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Abstract

This paper puts forward a model of the role of phosphorus in crop production, soil phosphorus dynamics and phosphorus loading that integrates the salient economic and ecological features of agricultural phosphorus management. The model accounts for the links between phosphorus fertilization, crop yield, accumulation of soil phosphorus reserves, and phosphorus loading. It can be used to guide precision phosphorus management and erosion control as means to mitigate agricultural loading. Using a parameterization for cereal production in southern Finland, the model is solved numerically to analyze the intertemporally optimal combination of fertilization and erosion control and the associated soil phosphorus development. The optimal fertilizer application rate changes markedly over time in response to changes in the soil phosphorus level. When, for instance, soil phosphorus is initially above the socially optimal steady state level, annually matching phosphorus application to the prevailing soil phosphorus stock produces significantly higher social welfare than using a fixed fertilizer application rate. Erosion control was found to increase welfare only on land that is highly susceptible to erosion.

Keywords: precision nutrient management, agricultural phosphorus loading, cereal production, soil phosphorus reserves, agricultural water pollution, dynamic programming

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1 Introduction

Phosphorus loading from agricultural land has been identified as one of the major causes of surface water quality problems in developed countries (Sharpley and Rekolainen 1997, Shortle and Abler 1999, HELCOM 2004, Ekholm et al. 2005). One fundamental cause of phosphorus loading is inefficiency in fertilizer use. The yield response to phosphorus consists of the impacts of the phosphorus fertilizer applied in the current year and the phosphorus accumulated in the soil. Soil phosphorus largely determines the crop response to phosphorus and in increasing quantities reduces the yield response to fertilizer. When phosphorus fertilization exceeds the removal of phosphorus by the crop, most of the surplus phosphorus will remain in the soil to add to the phosphorus reserve (Hooda et al. 2001). Soils with excessive phosphorus reserves in turn pose the highest risk to the environment (Yli-Halla et al. 1995).

Phosphorus loading can be mitigated by applying phosphorus fertilizers to the production process with greater precision as well as by reducing soil loss (e.g. catch crops, vegetative filter strips, reduced tillage). Precision phosphorus management requires knowledge about links between phosphorus fertilization, crop yield, accumulation of soil phosphorus reserves, and phosphorus loading. Efficient policies to control agricultural phosphorus loading should in turn weigh the trade-off between profits from production and the environmental damage from phosphorus loading over time.

The literature on the optimal management of phosphorus has not fully accounted for the complex dynamics governing phosphorus loading.[1] Schnitkey and Miranda (1993) analyzed optimal steady state manure and mineral fertilizer application rates under alternative phosphorus control policies and found that soil phosphorus reserves affect crop yield only. The study does not model the link between phosphorus loss and phosphorus reserves explicitly; rather, pollution is controlled through exogenous limits on annual manure and commercial fertilizer application. Goetz and Zilberman

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[1] There is a more extensive literature on optimal phosphorus fertilization from the point of view of soil fertility alone, that is, where the externality arising from phosphorus loss to the environment is not considered. Kennedy (1986) provides an overview of the numerical dynamic methods applied to such problems. Lambert et al. (2007) is a recent study that estimated site-specific crop response functions with a nutrient carry-over equation using on-farm agronomic data. Estimates were used in a dynamic programming model to determine optimal site-specific fertilizer policies and soil phosphorus evolution.

(2000) determined spatially and intertemporally optimal mineral fertilizer application rates, number of animal units, proportion of total manure applied to the soil, and phosphorus concentration in the receiving body of water. While their model accounts for the complex dynamic and spatial characteristics of phosphorus loading, it makes no provision for the dynamic development of soil phosphorus reserves in response to fertilizer application; the initial soil phosphorus level in each location is incorporated into a fixed phosphorus index. Goetz and Keusch (2005) analyzed farmers' choices of crop rotation, fertilizer type and tillage practice. They consider soil loss as the primary externality associated with agricultural production. Phosphorus loss is determined by soil loss alone, with the impact of soil phosphorus level on phosphorus loading not accounted for. Iho (2007) incorporated the control of erosion and soil phosphorus reserves as means to mitigate phosphorus loading. He derived optimal steady state policies for phosphorus fertilization and erosion control but did not analyze how soil phosphorus and optimal policies evolve over time, leaving open the question of how to adjust phosphorus application in response to changing field conditions.

This article extends the previous research on agricultural phosphorus management by considering the optimal development of soil phosphorus reserves over time; in particular, it accounts for the dual role of soil phosphorus in accelerating crop growth and phosphorus losses. We develop a framework for analyzing the intertemporally optimal combination of fertilization and erosion control policies and the associated soil phosphorus development. To study precision phosphorus management in a realistic setting, we employ a numerical example that allows us to work with a state-of-the-art ecological description. The example is based on cereal production in southern Finland, but the elements of the application are common to phosphorus management in agriculture worldwide. With the empirical components matched to local conditions, the model can be used in combination with soil phosphorus testing to guide precision phosphorus management and reduce the generation of polluting phosphorus residues. Our results indicate that adjusting phosphorus fertilization in response to changes in soil phosphorus levels is crucial in designing efficient phosphorus policies. The optimal fertilization rate changes markedly over time in response to changes in the soil phosphorus status. Matching fertilizer application to field conditions was found to be especially important when depleting particularly high soil phosphorus reserves: even following a privately optimal depletion path, which does not account for environmental damage from phosphorus loading, may produce smaller efficiency losses than following a fixed fertilizer application rate set equal to a socially optimal steady state level.

2 The modeling framework

Consider a field parcel bordering a waterway. For simplicity, we assume that the parcel is square in shape and measures one hectare. A single crop is produced using phosphorus fertilizer as a variable input. The per hectare production function is $Y(s_t, x_t)$, where s_t denotes accumulated soil phosphorus and x_t phosphorus fertilizer applied in the current period. The soil phosphorus level changes from one period to the next according to the state transition function $s_{t+1} = \Gamma(s_t, x_t)$. The product and input prices are denoted by *p* and *w*, respectively, and are assumed to be constant. Operational costs per hectare are denoted by *FC* and include costs such as seeds, labor and the rental or annualized cost of machinery.

Accumulated soil phosphorus and soil loss through erosion cause phosphorus loading from the field to the adjacent waterway. Phosphorus transport from fields to surface waters occurs in two main forms: dissolved phosphorus (DP) and particulate phosphorus (PP). The main determinant of DP loss is accumulated soil phosphorus, whereas PP loss is governed by erosion. Soil phosphorus also affects the bioavailability of PP (Sharpley 1993, Uusitalo et al. 2003).[2] In our model, total phosphorus load per hectare includes DP load and the bioavailable fraction of PP load. We consider two means of reducing phosphorus loading at source: precision phosphorus fertilizer application, where application rates are adjusted annually based on the current soil phosphorus level, and vegetative filter strips (VFS) as a measure to mitigate erosion. The erosion susceptibility of land is indexed by field slope γ (see e.g. Wischmeier and Smith 1978). The total phosphorus load is then given by $L(s_t, b_t, \gamma)$, where b_t is the VFS width, which by assumption can be chosen annually. For the hectare-sized square parcel considered here, the VFS width also determines

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[2] Bioavailability describes the fraction of phosphorus that can be used by algae and that thus contributes to eutrophication. The bioavailability of PP has been estimated to range from 20 to 60 percent (see e.g. Sharpley 1993), while DP is considered fully bioavailable (see e.g. Ekholm and Krogerus 2003).

the area of the VFS. The cost of planting and maintaining filter strips is given by $C(b_t)$. This includes the costs of seed as well as the machinery and labor required for planting and for removing plant residues. The per-period monetary damage resulting from phosphorus loading is denoted by $D\big(L(s_t,b_t,\gamma)\big)$. The per-period, per-hectare profit from crop production is given by

$$\pi\left(s_t,x_t,b_t\right)=\left[pY(s_t,x_t)-wx_t-FC\right](1-b_t)-C(b_t)\,. \tag{1}$$

Multiplication by the term $(1-b_t)$ in (1) accounts for the fact that conversion of a fraction of arable land b_t into a vegetative filter removes that area from production.

3 Dynamics of the phosphorus management problem

We are concerned with socially efficient fertilization and filter strip policies over time. Other inputs are assumed to be fixed. We assume that a social planner exists. The social planner's problem is to maximize the present discounted value of rewards from production, equal to profits net of environmental damage. The farmer's problem is limited to the present discounted value of profits. The social planner's discrete-time, continuous-state decision problem is given by

$$\max_{x_t,b_t}\sum_{t=0}^{\infty}\left\{\beta^t\left[\pi\left(s_t,x_t,b_t\right)\right]-D\big(L(s_t,b_t,\gamma)\big)\right\} \tag{2}$$

subject to

$$s_{t+1}=\Gamma(s_t,x_t),\qquad s_0{=}S_0,$$
$$x_t\geq 0,\quad 0\leq b_t\leq 1,$$

where β is the discount factor corresponding to the social discount rate δ, with $\beta=\dfrac{1}{1+\delta}$, and the parameter S_0 denotes the initial soil phosphorus level. The farmer's intertemporal optimization problem is identical to that described by equations (2) and (3) with the exception of the term $D\left(L(s_t, b_t, \gamma)\right)$.

Consider first the social planner's problem. Denote by $V(s)$ the maximum attainable sum of current and future net benefits given a current soil phosphorus level of *s*. Bellman's (1957) principle of optimality implies that the optimal policy must satisfy the functional equation

$$V(s) = \max_{x,b} \left\{ \pi(s,x,b) - D\Big(L\big(s,b,\gamma\big)\Big) + \beta V\Big(\Gamma\big(s,x\big)\Big) \right\}. \tag{3}$$

The optimal fertilization rate *x* and filter strip width *b* for each level of soil test phosphorus *s* must satisfy

$$\pi_x(s,x,b) + \beta\lambda\big(\Gamma(s,x)\big)\Gamma_x(s,x) = 0 \tag{4}$$

$$\pi_b(s,x,b) - D_L\big(L(s,b,\gamma)\big)L_b(s,b) = 0. \tag{5}$$

The envelope theorem applied to the same problem implies

$$\lambda(s) = \pi_s(s,x,b) - D_L\big(L(s,b,\gamma)\big)L_S(s,b,\gamma) + \beta\lambda\big(\Gamma(s,x)\big)\Gamma_s(s,x). \tag{6}$$

The equilibrium conditions do not involve the value function but its derivative $\lambda(s) \equiv V'(s)$, the shadow value of the soil phosphorus reserves. The first-order condition (4) states for an interior solution that at every soil phosphorus level fertilizer should be applied to the point where the sum of its marginal impact on profits in the current period and the marginal impact on the discounted value of the phosphorus reserve in the next period equals zero. Because the VFS does not affect the transition process, the partial derivative Γ_b is zero and the first-order condition (5) for VFS width collapses into a static optimality condition. The filter strip width should be chosen so that the marginal reduction in profits from production equals the marginal reduction in the damage costs associated with phosphorus loading. Equation (6) indicates that the shadow value of soil phosphorus in the current period equals the sum of its marginal impact on the current period profits, net of the marginal impact on the costs of generated runoff, and the discounted value of the marginal increase in the phosphorus reserve in the following period.

The solution to the private farmer's problem is defined by equations (4) to (6) with the terms describing marginal damage set equal to zero. The shadow value of soil phosphorus in (6) now only accounts for the marginal impact of soil phosphorus on profits from production and on the phosphorus reserve in the following period. Thus, assuming that crop yield is concave in its arguments, the farmer would apply more fertilizer than the social planner. Furthermore, the marginal benefit of a vegetative filter strip is negative for the farmer, and the non-negativity constraint becomes binding. Hence, a private farmer will not construct filter strips without policy intervention.

The long-term development of rewards from production, soil phosphorus level, phosphorus losses and environmental damage can be characterized by a steady state towards which the process converges over time. The steady state for the social planner's problem is characterized by the fertilization rate x^*, filter strip width b^*, soil phosphorus s^* and shadow price λ^*, which solve the equation system

$$\begin{aligned}
&\pi_x(s^*,x^*,b^*)+\beta\lambda^*\Gamma_x(s^*,x^*)=0\\
&\pi_b(s^*,x^*,b^*)-D_L\left(L(s^*,b^*,\gamma)\right)L_b(s^*,b^*,\gamma)=0\\
&\lambda^*=\pi_s^f(s^*,x^*,b^*)-D_L\left(L(s^*,b^*,\gamma)\right)L_S(s^*,b^*,\gamma)+\beta\lambda^*\Gamma_s(s^*,x^*)\\
&s^*=\Gamma(s^*,x^*).
\end{aligned} \tag{7}$$

The solution to the farmer's problem is defined by equations (7), with the terms describing marginal damage set equal to zero.

The characteristics of phosphorus raise interesting empirical questions regarding optimal dynamic phosphorus policies. The optimal path of phosphorus reserves over time has to accommodate the trade-off between the roles that soil phosphorus plays in both crop growth and environmental degradation. If the initial soil phosphorus level is above the socially optimal level, what is the optimal mix of abatement through depletion of soil phosphorus reserves and erosion control? Does the ranking of the two abatement measures change along the optimal path, and how is this ranking influenced by the key ecological characteristics of the site, such as susceptibility to erosion? To study these questions in a realistic setting, we construct a detailed bioeconomic model of crop production and phosphorus loading, with barley as the sample crop, and empirically evaluate optimal dynamic phosphorus policies.

4 Bioeconomic model and empirical illustration

Matching fertilizer application rates to soil phosphorus levels requires knowledge about the crop production and pollution generation processes. Our bioeconomic model considers the impact of soil phosphorus and phosphorus fertilization on yield and the accumulation of soil phosphorus as well as the link from soil phosphorus to phosphorus loading. The model is parameterized for sandy clay soils in southern Finland. We consider three representative field slopes: 0.5%, 2% and 7%. The average slope is 0-1% for some 57% of parcels in Finland; 1-3% for 26% of parcels; and greater than 7% for 3% of parcels (Puustinen et al. 1994). While the proportion of steeply sloped parcels is small, we include a steep slope in the analysis as an example of land with particularly high runoff potential. Throughout the empirical illustration, soil phosphorus level is expressed as agronomic soil test phosphorus (STP).

4.1 Crop production function

The yield response to phosphorus consists of the impacts of the fertilizer applied and the phosphorus accumulated in the soil. Following Myyrä et al. (2007), we specify the phosphorus response function for barley as

$$Y(s,x) = \alpha_1^Y(1-\alpha_2^Y e^{-\alpha_3^Y s}) + (\alpha_4^Y - \alpha_5^Y s)\sqrt{x} + \frac{(\alpha_6^Y - \alpha_7^Y x)x}{s} + \alpha_8^Y . \qquad (8)$$

From Myyrä et al. (2007), the parameter values for barley production in southern Finland are $\alpha_1^Y = 3367$, $\alpha_2^Y = 0.74$, $\alpha_3^Y = 0.37$, $\alpha_4^Y = 21.7$, $\alpha_5^Y = 0.414$, $\alpha_6^Y = 17.01$, $\alpha_7^Y = 0.1817$ and $\alpha_8^Y = 5.856$.

4.2 Transition function for soil phosphorus

Ekholm et al. (2005) model the relationship between the development of soil phosphorus and the phosphorus surplus, that is, the fertilizer applied to the land but not utilized by the crop. The phosphorus surplus is defined by $P_{bal}\left(s,x\right) = x - \Lambda(s)Y(s,x)$, where $\Lambda(s)$ is the phosphorus concentration of the crop yield. Saarela et al. (1995) provide information that allows specification of the phosphorus concentration of crop yield as a logarithmic function of the soil phosphorus. Following Ekholm et al. (2005), the change in soil phosphorus from one year to the next is then specified as follows:

$$\Gamma\left(s,x\right) \;=\; \alpha_1^\Gamma s \;+\; \left(\alpha_2^\Gamma + \alpha_3^\Gamma s\right)\;\left[x - \left(\alpha_4^\Gamma \ln(s) + \alpha_5^\Gamma\right)Y(s,x)\right], \qquad (9)$$

where the term $\left[x - \left(\alpha_4^\Gamma \ln(s) + \alpha_5^\Gamma\right)Y(s,x)\right]$ is the phosphorus surplus and the term $\alpha_4^\Gamma \ln(s) + \alpha_5^\Gamma$ defines the phosphorus concentration of the crop yield. The parameter estimates $\alpha_1^\Gamma = 0.9816$, $\alpha_2^\Gamma = 0.0032$ and $\alpha_3^\Gamma = 0.00084$ were obtained directly from Ekholm et al. (2005).[3] The parameter estimates $\alpha_4^\Gamma = 0.000186$ and $\alpha_5^\Gamma = 0.003$ were obtained from data in Saarela et al. (1995) through ordinary least squares estimation.[4]

4.3 Phosphorus load and abatement using vegetative filter strips

The phosphorus load function $L(s_t, b_t, \gamma)$ expresses DP load and the bioavailable fraction of PP load net of VFS abatement. Following Uusitalo and Jansson (2002), the

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[3] The transition function presented by Ekholm et al. (2005) depicts changes in STP with a time step of 10-15 years with a constant phosphorus surplus over the period. Using a one-year time step predicts STP values in the long run that differ slightly from those predicted by the Ekholm et al. (2005) equation. For initial STP levels ranging from 2 to 40 mg l^{-1} and P surpluses from -5 to 25 kg ha^{-1} y^{-1}, the differences in STP values for year 30 predicted by equation (10) with a constant phosphorus surplus and one- and ten-year time steps were 0 to 8%.

[4] The phosphorus concentration data in Saarela et al. (1995) were measured from dry matter. Their data were made commensurate with storage weight yield prior to the estimation.

annual DP load (kg ha^{-1}) from crop production is specified as a linear function of the soil phosphorus level:

$$L_{DP}(s) = \alpha_1^{DP} s - \alpha_2^{DP}. \tag{10}$$

In line with the universal soil loss equation (Wischmeier and Smith 1978), annual PP loss (kg ha^{-1}) is specified in turn as a quadratic function of field slope:

$$L_{PP}(\gamma) = \alpha_1^{PP}\gamma^2 + \alpha_2^{PP}\gamma + \alpha_3^{PP}. \tag{11}$$

As vegetative filter strips only retain nutrients in surface runoff, we distinguish PP load through surface runoff and through drainage water. We interpret the constant term in (11) as PP load in drainage, which should be independent of field slope. Accordingly, PP load via surface runoff is given by $L_{PP,S}(\gamma) = \alpha_1^{PP}\gamma^2 + \alpha_2^{PP}\gamma$ and PP load via drainage by $L_{PP,D} = \alpha_3^{PP}$.

Following Lankoski et al. (2006), the retaining of PP by filter strips is described by the function

$$R(b) = b^{\alpha_1^R}, \tag{12}$$

where $\alpha_1^R < 1$. Vegetative filter strips also mitigate PP loss by placing erodible field area under a stable vegetative cover (see e.g. Dosskey 2001). In other words, no PP loss occurs in the VFS area *b*.

Finally, only a proportion of PP contributes to the bioavailable phosphorus load. For simplicity, we assume a linear relationship between PP bioavailability and soil phosphorus level:

$$B(s) = \alpha_1^B s + \alpha_2^B. \tag{13}$$

From (10)-(13), the total bioavailable phosphorus load is given by

$$L(s,b,\gamma) = L_{DP}(s) + B(s)(1-b)\left[L_{PP,D} + (1-R(b))L_{PP,S}(\gamma)\right]. \tag{14}$$

The parameter estimates $\alpha_1^{DP} = 0.0567$ and $\alpha_2^{DP} = 0.0405$ for equation (10) were obtained by multiplying the estimates of DP concentration in mg l^{-1} in Uusitalo and Jansson (2002) by an estimated runoff volume of 270,000 l ha^{-1} (Ekholm et al. 2005) and converting the units to kg ha^{-1}. The data in Uusitalo et al. (2007) produce parameter estimates of $\alpha_1^{PP} = 0.035$, $\alpha_2^{PP} = 0.12$ and $\alpha_3^{PP} = 0.37$ for equation (11). The parameter value $\alpha_1^{R} = 0.3$ was obtained from Lankoski et al. (2006), who used results from a Finnish study on grass filter strips (Uusi-Kämppä and Kilpinen 2000) in calculating their estimate. The data in Uusitalo et al. (2003) yield parameter estimates of $\alpha_1^{B} = 0.48$ and $\alpha_2^{B} = 19.7$ for equation (13).

4.4 Damage from phosphorus loading

Following Gren and Folmer (2003), damage from phosphorus loading is described by the function

$$D(L(s,b,\gamma)) = \alpha_1^{D} L(s,b,\gamma). \tag{15}$$

Gren and Folmer estimated the constant marginal damage from nitrogen loading in the Baltic Sea countries. We use their estimate and multiply phosphorus loading by the Redfield ratio of 7.2 to convert it into nitrogen equivalents, which yields the parameter value $\alpha_1^{D} = 47$ (EUR kg^{-1}).

4.5 Prices and costs

Product price p and input price w, obtained from Myyrä et al. (2007), are 0.11 € kg^{-1} and 1.22 € kg^{-1}, respectively. The annual fixed costs of production, *FC*, were obtained from Helin et al. (2006) and equal 113 € ha^{-1}. The costs of establishing a vegetative filter strip derive primarily from removing plant residue each year in order to prevent phosphorus in the residue from leaching into the environment. The VFS cost function thus takes the form

$$C(b) = \alpha_1^C b. \tag{16}$$

Palva (2003) and Pentti and Laaksonen (2005) estimated the costs in Finland to be 31 € ha^{-1} for mowing and 65 € ha^{-1} for baling and transportation, yielding a total of $\alpha_1^C =$ 96 € ha^{-1}. Finally, the discount rate was set at 5%.

4.6 Solution method

To determine the optimal phosphorus control policies over time, the dynamic program in (3) was solved numerically using the collocation method. This technique involves writing the value function approximant as a linear combination of n known basis functions $\varphi_1, \varphi_2, \ldots, \varphi_n$ whose coefficients $c_1, c_2, \ldots, c_n$ are determined by the equation

$$V(s) \approx \sum_{j=1}^{n} c_j \phi_j(s) \tag{17}$$

The coefficients $c_1, c_2, \ldots, c_n$ are defined by requiring the value function approximant to satisfy the Bellman equation (3) at a finite set of collocation nodes. The solution was implemented using the CompEcon Toolbox for Matlab.[5] The solution produces policy functions for $x(s)$ and $b(s)$ that provide a mapping from the current soil phosphorus level to the optimal fertilization and VFS policies.

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[5] The Matlab code is available from the authors upon request. The CompEcon Toolbox is a library of Matlab functions, developed to accompany Miranda and Fackler (2002), for numerically solving problems in economics and finance . The library is downloadable at http://www4.ncsu.edu/~pfackler/compecon/toolbox.html.

5 Results

5.1 Base parameterization

Optimal steady state outcomes

Table 1 displays the socially and privately optimal steady state outcomes. The socially optimal steady state soil phosphorus level and phosphorus application rate are almost equal across the three field slopes considered. By contrast, the socially optimal VFS width differs notably across slopes. For the moderately sloped and level fields, the optimal steady state VFS width is practically zero (0.06 meters). For the steepest slope, the optimal VFS width is 0.6 meters, which produces an overall abatement of 15% in bioavailable phosphorus loss. As slope only affects phosphorus runoff, which does not enter the farmer's objective function, it has no impact on the privately optimal solution. The steady state phosphorus application rate is 5-6% above the socially optimal level, and no VFSs are constructed. The privately optimal soil phosphorus level exceeds the socially optimal value by 16-18%, depending on field slope. The impact of field slope on phosphorus loading is apparent in the results. Even with erosion control, the bioavailable phosphorus loading from the steepest slope is twice as high as that from the most level slope. The bioavailable fraction of PP is not very sensitive to soil phosphorus level and remains at approximately 23% for all the outcomes reported in Table 1.

Table 1. Socially and privately optimal steady state outcomes for the base parameterization.

	Fertilization kg/ha	VFS m	Soil P mg/l	PP load kg/ha	DP load kg/ha	Bioavailable P load kg/ha/year
Social optimum						
Slope 0.5%	24.4	0	6.1	0.44	0.31	0.41
Slope 2%	24.4	0.06	6.1	0.71	0.31	0.47
Slope 7%	24.1	0.60	6.0	2.35	0.30	0.83
Private optimum						
Slope 0.5%	25.6	0	7.3	0.44	0.37	0.47
Slope 2%	25.6	0	7.3	0.75	0.37	0.55
Slope 7%	25.6	0	7.3	2.97	0.37	1.0

Optimal policy functions

Figure 1 shows the socially optimal fertilization rate and filter strip width as functions of the current soil phosphorus level. For all three slopes considered, the optimal fertilization rate is sensitive to the current soil phosphorus status, ranging from 0 to 70 kg/ha/year for soil phosphorus levels between 2 and 26 mg/l. By contrast, slope has no noticeable effect on the rate. The result reflects the role of phosphorus in crop production: crop response is dependent primarily on soil phosphorus level, while the annual fertilization rate is chosen mainly to control soil phosphorus reserves. Field slope is a determinant of optimal VFS width, however: for the small and moderate slopes (0.5 and 2%), the optimal VFS width is practically zero regardless of the soil phosphorus level. Erosion control becomes important in the case of the steep slope (7%), with the optimal VFS width ranging from 0.6 to 1.2 meters. The results for the private farmer are qualitatively similar to those in Figure 1, but the VFS width remains zero for all soil phosphorus levels and all field slopes. Nevertheless, the optimal fertilizer application rate changes markedly as field phosphorus status changes, even with environmental damage excluded from the objective function.

The optimal VFS policy on a steeply sloped field is non-monotonic in soil phosphorus level: filter strips are wider when the phosphorus reserves are either very low or very high. The two opposite forces producing this result are the opportunity cost of land and the bioavailability of PP load. Constructing VFSs requires setting land aside from production. When the soil phosphorus reserve is low, land is relatively unproductive and the opportunity cost of establishing VFSs is low. At the same time, the optimal fertilization rate is high, increasing the need to reduce runoff by planting VFSs where effective. As the soil phosphorus level increases, fertilization decreases along the optimal path, and the cost of VFSs increases, resulting in an initial decrease in the optimal VFS width. On the other hand, the bioavailability of PP increases linearly with soil phosphorus, heightening the importance of erosion control on steeply sloped fields and resulting in an eventual increase in the optimal VFS width. For the small and moderate slopes, the optimal VFS width decreases with soil phosphorus for the range considered here.

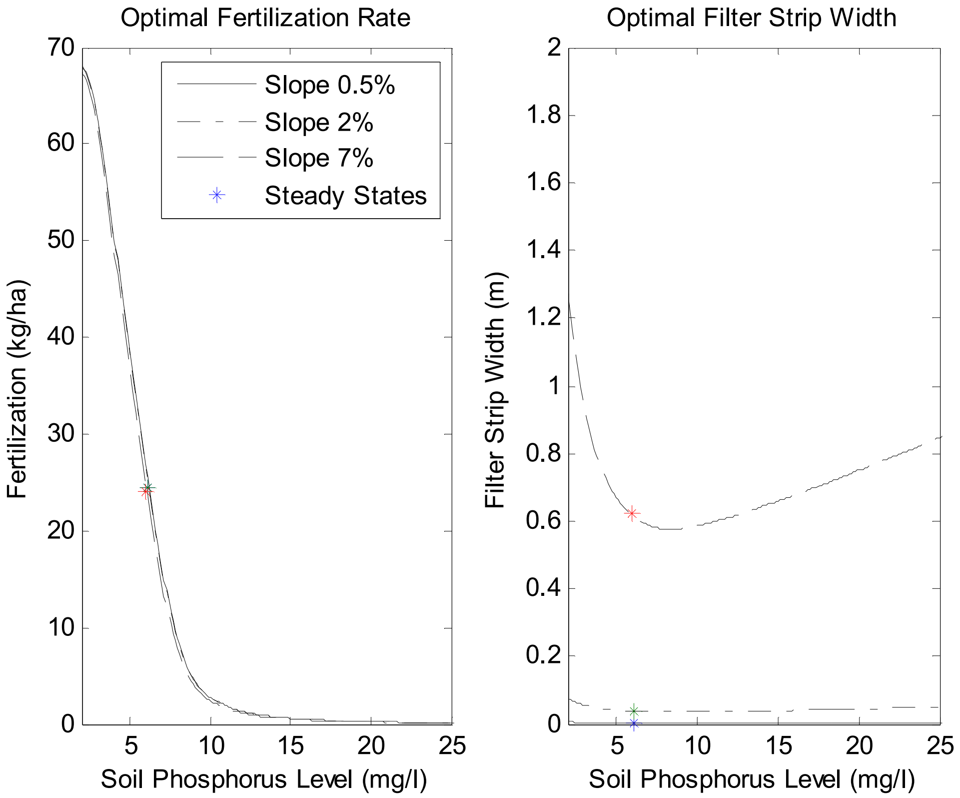

Figure 1. Optimal fertilizer application rate and vegetative filter strip width as functions of soil phosphorus.

Optimal paths to the socially optimal steady state

The previous section discussed optimal fertilization rate and VFS width as functions of the current soil phosphorus level. We next consider optimal depletion of excessive soil phosphorus reserves where the social planner takes over the management of land previously managed by a private farmer. Here the focal questions to be addressed are: How fast is soil phosphorus driven towards its new steady state value, and how do the optimal fertilization rate and VFS width evolve over time? What would be the associated reduction in phosphorus loading, relative to the load in the privately optimal solution? Figure 2 illustrates the optimal state and policy paths, with the initial soil phosphorus level set equal to the privately optimal steady state value of 7.3 mg/l.

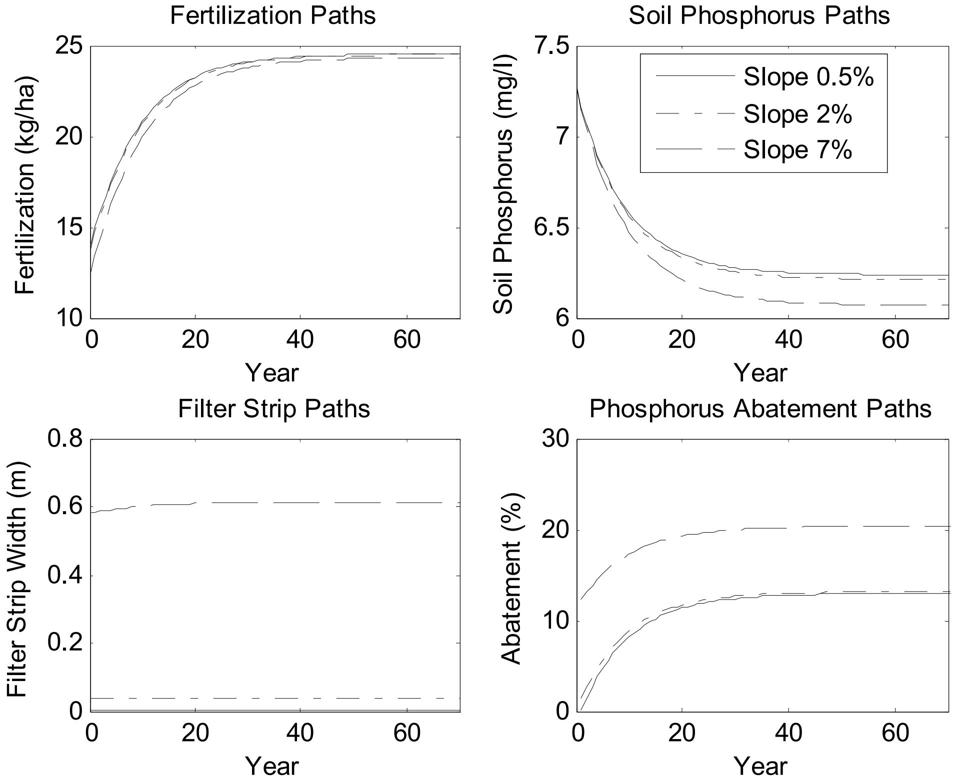

Figure 2. Socially optimal state and control paths and the associated phosphorus abatement. The initial soil phosphorus level is equal to the privately optimal steady state value of 7.3 mg/l.

While the socially and privately optimal steady state fertilization levels differ little, the privately optimal steady state soil phosphorus level is 16-18% above the socially optimal level; the discrepancy between the two solutions increases with field slope. In the initial years of depletion, the socially optimal policy actively drives the soil phosphorus reserve towards its socially optimal level. Fertilization is first reduced to less than 15 kg/ha/year to induce a rapid decline in the phosphorus reserves. As the excessive reserves are depleted, the fertilization rate increases towards its optimal steady state level. Depleting the soil phosphorus reserves nevertheless takes time, even though in absolute terms the difference between the socially and privately optimal steady state soil phosphorus levels is on the order of 1 mg/l. The socially optimal steady state reserve level is only reached some 20 to 40 years after the beginning of depletion. A VFS would in practice only be planted on the steep slope of 7%, and the optimal VFS width remains almost unchanged for the period considered. If a VFS is implemented, the environmental benefits of intensified erosion control are realized immediately, with abatement at approximately 12%. Contrastingly, abatement through depleted soil phosphorus is achieved slowly; there are no fast ways to reduce phosphorus loading from level or moderately sloped fields.

Comparison of the socially optimal dynamic solution, fixed fertilizer application rates, and the privately optimal dynamic solution

There are many ways to deplete the soil phosphorus reserve towards its socially optimal level. To illustrate the impact of optimally adjusting fertilizer application in response to changes in the soil phosphorus level, we considered a simple fixed policy rule as an alternative: in all periods, apply fertilizer at a rate equal to the socially optimal steady state level. The fixed policy rule reflects a situation where the social planner has the knowledge to determine the optimal steady state, but not the optimal adjustment path. Adjusting the policy may also be politically infeasible. Table 2 displays the present value of social welfare for the first hundred years of the socially optimal dynamic solution, the fixed fertilization rule, and the privately optimal dynamic solution; the initial soil phosphorus level is set equal to the privately optimal steady state value of 7.3 mg/l. Efficiency losses from following a suboptimal policy were computed as a percentage change relative to the welfare from the socially optimal solution. The differences in the overall level of welfare between the optimal dynamic solution and the fixed policy rule are negligible, due to the small difference between the initial and the optimal steady state soil phosphorus levels. Thus, where the existing soil nutrient stock is close to the target level, a fixed fertilizer application rate performs relatively well. The difference between the socially and privately optimal solutions is also relatively small.

Table 2. Net present value of social welfare (EUR) when initial soil phosphorus is 7.3 mg/l. Values in parentheses give the efficiency loss, measured as the percentage change in welfare relative to the socially optimal solution.

	Socially optimal dynamic solution	Fixed policy rule	Privately optimal dynamic solution
Slope 0.5%	4022	4008 (-0.3)	3995 (-0.7)
Slope 2%	3959	3945 (-0.3)	3928 (-0.7)
Slope 7%	3571	3553 (-0.5)	3456 (-3.2)

5.2 Sensitivity analysis

Optimal depletion of high soil phosphorus reserves

The soil phosphorus level is an important factor determining the runoff potential of a field. In the previous sections we considered socially optimal depletion of soil phosphorus reserves when the initial soil phosphorus was at the privately optimal level, which proved to be no more than 18% above the socially optimal level. Changes in land use, such as converting land previously used to produce sugar beets to grow grains, might bring about situations where both the private farmer and social planner would like to deplete the soil phosphorus reserve significantly.[6] We next analyze to what extent the optimal state and policy paths and the performance of the alternative policy approaches change if we consider land very rich in phosphorus, for example, with an initial soil phosphorus level of 20 mg/l.

Figure 3 shows the outcomes of the socially optimal solution, the fixed fertilization rule, and the privately optimal solution for a slope of 7%. For the first decades, the socially and privately optimal fertilization rates are low, starting from close to zero and gradually increasing to their steady state levels of 24.1 and 25.6 kg/ha/year, respectively. The privately optimal fertilization rate always remains slightly above the socially optimal one. Depleting the soil phosphorus to the optimal steady state level, whether social or private, takes decades. The notable cuts in fertilizer application in

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[6] In the European Union, for example, the arable land allocated to sugar beet production has decreased by about 20% between 2004 and 2007 (Eurostat 2009).

the first years nevertheless provide a fast reduction in soil phosphorus when compared to the fixed policy rule, which would only approach the socially optimal steady state level in hundreds of years. The environmental impacts of the dynamically optimal and fixed solutions differ notably. Over the first hundred years, phosphorus loads produced by the fixed fertilization rate remain at more than twice the level produced by the socially optimal solution. The results for 0.5% and 2% slopes are qualitatively similar to those shown in Fig. 3, but the optimal VFS width is essentially zero for all solutions and all soil phosphorus levels.

The discounted present value of social welfare generated by the three alternative solutions also differs markedly (Fig. 4). Both the socially optimal and the privately optimal solutions deplete the phosphorus stock and reduce phosphorus loading much more rapidly than the fixed fertilization rule. Here, the depletion path for a private farmer with the same information as the social planner is much closer to the socially optimal one than is the path associated with the fixed fertilization rule. As a consequence, the efficiency loss associated with the fixed policy rule is substantially higher than that associated with the privately optimal solution. The results emphasize the importance of taking changes in the soil phosphorus stock into account in choosing fertilizer application rates when the initial soil phosphorus level is significantly above the optimal steady state value. This result holds even when damage from phosphorus losses is not included in the objective function.

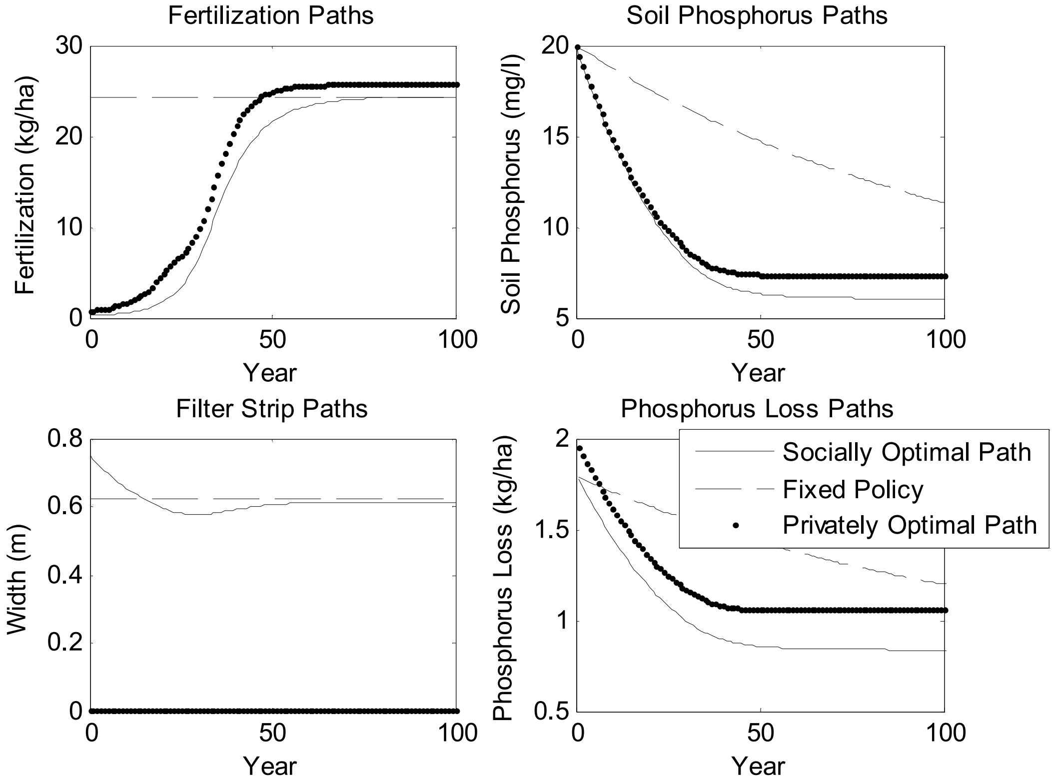

Figure 3. Depletion of high soil phosphorus reserves through the socially optimal policy, the fixed policy rule, and the privately optimal policy. Field slope 7%, initial soil phosphorus 20 mg/l.

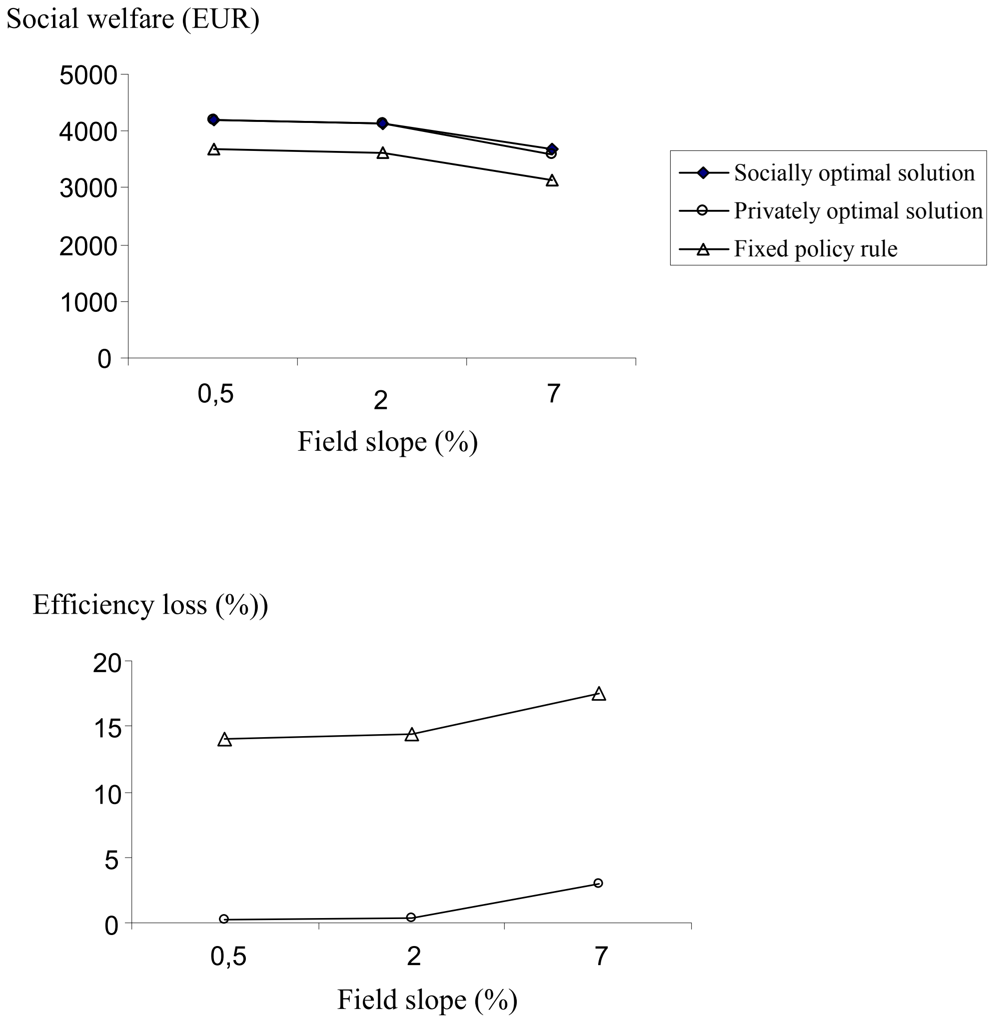

Figure 4. Social welfare and efficiency loss for the alternative depletion paths when initial soil phosphorus level is 20 mg/l

Alternative damage parameterizations

One would expect the environmental damage measure to be an important factor determining the divergence between the privately and socially optimal outcomes. However, monetizing the damage from agricultural phosphorus losses poses a key challenge in the model parameterization. In order to analyze to what extent the damage measure affects the socially optimal outcome and the welfare produced by the different solutions, we solved the model for two alternative damage parameterizations. Even significant increases in the marginal damage have relatively little impact on the socially optimal steady state soil phosphorus level and fertilization rate; instead, phosphorus loading is best controlled through substantial increases in the

VFS width on moderately and steeply sloped fields. Again, the most level fields remain without VFSs. (Table 3).

Table 3. Socially optimal steady state outcomes for alternative damage parameterizations. Values in parentheses are the percentage change relative to the base parameterization.

	Fertilization kg/ha		VFS width m		Soil P mg/l		Bioavailable P loss kg/ha	
Marginal damage 94 €/kg (+100%)								
Slope 0.5%	23.4	(-4.1)	0.0	(0)	5.4	(-13)	0.36	(-12)
Slope 2%	23.3	(-4.5)	0.1	(+150)	5.4	(-12)	0.42	(-11)
Slope 7%	22.8	(-5.8)	1.9	(+217)	5.2	(-13)	0.72	(-13)
Marginal damage 141 €/kg (+200%)								
Slope 0.5%	22.5	(-7.8)	0.0	(0)	4.8	(-23)	0.33	(-20)
Slope 2%	22.4	(-8.2)	0.2	(+400)	4.8	(-21)	0.38	(-19)
Slope 7%	21.5	(-11.2)	3.9	(+550)	4.5	(-25)	0.63	(-24)

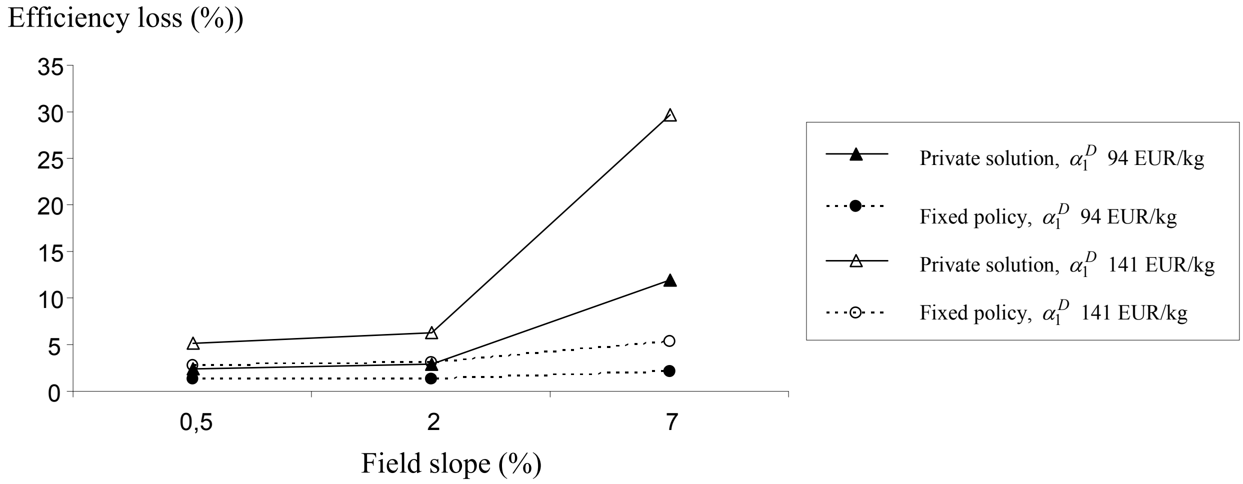

Figure 5. Efficiency losses from the private solution and the fixed fertilization rule for alternative values of the marginal damage parameter α_1^D *(initial soil phosphorus equal to the privately optimal level 7.3 mg/l)*

For both alternative damage parameterizations, the fixed fertilizer application rule outperforms the privately optimal outcome in terms of social welfare, with an efficiency loss of less than 6% even for the steeply sloped parcel (Figure 5). Here, the need for depletion is relatively small compared to the high initial phosphorus reserve discussed in the previous section: The difference between the socially and privately optimal steady state phosphorus levels is on the order of 2-3 mg/l. The result still holds for the alternative damage parameterizations that the privately optimal path produces higher welfare than the fixed fertilization rate when the initial soil phosphorus is at a high level.

6 Policy implications

The results show that efficient policies to reduce phosphorus loading from agricultural land should adjust fertilization application rates in response to changes in the soil phosphorus level, in particular where the initial soil phosphorus stock is notably above the target level. The empirical illustration shows that adjustments can be significant: the optimal fertilizer rate ranged from 0 to almost 70 kg/ha/year for an empirically reasonable range of soil phosphorus levels. The results also indicate that erosion control policy should focus on land most susceptible to erosion and is not very sensitive to changes in soil phosphorus level. A similar result was discussed by Iho (2007) who, however, considered only steady state results. The modest interlinkage between soil phosphorus level and erosion control is somewhat surprising, given that the bioavailability of particulate phosphorus in the empirical model considered here, and hence the environmental damage from erosion, increases with soil phosphorus.

The sensitivity of optimal fertilization rate to soil phosphorus level indicates that matching fertilizer application with the existing soil phosphorus stock has an important impact on welfare. We quantified this impact by comparing the welfare generated by the socially optimal path to that generated by a fixed fertilization rule and the privately optimal path. When the initial soil phosphorus level is very high – significantly above the privately optimal steady state – adjusting phosphorus application based on the soil phosphorus level, even without accounting for environmental damage from phosphorus loading, produces notable efficiency gains relative to following a fixed application rate. If, for example, land previously in sugar beet production is allocated to grains, phosphorus reserves are likely to be markedly above the level optimal for grains. Our empirical example suggests that as long as a private farmer knows the role of phosphorus in crop production and how the soil phosphorus reserves develop in response to phosphorus fertilization, the social welfare produced by the privately optimal depletion path can be very close to that of the socially optimal path. In our example, where the private farmer employed precision phosphorus management, the efficiency loss from no policy intervention was less than 5%. The efficiency loss from a simple fixed policy rule was markedly higher, close to 20%. These findings emphasize the importance of improved precision in phosphorus application as a means to reduce phosphorus loading from agricultural land. Furthermore, information provision through farmer education and training may offer a means to manage phosphorus loading at a relatively low cost.

7 Discussion

This paper presents a bioeconomic model for efficient phosphorus management in agriculture. The model tailors phosphorus application to existing soil phosphorus stock and accounts for environmental damage from phosphorus loading. It considers depletion of soil phosphorus reserves and erosion control through vegetative filter strips as measures to reduce phosphorus loading. The proposed dynamic programming approach and numerical solution method make it possible to incorporate state-of-the-art descriptions of crop production and agricultural phosphorus loading into the dynamic optimization framework. The model provides guidelines for the timing and intensity of phosphorus application and erosion control in different conditions. We calibrated the model for barley production in southern Finland in order to provide an empirical illustration of the importance of precision phosphorus management in reducing agricultural phosphorus loading.

The empirical results indicate that optimal dynamic adjustments in phosphorus application rates can have an important impact on social welfare. In fact, when starting from initially very high phosphorus levels, even a privately optimal solution matching fertilization to the existing soil phosphorus stock outperformed a fixed fertilization rule based on the socially optimal steady state. The results also confirm that reducing agricultural phosphorus loading requires long-term efforts: for soils initially very rich in phosphorus, phosphorus losses remain elevated for decades even when fertilization is reduced markedly.

The analysis presented here focused on determining optimal dynamic policy rules for agricultural phosphorus management. We considered a field parcel homogenous in soil characteristics and assumed perfect information about the soil phosphorus level. An important extension to this study would be to address the implications for phosphorus management of spatial variability in soil characteristics, which entails uncertainty about existing soil phosphorus. As pointed out by Lichtenberg (2002), appropriate sampling can reduce such uncertainty or even eliminate it, but there have been few studies investigating optimal sampling or testing strategies in an economic context. Regulations and incentive mechanisms to correct the externality associated with phosphorus loading were also not considered. An important focus for future work would be to investigate regulatory policies such as taxes and subsidies, and, as Xabadia et al. (2008) have done, to assess the gains from adjusting policies dynamically in response to soil phosphorus levels and from targeting policies to areas susceptible to erosion. While our analysis showed that tailoring fertilizer application to the prevailing field conditions produces the highest welfare, policy makers need to know whether the welfare gains suffice to offset the costs of investments in human and physical capital and increased monitoring that such policies may entail. Khanna and Zilberman (1997) provide a framework for studying the adoption of precision technology that could be combined with our model of crop production and pollution generation in order to design policies that would encourage more precise phosphorus management. Extending the model to optimal dynamic control of phosphorus loading from animal farms is also left for illumination by future research.

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