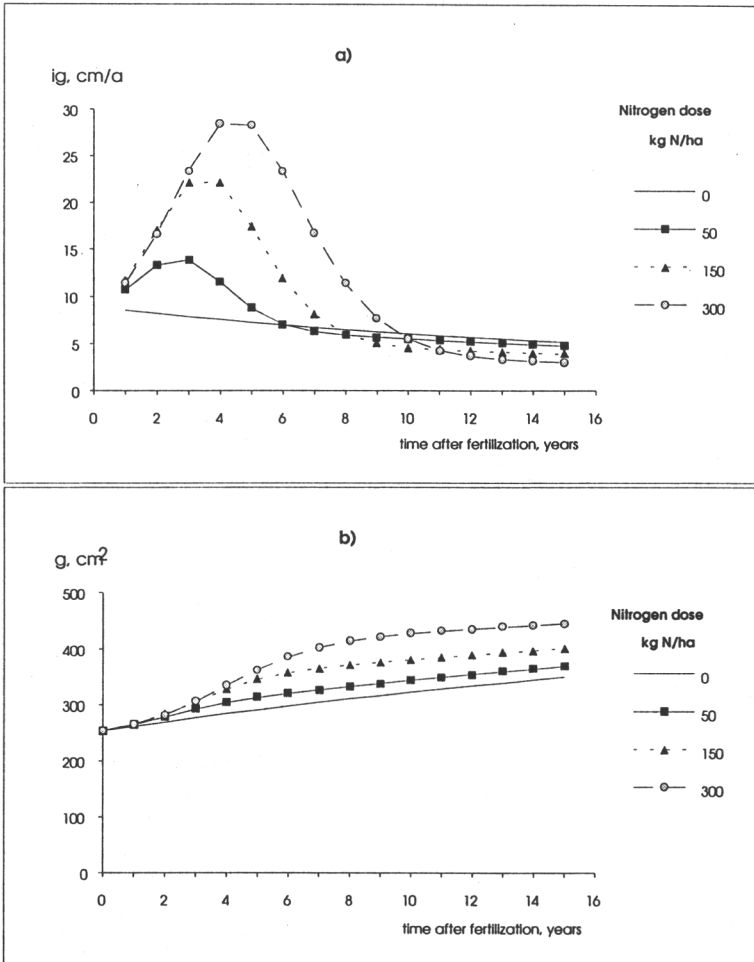


Figure 1. An example of simulated development of a fertilized tree. Annual tree basal area growth (a) and basal area development (b) following fertilization with varying nitrogen doses. Reference growth is predicted using basal area growth model for exp. 336.

VALIDATION OF THE MODELS

Because the response models were based on sample trees in experimental plots, stand-level simulations were done to test the models against data including all trees in experimental stands. Furthermore, the models were tested against the independent data. The list of tree diameters at the time of fertilization was used as the input data for simulation. Annual stand basal areas and mean diameters (weighted with basal area) of the sample plots were calculated from the predicted basal areas of trees. For every

fertilized plot, predicted stand basal area growth and mean diameter increment were compared with the observed values. The following characteristics were calculated to describe the reliability of the model:

$$b = \sum_{i=1}^n (y_i - \hat{y}_i) / n \quad (\text{systematic error}) \quad [7]$$

$$b_r = \sum_{i=1}^n [(y_i - \hat{y}_i) / \hat{y}_i] / n \quad (\text{relative systematic error}) \quad [8]$$

$$RMSE = \left[\sum_{i=1}^n (y_i - \hat{y}_i)^2 / n \right]^{0.5} \quad (\text{root mean square error}) \quad [9]$$

$$RMSE_r = \left[\sum_{i=1}^n [(y_i - \hat{y}_i) / \hat{y}_i]^2 / n \right]^{0.5} \quad (\text{relative RMSE}) \quad [10]$$

where y_i = observed value in stand i

\hat{y}_i = predicted value in stand i

At stand-level, simulations with the study material resulted in a relative bias of 1.01 % in the estimated annual stand basal area growth and a relative bias of -0,59 % in the annual mean diameter increment. The test against independent stand data showed that the models underestimated annual stand basal area growth by 6.8 % and mean diameter increment by 5.4 % on average (Table 4).

TABLE 4. Statistics describing the reliability of the models in stand-level growth prediction as tested against the study material and independent test material.

	Stand basal area growth, m ² /ha/a		Mean diameter increment, cm/a	
	Study material	Test material	Study material	Test material
<i>Mean of the estimate</i>	0,394	0.705	0,339	0.342
<i>b</i>	-0.001	0.003	-0.006	0.011
<i>b_r, %</i>	-1.01	6.79	-0.58	5.45
<i>RMSE</i>	0.049	0.091	0.053	0.036
<i>RMSE_r, %</i>	13.33	13.35	13.82	10.50

Most of the sample plots included in the test material were located in considerably younger and denser stands with faster basal area growth than the stands included in the study material. It was noticed that the model underestimated growth response most in youngest stands with rapid basal area growth (Fig. 2). As regards mean diameter increment, no such trend was to be noticed.

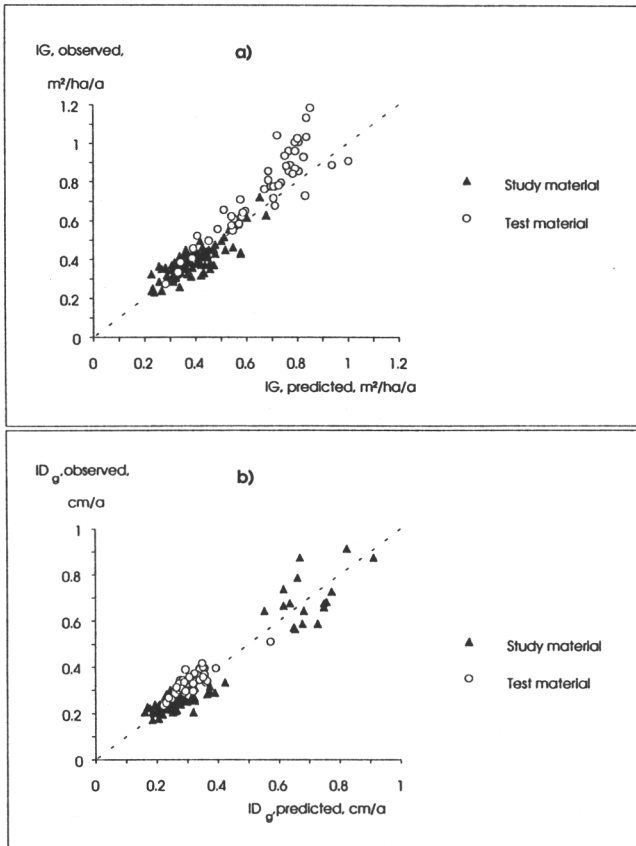


Figure 2. Observed and predicted mean annual stand basal area growth (a) and mean annual diameter increment (b).

At tree-level, the models were tested against independent test material. The residual variances of the predicted basal area growth responses were examined with regard to the variables of model [6] as well as tree and stand characteristics. The results showed that the model slightly underestimated the growth response in general, and especially in the stands fertilized with nitrogen doses over 450 kg N/ha. The residuals plotted against stand density, stand age and site index were evenly distributed. In young stands, after the growth response had levelled off, the models overestimated tree growth. This result indicates that there might be a negative aftereffect of nitrogen fertilization on tree growth. Furthermore, in young pine stands, the relative growth response of the smallest trees in the stands were slightly underestimated, while simulation resulted in overestimation of growth among the biggest trees.

DISCUSSION

Tree basal area growth response was modelled using the Weibull function. The parameters of the Weibull-function describe the size, location and the shape of the distribution. Whereas the Weibull parameters were expressed as functions of stand variables, the relationships between the characteristics of the distribution and stand variables are easy to interpret. Scaling parameter, β , was expressed as a function of coefficient parameter k because of the high correlation between k and β . The connection between these parameters in model [6] prevents irrational combination of the parameter values, and thereby the unrealistic shape of the distribution, even if the model is applied into stands with characteristics beyond the stands of the study material.

The growth response models developed in this study are based on data collected from managed, even-aged pine stands between the stand ages of 60 - 120 years. The study material represents the stands at that stage of development when fertilization is most profitable. Today, forest fertilization in Finland is mostly concentrated in mature pine stands with large-sized growing stock. The findings concerning the features of growth response in mature Scots pine stands with varying nitrogen doses and stand characteristics support the results of the earlier studies (Gustavsen and Lipas 1975, Rosvall 1980, Kukkola and Saramäki 1983).

The results obtained using the test material suggest that there are some uncertainties as to behaviour of the response model in young and dense pine stands. Firstly, applying the Weibull function to describe the temporal distribution of the response following nitrogen fertilization presupposes that the response is always positive. Therefore, the model fails to predict correctly those situations where fertilization has negative growth effects. It has been observed that nitrogen fertilization alters a trees internal growth ratios by decreasing the root:shoot ratio (e.g. Köstler et. al. 1968). It can be supposed that when the effect of fertilization terminates, the root:shoot ratio will be restored, and the growth will be allocated into roots at the expense of growth in the stem and crown. The results based on the test material showed that this kind of negative aftereffect is possible in young trees. Secondly, according to the basal area growth response model, the relative response is independent of tree size and stand age. This assumption is likely to hold in middle-aged and mature stands, where trees have enough growing space. Nevertheless, the results on young and dense stands included in the test material indicate that the assumption may not be valid. Therefore, the models should be applied very cautiously in young and dense Scots pine stands.

Nitrogen fertilization accelerates the development of stand density, which leads to increased competition for growing space between trees. While stand density increases, the growth rate of single trees may decrease. In a fertilized stand, once the trees have used up the applied nitrogen, individual trees may grow more slowly than trees in an unfertilized stand because of increased stand density. In this study, the effect of stand density has been taken into account in the reference growth model. Consequently, the negative aftereffect of fertilization caused by increased stand density can be described with the models developed in this study (see Fig. 1).

The growth response model can be included in a simulator in which basal area growth is predicted using individual-tree growth model. To fulfill the presumptions underlying the response model, the basal area growth model should meet certain requirements. According to the response model, relative growth response is independent of stand density and tree size. Nevertheless, these variables affect absolute tree growth and so they should be included in the tree basal area growth model.

In this study, only the model for tree basal area growth response was developed. To predict the volume growth in fertilized stands, also height growth response model should be developed. Such a model could not be developed in this connection, because the study material did not contain annual height growth data. Also, the combined effect of fertilization and thinning, as well as the effect of repeated fertilizations, on tree growth await further study. A method introduced by Kukkola and Saramäki (1983) for predicting stand-level growth response in repeatedly fertilized stands is likely to find use to individual-tree growth response modelling as well.

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Modeling tree growth in fertilized midrotation loblolly pine plantations

by

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Abstract

Hynynen, J., Burkhardt, H. E., and Allen, H. L. 1994. Modelling tree growth in fertilized midrotation loblolly pine plantations.

Diameter and height growth models for fertilized loblolly pine stands were developed using data from midrotation loblolly pine plantations across southeastern United States. Tree growth in fertilized stand was predicted with a reference growth model multiplied by an equation predicting the relative growth response following fertilization. The temporal distribution of the growth response was modeled by applying the Weibull function. These equations for fertilizer growth response were developed to be compatible with individual-tree simulation models. Information about dose, nutrient elements and time elapsed since fertilization are needed to predict the relative growth response following fertilization.

Additional keywords: *Pinus taeda* L., nitrogen, phosphorus, yield, simulation, growth response, Weibull function

Introduction

Forest fertilization is an important silvicultural practice to improve stand productivity in loblolly pine plantations in the southeastern United States. Results from studies based on field trials demonstrate that nitrogen and/or phosphorus fertilization will give significant growth responses in midrotation loblolly pine plantations. By 1992, more than 700,000 acres of midrotation loblolly pine stands were fertilized with nitrogen or nitrogen and phosphorus. Treated areas represent less than 10 percent of the 7.5 million acres of loblolly pine plantations in the southeastern United States.

In forest management planning, reliable growth and yield models are needed to assess the growth effects of silvicultural treatments. Purpose-designed fertilization experiments have been established for several tree species in order to provide information about growth effects of fertilization. Methods used to analyze these fertilization data have included analysis of variance and analysis of covariance (e.g. Paavilainen and Simpanen 1975, Miller and Tarrant 1983, Opalach and Heath 1988, Hynynen and Kukkola 1989) and regression analysis (e.g. Gustavsen and Lipas 1975, Wells et al. 1976, Rosvall 1980, Kukkola and Saramäki 1983, Miller et al. 1988, Shafii et al. 1990, Hynynen 1993).

In the evaluation of the fertilizer growth response and its change over time, the total response can be partitioned into direct and indirect effects (Miller and Tarrant 1983, Auchmoody 1985, Opalach and Heath 1988). Direct fertilization effect refers to the growth response

that is directly due to improved nutrition. Indirect effect can be defined as the growth response that is due to altered stocking brought on by fertilization. In analyses of fertilization data, various methods have been applied to determine these two effects or their combination. First, indirect effect can be removed by subtracting the mean growth of the control plots from that of the fertilized plots (Peterson et al. 1984, Opalach and Heath 1988, Peterson and Hazard 1990). Second method is to develop a growth model for unfertilized plots, and use that model to obtain predicted growth for fertilized plots. Direct fertilization effect can then be calculated as the difference between actual growth of fertilized plots and predicted growth (e.g. Kukkola and Saramäki 1983, Peterson et al. 1984, Miller et al. 1988, Hynynen 1993). Third method is to develop a model for total effect of fertilization that includes both direct and indirect components in its formulation (e.g. Stegemoeller and Chappel 1990, Schafii et al. 1990).

Although there are published studies that provide information about duration and magnitude of the growth response following fertilization, there is no complete prediction system for evaluating growth response in loblolly pine plantations. Stand-level models predicting the growth response after nitrogen fertilization for loblolly pine stands have been published by Wells et al. (1976), Ballard (1982), Duzan et al. (1982) and NCSFNC (1992a). Bailey et al. (1989) developed stand structure and yield prediction models for midrotation slash pine stands including prediction equations for stand basal area, dominant height, individual tree height, diameter distribution and survival. So far, individual-tree prediction models for fertilization response have not been available. Lack of suitable tree-level models has made it difficult to obtain reliable forecasts using growth simulators that are based on individual-tree growth models.

The objectives of this study were 1) to quantify the effects of nitrogen and phosphorus fertilization on tree diameter and height growth and 2) to develop models for predicting individual-tree growth in fertilized midrotation loblolly pine plantations for forest management planning purposes.

Study material

The analysis is based on data from the North Carolina State Forest Nutrition Cooperative's (NCSFNC) Regionwide 13 Study. Study material included eight-year data from 13 installations located across the southeastern United States. Experiments were established in 1984 and 1985. All the stands were midrotation loblolly pine plantations,

with an age of 11 to 14 years at the time the experiments were established (Table 1).

At each study location, four levels of nitrogen (0, 100, 200, 300 lbs N/acre) and three levels of phosphorus (0, 25, 50 lbs P/acre) were examined using a factorial experimental design. At each location, two or four replicates (blocks) of the basic twelve treatment matrix were established. Using the terminology of Milliken and Johnson (1984), experimental design has two-way treatment structure with randomized complete block design.

Rigorous guidelines for selection of candidate stands and blocking of plots were used to minimize within site variation for stand characteristics and soil type. Plots within a block generally did not vary more than 3 feet in dominant height, 10 ft² per acre in stand basal area, and 80 stems per acre at study establishment.

The observed rates of mortality during the eight-year study period were 5.8 % of the total stem number in non-fertilized plots, 6.1 % in the plots fertilized with 100 lbs N/acre, 7.1 % in the plots fertilized with 200 lbs N/acre and 8.6 % in the plots fertilized with 300 lbs N/acre. Mortality models were not developed in this study.

Measurement plots included a minimum of 30 to 40 trees surrounded by a treated buffer zone of at least 30 feet. Size of the rectangular plots varied between 0.045 to 0.516 acres, with an average of 0.09 acres. Diameter at breast height and total height were measured on all trees in each plot. Tree diameters were measured to an accuracy of 0.1 in., and tree heights to that of 1 ft. Measurements were carried out at 2-year intervals during the dormant season. Data included 17,900 trees from 432 sample plots. Total number of two-year growth observations was 57,900.

Table 1. Stand and tree characteristics of the study material.

	Mean	St. dev.	Min.	Max.
Stand Age, years	15.7	2.6	11	20
Site Index, ft	61.6	5.8	53	74
Trees/acre	516	165	205	939
Basal area,ft ² /acre	110.9	24.2	46.1	175.9
D _q , in.	6.65	1.09	4.14	10.44
I _{D_q} , in./year	0.28	0.10	0.08	0.52
H _{dom} , ft	44.8	7.8	25.2	68.6
I _{H_{dom}} , ft/year	2.39	0.74	0.10	5.57
DBH, in.	6.65	1.54	1.30	13.70
I _{DBH} , in./year	0.25	0.15	0.05	1.20
h, ft	42.4	8.4	14.0	74.0
i _h , ft/year	2.3	1.0	0.5	9.0

Regionwide 13 Study included foliar nutrient data measured from the sample plots (NCSFNC 1992b). That data were used in this study to examine the applicability of foliar nutrient information in prediction of fertilizer response. Foliar samples were collected by taking samples of the first flush of the most recent year's foliage during December or January prior to one, two, four and six years following the fertilization. At each sample period, twenty fascicles were collected from the terminal of a primary branch from the upper crown on each of five trees per plot. The sample trees were randomly selected among the dominant and codominant trees. Samples were combined by plot and returned to the laboratory for nutrient concentration and fascicle weight determinations. To determine an average level of foliar nutrient content for each study location, an average of foliar nutrient contents measured at two, four and six years after establishment was calculated for each non-fertilized plot, and that mean value was used in the analysis of this study.

Before analysis, the data were randomly split into two parts of equal sizes. For diameter growth models, half of the trees of each sample plot were randomly selected into the data set D1, the rest being included in data set D2. In the model development, only data set D1 was used. Models were tested with double cross-validation, i.e. the models were first fitted to data set D1 and were tested using data set D2, after which the procedure was repeated in reverse order. For modeling stand dominant height increment, data was splitted randomly so that for each location, half of the sample plots were included in the modeling data (data set H1), the rest being included in the test data (data set H2).

The data included successive observations from each tree and sample plot. Thus, there will be autocorrelation between the successive observations of a tree and plot. The effect of autocorrelation on the parameter estimates was studied by generating new data sets (data sets D3 and H3) from the study material. Data set D3 that was used in diameter growth modeling contained only one randomly selected observation for each tree. Data set H3 used in modeling dominant height increment, included one observation from each sample plot.

Analysis of the growth response on stand-level

Before the development of individual-tree growth models, growth responses to different fertilization treatments were studied on a stand-level with analysis of covariance. The aim of this study, as regards

model development, was to construct models for tree diameter and height growth for fertilized loblolly pine plantations. Therefore in the stand level analysis, the effect of fertilization on the increment of quadratic mean diameter and dominant height increment were studied.

The following model structure for increment of stand quadratic mean diameter and for stand dominant height increment over the 8-year study period was applied:

$$Y_{ijkm} = \mu + T_{Ni} + T_{Pj} + T_{NixPj} + B_m + \beta(S_{ijkm} - S_{...}) + e_{ijkm} \quad [1]$$

where Y_{ijkm} = The growth during the 8-year study period with nitrogen fertilization level i , phosphorus fertilization level j of the k th replicate within installation m

μ = Overall mean increment

T_{Ni} = Main effect of nitrogen fertilization level i

T_{Pj} = Main effect of phosphorus fertilization level j

T_{NixPj} = Nitrogen x phosphorus fertilization interaction effect

B_m = Block effect of installation m

β = Regression covariate

S_{ijkm} = Initial stocking covariate

$S_{...}$ = Mean stocking

e_{ijkm} = Error term

In the model for quadratic mean diameter increment, stand basal area at the time of the establishment of the experiments was used as a covariate. Correspondingly, in the model for dominant height increment response, stand dominant height at the time of the establishment of the experiments was tested to be used as a covariate, but it proved to be nonsignificant. SAS GLM procedure (SAS Institute Inc.1989) was used to obtain the analysis.

In the model for increment of quadratic mean diameter, main effects and interaction of N- and P-fertilization treatments, as well as covariate proved to be significant. Two-way least square means were examined to compare the responses to different fertilization treatments and examine the significance of the differences in growth following the treatments. Nitrogen fertilization resulted in a significant growth response. Growth increased with increasing doses of nitrogen within the range of applied nitrogen doses. The response to phosphorus fertilization was significant only, when applied together with nitrogen. In the plots fertilized with phosphorus, the increase in phosphorus dose from 25 to 50 lbs per acre did not significantly increase growth (Figure 1).

In the analysis of dominant height increment, the main effects of N- and P-fertilization proved to be significant, but their interaction as well as covariate were nonsignificant. Nitrogen applied alone resulted only in slight increase in dominant height increment, but it was statistically

significant with doses 200 and 300 lbs per acre. Phosphorus increased growth only, when applied together with nitrogen. The increase in phosphorus dose from 25 to 50 lbs per acre did not significantly increase growth (Figure 2).

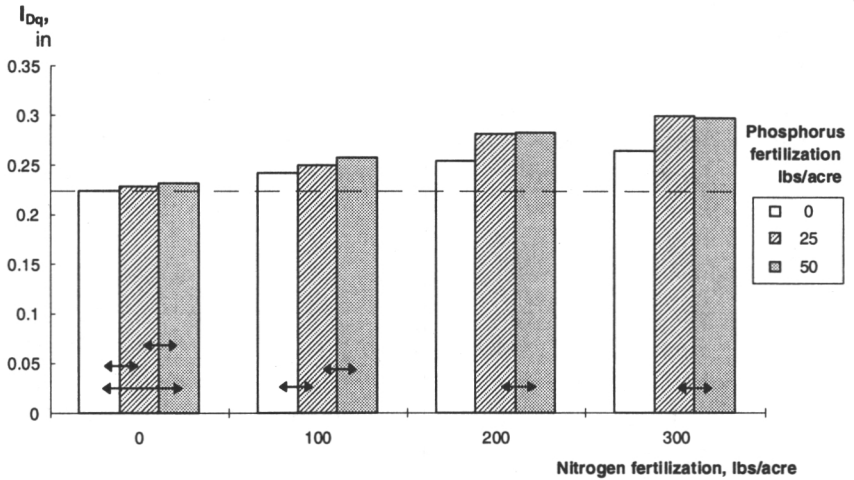


Figure 1. Adjusted mean annual increment of quadratic mean diameter over the 8-year study period after different fertilization treatments. Growth bars connected with arrows are not significantly different.

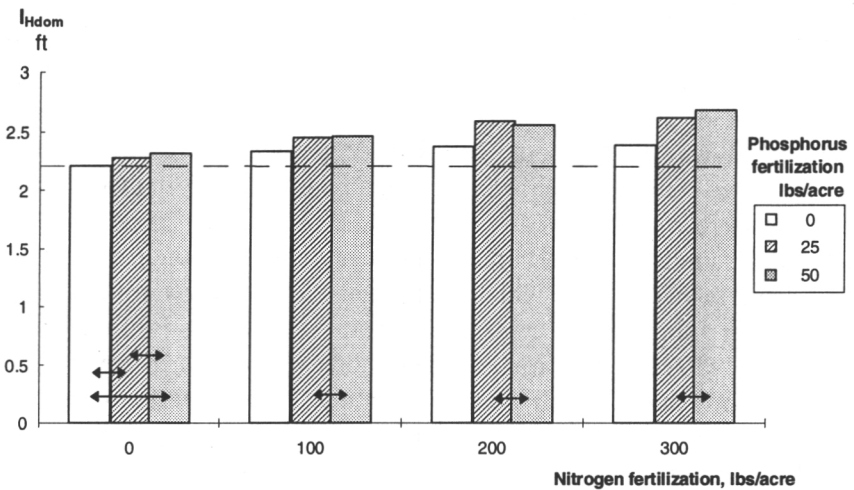


Figure 2. Adjusted mean annual increment of stand dominant height over the 8-year study period after different fertilization treatments. Growth bars connected with arrows are not significantly different.

Model development

Diameter growth model

Two-year tree diameter growth was used as a dependent variable in the models, because measurements were made at two-year intervals. The strategy adopted in modeling tree diameter growth following fertilization was to incorporate the effects of fertilization into a base growth model. Thus, a single growth model was developed using data from both non-fertilized and fertilized stands. The following multiplicative model form was chosen as a basic model structure:

$$\hat{i}_{dbh} = F_1(ref) \cdot F_2(fert) \quad [2]$$

The first part of the model ($F_1(ref)$) predicts tree growth without fertilization (reference growth). It includes the effects of site and climatic variation, as well as tree and stand characteristics on tree growth. In the reference growth model, tree growth is assumed to be affected by site fertility, size of the growing biomass and the efficiency of biomass at net assimilation (Jonsson 1969, Hägglund et. al. 1981). Growth factors are expressed as functions of measured tree and stand characteristics. It is further assumed that the effects of different growth factors interact multiplicatively (Baule 1917).

The second part of the model ($F_2(fert)$) includes the direct fertilization effects on tree growth. It will predict the relative growth response following fertilization. Thus, $F_2(fert)$ is a multiplier, with which reference growth is multiplied to predict the growth of fertilized trees. One of the main objectives of this study was to develop $F_2(fert)$ in such a way that it could be used independently from the present model and could be applied as a multiplier in other individual-tree diameter growth models.

Information about the duration and temporal distribution of growth response in midrotation loblolly pine stands following fertilization is documented in NCSFNC studies reported by Ballard (1982) and NCSFNC (1992b). According to these studies, growth increases during the first growth period following fertilization. The peak response occurs during the first 4 years after fertilization, and thereafter response starts to decline rapidly.

In the present study, the Weibull probability density function was applied in modeling the temporal distribution of the response to fertilization. The three-parameter Weibull distribution can be expressed as

$$f(\text{time}) = \frac{\chi}{\beta} \left(\frac{\text{time} - \alpha}{\beta} \right)^{(\chi-1)} \exp \left[- \left(\frac{\text{time} - \alpha}{\beta} \right)^\chi \right], \text{ when } (\alpha \leq \text{time} < \infty) \quad [3]$$

$$= 0, \text{ otherwise}$$

where α = location parameter

β = scaling parameter (> 0)

χ = shape parameter (> 0)

It has been observed that tree growth in loblolly pine stands responds to fertilization without any delay. Thus, the location parameter (α) was set to zero, resulting in a two-parameter Weibull p.d.f. The Weibull function has some desirable properties that make it suitable to apply in this kind of modeling. It is known to be a very flexible function that can describe a large variety of distribution forms as a result of modifying the scaling (β) and shape (χ) parameters. Like other probability density functions, the integral of the Weibull p.d.f. equals to one. To get a varying magnitude of response as a result of different fertilization treatments, the Weibull p.d.f. is scaled by multiplying it with a variable expressed as a function of the dose and the elements.

The results from stand-level analysis of the data together with the information from prior fertilization studies were used in formulation of the equations to predict the magnitude of the fertilizer response. Magnitude of growth response varies according to the fertilizer dose and the fertilizer elements applied. Growth response increases with increasing amount of added nitrogen up to 300 lbs N/acre. The response curve has typically found to follow a decreasing exponential trend (Wells et al. 1976, Ballard 1982, NCSFNC 1992a). Fertilization with both phosphorus and nitrogen results on most sites in a greater response than the additive effects of nitrogen or phosphorus alone. Both fertilizer dose (lbs/acre) and fertilizer elements (N, P) were included into the function, with which the Weibull p.d.f. was multiplied.

Magnitude of fertilization response is affected by site, as well as stand and tree characteristics. According to Duzan et al. (1982) absolute growth response in loblolly pine stands increases with increasing site index, while it decreases with increasing stand basal area. Also for other tree species, stand-level attributes such as stand age, site index, and stand basal area have found to affect the magnitude of growth response (e.g. Kukkola and Saramäki 1983, Peterson et al. 1984, Peterson and Hazard 1990). The effect of tree size is also obvious, absolute growth response being the greatest among the largest trees in the stand. However, the relative growth response has been found to be fairly independent of tree size as documented by Hynynen

(1993) for Scots pine and by Moore et al. (1994) for Douglas-fir. In the present model, the effects of site index, stand density and tree size on tree growth were included in the reference growth model ($F_I(ref)$). It was assumed that they do not affect the relative growth response following fertilization.

Data analysis resulted in the following model for tree diameter growth prediction in fertilized midrotation loblolly pine stands:

$$\hat{i}_{DBH} = a_0 \cdot (DBH/BA)^{a_1} \cdot \exp(a_2 BA + a_3 \ln(H_{dom})/Age^2) \cdot \exp(d_1 D85 + d_3 D87 + d_4 D88 + d_5 D89 + d_6 D90 + d_7 D91) \cdot F_2(fert) \quad [4]$$

$$F_2(fert) = 1 + ((a_4 + a_5 P) N^{a_6}) \left(\frac{c}{b} \right) \left(\frac{Time}{b} \right)^{c-1} \exp\left(-\left(\frac{Time}{b} \right)^c \right), \text{ where}$$

\hat{i}_{DBH} = Predicted two-year tree diameter growth, in

DBH = Tree diameter at the beginning of the growth period, in

BA = Stand basal area at the beginning of growth period, ft²/acre

H_{dom} = Stand dominant height at the beginning of growth period, ft

Age = Stand age at the beginning of growth period, years

$Time$ = Time after fertilization, years

N = Nitrogen dose, lbs/acre

P = Categorical variable referring to phosphorus fertilization;

$P = 1$, if fertilized with phosphorus, otherwise $P = 0$

$D85, \dots, D91$ = Categorical variables referring to the growth periods of 85-86, ..., 90-91, respectively; e.g. $D85 = 1$, if growth period is 1985-1986, otherwise $D85 = 0$

$a_0, \dots, a_6, d_1, \dots, d_7, b, c$ = parameters

To account for time-dependent factors, such as climatic variation, during the 8-year growth period fixed annual effects (dummy-variables) referring two-year growth periods, were added into the reference growth part of model (4).

All the parameters were fitted simultaneously using data from both non-fertilized and fertilized plots. Nonsignificant parameters were screened on the basis of their asymptotic standard errors. Residual sum of squares of full and reduced models were tested for significant differences ($p < 0.05$) as a basis for parameter rejection. The model was fitted using SAS derivate-free algorithm DUD with convergence criterion set to 10^{-5} (SAS Institute Inc. 1989).

To confirmate the model, double cross-validation was done using both data sets D1 and D2. The model fitted to data set D1 was tested by predicting tree diameter growth in data set D2. Finally, the procedure was repeated in reverse order by obtaining parameter estimates using data set D2 and testing the model with data set D1 (Table 2).

There were no significant differences between parameter estimates obtained from the two data sets indicating high stability of the estimated parameter values. Therefore, the final parameter estimates were obtained by fitting the model (4) to the combined data (data sets D1 and D2) (Table 3). The relative standard error of the model (4) fitted to the combined data was 34.8 %.

The effect of autocorrelation between successive observations of each tree was tested by fitting the model (4) to data that contained only one randomly selected observation from each tree (data set D3). There were no major differences in parameter estimates of the model fitted to the combined data, and data set D3. All the parameter estimates of the model (4) fitted to the data set D3, except for parameter a_1 , were within 95 % asymptotic confidence interval of parameter estimates of the model fitted to the combined data (Table 3).

Table 2. Parameter estimates and confirmation of model (4) fitted to datasets D1 and D2.

Parameter	Dataset D1		Dataset D2	
	Estimate	Asymptotic Std. Dev.	Estimate	Asymptotic Std. Dev.
	Model estimation			
	<i>Reference growth - $F_1(\text{ref})$:</i>			
a_0	6.4200	0.1490	6.1506	0.1335
a_1	1.4800	0.0106	1.4699	0.0105
a_2	0.0049	0.0002	0.0049	0.0002
a_3	44.7107	0.5589	45.4996	0.3417
d_1	0.0233	0.0059	0.0263	0.0063
d_2	0.1582	0.0069	0.1703	0.0083
d_4	0.1659	0.0099	0.1633	0.0102
d_5	0.1452	0.0115	0.1394	0.0121
d_6	0.2102	0.0125	0.2142	0.0190
d_7	0.3641	0.0201	0.3629	0.1557
	<i>Response to fertilization - $F_2(\text{fert})$:</i>			
a_4	0.0257	0.0054	0.0124	0.0029
a_5	0.0245	0.0048	0.0118	0.0004
a_6	0.6561	0.0334	0.8022	0.0415
b	4.2289	0.1322	4.3876	0.2177
c	2.0688	0.0612	1.9435	0.0642
\hat{i}_{DBH}	0.4958		0.5007	
MSE	0.0297		0.0301	
Observations	28969		28759	
	Model confirmation			
	Residuals (observed - predicted)			
Mean	0.0012		-0.0049	
MSE	0.0301		0.0297	

Table 3. Parameter estimates of diameter growth model (4) fitted to the combined data (D1 + D2), and to data set (D3) including only one observation from each tree.

Parameter	<i>Combined data (D1 + D2)</i>				<i>Data set D3</i>	
	Estimate	Asymptotic Std. Dev.	Asymptotic 95 % confidence interval		Estimate	Asymptotic Std. Dev.
			Lower	Upper		
	<i>Reference growth - F₁(ref):</i>					
a_0	6.2695	0.1100	6.0540	6.4850	6.0581	0.2026
a_1	1.4774	0.0043	1.4635	1.4913	1.4613	0.0150
a_2	0.0049	0.0001	0.0046	0.0051	0.0049	0.0002
a_3	45.2754	0.4192	44.4537	46.0971	45.4245	0.7936
d_1	0.0215	0.0043	0.0170	0.0339	0.0265	0.0085
d_3	0.1633	0.0143	0.1353	0.1913	0.1611	0.0105
d_4	0.1652	0.0087	0.1482	0.1822	0.1588	0.0141
d_5	0.1430	0.0103	0.1227	0.1633	0.1481	0.0164
d_6	0.2147	0.0085	0.1980	0.2314	0.2132	0.0174
d_7	0.3685	0.0322	0.3054	0.4316	0.3727	0.0203
	<i>Response to fertilization - F₂(fert):</i>					
a_4	0.0216	0.0028	0.0161	0.0270	0.0190	0.0059
a_5	0.0202	0.0025	0.0154	0.0251	0.0155	0.0046
a_6	0.6978	0.0230	0.6528	0.7429	0.7231	0.0481
b	4.3000	0.0675	4.1677	4.4322	4.2631	0.1331
c	2.0070	0.0475	1.9138	2.1002	2.0572	0.0977
\hat{i}_{DBH}		0.4982			0.4983	
MSE		0.0299			0.0299	
Observations		57726			14405	

After obtaining the parameter fitted to the combined data, residuals were analyzed with respect to all the regressor variables in the model (4). Model behavior was satisfactory with relatively unbiased prediction. Model residuals showed no meaningful trends, or lack of homogeneity or normality of variances when plotted against the predicted diameter growth and against the regressor variables (Figure 3).

To study whether the foliar nutrient information would improve the prediction of tree diameter growth, nutrient concentrations of nitrogen, phosphorus and potassium in foliage were included into the model as a part of the reference growth regressors. The study material contained foliar N, P and K concentrations measured from non-fertilized plots. The resulting model (5) is as follows:

$$\hat{i}_{DBH} = a_0 \cdot (DBH/BA)^{a_1} \cdot \exp(a_2 BA + a_3 \ln(H_{dom}) / Age^2 + a_7 N_f + a_8 P_f + a_9 K_f) \cdot \exp(d_1 D85 + d_3 D87 + d_4 D88 + d_5 D89 + d_6 D90 + d_7 D91) \cdot F_2(fert), \text{ where} \quad [5]$$

$F_2(\text{fert})$ as in model (4)

N_f = Nitrogen concentration in foliage, %

P_f = Phosphorus concentration in foliage, %

K_f = Potassium concentration in foliage, %

Other symbols like in model (4)

Including the foliar nutrient information did not improve the predictive capability of the model. Parameter estimates related to foliar nutrient contents in model (5) were nonsignificant.

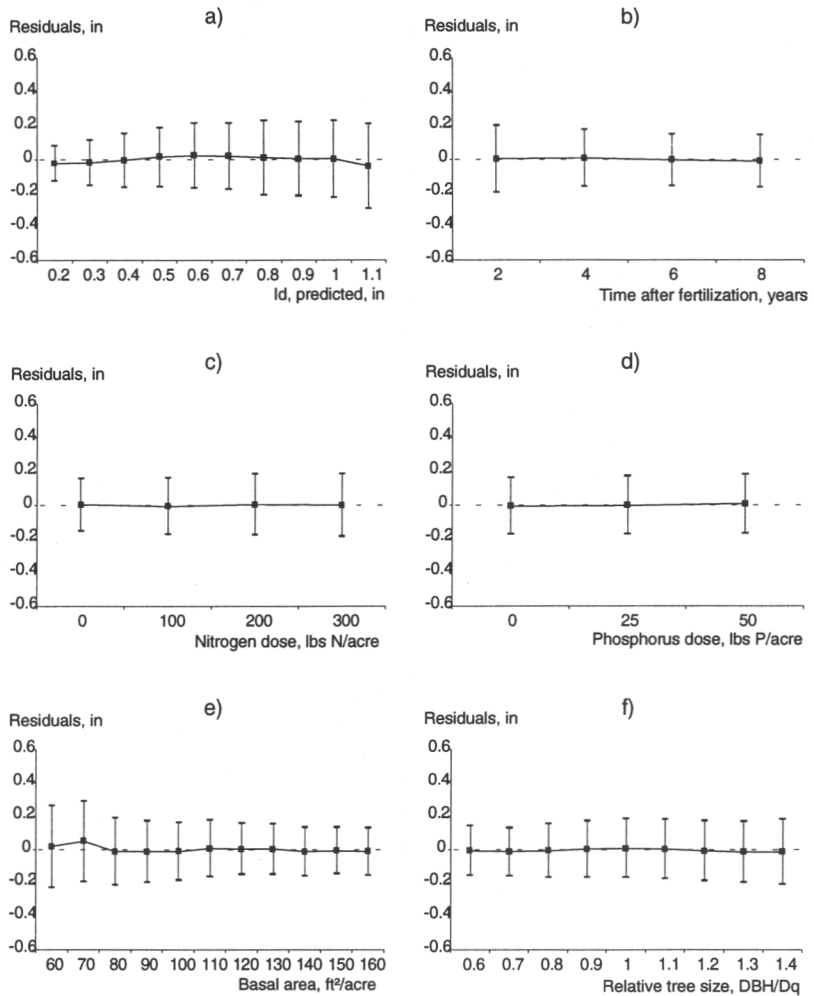


Figure 3. Residuals (means \pm SD) of diameter growth model (4) with respect to predicted diameter growth (a), time after fertilization (b), nitrogen dose (c), phosphorus dose (d), stand basal area (e) and relative tree size (f)

Height growth model

Tree height growth was modeled as the product of potential height growth multiplied by a modifier function. It is a commonly applied model structure in individual-tree growth models (e.g. Ek and Monserud 1974, Arney 1985, Burkhart et al. 1987). Increment of dominant height was assumed to represent the potential height growth. Growth of an individual tree was regarded to be either smaller or greater than dominant height increment depending on the tree's competitive status and vigor. In modeling growth response following fertilization, a similar strategy to that applied in developing the diameter growth model was adopted. The aim was to develop a single model that could predict tree height growth with and without fertilization.

To study the effect of relative tree size on the growth response, the following model was fitted to both fertilized and non-fertilized trees (Table 4).

$$\hat{i}_h = IH_{dom}(DBH/D_{dom})^{a_1} IH_{dom} e^{(-a_2 BA)}, \text{ where} \quad [6]$$

\hat{i}_h = Predicted two - year height growth of a tree, ft

IH_{dom} = Two-year dominant height increment, ft

DBH = Tree diameter, in

D_{dom} = Mean diameter of dominant and codominant trees, in

BA = Stand basal area, ft² / acre

a_1, a_2 = Parameters

In model (6), observed dominant height growth of the prediction period was used as a regressor in the model. Thus, it already included the effect of fertilization on dominant height increment. Therefore, model (6) does not include any variable expressing fertilization. It was assumed that if relative growth response varied with tree size, model (6) would result in biased prediction with respect to tree size. Analysis of model residuals showed, however, no bias with respect to absolute or relative tree size except for all the smallest, suppressed trees in the stand (Figure 4). On the basis of residual analysis, it was concluded that relative growth response was not notably affected by tree size. As a conclusion of this result, in further model development it was assumed to be sufficient to modify only dominant height growth to predict tree height growth in fertilized stands.

Dominant height growth was modeled using similar basic model structure than in diameter growth model (2). Reference growth was modeled as a function of stand age and site index. Again, time-dependent variation was incorporated into the model by adding categorical variables referring to two-year growth periods. Relative

Table 4. Parameter estimates of model (6) fitted to the combined data (D1 + D2).

Parameter	Estimate	Asymptotic Std. Dev.	Asymptotic 95 % confidence interval	
			Lower	Upper
a_1	0.06933	0.00131	0.06676	0.07191
a_2	-0.00013	0.00001	-0.00016	-0.00011
\hat{i}_h	4.5161			
MSE	2.1770			
Observations	57726			

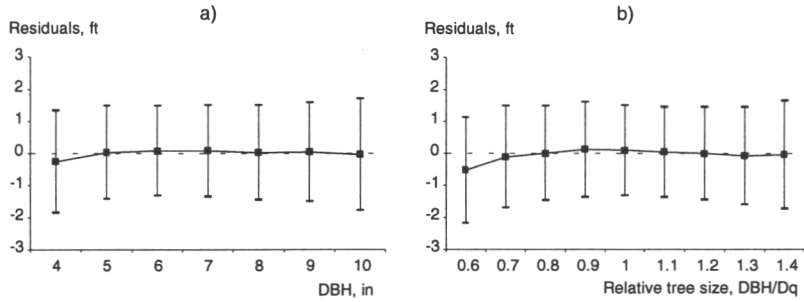


Figure 4. Residuals (means \pm SD) of the model (6) with respect to absolute (a) and relative (b) tree size.

growth response was modeled as a function of added nutrient element, dose, and time elapsed since fertilization. As in the diameter growth model, temporal distribution of growth response was modeled using Weibull function. The results from stand-level analysis were used in formulating the equations for the magnitude of growth response. A model formulation for relative fertilizer response (F_2) similar to that of diameter growth model (4) was applied as an original model structure. As the result of model development, dominant height growth model can be expressed as follows:

$$\hat{I}_{Hdom} = a_0 \cdot SI^{a_1} \cdot \exp(a_2 AGE^{-1} + d_1 D85 + d_2 D86 + d_5 D89 + d_6 D90 + d_7 D91) \cdot F_2(fert) \quad [7]$$

$$F_2(fert) = 1 + P(a_3 N) \left(\frac{c}{b} \right) \left(\frac{Time}{b} \right)^{c-1} \exp \left(- \left(\frac{Time}{b} \right)^c \right), \text{ where}$$

- \hat{I}_{Hdom} = Predicted two-year dominant height increment, ft
- SI = Site index, ft
- Age = Stand age, years
- $Time$ = Time after fertilization, years
- N = Nitrogen dose, lbs /acre

$P = 1$, if fertilized with phosphorus, otherwise = 0
 D_{85}, \dots, D_{91} = Dummy variables referring the growth periods of
 1985-1986, ..., 1991-1992, respectively, e.g. $D_{85} = 1$, if
 growth period is 1985-1986, otherwise $D_{85} = 0$
 $a_0, \dots, a_3, d_1, \dots, d_7, b, c$ = parameters

Model (7) was fitted to the stand data in a similar manner as in modeling tree diameter growth model. Model was fitted to data including half of the sample plots (data set H1), the rest were used to test the model (data set H2). Again, parameter estimates obtained from the both parts of the splitted data were quite similar (Table 5). Therefore, the final model was fitted to combined data that included dominant height increment observations of every growth period from all the sample plots in the studied stands, total of 1714 observations (Table 6). The relative standard error of the model was 24.9 %. Residual analysis showed no bias with respect to predicted dominant height increment, or any of the regressors (Figure 5).

Table 5. Parameter estimates and confirmation of model (7) fitted to datasets H1 and H2.

Parameter	Data set H1		Data set H2	
	Estimate	Asymptotic Std. Dev.	Estimate	Asymptotic Std. Dev.
	Model estimation			
	<i>Reference growth - $F_1(ref)$:</i>			
a_0	0.00887	0.00354	0.00935	0.00355
a_1	1.31489	0.09366	1.31686	0.09425
a_2	10.8910	1.12009	10.0133	1.14703
d_1	0.07191	0.02969	0.08863	0.03020
d_2	0.07839	0.02703	0.08513	0.02624
d_5	0.20992	0.02957	0.23218	0.03217
d_6	0.15551	0.03689	0.16657	0.03637
d_7	0.36663	0.04041	0.33303	0.03798
	<i>Response to fertilization - $F_2(fert)$:</i>			
a_3	0.00369	0.00059	0.00467	0.00083
b	4.91266	0.43034	5.03949	0.62405
c	2.69927	0.56165	2.09972	0.41007
\hat{I}_{Hdom}	4.7471		4.8220	
MSE	1.3654		1.4679	
Observations	857		857	
	Model confirmation			
	Residuals (observed - predicted)			
Mean	0.07488		-0.07817	
MSE	1.46466		1.36213	

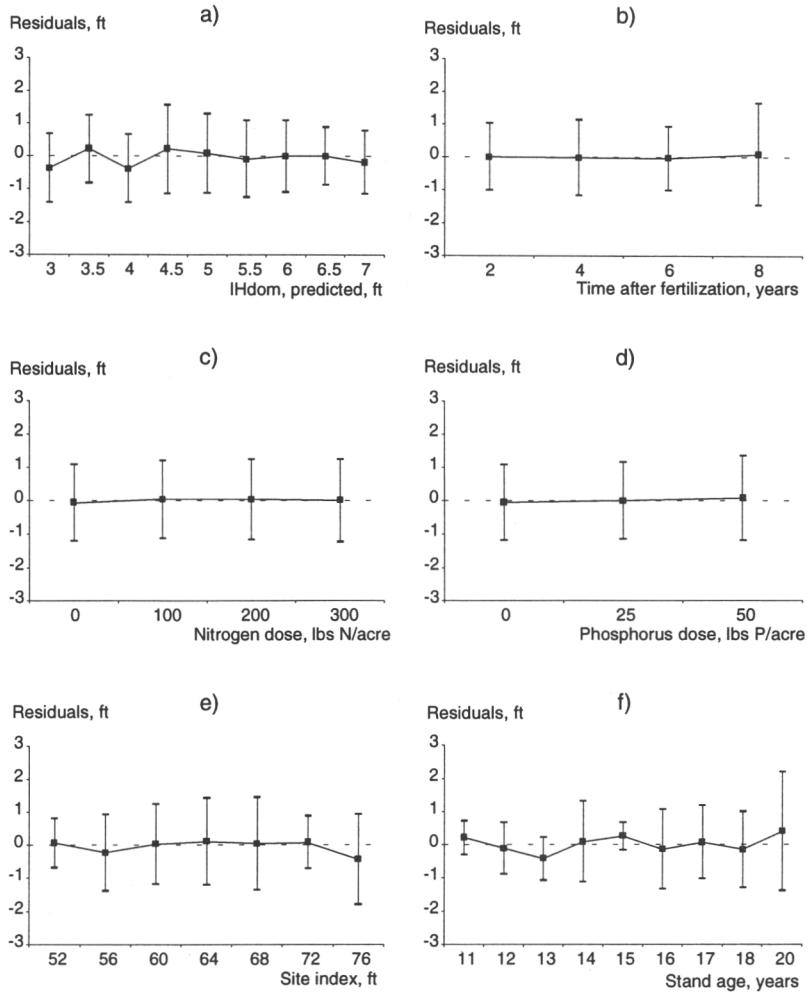


Figure 5. Residuals (means \pm SD) of dominant height increment model (7) with respect to predicted dominant height increment (a), time after fertilization (b), nitrogen dose (c), phosphorus dose (d), stand site index (e) and stand age (f).

To examine the effects of autocorrelation between successive measurements of a sample plot on the parameter estimates of model (7), model was also fitted to data set H3 including only one observation from each sample plot. The effect of autocorrelation proved to be negligible; all the parameter estimates of the model fitted to data set H3, except for parameter a_2 , were within 95 % confidence interval of the parameter estimates obtained from fitting the model (7) to combined data (Table 6).

Table 6. Parameter estimates of stand dominant height increment model (7) fitted to the combined data (H1 + H2), and to data set (H3) including only one observation from each sample plot.

Parameter	<i>Combined data (H1 + H2)</i>				<i>Data set H3</i>	
	Estimate	Asymptotic Std. Dev.	Asymptotic 95 % confidence interval		Estimate	Asymptotic Std. Dev.
			Lower	Upper		
	<i>Reference growth - $F_1(ref)$:</i>					
a_0	0.00789	0.00205	0.00386	0.01191	0.00528	0.00282
a_1	1.35162	0.10988	1.13610	1.56710	1.47495	0.13143
a_2	10.4297	0.86994	8.72340	12.1360	8.69564	1.70267
d_1	0.08078	0.02166	0.03830	0.12627	0.11087	0.05812
d_2	0.07798	0.01784	0.04299	0.11297	0.07318	0.03482
d_5	0.22261	0.02276	0.17796	0.26725	0.23571	0.03860
d_6	0.16378	0.02786	0.10913	0.21843	0.13162	0.05194
d_7	0.33706	0.02840	0.28135	0.39277	0.33390	0.05688
	<i>Response to fertilization - $F_2(fert)$:</i>					
a_3	0.00395	0.00053	0.00290	0.00499	0.00387	0.00109
b	4.76922	0.28779	4.20476	5.33368	4.43411	0.61966
c	2.50965	0.30395	1.91348	3.10583	2.29272	0.71005
\hat{I}_{Hdom}		4.7789			4.7015	
MSE		1.41268			1.35839	
Observations		1714			432	

The effect of including foliar nutrient information on dominant height growth prediction was studied by adding foliar N, P and K concentrations to the model (7) as regressors in the same manner as in the diameter growth model (5). Adding the foliar nutrient information to the model did not result in any improvement in prediction, while all the parameter estimates of foliar nutrient variables were nonsignificant.

Discussion

Behavior and performance of the models

In fitting models (4) and (7), tree and stand characteristics were obtained at the beginning of each two-year growth period. When applying the models in the growth simulation, the values of all the regressor variables must be updated after each two-year simulation period before the growth prediction of the next period. Therefore, the indirect fertilizer effect from the previous growth periods is already included in the tree and stand variables with increased tree diameter and

stand density. The fertilization response function, $F_2(fert)$ of models (4) and (7) that accounts for direct fertilization response, is conditioned to be greater or equal to one (Figures 6 and 7). It predicts the additional growth increase caused by fertilization compared to the situation where fertilization would not give any response during the growing period in question.

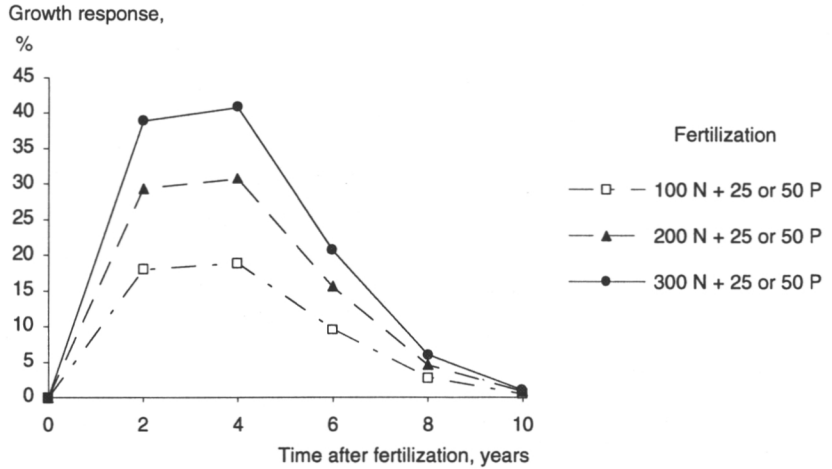


Figure 6. Relative diameter growth response surface with varying fertilization treatment predicted by fertilization response function of model (4).

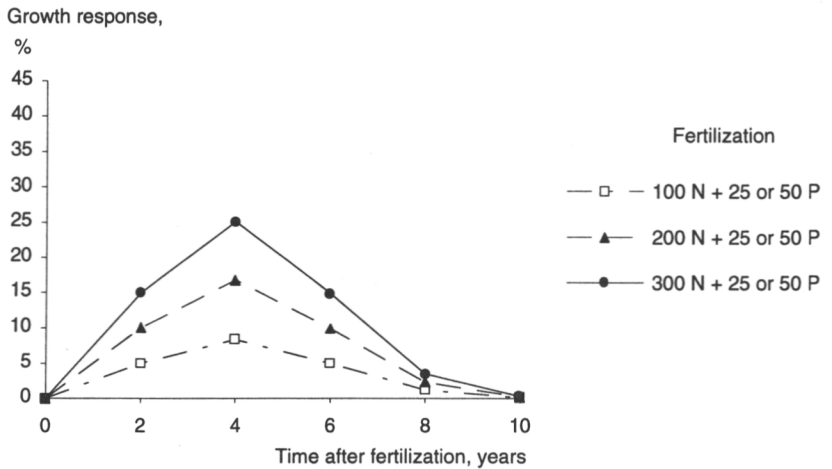


Figure 7. Relative response surface in stand dominant height increment with varying fertilization treatment predicted by fertilization response function of model (7).

The fertilization response function by itself will not give the final, absolute growth response to fertilization, which is affected by predicted reference growth as well. Because fertilization changes the patterns of stand development, e.g. the development of stand density, it has a strong effect on the reference growth prediction. The magnitude and duration of the total absolute growth response can only be obtained by simulating the stand development using the reference growth model and fertilization response function together.

Although the fertilization response function is always ≥ 1 , it is possible to obtain negative absolute growth responses in simulation of individual-tree development. Especially, after the fertilization response has diminished, only the reference growth model affects growth prediction. Actual growth of a fertilized tree can then be either greater or smaller than what it would be if the tree had not been fertilized, depending on relative tree size and stand density. Therefore, the models can take into account the effects of changed patterns of stand development in fertilized stands in long-term simulations.

The performance of the models in predicting tree diameter, stand basal area and stand dominant height growth was studied by simulating the development of all the sample plots in the data set over the eight-year study period. In simulation, data from the first measurements were taken as a starting point of the simulation. Growth of each tree was predicted with tree diameter growth model (4), stand dominant height and tree height growth with models (6) and (7). After each two-year simulation period, tree and stand variables were updated. Mortality was taken into account by removing those trees from the tree list that actually had died in the study plots during the growth period in question.

Predicted absolute growth responses in basal area and height were obtained by simulating the development of fertilized plots twice. First, only the reference growth function of the models ($F_J(ref)$) was used in order to simulate the development without fertilization. Thereafter, simulation was done again with models including the fertilization response equation to obtain stand development with fertilization. The total fertilization growth response for each plot was calculated as a difference between these simulated growths.

Temporal pattern of the growth response to fertilization showed that stand basal area growth response reaches its peak around two years after fertilization. Thereafter the response starts to decrease and will level off around eight years after fertilization (Figure 8). For stand dominant height, growth response reaches its maximum level somewhat later than basal area, around 2 to 4 years after fertilization (Figure 9).

Magnitude of the growth response is greater for diameter growth than height growth. According to the models, fertilization with 200 lbs

of nitrogen and 25 or 50 lbs of phosphorus increased basal area growth by 26.0 % in the eight-year period following fertilization. For dominant height increment, the response was 9.0 %.

The magnitude of growth response is strongly affected by the dose and the elements added. According to the models, response increases with increasing dose of nitrogen between 0 and 300 lbs/acre. Fertilization with phosphorus and nitrogen results in much greater response than nitrogen application alone (Figure 10). In the fertilized

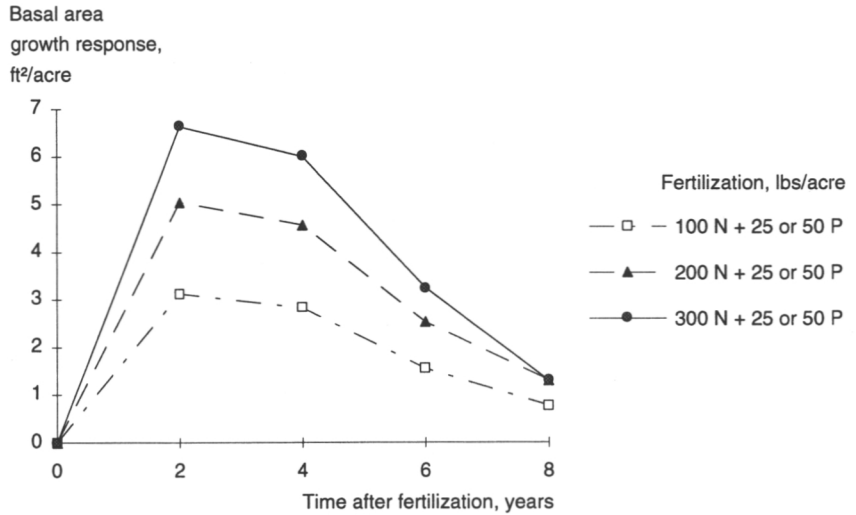


Figure 8. Predicted stand basal area growth response with varying fertilization treatments.

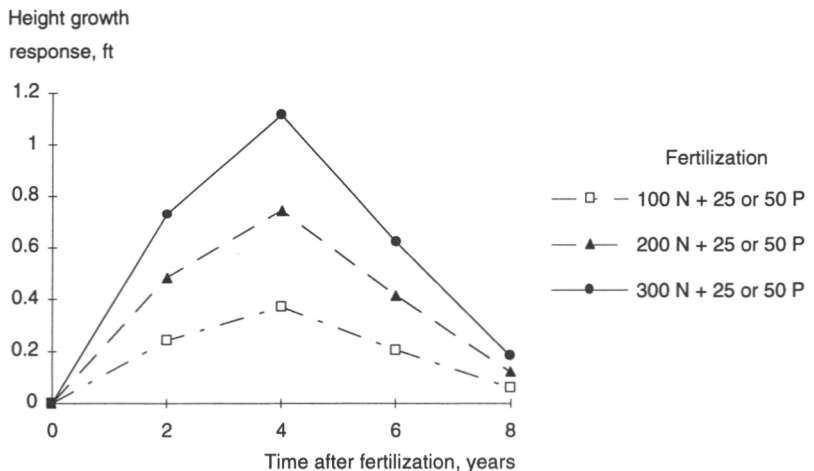


Figure 9. Predicted response in stand dominant height increment with varying fertilization treatments.

stands included in the study material, phosphorus application gave significant growth response only when added with nitrogen. Therefore, the models will predict no growth response after fertilization with only phosphorus. Furthermore, the increase in phosphorus dose from 25 to 50 lbs/acre had no significant effect on the response. Consequently the effect of phosphorus was included into the models (4) and (7) using a categorical variable.

Both nitrogen and phosphorus were needed in the fertilizer to give a growth response in dominant height. The response was found to increase linearly with increasing amount of nitrogen in the range of nitrogen doses included in the study. As in diameter growth, increasing the phosphorus dose from 25 to 50 lbs/acre did not affect response. In the growth simulations, fertilization with 100, 200 and 300 lbs of N with 25 or 50 lbs of P resulted in eight-year growth response of 0.9, 1.8 and 2.6 feet, respectively.

Reliability of model prediction was tested by comparing actual and simulated development of the stands in the eight-year study period. Average bias of the simulated eight-year basal area growth was -1.88 ft²/acre (-3.11%), i.e. the model slightly overpredicted stand basal area increment (Table 7, Fig 11). In stand dominant height prediction bias was 0.13 ft (0.68%) in eight-year increment (Table 8). In both basal area and dominant height simulation, overprediction was greatest in non-fertilized plots. It is possible that overprediction of non-fertilized stands, i.e. slightly biased behavior of the reference growth function $F_1(ref)$, will be present in prediction of fertilized stand development.

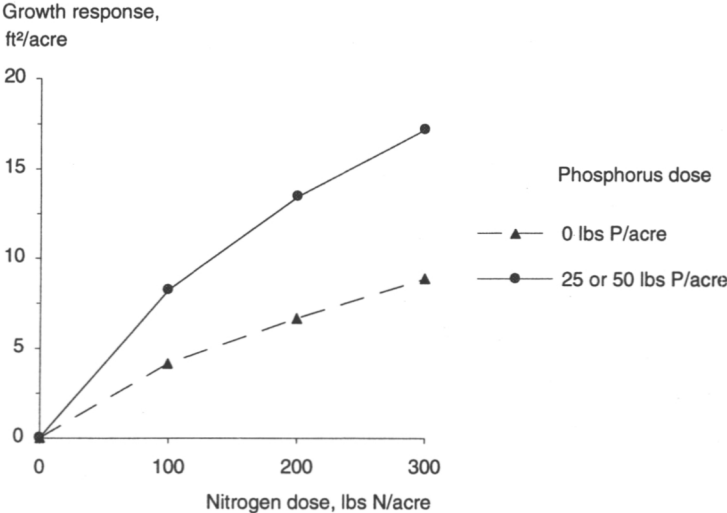


Figure 10. The effect of the dose and type of fertilizer on basal area growth response in eight-year study period.

Being so, the fertilizer response function itself, ($F_2(fert)$) is likely not to overpredict growth response to fertilization. However, validity of this assumption could not be verified based on the data of this study.

Table 7. Model performance in stand basal area growth prediction over eight-year study period.

Treatment	Observed ft ² /acre	Predicted ft ² /acre	Bias ft ² /acre	Rel. Bias %	RMSE ft ² /acre
Control	51.96	54.87	-2.92	-5.32	9.33
P	52.62	53.52	-0.90	-1.68	8.00
N	56.37	58.24	-1.87	-3.21	10.82
N+P	62.79	64.82	-2.03	-3.13	9.17
Average	58.60	60.47	-1.88	-3.11	9.44

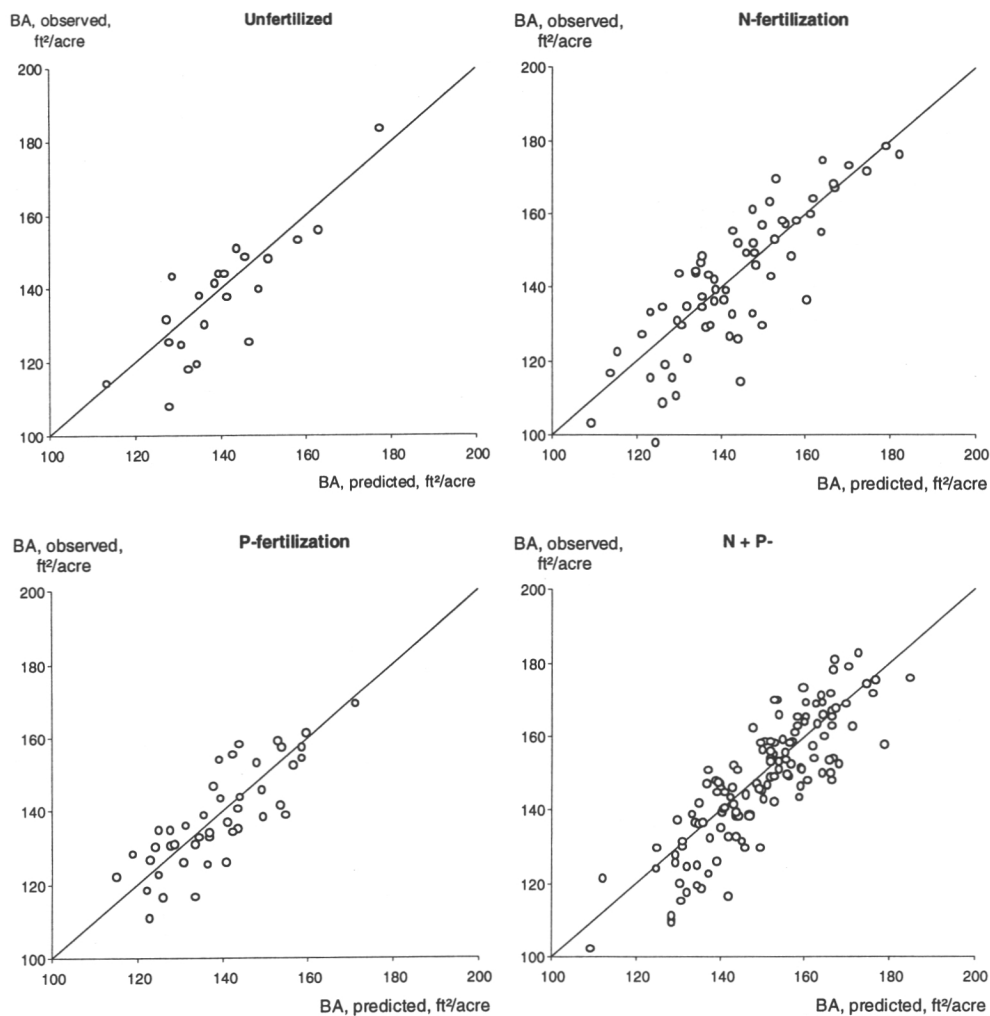


Figure 11. Observed and simulated stand basal areas of the sample plots in the end of eight-year study period grouped by fertilization treatments.

Table 8. Model performance in stand dominant height increment prediction over eight-year study period.

Treatment	Observed ft	Predicted ft	Bias ft	Rel. Bias %	RMSE ft
Control	17.46	18.20	-0.74	-4.07	4.78
P	18.26	18.20	0.06	0.33	6.61
N	18.52	18.20	0.32	1.76	6.86
N+P	20.17	19.97	0.20	1.00	5.23
Average	19.22	19.09	0.13	0.68	5.83

The lack of significance of foliar nutrient effects on fertilizer response indicates that in this data set an "average" response is satisfactory (i.e., fertilizer response is not site specific). This was not found with the larger four-year data set of 31 studies of which these thirteen studies are a subset (NCSFNC 1992a). If more sites would have been included in the modeling data of this study, or if more comprehensive foliar nutrient data in study material would have been available, it is probable that a site specific foliar data would have better explained the variation in growth response following fertilization.

The effect of stand characteristics, such as site index, stand age and stand basal area, were included in the models for reference growth (F_I). It was assumed that direct, relative growth response to fertilization is not affected by these factors. Model performance confirmed that the assumption is valid in midrotation loblolly pine plantations used in this study.

Conclusions

In this study, growth response to fertilization was analyzed by applying two different methods. First, analysis of covariance was applied in order to analyze the magnitude of the total stand-level growth responses to applied fertilization treatments. Second, tree-level simulation models were developed using nonlinear regression analysis. The main results of both analyses were alike, and similar to the results of the earlier forest fertilization studies based on the data from fertilized loblolly pine plantations concerning the duration and magnitude of the growth response (Wells et al. 1976, Ballard 1982, NCSFNC 1992b). The results of the model development show that for forest management planning purposes, it is possible to predict tree growth response following fertilization by using the general stand variables usually included in forest inventory data.

The study material included repeated measurements of permanent sample plots. Applying Ordinary Least Squares in the analysis of the

data with correlated observations may result in models in which estimates of parameter variance is biased. In this study, the effect of autocorrelation on the parameter estimates was studied by generating new data sets from the study material contained only one randomly selected observation for each tree (diameter growth model) and for each sample plot (model for dominant height increment). The results showed that the effects of serial correlation between successive measurements on the parameter estimates of the models were negligible in the study material.

There are some limitations in the data of this study, that need to be taken into account in assessing the applicability of the models. Models are based on data including thirteen installations that is relatively small sample size. The range in stand age at the time the experiments were established and fertilized was quite narrow (11 to 14 years). Further, the variation in site indices between locations was rather small (from 53 to 74 feet).

In the models for diameter and height growth, the effect of fertilization was incorporated in the model as a growth multiplier equation that predicts the relative growth response following fertilization. Because of the model structure, fertilization response equations in models (4) and (7) can also be used with growth models other than those used in this study. The response functions can be applied as growth multipliers in individual-tree based stand growth simulators, such as PTAEDA2 (Burkhart et al. 1987). However, further simulation studies are needed to obtain reliable information about the predictive capability of the fertilizer response equations when they are applied with tree growth models based on data sources different from that of the response equations of the present study.

Models for predicting tree mortality in fertilized stands were not developed in this study. The observed mortality rates in the data suggest that fertilization will slightly increase tree mortality. One possible reason for this increase is the enhanced stand density development in fertilized stands that leads to increased competition among the trees, and therefore the increased mortality rate among the suppressed trees. In simulation of stand development, separate mortality models may not be needed for fertilized stands if mortality is predicted using models that include the effects of stand density and relative tree size on mortality.

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Predicting tree crown ratio for unthinned and thinned Scots pine stands

Jari Hynnen

Abstract: A nonlinear model that yields logical predictions for tree crown ratio is presented. The model is based on data from permanent experimental plots located in even-aged Scots pine (*Pinus sylvestris* L.) stands in southern and central Finland. Regressor variables in the model are stand dominant height, stand basal area, tree diameter, and tree height. The effect of thinning on tree crown ratio is modelled by incorporating a thinning response variable into the model. Thinning effect is dependent on thinning intensity and time elapsed from thinning, the latter of which is represented by the difference between current stand dominant height and dominant height at the time of thinning.

Résumé : Cet article décrit un modèle non linéaire de prédiction de la longueur relative du houppier sur la hauteur de l'arbre. Le modèle s'appuie sur les données des placettes expérimentales installées dans les peuplements équiennes de pin sylvestre (*Pinus sylvestris* L.) au sud et au centre de la Finlande. Les variables explicatives du modèle de régression sont la hauteur dominante et la surface terrière du peuplement, le diamètre et la hauteur de l'arbre. Le modèle rend compte de l'effet de l'éclaircie sur la longueur relative du houppier en faisant appel à l'intensité d'éclaircie et à la différence entre la hauteur dominante actuelle et celle au moment de l'éclaircie.

[Traduit par la Rédaction]

Introduction

Crown size is of major importance to tree growth and survival. Crown ratio or crown length are commonly measured tree characteristics describing crown size. They have been widely used in growth and yield models for predicting growth and survival. In practical silviculture, crown ratio is often used as a criterion for determining the timing of a thinning, and it is also used as an aid in assessing tree growth response following thinning.

Crown ratio or crown height has been predicted using both linear and nonlinear allometric models that include tree and stand characteristics as regressor variables. Multiple linear models have been developed by Ward (1964), Daniels and Burkhart (1975), Wykoff et al. (1982), Kilkki (1983), and Mielikäinen (1985). In modelling tree crown ratio, prediction should always be between 0 and 1 for logical model behavior. In many nonlinear models, unlike most of the linear regression models, prediction of crown ratio inherently results in values between 0 and 1 because of the model structure (Ek and Monserud 1975; Dell et al. 1979; Feduccia et al. 1979; Van Deusen and Biging 1985; Dyer and Burkhart 1987). In the computer simulations, these static crown models are used to predict crown height or crown ratio at the end of the simulation period. The

development of crown size can be obtained as the difference between crown size estimates at the beginning and the end of the growth period. An alternative approach to allometric models in predicting the development of crown ratio is to model directly change in crown ratio or crown height. Nonlinear models predicting crown-height increment have been developed by Maguire and Hann (1990a, 1990b) and Short and Burkhart (1992).

Tree crown ratio is affected by stand density. Thinning abruptly changes stand density, in turn producing a strong effect on the crown development (Assmann 1970). In modelling tree crown ratio in thinned stands, the model should include a variable referring to the intensity and time of thinning. During the first years after thinning, tree crown ratio is affected not only by the actual stand density, but also by stand density before thinning. In this situation, an allometric model including only the actual stand density among the regressor variables will result in biased prediction by predicting crown base lower than it actually is. The effect of thinning on tree crown development has been incorporated in the crown height increment model developed by Short and Burkhart (1992). In their model, the thinning response is modelled by using a variable that accounts for thinning intensity and time interval since thinning.

The purpose of this study is to develop an allometric model to predict tree crown ratio in both unthinned and thinned Scots pine (*Pinus sylvestris* L.) stands. The aim is to develop a model for forest management planning purposes. Therefore, crown ratio is predicted using tree and stand variables that can be measured in practical forest inventories.

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Data

Data were obtained from permanent sample plots established in the experimental stands of Scots pine. The study material consisted of 11 even-aged, naturally regenerated or seeded stands located in southern and central Finland. The experiments were established by the Finnish Forest Research Institute in the early 1970s. The purpose of the experiments was to study the effects of varying thinning intensities and nitrogen fertilization on the growth and yield of Scots pine stands.

Before the establishment of the experimental plots, each stand had been thinned at the seedling stage to a density of 2300 trees/ha on average. At the time the experiments were established, all of the stands were in the commercial thinning stage. In the study material stand age varied between 29 and 80 years, and dominant height was 15.4 m on average (Table 1). The variation of site indices in the study material covered most of the natural range of Scots pine site types.

The effects of three thinning intensities and three levels of nitrogen fertilization were studied using a factorial experimental design. Only the unfertilized sample plots were included in the analysis of this study. Sample plot size was 1000 m². After the initial measurement, one-third of the sample plots were left unthinned, one-third were thinned moderately (30% of the stems were removed), and one-third were thinned heavily (60% of the stems were removed). Ten years after establishment, another thinning was done in the moderately thinned sample plots, again removing 30% of stems.

All the trees in the sample plots were measured for breast-height diameter. From every sample plot an average of 42 sample trees were selected. In the selection of sample trees, the probability that a tree would be selected was proportional to its diameter, but the sample trees were randomly located on the sample plot. For every sample tree, height and crown height were measured. Crown height was defined as the height of the lowest live contiguous whorl.

The study period covered 15 years following establishment. The stands were measured at 5-year intervals. Thus, observations from four measurements from every stand were available for the analysis. Because of the second thinning in moderately thinned sample plots, observations from the last remeasurement of those plots were not included in the final data.

The study material included 4655 observations from 1579 trees measured from 34 sample plots. Before analysis, the data were randomly split into two parts of equal size (data sets 1 and 2). For every plot, half of the trees were randomly selected for data set 1, and the rest were included in data set 2. Splitting was done separately for each measurement date. In the model development only data set 1 was used. Models were tested with double cross-validation, i.e., the models were first fitted to data set 1 and were tested using data set 2, after which the procedure was repeated in reverse order.

The data included successive observations from each tree. Thus, there was autocorrelation between the successive observations of a single tree. The effect of autocorrelation

Table 1. Mensurational characteristics of the study material.

Variable	Mean	SD	Min.	Max.
Stand characteristics				
Site index (H_{100}), m	22.5	3.2	16.9	28.8
Stand age, years	52	11	29	80
Basal area, m ² /ha	23.3	4.6	11.3	33.5
Stem number, no./ha	1834	857	440	3640
Dominant height, m	15.4	2.2	10.4	20.8
Tree characteristics				
Mean diameter (d), cm	15.0	4.6	3.6	29.3
Mean height (h), m	13.8	2.8	4.5	22.1
d/h	1.08	0.21	0.54	2.00
Mean crown ratio, %	50.0	8.9	17.6	80.6

NOTE: Site indices (base age 100 years) were calculated with the models of Vuokila and Väliho (1980).

on the parameter estimates was studied by generating a new data set (data set 3) from the study material containing only one randomly selected observation for each tree and by fitting the model to these data.

The possible effects of spatial autocorrelation were not taken into account in the analysis. On average, every fourth tree was selected as a sample tree. The average distance between sample trees was 4.6 m, assuming them to be evenly distributed over the plot. Correspondingly, the average distance between all the trees in a plot was 2.3 m. Although spatial autocorrelation is likely to exist to some extent among the sample trees in a plot, the effect is considerably smaller than if all the trees in the plot had been included in the analysis.

Model development

To behave logically, the crown ratio model must produce predictions between 0 and 1 in all circumstances. The following nonlinear model structure was used as a basic model in this study:

$$[1] \quad CR = 1 - \exp[-\Phi(x)]$$

where CR is tree crown ratio and $\Phi(x)$ is a function of tree and stand characteristics. CR will remain within its range if $\Phi(x)$ is positive. This model structure was used by Ek and Monserud (1975), Dell et al. (1979), Feduccia et al. (1979), Van Deusen and Biging (1985), and Dyer and Burkhart (1987).

The function $\Phi(x)$ was first determined using data from unthinned sample plots. The model developed by Dyer and Burkhart (1987) was employed as an initial model candidate:

$$[2] \quad CR = 1 - \exp\left[-(a_0 + a_1 A^{-1}) \frac{d}{h}\right]$$

where A is stand age, d is tree diameter at breast height, and h is tree height. Dyer and Burkhart (1987) assumed that the effect of stand density on crown ratio is accounted for in the d/h variable in the model [2]. However, in the Scots pine stands studied, variable d/h , referring to tree form, proved to be fairly insensitive to changes in stand density. Therefore, the effect of stand density described

by basal area (G) was included in the model. Further, an exponent was added to d/h for improving flexibility of the response relative to this variable.

Dominant height of the stand (H_{dom}) was preferred over stand age because it is a better surrogate for stage of stand development across varying site qualities. The stands in the study material were naturally regenerated or seeded. Thus, there is a large variation among the ages of trees compared with the age variation in pine plantations. Furthermore, in practical forest inventories stand age in naturally regenerated stands is not always measured, unlike dominant height. By introducing H_{dom} into the model, both the effects of stand age and site can be taken into account.

The analysis of the relationships between crown ratio and tree and stand characteristics resulted in the following crown ratio model form for unthinned stands:

$$[3] \quad CR = 1 - \exp\left\{-[a_0(\exp(-a_1G)) + a_2H_{dom}^{-1}]\left(\frac{d}{h}\right)^{a_3}\right\} + e$$

where

G is stand basal area, m^2/ha

H_{dom} is dominant height, m

d is tree diameter at breast height, cm

h is tree height, m

a_0, a_1, a_2, a_3 are parameters

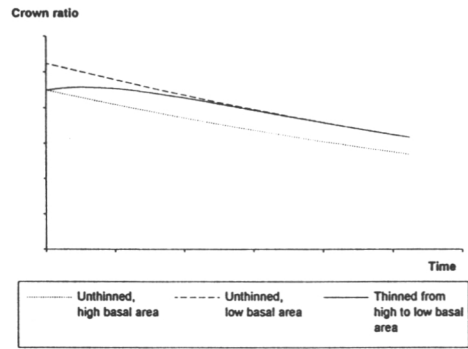
e is an error term

Thinning changes stand density, which has a strong effect on the development of tree crown ratio (e.g., Kramer 1966; Short and Burkhart 1992). At the time of thinning, crown ratio of a tree will still be equal to the crown ratio of a similar tree in an unthinned stand. After thinning, crown recession for most trees in thinned stands is temporarily arrested because of the increased growing space, and tree crown ratios start to build back up by height increment. In developing a model for thinning response, it was assumed that in thinned stands, tree crown ratio approaches the crown ratio of a tree growing in an unthinned stand with initial basal area equal to the basal area of the thinned stand after thinning (Fig. 1).

The effect of thinning on crown ratio was introduced into the model by modifying the variable referring to stand density, i.e., basal area. Thinning response function accounting for the change in the effect of stand density on crown ratio after thinning was incorporated in the model in connection with the stand density variable (stand basal area, G). An exponential function similar to that in the cumulative Weibull function was applied in describing the stand density trajectory.

It was assumed that the effect of thinning on crown ratio is affected by thinning intensity and time elapsed after thinning. The effect of thinning intensity was described by the difference between basal area before thinning (G_b) and stand basal area after thinning (G_a), which modifies the thinning response function. The difference between currently dominant height (H_{dom}) and dominant height at

Fig. 1. Development of tree crown ratio in a thinned stand and in unthinned stands with different initial stand basal areas.



the time of thinning (H_{domt}) was applied to describe the effect of time after thinning. The thinning response function is expressed as

$$[4] \quad THIN = (G_b - G_a) \exp\left[-\left(\frac{H_{dom} - H_{domt}}{a_4}\right)^{a_5}\right]$$

where

G_b is stand basal area before thinning, m^2/ha

G_a is stand basal area after thinning, m^2/ha

H_{dom} is dominant height, m

H_{domt} is dominant height at the time of thinning, m

a_4, a_5 are parameters

Function [4] was placed in model [3] in connection with stand basal area, resulting in the crown ratio model for thinned stands (full model):

$$[5] \quad CR = 1 - \exp\left\{-[a_0 \exp(-a_1(G + THIN)) + a_2H_{dom}^{-1}]\left(\frac{d}{h}\right)^{a_3}\right\} + e$$

where

$$THIN = (G_b - G_a) \exp\left[-\left(\frac{H_{dom} - H_{domt}}{a_4}\right)^{a_5}\right]$$

Results

The full model [5] was first applied to data set 1. To study the possibility of achieving a model with fewer parameters, the parameters were screened from the full model on the basis of their asymptotic standard errors. Residual sum of squares of full and reduced models were tested for significant differences ($p < 0.05$) as a basis for parameter rejection. On the basis of this analysis, parameters a_0 and

Table 2. Parameter estimates and validation of model [6] fitted to data sets 1 and 2.

(A) Parameter estimates				
Parameter	Data set 1		Data set 2	
	Estimate	Asymp. SD*	Estimate	Asymp. SD*
a_1	0.0309	0.0015	0.0319	0.0016
a_2	3.0754	0.2688	3.1301	0.2640
a_3	0.4161	0.0231	0.4808	0.0220
a_4	1.8616	0.3123	1.8483	0.2996
CR	0.502		0.499	
MSE	0.005 06		0.004 87	
Observations	2327		2328	

(B) Model validation

	Residuals (observed - predicted)	
	Data set 1	Data set 2
Mean	-0.002 58	0.002 46
MSE	0.004 89	0.005 19

*Asymptotic SD.

a_5 in model [5] were eliminated, resulting in the final model as follows:

$$[6] \quad CR = 1 - \exp\left\{-[\exp(-a_1(G + THIN)) + a_2 H_{dom}^{-1} \left(\frac{d}{h}\right)^{a_3}]\right\} + e$$

where

$$THIN = (G_b - G_a) \exp\left[-\left(\frac{H_{dom} - H_{domt}}{a_4}\right)\right]$$

To validate the model, double cross-validation was done using data sets 1 and 2. Model [6] was fitted separately to data sets 1 and 2, and its predictive capability was studied (Table 2). In validation, the model fitted to data set 1 was tested by predicting tree crown ratios in data set 2 and vice versa. The analysis confirmed that coefficient estimates were quite stable. There were no major differences in parameter estimates or the asymptotic standard deviations of the models obtained from the two data sets. Validation of the model showed satisfactory predictions for both parts of the data, with negligible bias and similar mean square errors.

On the basis of the results from double cross-validation, final parameter estimates were obtained by fitting the model to the combined data (data set 1 + 2) (Table 3). There were no trends in the residuals with respect to predicted crown ratios or regressor variables. Model behavior

Table 3. Parameter estimates of model [6] fitted to the full data (data sets 1 and 2) and to data set 3.

Parameter	Data sets 1 and 2		Data set 3	
	Estimate	Asymp. SD*	Estimate	Asymp. SD*
a_1	0.0314	0.0011	0.0311	0.0018
a_2	3.0994	0.1883	3.0273	0.3053
a_3	0.4496	0.0159	0.4361	0.0265
a_4	1.8491	0.2158	1.4178	0.3137
CR	0.500		0.503	
MSE	0.0050		0.0048	
Observations	4655		1579	

*Asymptotic SD.

was also satisfactory with respect to varying thinning intensity and to time since thinning (Fig. 2).

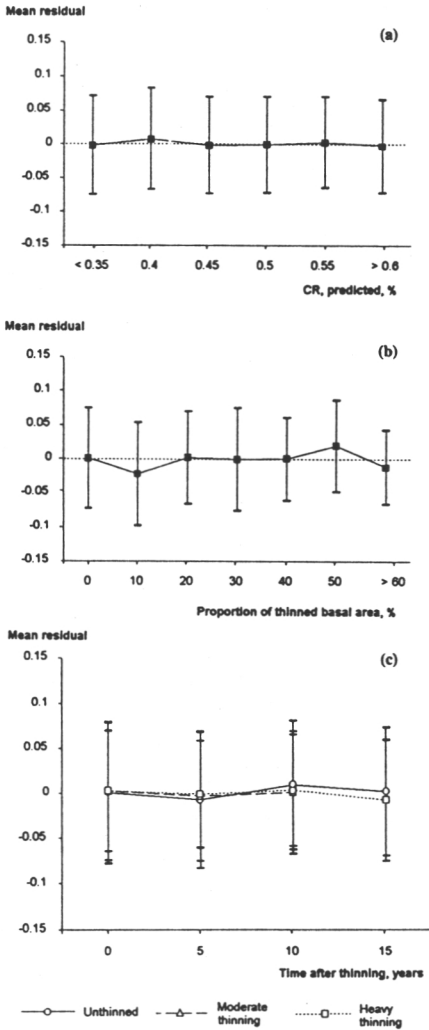
The effect of autocorrelation between successive observations of each tree was tested by fitting the crown ratio model to data set 3 that contained only one randomly selected observation from each tree. Parameter estimates obtained from data set 3 did not differ significantly from the estimates based on the full data except for parameter a_4 . Even for that parameter the estimated value obtained from the full data was within the asymptotic 95% confidence interval of the estimated value from data set 3.

Discussion

The new allometric crown ratio model presented here is based on the tree data that include various thinning treatments. Because of the thinning response function, the dynamics of the crown recession in thinned stands is also taken into account. Therefore, the model produces reliable prediction in thinned stands as well as in unthinned stands, unlike those allometric crown models based on the data collected from unthinned stands only.

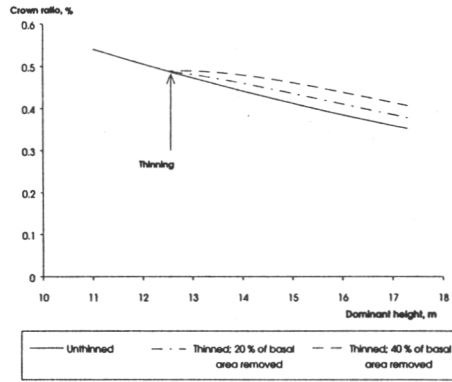
The model presented in this study can describe the most important patterns that are characteristic for the development of crown ratio. Because of its structure the model behaves logically regardless of the values of tree and stand variables; that is, predicted value of crown ratio will always be between 0 and 1. Tree crown ratio is known to change with stand age and stand density (e.g., Assmann 1970). According to this model, crown ratio will decrease with increasing stand height and thus with increasing age. Because of the reciprocal form of H_{dom} in the model, crown recession will slow with increasing dominant height and age. The increase in stand density will result in decrease of tree crown ratios. In the model of Dyer and Burkhart (1987) the effect of stand density was described through change in tree form. In the Scots pine data used here, change in tree form with varying stand density was too small to describe the effect of stand density on tree crown ratio; hence, stand density (G) was treated explicitly in the model. The variable d/h in the model ensures that trees with more taper will have higher crown ratios.

Fig. 2. Model residuals (means \pm SE) plotted against predicted crown ratio (a), thinning intensity (b), and time after thinning with various thinning treatments (c).



The effect of thinning was described with a thinning response variable. It is based on the assumption that the crown ratio of residual trees in a thinned stand will converge to the crown ratio of trees growing in an unthinned stand with equal stand basal area. Although the effect of stem number on tree crown ratio is not explicitly present in the model [6], it will affect crown ratio through the stem form variable (d/h). Therefore, the general model assumptions are similar to those of Pienaar and Rheney (1993) in their models predicting stand development after thinning.

Fig. 3. An example of the predicted development of tree crown ratio with various thinning treatments.



The thinning response function modifies the effect of stand density on crown ratio using the information about thinning intensity and time since thinning (Fig. 3). At the time of thinning, tree crown ratio will be the same as just before thinning. After thinning, crown recession decreases and crown ratio begins to approach to the level that it would be in the unthinned stand with initial basal area equal to the basal area of the growing stock after thinning. According to the model the rate of approach is fastest, i.e., the thinning response is greatest, just after thinning and it decreases with increasing time after thinning. In reality, thinning response is more likely to first increase to its maximum level and then decrease with time. However, in these data the stands were not measured until 5 years after thinning, so there was no information on the actual behavior of the crown ratio development during the first years after thinning.

In the thinning response function, time elapsed after thinning was described by using the difference between actual dominant height and dominant height at the time of thinning instead of using years after thinning. Therefore, it will take longer for tree crowns to adjust to the increased growing space on poor sites than on fertile sites, where height growth is faster. Furthermore, in older stands tree crown adjustment after thinning takes longer than in fast-growing younger stands.

Many individual-tree growth models include tree crown ratio as a regressor variable. In practice, forest inventory data do not necessarily include measures of crown length or crown ratio. The allometric crown ratio model can be used to predict crown ratios in those cases. The model includes only those stand and tree variables that are measured in most forest inventories. The crown ratio model can also be applied when making decisions concerning thinnings of stands. As a measure of tree vigor, crown ratio is often used as a criterion in determining the timing of thinnings. Using the allometric model it is possible to predict the effects of different thinning schedules on the development of tree crown ratios.

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Predicting the growth response to thinning for scots pine stands using individual-tree growth models

Jari Hynynen

Abstract

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Individual-tree growth models for diameter and height, and a model for the cylindrical stem form factor are presented. The aims of the study were to examine modelling methods in predicting growth response to thinning, and to develop individual-tree, distance-independent growth models for predicting the development of thinned and unthinned stands of Scots pine (*Pinus sylvestris* L.). The models were constructed to be applicable in simulation systems used in practical forest management planning. The models were based on data obtained from eleven permanent thinning experiments located in even-aged Scots pine stands in southern and central Finland.

Two alternative models were developed to predict tree diameter growth in thinned and unthinned stands. In the first model, the effect of stand density was described using stand basal area. In the alternative model, an explicit variable was incorporated referring to the relative growth response due to thinning. The magnitude of the growth response was expressed as a function of thinning intensity. The Weibull function was employed to describe the temporal distribution of the thinning response. Both models resulted in unbiased predictions in unthinned and in moderately thinned stands. An explicit thinning variable was needed for unbiased growth prediction in heavily thinned stands and in order to correctly predict the dynamics of the growth response.

In the height growth model, no explicit thinning variable referring to thinning was necessary for growth prediction in thinned stands. The stem form factor was predicted using the model that included tree diameter and tree height as regressor variables. According to the results obtained, the information on the changes in the diameter/height ratio following thinning is sufficient to predict the change in stem form.

Keywords: growth modelling, individual-tree, distance-independent, thinning, stem form, *Pinus sylvestris*

List of symbols and definitions

Stand variables

H_{dom}	= Stand dominant height defined as: average height of the 100 thickest trees per hectare, m
IH_{dom}	= Five-year increment of dominant height, m
H_{100}	= Site index, m (base age 100 years, calculated using models developed by Vuokila and Väliaho (1980))
H_g	= Mean height, weighted with basal area, m
D_g	= Mean diameter, weighted with basal area, cm
D_{dom}	= Stand dominant diameter defined as: average diameter of 100 thickest trees per hectare, cm
G	= Stand basal area over bark, m^2ha^{-1}
I	= Thinning intensity, defined as: (G , pre-thinning - G , post-thinning)/ G , post-thinning
T	= Time elapsed from thinning, years
$D_{502}... D_{558}$	= Categorical variables referring to experimental stands

Tree variables

d	= Diameter at breast height, over bark, cm
$d_{6.0}$	= Diameter at 6 m height, over bark, cm
i_{d5}	= Five-year increment in tree diameter, cm
g	= Tree basal area at breast height, over bark, cm^2
h	= Height, m
i_{h5}	= Five-year increment in tree height, m
v	= Tree volume, dm^3
GL	= Basal area of trees (over bark) larger than subject tree, m^2ha^{-1}
cr	= Tree crown ratio, defined as: length of live crown/total tree height
$f_{1.3} = v/gh$	= Cylindrical stem form factor

Other definitions

$a_0, a_1...a_7, b, c$	= Parameters
e	= Error term
MSE	= $\sum_{i=1}^n (y_i - \hat{y}_i)^2 / n$ (Mean square error)
RMSE	= $\left[\sum_{i=1}^n (y_i - \hat{y}_i)^2 / n \right]^{0.5}$ (Root mean square error)
RMSE _r	= $\left[\sum_{i=1}^n [(y_i - \hat{y}_i) / \hat{y}_i]^2 / n \right]^{0.5}$ (Relative root mean square error)
Absolute bias	= $\sum_{i=1}^n (y_i - \hat{y}_i) / n$
Relative bias	= $\sum_{i=1}^n [(y_i - \hat{y}_i) / \hat{y}_i] / n$

1 Introduction

In Finnish forestry, thinning from below is the most widespread treatment applied in silviculture. The regulation of stand density with the help of intermediate thinnings has been based on both silvicultural and economical aspects. During the recent years, thinning costs have increased. Especially the first commercial thinning of young stands has become less profitable, and this has resulted in changes in thinning schedules. Therefore, it has become increasingly important to be able to forecast the impacts of alternative thinning schedules on the future development of forests. The simulation systems applied in forest management planning should be capable of reliably predicting stand development regardless of the thinning treatment applied.

Tree diameter growth is known to be affected by stand density. Thinning decreases stand density abruptly, and this has a strong impact on tree growth. Growth responses following thinning are the result of (i) increased growing space, (ii) the fertilization effect provided by the non-harvested parts of felled trees, and (iii) the selection effect (Hägglund 1981). The latter effect means that trees retained in stands subjected to thinning from below have grown better before thinning compared to those removed in thinning.

In many growth simulators used in forest management planning, tree growth is predicted using models that do not include any explicit thinning effect (e.g. Belcher et al. 1982, Wykoff et al. 1982, Burkhart et al. 1987, Ojansuu et al. 1991). Such models are based on the assumption that the thinning response can be described through stand characteristics, which are affected by stand density and will change due to thinning.

An alternative method in predicting the effect of thinning is to incorporate an explicit thinning variable in the growth model. This approach has been justified by a hypothesis, according to which an abrupt change in stand density, caused by thinning, changes the effect of stand density on tree growth. Consequently, the effect of stand density in two stands of equal stand density is different in the stand that has been recently thinned compared to the stand where the trees have initially been more widely spaced. There is a group of models in which the thinning response is expressed explicitly in terms of categorical variables (Harrison et al. 1986, Söderberg 1986, Shafii et al. 1990). These models are capable of predicting the magnitude of the total growth response to thinning over the predicted growth period. This approach is not, however, flexible enough to give any information about the temporal distribution of the response. Jonsson (1974) has developed a model for the relative thinning response in tree diameter

growth that is capable of predicting both the magnitude and the temporal distribution of the response. Jonsson used information about diameter increment in unthinned and thinned stands during the growth period prior and subsequent to thinning in modelling the thinning response. In addition to growth models, the magnitude and duration of the thinning response has been incorporated in models predicting the crown ratio of trees (Short and Burkhart 1992, Hynynen 1995).

Increment in dominant height is known to be fairly insensitive to stand density and intermediate thinnings as long as stands are thinned from below (e.g. Assmann 1970, Hägglund 1974, Clutter et al. 1983, Vuokila and Väliäho 1980). In individual-tree growth models, height growth is generally expressed as a function of the increment in the stand dominant height and the relative, or absolute, tree size, without any explicit thinning response variable (e.g. Wykoff et al. 1982, Arney 1985, Burkhart et al. 1987, Ojansuu et al. 1991).

Stem form is strongly affected by stand density. Differences in the thinning response in tree diameter and height growth result in changes in the stem form; this is well documented in many growth and yield studies (Vuokila 1960, Assmann 1970, Söderberg 1986, Valinger 1990). Most growth simulators based on individual-tree models predict tree growth either by means of tree diameter/basal area growth models (Belcher et al. 1982) or diameter/basal area and height growth models (Wykoff et al. 1982, Arney 1985, Burkhart et al. 1987, Ojansuu et al. 1991). In all these simulators, tree volume is predicted with static volume equations. This kind of simulation procedure implies two assumptions concerning the prediction of stem form and stem volume. First, volume equations are assumed to be applicable in volume prediction for all trees regardless of the thinning treatment applied. Second, the change in stem form due to thinning, and in more general due to change in stand density, can be explained by the change in the d/h ratio.

The aims of this study were to examine the modelling methods used in predicting the growth response to thinning, and to develop individual-tree, distance-independent growth models for predicting the development of thinned and unthinned stands of Scots pine. The models were constructed to be applicable in the simulation systems used in practical forest management planning. The input of the models were determined to be consistent and compatible with the information available in practical forest inventory data.

In modelling the growth response to thinning, there were three specific areas of interest. First, in modelling diameter growth, the goal was to determine whether an explicit thinning variable needs to be incorporated in the model for unbiased growth prediction in thinned stands. Second, the effects of thinning on the development of dominant

height increment as well as on the growth of individual tree were examined. Third, a further aim was to analyse whether the effect of thinning on tree stem form development can be explained by changes in the diameter/height ratio.

2 Study material

2.1 Modelling data

Data were obtained from permanent sample plots established in experimental Scots pine stands (*Pinus sylvestris* L.). The study material consisted of eleven even-aged stands growing on mineral soils and located in southern and central Finland (Fig. 1). The experiments were established by the Finnish Forest Research Institute in the early 1970s with the purpose of studying the effects of varying thinning intensities and nitrogen fertilization on the growth and yield of Scots pine stands.

Prior to the establishment of the experimental plots, each stand had been thinned to an average density of 2 355 trees/ha at the seedling stage. At the time the experiments were established, all the stands had reached the stage of the first commercial thinning. Stand age in the study material varied between 29 and 56 years, and the mean stand height varied between 10.0 m and 15.2 m (Table 1).

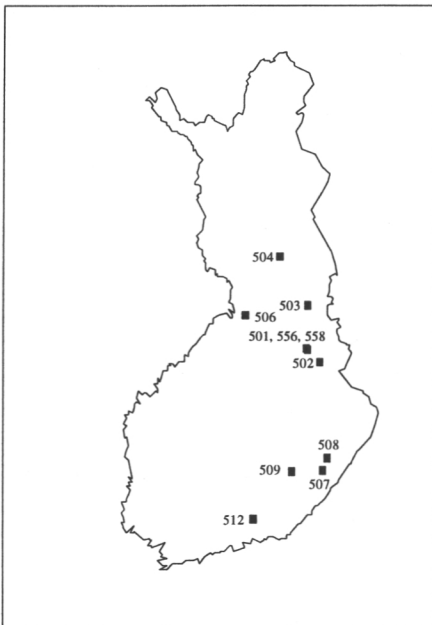


Figure 1. Location of the experimental stands.

Table 1. Mensurational characteristics of the study material.

Exp no	Age, years	Site type ¹⁾	H ₁₀₀ m	H _g m	D _g cm	G m ² ha ⁻¹	Stem number no ha ⁻¹	No. of sample plots	No. of sample trees / sample plot	
									Mean	(Min. - Max.)
501	40	EVT	23.4	10.0	10.4	18.2	3003	3	42.7	(40 - 46)
502	55	EVT	20.5	10.9	12.4	17.9	2084	3	48.7	(47 - 51)
503	41	VMT	24.2	11.8	14.3	23.9	1858	3	45.7	(44 - 48)
504	45	VMT	21.5	11.9	14.1	21.7	1800	3	39.3	(29 - 49)
506	56	VT	22.1	14.7	17.6	21.7	1118	4	43.2	(32 - 49)
507	38	VT	25.6	10.7	11.3	22.8	3070	3	39.0	(21 - 51)
508	39	VT	24.9	10.7	11.0	22.9	3148	3	39.0	(22 - 52)
509	29	MT	28.8	10.7	13.3	25.6	3081	3	42.3	(32 - 50)
512	48	VT	21.0	10.6	11.8	20.1	2423	3	35.0	(29 - 42)
556	40	EVT	23.8	10.3	10.8	18.7	2762	3	47.3	(46 - 49)
558	44	EVT	26.5	15.2	15.9	25.9	1560	3	37.3	(29 - 45)

¹⁾ According to Cajander (1909)

The effects of three levels of thinning intensities and three levels of nitrogen fertilisation were studied using a factorial experimental design. In the experimental stands, each treatment was applied on one rectangular sample plot 1000 m² in size except for one stand, in which two control plots were established.

Only the unfertilized sample plots were included in the analyses conducted in the course of this study. Once the initial measurements had been carried out, one third of the sample plots was left unthinned, one third was thinned moderately (30% of the stem number were removed) and one third was thinned heavily (60% of the stem number were removed). In moderately thinned plots, the second thinning was carried out ten years after the first thinning by again removing 30% of the initial stem number. On these sample plots, only the data from the measurement instances prior to the second thinning were included in the analyses.

The stands were measured at five-year intervals over a study period of 15 years. All the trees on the sample plots were measured for their breast height diameter. In the first measurement instance, an average of 42 sample trees were selected from each sample plot and used throughout the study period. During the stage of selecting the sample trees, the probability of a tree to be selected was proportional to its diameter and independent of its location on the sample plot. Two thirds of the sample trees were thicker than the stand's average diameter. The height and crown height were measured for every sample tree. Crown height was defined as the height above ground of the lowest live contiguous branch whorl. In addition to breast height diameter, also diameter at six metres and diameters at the relative heights of 2.5%, 10%, 30% and 50% along the stem were measured.

The stand-level characteristics of the growing stock were calculated using a software package for computing stand and tree characteristics (KPL), developed at the Finnish Forest Research Institute (Heinonen 1994). Height information obtained for sample trees was generalised with help of Näslund's (1937) height curve to apply to the tallied trees. Sample tree volumes were calculated using the simultaneous equations developed by Laasasenaho (1982). By using these functions, all the available tree diameter observations at absolute and relative heights along the stem could be used in the stem volume calculations. Volumes for the tallied trees were computed from the sample tree volumes by using smoothing functions. Increments for the tree and stand variables were calculated as differences between the values of the variables at the end and the beginning of the five-year growth periods.

Only sample trees were used in the model development. The study material included 4 634 measurements of tree characteristics involving 1 579 sample trees located on 34 sample plots. The number of tree-diameter growth observations (five-year growth periods) was 3 479, and the number of tree-height growth observations was 3 406.

The effect of annual climatic variation on tree growth was taken into account with the help of annual growth indices provided by Mielikäinen & Timonen (1995). For every five-year growth period, an average index was calculated from the annual growth indices, with which the observed diameter and height growth was divided.

2.2 Test data

Data from permanent thinning experiments established by the Finnish Forest Research Institute (Vuokila 1987) were used as the independent test material in model validation. The test data included 3 551 trees with 5-year growth observations covering 24 sample plots located in six stands in southern Finland (Table 2). Nine of the sample plots were unthinned. The thinning intensity among the thinned sample plots varied between 15–55% (of the stand basal area removed). The average thinning intensity was 27%. The study period covered 0–13 years after thinning.

The sample plots providing the test data were measured, and the sample trees on these plots were selected in the same manner as with the modelling data. Also, the calculation of tree and stand variables was done in a similar manner, except for the calculation of sample tree volumes. These were calculated using the volume equations provided by Laasasenaho (1982), based on tree diameters at breast height and at 6 m height, and on tree height, because the sample trees were not measured for their diameters at relative heights.

Table 2. Mensurational characteristics of the test material.

Exp no	Age, years	Site type ¹⁾	H ₁₀₀ m	H _g m	D _g cm	G, m ² ha ⁻¹	Stem number no ha ⁻¹
6	77	VT	20.3	17.9	20.3	23.0	795
42	35	VT	27.5	13.0	16.1	19.6	1075
63	24	MT	26.6	7.9	12.2	16.7	1703
65	36	VT	30.7	14.5	16.0	30.0	1919
541	52	VT	26.0	17.7	21.9	18.6	527
542	62	VT	24.0	17.2	20.3	16.5	551

¹⁾ According to Cajander (1909)

3 Diameter-growth model

3.1 Modelling approach

Tree diameter growth was assumed to be affected by site fertility, the amount of the growing biomass, and the net-assimilation efficiency of the biomass (Jonsson 1969, Hägglund et. al. 1979). It was further assumed that the effects of the different growth factors interact multiplicatively (Baule 1917).

Site fertility was expressed using the site index (H_{100}) calculated with the equations provided by Vuokila and Väliäho (1980). The amount of the growing biomass was described in terms of tree diameter and the crown ratio. The net assimilation efficiency of the growing biomass was assumed to be affected by stand density, described in terms of the stand basal area, relative positions of trees in the stand, described in terms of the basal area of trees larger than the subject tree, and the phase of stand development, described in terms of the stand dominant height.

Two separate diameter growth models were developed. In the first model, the effect of thinning on tree growth was assumed to be taken into account by including the basal area of the growing stock as a regressor variable referring to the actual stand density. In the second model, the effect of thinning on tree growth was incorporated explicitly in the model by using a variable accounting for the thinning intensity and time interval since thinning. In both models, five-year diameter growth, over bark, was used as a dependent variable.

Because of the hierarchical data structure, there was temporal autocorrelation between successive observations made of a single tree, and there was spatial autocorrelation between observations made of trees on the same sample plot. The effect of autocorrelation was not taken into account in the parameter estimation of the models, because autocorrelation does not generally affect the unbiasedness of models.

Ordinary Least Squares (OLS) estimation was applied in the parameter estimation of all the models. Parameters of the nonlinear regression models were estimated using the NLIN program of the SAS software package (SAS Institute Inc., 1989) and applying Marquardt's method with the convergence criterion set to 10^{-8} .

3.2 Model without explicit thinning response variable

In the first diameter growth model, it was assumed that the thinning effect would be reflected in tree growth through the actual stand basal area and through the variables affected by stand density. Therefore, no explicit thinning variable was included in the model. The analysis of the data resulted in the following model

$$i_{d5} = a_0 d^{a_1} cr^{a_2} \exp(a_3 d^2 + a_4 GL^2) H_{dom}^{a_5} H_{100}^{a_6} G^{a_7} + e \quad [1]$$

where

- i_{d5} = Five-year increment of tree diameter, cm
- d = Tree diameter at breast height, over bark, cm
- cr = Tree crown ratio, defined as:
length of live crown/total tree height
- GL = Basal area of trees (over bark) larger than the subject tree, $m^2 ha^{-1}$
- H_{dom} = Stand dominant height defined as:
average height of 100 thickest trees per hectare, m
- H_{100} = Site index, m (base age 100 years, calculated using models provided by Vuokila and Väliäho (1980))
- G = Stand basal area, over bark, $m^2 ha^{-1}$
- a_0, a_1, \dots, a_7 = Parameters
- e = Error term

The formulation of the effect of the stand basal area (G^{a_7}) was chosen, although it leads to illogical model behaviour when the stand basal area is close to $0 m^2 ha^{-1}$. Despite this structural weakness, the applied expression proved to describe the effect of the stand basal area in the modelling data better than the other examined transformations of the stand basal area. Model behaviour is logical within the range of basal area variation of the modelling data ($G > 9.5 m^2 ha^{-1}$).

The parameter estimates of model [1] were obtained with the OLS estimation (Table 3). The autocorrelation between the observations of the modelling data does not affect the parameter estimates. However, the standard error of the estimates obtained with OLS are likely to be too small.

Table 3. Parameter estimates of diameter growth model [1].

Parameter	Estimate	Asymptotic Std. Dev.
a_0	0.0504	0.0137
a_1	0.7917	0.0733
a_2	0.5557	0.0439
a_3	-0.0010	0.0001
a_4	-0.00075	0.00008
a_5	-0.6470	0.0654
a_6	1.4995	0.0606
a_7	-0.4349	0.0256
\widehat{i}_{d5}	1.1479	
RMSE	0.4046	
Observations	3479	

Residual analysis of model [1] showed a slight increase in the error variance with increasing predicted growth (Fig. 2a). However, there were no trends in the residuals with respect to the predicted diameter growth (Fig. 2a) or with respect to the regressor variables of the model. Residuals plotted against thinning intensity showed that model behaviour in general was satisfactory in unthinned and moderately thinned stands (Fig. 2b). However, in heavily

thinned stands, with more than 50% of stand basal area removed, significant biases were observed. In these stands, the mean growth was underpredicted on average by 17.1% over the entire 15-year study period.

During the first five-year growth period following thinning, model [1] overpredicted growth, except in the case of the heavily thinned stands, in which the model resulted in a small underprediction (Fig. 2b). During the second and third growth periods, the model resulted in a noticeable underprediction in heavily thinned stands, but also in a slight underprediction in unthinned and moderately thinned stands. In heavily thinned stands, the bias was at its highest during the second growth period, 5–10 years after thinning.

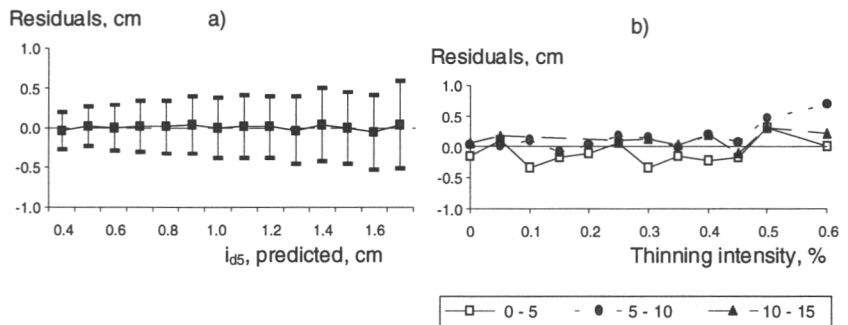


Figure 2. Mean residuals (\pm standard deviation of the residuals) of the diameter growth model [1] with respect to predicted diameter growth (a), and mean residuals of the five-year growth periods with respect to thinning intensity (b).

The predictive capability of the tree crown ratio was examined by fitting a model similar to [1], but from which the crown ratio (cr) had been excluded. The root mean square error (RMSE) of the model after excluding cr was 0.4140, i.e. 2.3% greater than the RMSE of model [1], which was 0.4046. Removing the crown ratio from model [1] did not change the model behaviour in regard to thinning intensity.

3.3 Model with thinning response variable

An alternative diameter growth model was developed with the effect of thinning explicitly incorporated in it. Tree diameter growth in a thinned stand can be expressed as a product of a reference growth and thinning response function

$$i_{d5} = F_1(ref) F_2(thin) \quad [2]$$

Reference growth ($F_1(ref)$) accounts for the factors affecting tree growth in unthinned stands. The thinning response function ($F_2(thin)$) predicts the relative growth response following thinning, and the reference growth is multiplied with this. The model structure is similar to that of the growth model for fertilized Scots pine stands developed by Hynynen (1993).

The Weibull function was applied in modelling the temporal distribution of the thinning response. It was assumed that tree growth responds to thinning without any delay. Thus, a two-parameter Weibull function was applied in the model. The integral of the Weibull function equals one. In order to get varying magnitudes of response as the results of different thinning intensities, the Weibull function was scaled by multiplying it with a variable expressed as a function of the thinning intensity.

As the result of analysis, the following model was developed

$$i_{d5} = a_0 d^{a_1} cr^{a_2} \exp(a_3 d^2 + a_4 GL^2) H_{dom}^{a_5} H_{100}^{a_6} \cdot F_2(thin) + e, \text{ in which} \quad [3]$$

$$F_2(thin) = 1 + a_7 I \left(\frac{c}{b} \right) \left(\frac{T}{b} \right)^{(c-1)} \exp \left(- \left(\frac{T}{b} \right)^c \right)$$

where

I = Thinning intensity, defined as:

$(G, \text{ pre- thinning} - G, \text{ post- thinning}) / G, \text{ post- thinning}$

T = Time elapsed from thinning, years

$a_0, a_1, \dots, a_7, b, c$ = Parameters

In model [3], the regressor variables used in predicting reference growth are the same as in model [1] except for the stand basal area.

The effect of the stand basal area is reflected in growth through the size of the tree crown ratio, and in thinned stands also through the thinning response function. According to the model, the relative response to thinning is not affected by site, and neither by any stand or tree variables.

Because model [3] predicts the five-year tree diameter growth, also the temporal distribution of thinning response is predicted by five-year periods. Thus, $F_2(thin)$ refers to the average relative growth response during the 5-year growth period in question. Consequently, variable T in model [3] refers to the last year of the 5-year growth period. According to model [3], thinning increases the relative diameter growth without any delay. The response reaches its maximum within a period of 5–10 years after thinning, and levels off by 30 years after thinning (Fig. 3).

Including the effect of thinning in the diameter growth model improved the accuracy of the model compared to model [1], in which the thinning effect is implicitly included through the actual stand basal. The root mean square error in model [1] was reduced from 0.4046 (Table 3) to 0.3841 (Table 4), i.e. by 5.1%. An improvement of the model was also observed in residual analysis (Fig. 4). No bias was observed with respect to any of the regressor variables. There were no trends in the residuals as regards thinning intensity during any of the successive growth periods (Fig. 4b). Therefore, the model's performance was improved compared to model [1].

The tree crown ratio (cr) was a significant regressor variable in model [3] as well as in model [1]. Removing the tree crown ratio from model [3] increased the RMSE from 0.3841 to 0.3910, i.e. by 1.8%.

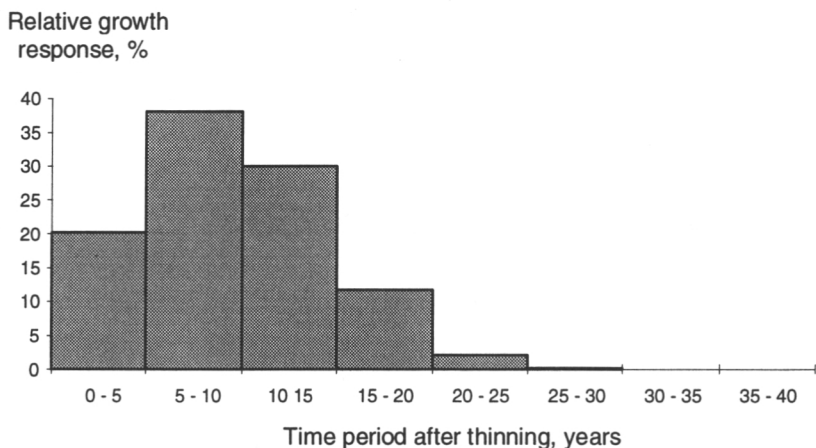


Figure 3. Temporal distribution of the relative diameter growth response according to thinning response function, $F_2(thin)$, of the model [3].

Table 4. Parameter estimates of diameter growth model [3].

Parameter	Estimate	Asymptotic Std. Dev.
a_n	0.1769	0.0454
a_1	0.5693	0.0659
a_p	0.4737	0.04242
a_n	-0.00070	0.00014
a_d	-0.00094	0.00008
a_f	-0.9694	0.0661
a_r	1.0796	0.0539
a_7	7.7395	0.5512
b	13.4054	0.5314
c	2.4828	0.1385
\hat{i}_{d5}	1.1473	
RMSE	0.3841	
Observations	3479	

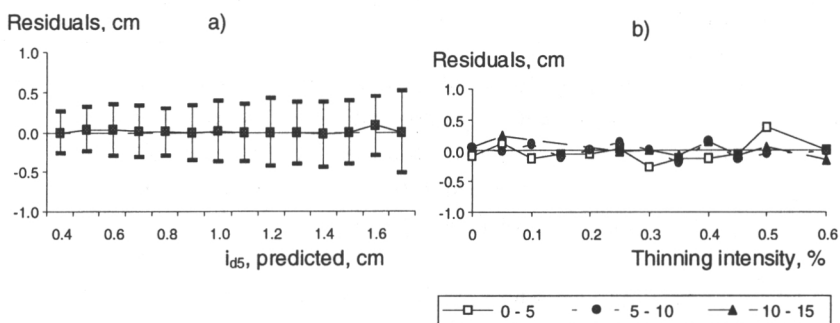


Figure 4. Mean residuals (\pm standard deviation of the residuals) of the diameter growth model [3] with respect to predicted diameter growth (a), and mean residuals of the five-year growth periods with respect to thinning intensity (b).

4 Height growth model

The tree height growth model was developed on the basis of an assumption according to which tree height growth can be represented as the product of potential height growth times a modifier function. This kind of a model structure has been widely applied in growth and yield modelling (e.g. Daniels and Burkhart 1975, Leary 1979, Arney 1985).

Increment in the stand dominant height was regarded as the potential height growth. With reference to earlier studies, it is generally assumed

that dominant height increment is not affected by thinning from below (Burkhart et. al. 1987, Vuokila and Väliaho 1980). To verify this assumption within the context of the modelling data, a simple regression model for dominant height increment was developed. In the model, dominant height increment was assumed to be affected by site quality, stand age and thinning intensity. The effects of site quality and stand age were taken into account by using categorical variables referring to the experimental stand. The effect of thinning was studied by incorporating a variable referring to thinning intensity into the model. The logarithm of the mean annual increment in the stand dominant height over the 15-year study period was employed as the dependent variable of model [4] below.

$$\ln(IH_{dom}) = a_0 + a_1I + a_2D_{502} + a_3D_{503} + \dots + a_{11}D_{558} + e \quad [4]$$

where

IH_{dom} = Mean annual increment of stand dominant height over 15-year study period, m

$D_{502} \dots D_{558}$ = Categorical variables referring to experimental stands

$a_0, a_1 \dots a_{11}$ = Parameters

Model [4] was fitted to the data including observations of the dominant height increments from every sample plot in the data. The effect of thinning intensity did not prove to be a significant regressor (Table 5). Therefore, it was concluded that thinning intensity does not have any significant effect on the increment of the stand dominant height, which was employed as the height growth potential of an individual tree in a stand.

In developing the height growth model for individual trees, it was supposed that tree growth can be faster or slower than potential growth depending on the relative size of the tree. Relative tree size was described by the ratio between tree diameter at breast height and the stand dominant diameter ($d_{1.3} / D_{dom}$), the latter being defined as the average diameter of the 100 thickest trees per hectare. Thus, the stand dominant diameter is the arithmetic mean diameter of the trees included in the calculation of the stand dominant height (H_{dom}).

Table 5. Parameter estimates of height growth model [4].

Parameter	Estimate	Std. Error	t-value	Prob. > T
<i>Intercept</i>	-1.194			
a_1	-0.103	0.129	-0.802	0.431
\hat{IH}_{dom}	-1.215			
R ²	0.901			
RMSE	0.129			
Observations	32			

Note: *Intercept* = Parameter a_0 + mean of the parameters a_2, \dots, a_{11} .

It was further assumed that thinning from below does not directly affect tree height growth, and that stand basal area affects only the growth of suppressed trees. The crown ratio (*cr*) can be considered as an expression of the tree's photosynthetic potential. Therefore, it can be assumed to have an effect on the realization of potential tree growth.

The parameters were estimated using the same method as when estimating the parameters of diameter growth models [1] and [3]. The analysis resulted in the following model for tree height growth (Table 6):

$$i_{h5} = IH_{dom} \left[\frac{d}{D_{dom}} \right] \left(a_1 IH_{dom} + a_2 \left(\frac{d}{D_{dom}} \right)^2 + a_3 \left(\frac{d}{D_{dom}} \right)^3 \right) + e \quad [5]$$

where

- i_{h5} = Five-year increment in tree height, m
- IH_{dom} = Five-year increment in dominant height, m
- D_{dom} = Stand dominant diameter defined as:
average diameter of 100 thickest trees per hectare, cm
- a_1, a_2, a_3 = Parameters

The effects of the tree crown ratio and the stand basal area on height growth proved to lack statistical significance as regressors. Thus, they were not included in the final model. Residual analysis showed satisfactory model behaviour in regard to predicted height growth, regressor variables and thinning intensity (Fig. 5). Although the effect of thinning was not incorporated in the model, there were no trends in the residuals with respect to thinning intensity during any of the growth periods.

In fitting the model, the measured increment in the stand dominant height (IH_{dom}) was used as the potential height growth. When applying the model, IH_{dom} can be obtained from the site index equation.

Model [5] is restricted so that trees with diameters equal to the average diameter of the dominant trees will have height growths equal to the increment of the dominant trees.

The relationship between tree height growth and relative tree size is of curvilinear form. Starting from the most suppressed trees in a stand, height growth increases with increasing relative tree size, until it reaches its maximum (Fig. 6).

After that, growth starts to decrease with increasing relative size. The position of maximum height growth depends on the rate of the dominant height increment.

According to the model, growth of a individual trees in stands with rapid dominant height increment (Ih_{dom}) is more

Table 6. Parameter estimates of tree height growth model [5].

Parameter	Estimate	Asymptotic Std. Dev.
a_1	0.2445	0.0151
a_2	-0.4710	0.0558
a_3	0.8045	0.1804
\hat{i}_{h5}	1.4794	
RMSE	0.4495	
Observations	3406	

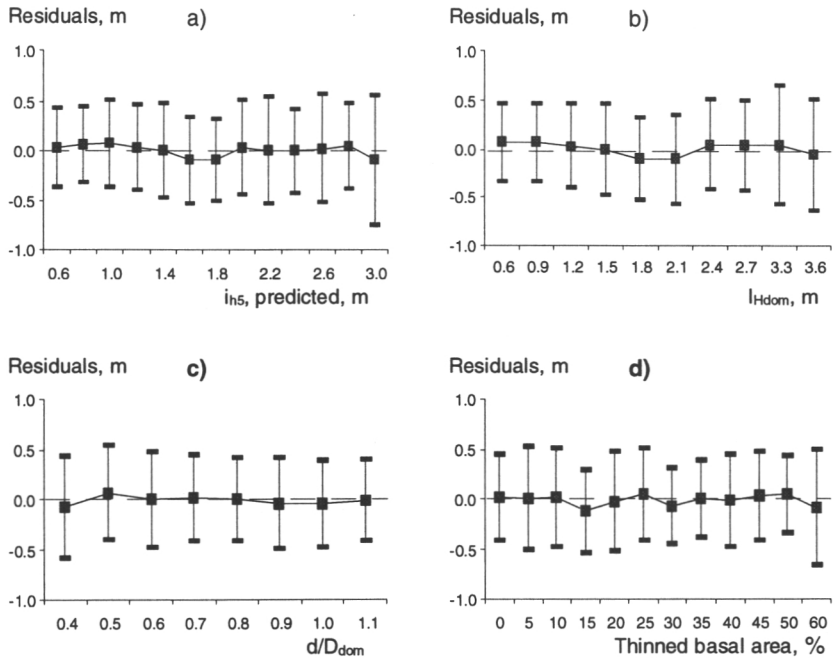


Figure 5. Mean residuals (\pm standard deviation of the residuals) of tree height growth model [5] with respect to predicted height growth (a), stand dominant height increment (b), relative tree size (c), and thinning intensity (d).

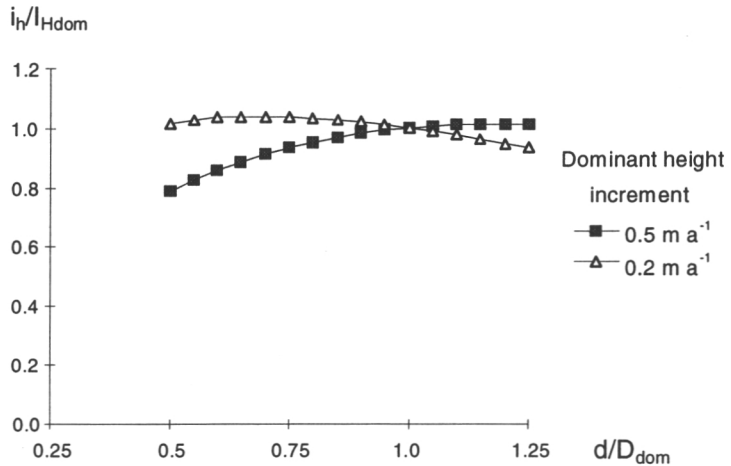


Figure 6. The relationship between relative height growth of a tree and relative tree size in stands with different rate of dominant height increment.

affected by relative tree size than in stands with slow dominant height increment (Fig. 6). In other words, differentiation in height growth among trees is greatest in stands with rapid height growth. With respect to stand growth dynamics, height growth differentiation is at its highest in young stands.

5 Model for predicting stem form factor

A measure of tree stem form is needed in addition to the tree diameter and height growth prediction in order to simulate the development of stand and tree volumes. The ratio between tree diameter and total height (d/h) is an indicator of the stem form. Thinning is known to change this ratio, which can be predicted with the help of diameter and height growth models ([3] and [5]).

In this study, the development of a model for predicting the stem form factor was of interest as the aim was to examine whether thinning affects stem form in a way not explained by the change in the d/h ratio. In order to be able to quantify these kinds of possible effects in the present study material, a model for the stem form was developed.

A static model for the cylindrical form factor was chosen to describe the stem form. The cylindrical form factor is a widely used expression for tree stem form. It is defined as the ratio of the total stem volume to the volume of a cylinder with diameter equal to tree diameter at breast height and height equal to the total height of the tree. The stem volume can be expressed as

$$v = f_{1.3}gh \quad [6]$$

where

v = Tree volume

$f_{1.3}$ = Cylindrical form factor

g = Tree basal area at breast height

h = Tree height

The absolute value of the form factor is restricted to between 0 and 1, but excluding small trees with heights close to 1.3 m. In the present modelling data, the smallest measured tree height was 4.5 m. Thus, the model for the stem form factor should inherently result in values between 0 and 1 to facilitate logical model behaviour. The following model structure was employed as the basic structure in model development.

$$f_{1.3} = 1 - \exp(-\Phi(x)), \quad \text{where } \Phi(x) > 0 \quad [7]$$

In model [7], $\Phi(x)$ is a function of measured tree and stand variables. A similar structure has been used earlier in numerous models for the tree crown ratio, which is also a variable restricted to values between 0 and 1 (e.g. Ek and Monserud 1975, Dell et. al 1979, Dyer and Burkhart 1987, and Hynynen 1995).

A simple model for stem form factor was developed based only on the information about tree diameter and height. After fitting the model, its behaviour was studied with respect to other stand and tree variables, including thinning intensity.

As the result of analysis, the following model for the stem form factor was constructed (Table 7):

$$f_{1.3} = 1 - \exp\left[-\left(a_0 + a_1(d/h)^{a_2} + a_3h^{a_4}\right)\right] + e \quad [8]$$

According to model [8], trees with more taper have smaller stem form factors. Furthermore, the form factor decreases with increasing tree size, and this is expressed with total tree height.

The residuals showed no trends with respect to the predictor variables (Fig. 7). Plotted residuals against stand basal area, stand dominant height, as well as against tree crown ratio showed slight trends, but the model resulted in unbiased prediction concerning thinning intensity (Fig. 7f).

In order to improve the model, the effects of the other measured stand and tree variables, in addition to d and h , was examined. Tree crown ratio, stand dominant height, and stand basal area proved to be significant regressors, resulting in the following model:

$$f_{1.3} = 1 - \exp\left[-\left(a_0 + a_1(d/h)^{a_2} + a_3h^{a_4} + a_5cr + a_6H_{dom} + a_7G\right)\right] + e \quad [9]$$

The inclusion of new variables in model [8] removed the biased behaviour with respect to these variables, but improved the precision of the model only slightly by reducing the RMSE by 1.2% (Table 8).

Table 7. Parameter estimates of model for form factor [8].

Parameter	Estimate	Asymptotic Std. Dev.
a_0	0.6390	0.0159
a_1	-0.0263	0.0040
a_2	3.0028	0.2742
a_3	3.7094	0.6233
a_4	-1.2138	0.1010
$\hat{f}_{1.3}$	0.5341	
RMSE	0.0251	
Observations	4634	

Table 8. Parameter estimates of model for form factor [9].

Parameter	Estimate	Asymptotic Std. Dev.
a_0	0.8217	0.0252
a_1	-0.0381	0.0065
a_2	2.4243	0.2733
a_3	6.5014	1.4381
a_4	-1.6114	0.1299
a_5	-0.0638	0.0111
a_6	-0.0021	0.0007
a_7	-0.0020	0.0002
$\hat{f}_{1.3}$	0.5341	
RMSE	0.0248	
Observations	4634	

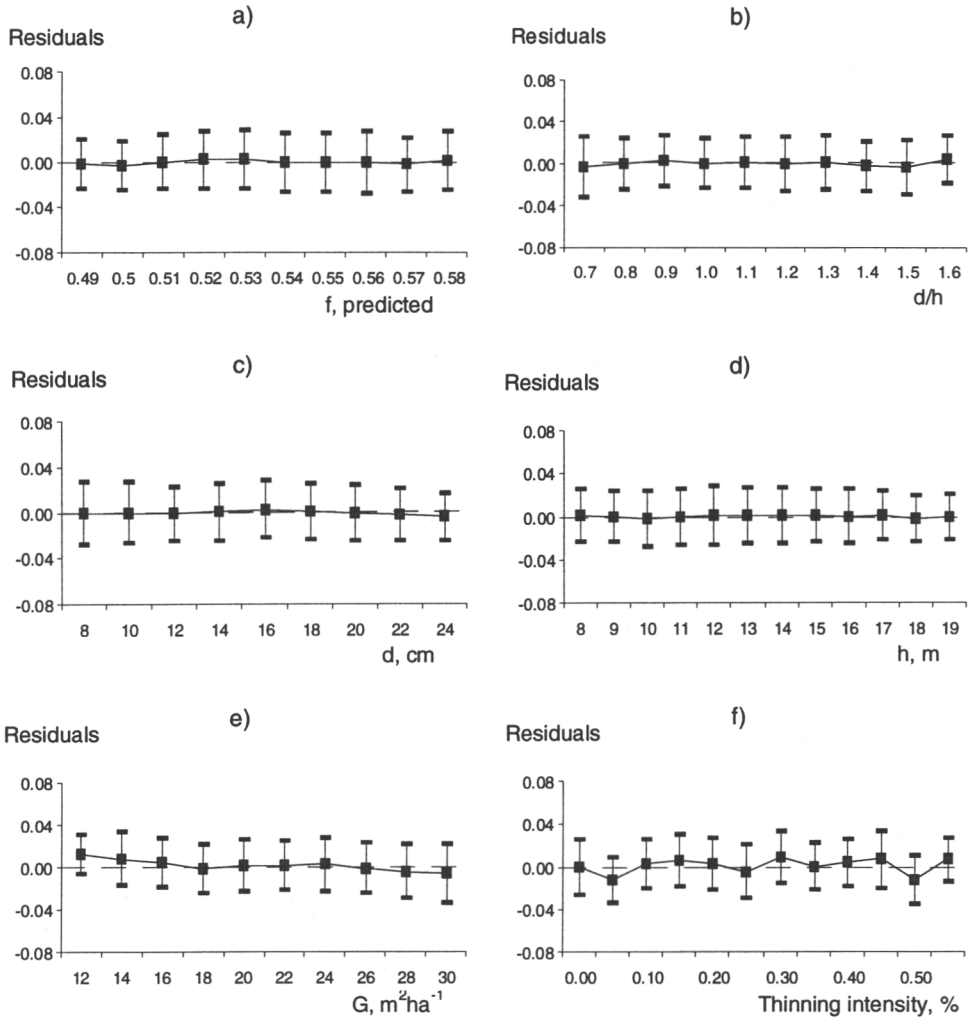


Figure 7. Mean residuals (\pm standard deviation of the residuals) of stem form model [8] with respect to predicted height growth (a), d/h -ratio (b), tree diameter (c), tree height (d), stand basal area (e), and thinning intensity (f).

According to model [9], increases in the tree crown ratio and the stand dominant height impair the stem form; this is consistent with the previous knowledge on these relationships. The slightly negative effect of the basal area ($a_7 < 0$) on the form factor is more difficult to interpret. However, the effect of the stand basal area will also be reflected in the tree crown ratio and the ratio d/h . As the basal area increases, the crown ratio and d/h decrease, and these in turn increase the form factor, and thus improve the stem form. Therefore, the effect of the stand basal area is also implicitly included in the other regressor variables.

On the basis of the stem form factor models [8] and [9], it can be

concluded that in the case of both unthinned and thinned stands, the development of the stem form can be predicted without bias by using those tree and stand variables that are used as regressors in models [8] and [9]. No additional information concerning thinning itself is needed. The results of this analysis suggest that the effect of thinning on tree form can be explained with adequate accuracy by the change in the d/h ratio.

6 Model validation

6.1 Reliability of models in predicting tree diameter growth, height growth and stem volume

All the models were tested against independent data as described in section 2.2. Validation was first carried out separately for each model to test their predictive capability. Then, the models were applied together in predicting the stand basal area and volume increment. The following characteristics were calculated to describe the reliability of the model prediction:

$$\text{Absolute bias} = \sum_{i=1}^n (y_i - \hat{y}_i) / n$$

$$\text{Relative bias} = \sum_{i=1}^n \left[(y_i - \hat{y}_i) / \hat{y}_i \right] / n$$

$$\text{RMSE} = \left[\sum_{i=1}^n (y_i - \hat{y}_i)^2 / n \right]^{0.5} = \text{Root mean square error}$$

$$\text{RMSE}_T = \left[\sum_{i=1}^n \left[(y_i - \hat{y}_i) / \hat{y}_i \right]^2 / n \right]^{0.5} = \text{Relative root mean square error}$$

where

y_i = Observed value of i :th observation

\hat{y}_i = Predicted value of i :th observation

n = Number of observations

In general, the tree growth models resulted in a slight underprediction when applied to the test material (Table 9, Fig. 8). Diameter growth model [1] resulted in a smaller average bias than

Table 9. Behaviour of the models against the independent test data.

	Diameter growth ¹⁾		Height growth ¹⁾	Tree volume (form factor)
	Model [1]	Model [3]	Model [5]	Model [8]
Observed, mean	1.302 cm	1.302 cm	1.949 m	150.09 dm ³
Predicted, mean	1.274 cm	1.221 cm	1.939 m	158.38 dm ³
Absolute bias	0.028 cm	0.877 cm	0.011 m	-8.296 dm ³
Relative bias	0.0387	0.0673	0.0426	-0.0577
RMSE	0.639	0.647	0.383	13.039
RMSE _r	0.716	0.761	0.307	0.067
No. of obs.	3551	3551	3523	8583

¹⁾ Five-year growth period

model [3], which included an explicit thinning response variable. Both models showed no biased behaviour as regards thinning intensity (Fig. 8b). However, among the test material, there were only a small number of heavily thinned sample plots; only on three of the sample plots was more than 35% of the basal area removed in thinning, and only on one plot more than 50%. Therefore, with the test material as the basis, it could not be reliably confirmed how necessary it would be to incorporate an explicit thinning response variable in the diameter growth model when predicting the development of heavily thinned stands.

The residual mean square errors of diameter growth models [1] and [3] in the test data were notably greater compared to those in the modelling data. This was probably due to larger overall variation in the test data compared to the modelling data. In the test data, the average five-year diameter growth was 1.278 cm with a standard deviation of 0.809 cm. In the modelling data, the corresponding values were 1.147 and 0.546 cm, respectively.

The validation of the height growth model confirmed that the applied model structure is feasible in height growth prediction in both thinned and unthinned stands (Fig. 9b). The model seemed to underpredict the height growth of trees with slow predicted height growth (Fig 9a). However, further data analysis showed that all the observations of trees with predicted height growths of less than 1.2 m were obtained from the one experimental stand only.

For validation of form factor model [8], the form factors for all the trees in the test material were first predicted using model [8], separately for every measurement instance. Thereafter, the stem volumes were calculated using formula [6]. Finally, the predicted stem volumes were compared with the stem volumes calculated on the basis of field measurements.

In general, tree volume prediction resulted in a 5.8% overprediction (Table 9), but there were no trends to be seen in the model prediction

regarding thinning intensity (Fig 10b). Nevertheless, there was a slight trend as regards tree diameter; overprediction was at its maximum among the smallest trees (Fig. 10a).

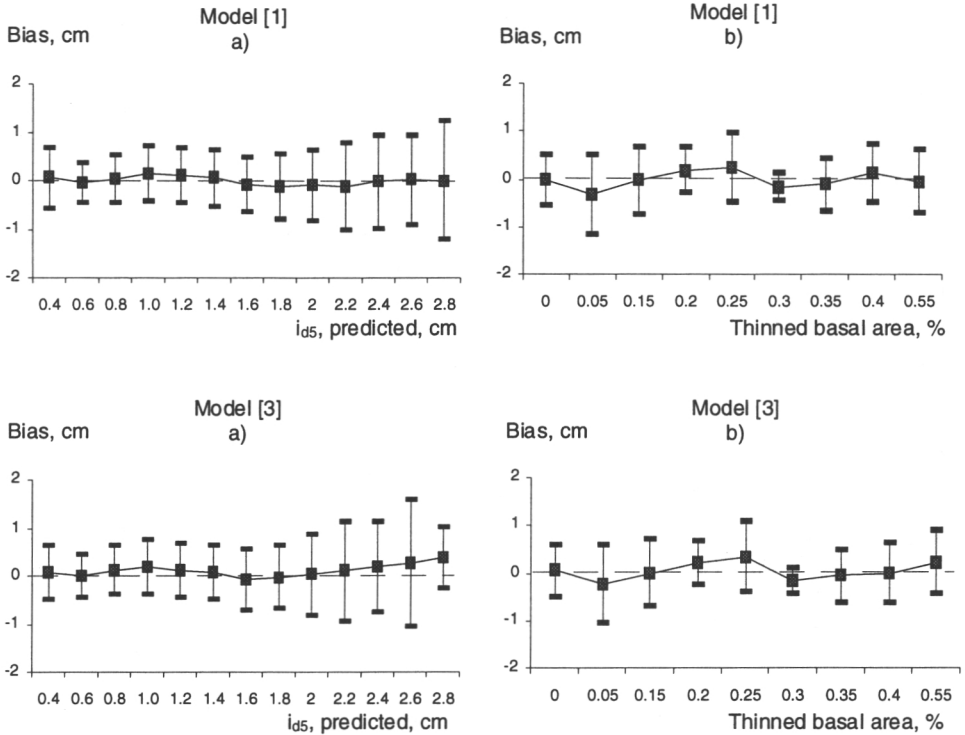


Figure 8. Average bias (\pm standard deviation of the residuals) of the diameter growth models [1] and [3] in the test data plotted against predicted tree diameter (a), and thinning intensity (b).

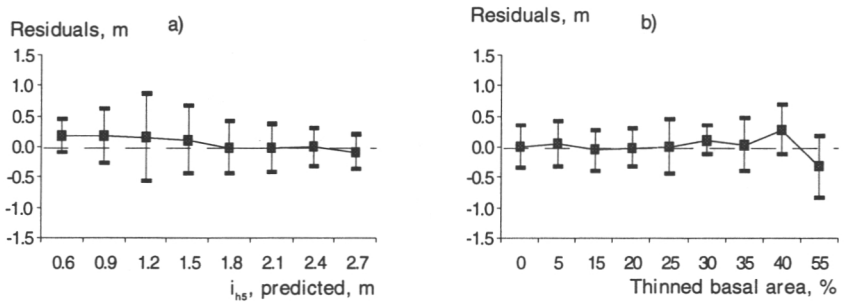


Figure 9. Average bias (\pm standard deviation of the residuals) of the height growth model [5] in the test data plotted against predicted tree height (a), and thinning intensity (b).

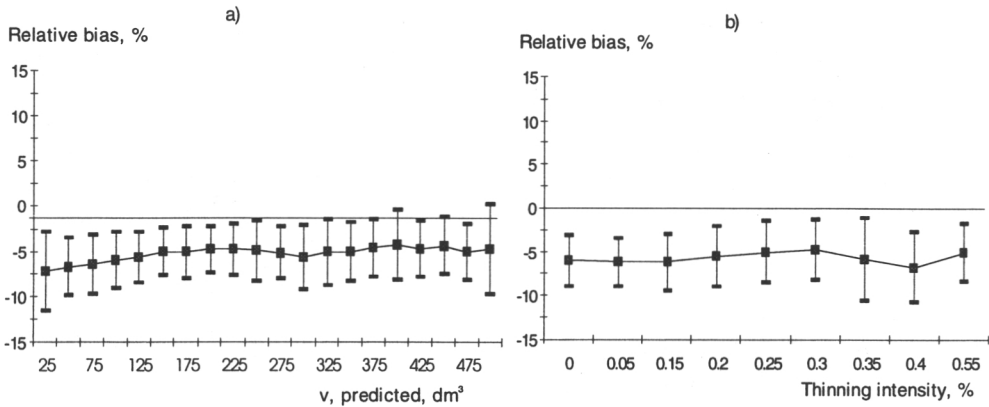


Figure 10. Average relative bias (\pm standard deviation of the residuals) of the predicted stem volume calculated with models [6] and [8] in the test data plotted against predicted tree volume (a), and thinning intensity (b).

In assessing the results concerning the reliability of stem volume prediction, it must be observed that stem volumes in the modelling data were calculated using simultaneous equations based on tree diameter observations at absolute and relative heights along the stem. In the test data, a three-parameter volume equation ($v = f(d_{1.3}, d_{6.0}, h)$) was applied.

The effect of the applied equation on volume calculation was tested using the modelling data, in which tree volumes were also calculated using the three-parameter volume equation, i.e. with the same equation as was used in the test data. It was observed that the simultaneous equation resulted, on average, in a 3.5% greater stem volume compared to the volume estimate calculated using the three-parameter volume equation. Because form factor model [8] is based on the data in which volumes were calculated using the simultaneous equations, it can be said that most of the bias (Table 9, Figure 10) can be explained by the differences in the volume estimates obtained using these two different volume equations.

6.2 Model reliability in predicting stand basal area and volume increment

The reliability of the models in predicting stand-level characteristics was examined by simulating the stand basal area and volume increment. Mortality was taken into account in the simulations by removing from the list the trees that had died during the simulation period, before any comparisons were made. The tree volumes at the beginning of the study period were estimated for all trees in the test

data using formula [6], in which the stem form factor was calculated using model [8]. Tree diameter increment was simulated using models [1] or [3], and tree height growth using model [5]. In the height growth prediction, the observed dominant height increments ($I_{H_{dom}}$) on the sample plots were employed as the height growth potential. At the end of the five-year simulation period, the tree volumes were recalculated. Volume growth was calculated as the difference between tree volume at the end and at the beginning of the simulation period. Total volumes of the growing stock and of the stand basal areas of the sample plots were obtained by summing up the tree volumes and tree basal areas, respectively.

Underprediction of the stand basal area increment was obtained with both diameter growth models (Table 10). The model behaviour was similar to that of the tree diameter growth model (Table 9). In stand volume growth prediction the bias was smaller, mainly because the underestimation of the basal area increment was offset by the overprediction of tree volumes. No dependence between bias and thinning intensity could be observed in basal area or in volume increments (Figs. 11 and 12).

Table 10. Statistics describing the reliability of the model in the prediction of stand basal area and volume increment of the test data during the 5-year growth period.

	Basal area increment		Volume increment	
	Model [1]	Model [3]	Model [1]	Model [3]
Observed	3.495 m ² ha ⁻¹	3.495 cm	42.40 m ³ ha ⁻¹	42.40 m ³ ha ⁻¹
Predicted	3.347 m ² ha ⁻¹	3.174 cm	43.38 m ³ ha ⁻¹	42.08 m ³ ha ⁻¹
Absolute bias	0.148 m ² ha ⁻¹	0.320 cm	-0.985 m ³ ha ⁻¹	0.315 m ³ ha ⁻¹
Relative bias	0.068	0.143	-0.009	0.038
RMSE	0.733	0.796	6.297	6.355
RMSE _r	0.303	0.351	0.154	0.178
No. of obs.	28	28	28	28

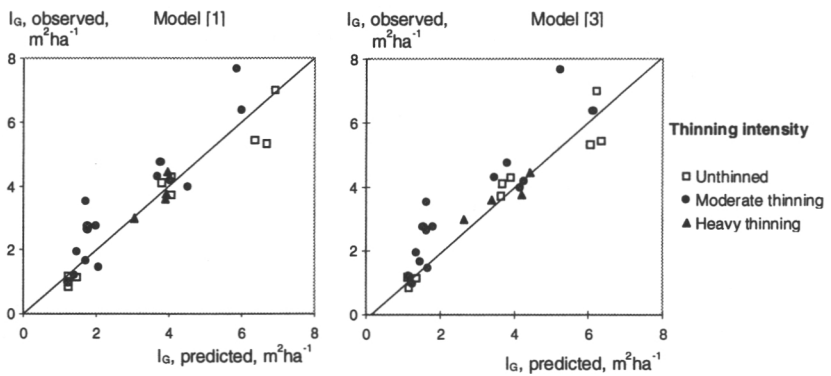


Figure 11. Observed and predicted stand basal area increments with varying thinning intensity in the sample plots of the test data. In moderate thinning < 30 %, and in heavy thinning > 30 % of stand basal area was removed.

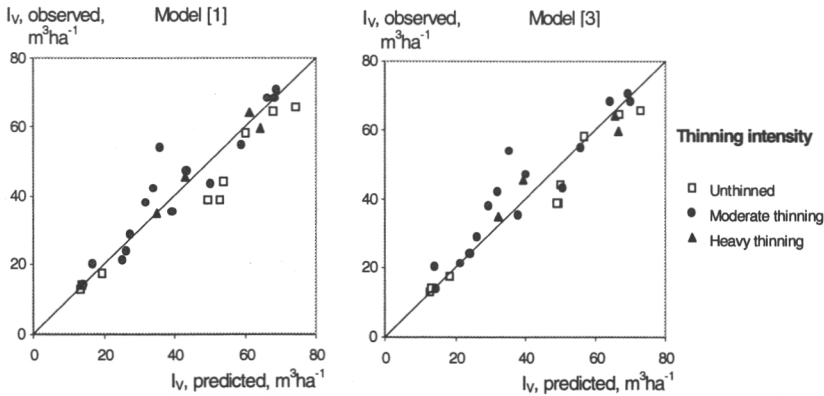


Figure 12. Observed and predicted stand volume increments with varying thinning intensity in the sample plots of the test data. In moderate thinning, < 30 %, and in heavy thinning > 30 % of stand basal area was removed.

7 Discussion

The main purposes of this study were to examine modelling methods used in predicting the growth response of Scots pine to thinning, and to develop a model structure that can be applied in growth simulations to thinned, as well as unthinned stands. Both the modelling data and the test data were obtained from intensively managed experimental stands. Therefore, the models are not directly applicable to the average commercial Scots pine stands in Finland. Nevertheless, the description of the interactions between growth factors, as well as the basic model structures developed in this study, are likely to be valid also when applied to more comprehensive and more representative data.

Due to the hierarchical data structure, there were both temporal and spatial correlations among the observations. Applying OLS estimation with this kind of data does not cause bias in the parameter estimates, but the standard errors of the parameter estimates are likely to be too small. The hierarchical data structure with the correlated observations can be taken into account in parameter estimation by applying the Generalized Least Squares (GLS) estimation. However, the main interest in this study was in examining the model behaviour and possible biases in the growth prediction obtained when using alternative growth models. Since model predictions are unbiased even when applying the OLS estimation, the main results obtained from the analyses were not influenced by the applied parameter estimation method. Furthermore, because most of the models were intrinsically nonlinear, fitting nonlinear models with the GLS estimation would have been statistically extremely complicated.

A diameter growth model was developed in which the effect of thinning was predicted using an explicit variable referring to the time and intensity of thinning. The analysis confirmed the suitability of the Weibull function in predicting the temporal variation of the growth response. A similar model structure has been employed previously in predicting growth response to nitrogen fertilization (Hynynen 1993). Model [3] proved to be capable of predicting the dynamics of the thinning response, and to perform satisfactorily in both unthinned and thinned stands. In accordance with model [3], the relative growth response to thinning was assumed not to be affected by tree size or by any stand-level variables. The residuals against the modelling and test data indicated unbiased model behaviour with respect to these variables, thus verifying the assumption to be valid in the data sets used in this study. The results comply with the earlier findings of Moore et al. (1994).

In general, both diameter growth models [1] and [3] resulted in unbiased prediction when applying varying thinning intensities. Only in heavily thinned stands did model [1] result in underprediction. Further examination of growth during successive 5-year growth periods revealed that model [1] failed to predict the dynamics of post-thinning diameter growth (Fig. 2b). During the first post-thinning 5-year period, model [1] overpredicted diameter growth, thus indicating that a reduction in the stand basal area through thinning did not immediately increase growth as much as was predicted by model [1]. Correspondingly, during the 5 - 10 years after thinning, when the thinning response was at its maximum (Fig. 4), model [1] resulted in notable underprediction of diameter growth, especially in heavily thinned stands (Fig. 2b).

The results obtained verified that an explicit thinning variable in a growth model is needed to reliably predict the dynamics of post-thinning diameter growth and to reliably predict tree growth in heavily thinned stands (with more than 50% of the basal area removed). However, there are some disadvantages in incorporating an explicit thinning variable in the model. First, including an explicit thinning variable in a growth model is likely to result in a rather complicated model structure. Second, the model will require detailed information about thinnings in order to be capable of predicting the growth response.

The diameter growth model [1] without any explicit thinning variable performed adequately in unthinned and moderately thinned stands. It is likely to be sufficient for most practical applications, because so far in practical forestry in Finland thinning intensities have rarely exceeded 50% of the stand basal area. Model [1] does not require any information about thinning, and this is an important

advantage considering the practical application of the model. Information about the timing and intensity of thinnings is seldom available in forest inventory data. In regard to parameter estimation, model [1] can be linearised by using logarithmic transformation, following which the parameters can be estimated with linear regression, and also by applying GLS estimation. In this study, model [1] was not linearised, because that would have complicated the comparisons with intrinsically nonlinear model [3].

Tree crown ratio has been widely applied in growth and yield models as the major driving variable (e.g. Belcher et al. 1982, Burkhart et al. 1987). It is the only variable measured in forest inventories that is directly related to the size of the photosynthetically active biomass. The efficiency of the tree crown ratio as a regressor in diameter and height growth models for Scots pine was tested in this study. Although the crown ratio is known to be correlated with variables referring to stand density (stand basal area), it proved to have a great impact on diameter growth prediction. Including the tree crown ratio in models [1] and [3] significantly improved their accuracy. However, it did not have any major effect on the behaviour of the model with respect to thinning intensity. In height growth prediction, the crown ratio did not prove to be a significant regressor in the present data.

In the model for tree height growth [5], a widely applied model structure was employed, and it proved to be suitable also for Scots pine stands. Height growth prediction in the case of an individual tree was bound to the stand dominant height increment, which was used as the potential height growth. In applying the model, dominant height increment can be obtained from site index equations (e.g. Vuokila and Väliäho 1980, Gustavsen 1980). Therefore, the final performance of the height growth model, when applied in practice, depends on the performance of the site index equation employed in the prediction of dominant height increment. The modelling data of this study were not comprehensive enough to be used in the development of new site index equations.

The dominant height increment was not affected by thinning from below. This is a result that markedly supported the findings of earlier studies (e.g. Hägglund 1974, Vuokila and Väliäho 1980). According to the results of this study, the height growth of an individual tree in a thinned stand can be adequately predicted without any explicit variable referring to thinning.

The aim of the analysis on tree form was to examine whether the well-known change in tree form following thinning can be explained only by the change in the d/h ratio. A relatively simple model for the cylindrical form factor was developed (model [8]) based only on the information about tree diameter and height. Adding other tree and stand

variables as regressors into the model improved the model performance only slightly. The model residuals, as well as the validation against an independent data, revealed no biased behaviour with respect to thinning intensity. This being the case, it is reasonable to conclude that, at least in these data sets, there was no change in stem form that could not be explained by change in the d/h ratio.

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