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Optimal choice between even- and uneven-aged forest management systems

Olli Tahvonen

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Abstract
Existing optimal rotation models include even-aged management exogenously into the model structure. As an economic model, this Faustmann framework is restrictive and a more general model should not include any preconditions concerning the forest management system. Even-aged management should follow endogenously as an optimal solution if it proves out to be superior to other systems, such as uneven-aged management. Without such a general model, the economically optimal choice between even-aged and uneven-aged forestry remains somewhat arbitrary. This study specifies such a model and shows how even-aged management follows endogenously and reveals what factors work in favor of each management alternative. Numerical analysis shows that even and uneven-aged systems may represent locally optimal solutions and may yield equal economic outcomes. Instead of the usual comparative statics results of the Faustmann model, changes in the rate of discount, timber price or planting cost may imply that the optimal solution shifts from even-aged management to uneven-aged management.

Keywords
Uneven-aged management, even-aged management, optimal harvesting, optimal rotation, size-structured models, age-structured models

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Olli Tahvonen

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1. Introduction

The economic analysis of forest management is heavily based on the optimal rotation model dating back to Faustmann (1849). This model has been fruitful from both theoretical and practical points of view. Its simplicity and clarity follows from a set of assumptions, such as perfect capital markets and certainty (Samuelson 1976). In addition, the model is restricted to consider even-aged management only. Under this system, trees are grown in cohorts of equal age. After planting, trees may be harvested in thinnings to control density and timber quality and finally the stand is clearcut before repeating the cycle. However, such a rotational system is only one possibility for managing an area of forest land. From the economics point of view, the design of the management system should be determined endogenously by optimization and by the underlying economic and biological factors. So far, it has not been shown how an even-aged system could result endogenously from a general forestry optimization model. This study aims to develop a model that leaves the forest management system open and that is able to determine the rotational framework when that proves to be optimal. Without such a general model, the theoretical basis for comparing different management systems like even and uneven management remains controversial.

The theoretical question of endogenous rotation model is closely related to the debate between even-aged and uneven-aged forestry. Even-aged forestry has proved to be successful in producing timber for industrial purposes and it has been the dominant forest management system around the world. It has nevertheless been subject to criticism. Even-aged management leads to plantation forestry and the resulting forest and landscape may differ considerable from the original forest environment. In addition, single species, even-aged plantations may not be suitable for preserving biodiversity. The main alternative to even-aged management is uneven-aged management. Within this management system, forest cover is maintained continuously, natural regeneration is typical and forests may consist multiple tree species. The debate between these forest management forms has been active (see e.g. Guldin 2002, O'Hara 2002, Siiskonen 2007) but the economic comparison of these alternative approaches has been far from straightforward. Seydarck (2002) emphasized the great potential of uneven-aged (or continuous cover) forestry in tropics and that the central challenge is to combine understanding of tropical forest dynamics and powerful simulation and optimization methods.

Uneven-aged models are much more complex than even-aged models. Compared to the rotation model, there is no simple generic, analytically solvable uneven-aged model available[1]. In the forest economic literature, a seminal paper by Adams and Ek (1974) utilizes a transition matrix model for tree-size classes originally proposed by Usher (1966). This model is closely related to the matrix model for describing the growth of age-structured populations (Leslie 1945). The model version by Adams and Ek (1974) is nonlinear due to density dependence effects in regeneration and growth. The eight state variable model is analyzed numerically by first specifying a (MSY -type) steady state with smooth timber harvest over time and then solving a transition path that must reach the steady state and the given size distribution within a given period. This approach has been applied in numerous studies and in addition to the MSY-type of endpoint requirement, the literature offers other steady states such as equilibrium or "investment efficient" endpoints.

However, as discussed by Getz and Haight (1989) this approach, nor the frequently applied simplified approach of comparing only the steady states of different management systems, cannot reveal true potential of the uneven-aged management. Instead of solving a transition path toward a predefined *ad hoc* steady state, the uneven-aged problem should be solved as an infinite horizon problem without restricting endpoints.

Haight (1987) and Getz and Haight (1989) were able to approximate the infinite horizon uneven-aged problem without *ad hoc* restrictions. This solution was then possible to compare to the outcome of converting any initial size class structure to even-aged plantation forestry. Haight and Monserud (1990) apply this same optimization approach together with a more detailed "single tree" growth model. Typically, these studies conclude that uneven-aged management is superior to even-aged system. An exception is Wikström (2000) who found that within the single tree framework for Norway spruce, even-aged management is superior to uneven-aged management. However, even in these studies, the optimal even-aged management is solved by applying the Faustmann modelling structure that predetermines the management system. Since plantation forestry brings additional restrictions to the optimization problem, the superiority of the uneven-aged system can be expected. The result of Wikström (2000) is therefore unexpected.

The aim of this paper is to examine whether it is possible to obtain the rotational even-aged system endogenously by a model that may also yield an optimal uneven-aged system. Without such a result, it will not be clear whether the even-aged Faustmann system is optimal in any broader sense. In addition, only a model where the management system is determined endogenously from the underlying economic and biological factors can give a theoretically coherent basis for comparing the superiority of these competing alternatives.

For the description of forest internal structure and growth this study employs the size-structured transition matrix model. This model has proved to be suitable for a number of extensions such as stochastic growth, stochastic timber prices, multiple tree species and environmental preferences (Buongiorno et al. 1995). In addition, the model has deep roots in population ecology (Caswell 2001) and as suggested by Getz and Haight (1989), and also more recent studies (Liang et al. 2005), the model is capable of accurately describing the development of various types of forests.[2]

After including an infinite horizon objective function, timber yield and prices, the nonlinear density dependent version of this model can be viewed as a (large scale) nonlinear programming problem. This study shows by numerical analysis that the steady state may either be an equilibrium with smooth yield and size class distribution or a stationary cycle. When the cycle has the property that the stand is clearcut after regular intervals, the solution represents an endogenously determined even-aged management. Along such a cycle, the optimal solution is a regeneration-thinning from below-thinning from above-clearcut rotation cycle. Whether the optimal solution is an even- or uneven-aged management system or some intermediate form depends on the regeneration and how density dependence affects tree growth. In addition, the choice is shown to depend on the rate of discount, on the regeneration cost and timber prices. Thus, instead of the usual effects, increasing the rate of discount may cause the optimal solution to switch from even-aged management to uneven-aged management. A numerical example shows that these two forest management alternatives may exists simultaneously as locally optimal solutions andyield equal economic outcomes.

These results are new contributions in the forest economics literature. They shead new light on both the theoretical basis of the Faustmann optimal rotation model and on the economics of optimal uneven-aged management. In addition, the results suggest that the debate and optimal choice between these forest management forms benefits from developing more general models with sound economic and ecological basis.

Besides to the theoretical analysis, the model is applied to an empirical example for Norway spruce. The optimal solution represents an intermediate case between the management systems and although trees are partly produced in cohorts, forest cover is maintained continuously. This management system is shown to yield about 30% higher economic output compared to a solution where the rotational system is predetermined. This result is surprising given that in some Nordic countries the uneven-aged management of Norway spruce has been practically forbidden.

The paper is organized as follows. Section 2 introduces the model and the solution method. Section 3 presents the numerical analysis and shows how different management systems follow endogenously depending on the underlying assumptions. Section 4 introduces and solves the empirical example. The final section extends the discussion and suggests that the choice between the management systems must also depend on uncertainty, possible capital market imperfections and on environmental preferences.

2. Theoretical model

The development of tree size-classes can be written as

$$\mathbf{x}_{t+1} = \mathbf{G}_t \mathbf{x}_t - \mathbf{q}_t, \tag{1}$$

where $\mathbf{x}_t$ is an n-dimensional vector for the number of trees in different size-classes, $\mathbf{G}_t$ is an $n \times n$ transition matrix and $\mathbf{q}_t$ is an n-dimensional vector for the regeneration and harvest of trees in each size class. The transition matrix takes the form

$$\mathbf{G}_t = \begin{bmatrix} \beta_1(\mathbf{x}_t) & 0 & \dots & 0 & 0 \\ \alpha_1(x_t) & \beta_2(\mathbf{x}_t) & \dots & 0 & 0 \\ 0 & \alpha_2(\mathbf{x}_t) & \dots & 0 & 0 \\ . & . & . & & . \\ . & . & . & \dots & . \\ . & . & . & & . \\ 0 & 0 & \dots & \beta_{n-1}(\mathbf{x}_t) & 0 \\ 0 & 0 & \dots & \alpha_{n-1}(\mathbf{x}_t) & \beta_n(\mathbf{x}_t) \end{bmatrix}, \tag{2}$$

where the dependence of transition coefficients on $\mathbf{x}_t$ reflects the effects of stand density on the growth of each size class. The elements $\alpha_s(\mathbf{x}_t) \leq 1,\ s = 1,\dots,n-1$ denote the share of trees that move to the next size class for period $t+1$ and the elements $\beta_s(\mathbf{x}_t)$ the share of trees that remain at their present size class for period $t+1$. The shares of trees that will die in each size class are given as $\sigma_s(\mathbf{x}_t) = 1 - \alpha_s(\mathbf{x}_t) - \beta_s(\mathbf{x}_t) \geq 0,\ \ s = 1,\dots,n-1$ and $\sigma_n(\mathbf{x}_t) = 1 - \beta_n(\mathbf{x}_t) \geq 0$. Thus, for the largest size class it is assumed that the share equal to $\beta_t(\mathbf{x}_t) \leq 1$ will exist among the largest trees for the next period. The remainder of the largest trees will die.

If ϕ_t denotes the regeneration or ingrowth of trees to the smallest size class, the vector $\mathbf{q}_t$ is given as

$$\mathbf{q}_t = [-\phi_t + h_{1t},\ h_{2t},\dots,h_{n-1,t},h_{nt}]'. \tag{3}$$

To clarify the timing between transition and harvest, it is useful to write the transition dynamics explicitly as a system of difference equations:

$$x_{1,t+1} = \phi_t + \beta_1(\mathbf{x}_t)x_{1,t} - h_{1,t}, \tag{4a}$$

$$x_{s+1,t+1} = \alpha_{s,t}(\mathbf{x}_t)x_{st} + \beta_{s+1,t}(\mathbf{x}_t)x_{s+1,t} - h_{s+1,t},\ s = 1,\dots,n-2, \tag{4b}$$

$$x_{n,t+1} = \alpha_{n-1}(\mathbf{x}_t)x_{n-1,t} + \beta_{nt}(\mathbf{x}_t)x_{nt} - h_{nt}. \tag{4c}$$

Thus, the number of trees in size class $s+1$ in the beginning of the next period equals the number of trees that will move from size class s plus the number of trees of size class $s+1$ that will remain in this size class minus the number of trees that will be harvested at the end of the period from size class $s+1$. Harvesting occurs therefore at the end of each period and after growth and transition of trees to the next size class (cf. Getz and Haight 1989, p. 44).

In transition matrix models for trees, the regeneration or ingrowth may depend on the basal area of the forest[3], on the number of trees or on the gap left by trees harvested from different age classes (e.g Usher 1966). In addition, trees can be planted. Denote the number of planted trees by g_t. This study applies the following four alternative ingrowth models

$$\phi_t = \phi(\mathbf{x}_t), \tag{5a}$$
$$\phi_t = g_t, \tag{5b}$$
$$\phi_t = \phi_t(\mathbf{x}_t) + g_t, \tag{5c}$$
$$\phi_t = \textstyle\sum_{s=1}^{n} \eta_s h_{st}, \tag{5d}$$

Ingrowth model (5a) specifies ingrowth as a function of the number of trees in different size classes. This dependence may first be increasing and later decreasing reflecting density dependence. Model (5b) is the case of artificial planting and model (5c) combines natural regeneration and artificial planting. Some empirical studies have found a relationship between new seedlings and the gap left by harvested trees. This regeneration model is given by (5d) where $\eta_s,\ s = 1,\ldots,n$ denote the number of seedlings per harvested tree.

In the transition matrix model, the transition coefficients may depend on the stand state $\mathbf{x}_t$. In existing models, it is assumed that transition coefficients depend on the stand basal area y_t. In this case, $\alpha_s(\mathbf{x}_t) = \alpha_s(y_t),\ s = 1,\ldots,n-1$ and $\beta_s(\mathbf{x}_t) = \beta_s(y_t),\ s = 1,\ldots,n$. If d_s denotes the tree diameter in size class s, the basal area is given by

$$y_t = \sum_{s=1}^{n} x_{st}\, \pi\, (d_s/2)^2. \tag{6}$$

The stand basal area measures simultaneously the number of trees and their size and for many species it represents a reasonable and easily obtainable measure of stand density. However, for shade intolerant tree species the growth of one particular size class tree may depend more sensitively on the number of trees in this and bigger size classes (see e.g. Vanclay 1994). For these cases, the transition coefficients depend on the size class specific basal area given by

$$y_{st} = \sum_{j=m_s}^{n} x_{jt}\, \pi\, (d_j/2)^2, \tag{7}$$

where $m_s \geq s,\ s = 1,\ldots,n.$ The number of trees that move to the next size class depend negatively on density, i.e. on the basal area. This dependence is linear in most existing studies (e.g. Liang et al. 2005) . One implication of the negative linear relationship is that since this fraction cannot be negative the function must have a kink at a level where the fraction has decreased to zero. Getz and Haight (1989) wrote that the linear relationship may be theoretically unsatisfactory and used a decreasing strictly convex function of the form $\exp(-a(y/b)\hat{\ }c)$, where a, b and c are positive constants and $c < 1$. However, they note that these functions may rather be sigmoid, decreasing and concave with lower levels and convex with higher levels of the basal area. A sigmoidic form with such a property would reflect the fact that intertree competition takes place only after some shading is present. Thus, in addition of the usual linear form this study applies a sigmoidic function of the form

$$\alpha_s(y_{st}) = [1 - e^{-\gamma_{s1}/(1+\gamma_{s2}y_{st})}], \tag{8}$$

where γ_{s1}, and γ_{s2} are positive constants. This function is decreasing in y_{st} and has the desired sigmoidic form.

The number of trees that do not reach the next diameter class will die or stay at their existing diameter class. The share of trees that remain alive and stay at their present size class are given by

$$\beta_s(y_{st}) = \tau_s[1 - \alpha_s(y_{st})],\ s = 1,\ldots,n, \tag{9}$$

where $0 \leq \tau_s \leq 1,\ s = 1,\ldots,n.$ If $\tau_s < 1,$ a fraction of trees die.

Harvested trees may yield different type of timber depending on the size of harvested tree. Let $\omega_{sk},\ s = 1,\ldots,n,\ k = 1,\ldots,m$ denote the amount of timber of type k from one harvested tree of size s and let $p_k,\ k = 1,\ldots,m$ denote the market price. Thus the revenues per period are given as

$$R_t = \sum_{s=1}^{n} h_{st} \sum_{k=1}^{m} \omega_{sk} p_k. \tag{10}$$

The economic criteria to be maximized, i.e. the present value of net timber income, takes the form

$$\sum_{t=0}^{\infty} [R_t - cg_t - C(Q_t)]\ b^t, \tag{11}$$

where $b = 1/(1+r)$ is the discount factor, r the real rate of interest, cg_t is the cost of artificial regeneration, C is a function for logging costs and Q_t denotes the total volume of harvest. The total harvest volume equals

$$Q_t = \sum_{s=1}^{n} \mu_s h_{st}, \tag{12}$$

where μ_s gives the volume of timber for size class s trees. When the logging cost is included in the analysis, it is assumed that $C = cQ_t{}^{\beta}$, where $c > 0$ and $\beta \geq 1$ are constants.

Obviously, any solution must satisfy the physical restrictions

$$0 \leq \mathbf{x}_t,\ t = 0,1,\ldots, \tag{13a}$$
$$0 \leq \mathbf{h}_t,\ t = 0,1,\ldots, \tag{13b}$$
$$0 \leq g_t,\ t = 0,1,\ldots. \tag{13c}$$

In addition, the initial state of the forest is given by

$$\mathbf{x}_0 = \underline{\mathbf{x}}_0. \tag{14}$$

2.1 The optimization procedure

The optimization problem can now be defined as maximizing the present value of net revenues defined by (11), (12), (10) subject to restrictions (1), (13a-c) and (14) and taking into account the definitions (2)-(9). This formulation does not include any ad hoc restrictions nor it is required that the solution should reach any a priori defined end point equilibrium state. Mathematically, the problem is a (large scale) nonlinear programming problem. Due to the dependence of the transition matrix coefficients on the basal area, the problem is potentially nonconvex and may yield multiple locally optimal solutions. It would be possible to write the Langrangian and derive the Karush-Kuhn-Tucker conditions and perhaps to obtain some analytical results. However, this would lead to somewhat tedious expressions. Instead, the analysis follows previous work in this area and investigates the problem numerically. The numerical analysis utilizes Knitro optimization software (Byrd et al. 1999, 2006) that applies gradient-based, state-of-the-art, interior point solution methods.[4] The initial guess can be chosen randomly and may repeated (e.g. 100 times) to find the globally optimal solution. The infinite horizon solutions are approximated by applying finite time horizons. These are increased (up to 1500 periods) until the solutions represent transitions toward some steady states or stationary cycles that do not vary with a further lengthening of the time horizon.

3. Theoretical analysis

The parameter values for the theoretical analysis are given in Table 1. There are ten size classes. The parameter values for the tree diameter and volume are roughly in line with the data for Norway spruce. Regeneration is either artificial and costly or natural. There are two possibilities for natural regeneration: the number of seedlings is either a function of the basal area or it depends on the gaps left by harvested trees. The regeneration function increases with low and decreases with high basal-area levels. The dependence of the transition on stand density has two main alternatives: the first alternative is a general density dependence assumption, where the transition depends negatively on the basal area of all the trees and the transition function has either a sigmoid or linear form. The second alternative is a shading specification, where the transition of a given size class is not affected by smaller size classes (see e.g. Vanclay 1994, p. 159). Since seedlings and smaller size class trees have rather narrow crowns their growth is affected only by larger trees.

However, the size classes $1, 2, 3$ are assumed to be more shade intolerant than larger trees. The growth of larger size classes ($s = 4, \ldots 9$) is affected by the given size-class itself and by all larger trees. The shading effect is weaker in the moderate alternative compared to the stronger shading alternative. In all cases, it is assumed that fifteen per cent of trees that do not move to the next size class will die.

Table 1. Parameter values for theoretical simulations.

n	10
d_i (cm)	2, 6, 10, 14, 18, 22, 26, 30, 34, 38
b	$0.99, 0.95, 0.90$ *or* 0.80
c	$12, 5$ *or* 0
μ_s	$0, 0, 0, 0.0745, 0.1742, 0.2928, 0.4856, 0.7019, 0.9671, 1.2192$
$\sum_{k=1}^{m} \omega_{sk} p_k$ (€)	$0,\ 0,\ 2,\ 3,\ 4,\ 10.7,\ 20.1,\ 29.2,\ 42.4,\ 45$
Regeneration	
ϕ_t	$a)\ g_t,\ b)\ 20 y_t e^{-y_t/10},\ c)\ 40 y_t e^{-y_t/2},\ d)\ \sum_{s=1}^{n} \eta_s h_{st}$
η_s	$0, 0, 0, 1, 2, 3, 5, 10, 15, 20$
Gen. density dependence:	
$\alpha_{st},\ s = 1, \ldots, n-1$	$a)\ linear : 1 - y_t/50,\ b)\ sigmoidic : (1 - e^{-7/(1+y_t)}),$
Strong shading:	
$\alpha_{st},\ s = 1, 2, 3$	$(1 - e^{-7/(1+30y_{2t})}),\ (1 - e^{-7/(1+30y_{3t})}),\ (1 - e^{-7/(1+20y_{4t})})$
$\alpha_{st},\ s = 4, \ldots, n-1$	$(1 - e^{-7/(1+y_{st})})$
Moderate shading:	
$\alpha_{st},\ s = 1, 2, 3$	$(1 - e^{-7/(1+10y_{2t})}),\ (1 - e^{-7/(1+10y_{3t})}),\ (1 - e^{-7/(1+10y_{4t})})$
$\alpha_{st},\ s = 4, \ldots, n-1$	$(1 - e^{-7/(1+y_{st})})$
Mortality:	
β_{st}	$0.85(1 - \alpha_{st}),\ s = 1, \ldots, n-1,\ \beta_{nt} = 0.85$

3.1 General density dependence outcomes

Fig. 1a and b show three optimal solutions based on general density dependence and linear transition (Table 1). This is the case most typically considered in existing studies (e.g. Buongiorno et al. 1995). When regeneration depends on the basal area (Table 1, case b), the optimal solution converges toward a steady state with smooth cutting and basal area levels. At the steady state, trees are harvested when they reach the dimensions of size class nine. Only during the initial transition are some trees permitted to reach dimensions of size class ten to increase natural regeneration.

When regeneration depends on the gaps left by harvested trees (Table 1, case d), the long run steady state becomes a stationary cycle (dashed line). In this case trees are harvested simultaneously from size classes 3 and 7. Trees are thinned from below, because regeneration is excessive. Thinning is concentrated on size class three since this is the smallest size class with direct economic value. However, the main yield is obtained from size class 7 and only during the initial transition are trees permitted to reach larger dimensions.

The third solution (dotted line) is based on artificial regeneration (Table 1, case a). Again the long run steady state is a stationary cycle. Trees are harvested when they reach size class nine. Planting is carried out every fourth period and at those periods when the basal area starts to decrease. All three solutions in Fig. 1a,b represent uneven-aged continuous cover forestry. This forest management system is typically obtained under the general density dependence specification. However, when regeneration is artificial and costly or depends on the gaps left by harvested trees, the optimal solutions consist of cycles. This implies that trees are partly produced in cohorts. Earlier studies have not presented this type of cyclical equilibria.

Fig. 2a and b are based on the general density dependence specification with sigmoidic transition and natural regeneration that depends on the basal area (Table 1, case b). Under these assumptions, the optimal solution is a transition toward a smooth steady state and uneven-aged forestry. Given $b = 0.99$, trees of size class 9 are harvested and under $b = 0.8$ trees are cut when they reach size class 7. As in Fig. 1, changing the rate of discount or other economic parameters, such as the value of the timber content per tree, does not change the optimal forest management system.

Compared to Fig. 2a,b, the only difference in Fig. 3a, b is that the regeneration is artificial and costly (c=12). Since the basal area and optimal yield remain positive, the solution still represents uneven-aged forestry inspite of the cyclical equilibrium. Fig. 3a shows that planting (dashed line) is carried out at the same periods when harvest reaches its maximum. Fig. 3b shows that at these periods, the basal area, i.e. stand density, is decreasing. This implies that it is optimal to synchronize harvest and planting and increase planting for those periods when the effect of density on growth will be lower. Thus, this solution, like the cyclical solutions in Fig. 1a,b, partly produces trees in cohorts and in this sense it represents an intermediate form between even and uneven-aged management.

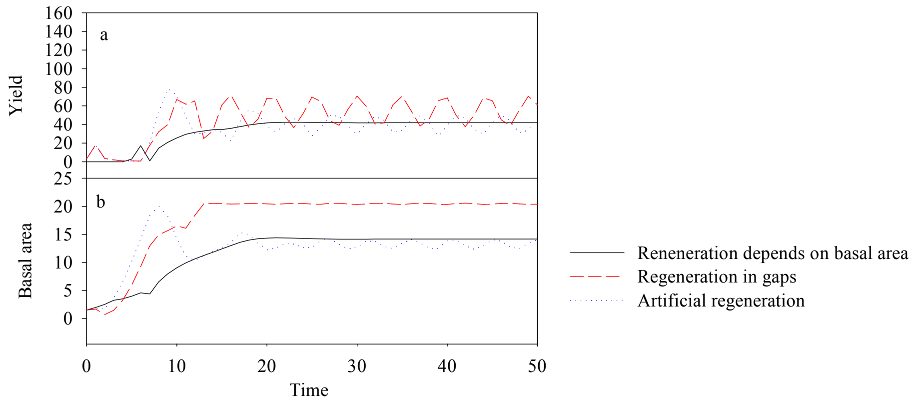


Figure 1. Uneven aged management under general density dependence and linear transition. Parameter values: b=0.99, c=0 or 12 others see Table 1.

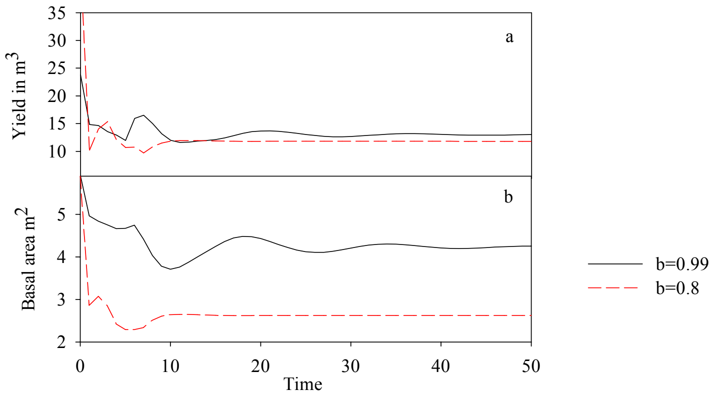


Figure 2a,b. Uneven-aged forestry under general density dependence, sigmoidic transition and natural regeneration. Parameter values: Table 1

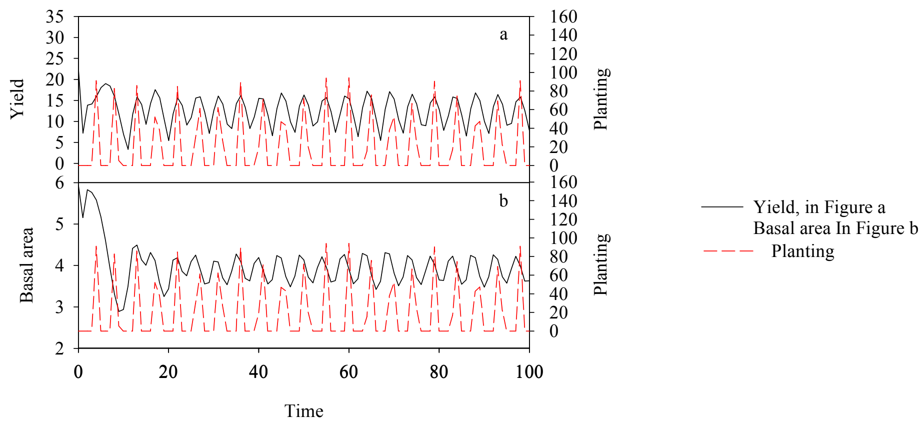


Figure 3a,b. Cyclical uneven-aged forestry under general density dependence and artifical regeneration. Parameter values: b=0.99, c=12 others see Table 1.

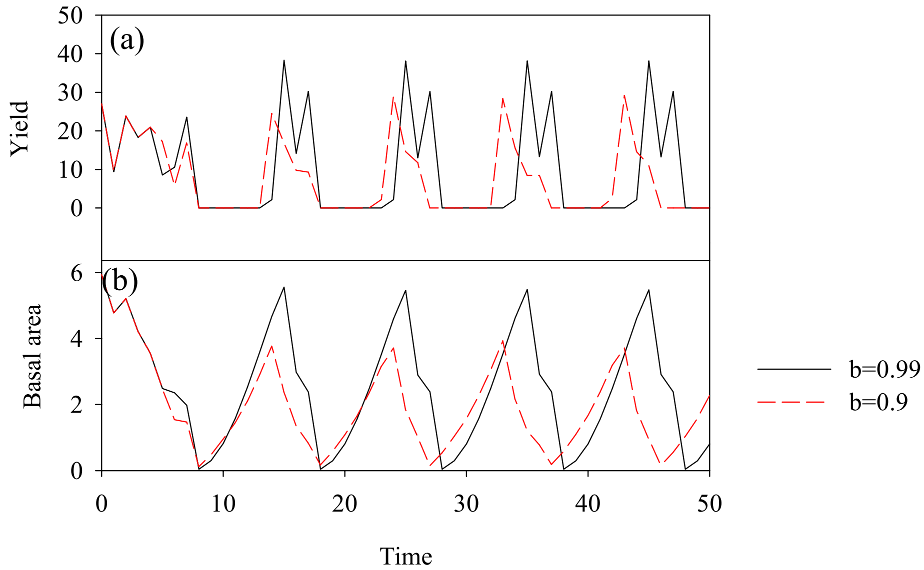


Figure 4a,b. Even-aged forestry under shading and natural regeneration

Development of total harvest and basal area

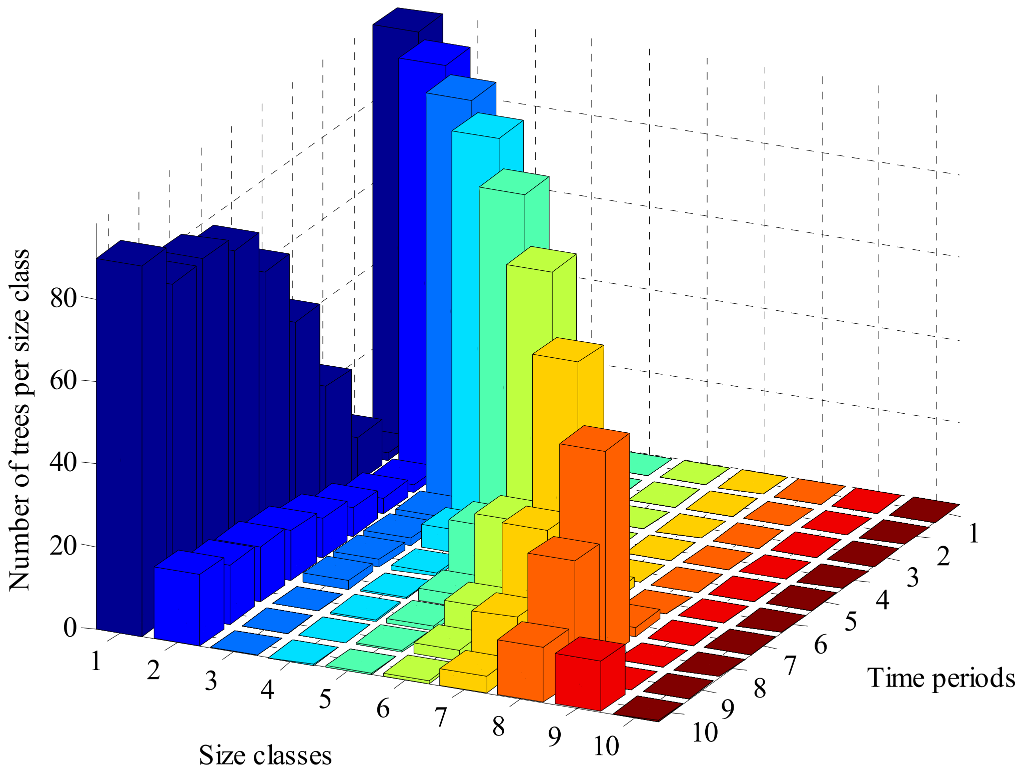


Figure 4c. Even-aged forestry under shading and natural regeneration

Development of size classes over the rotation, b=0.99

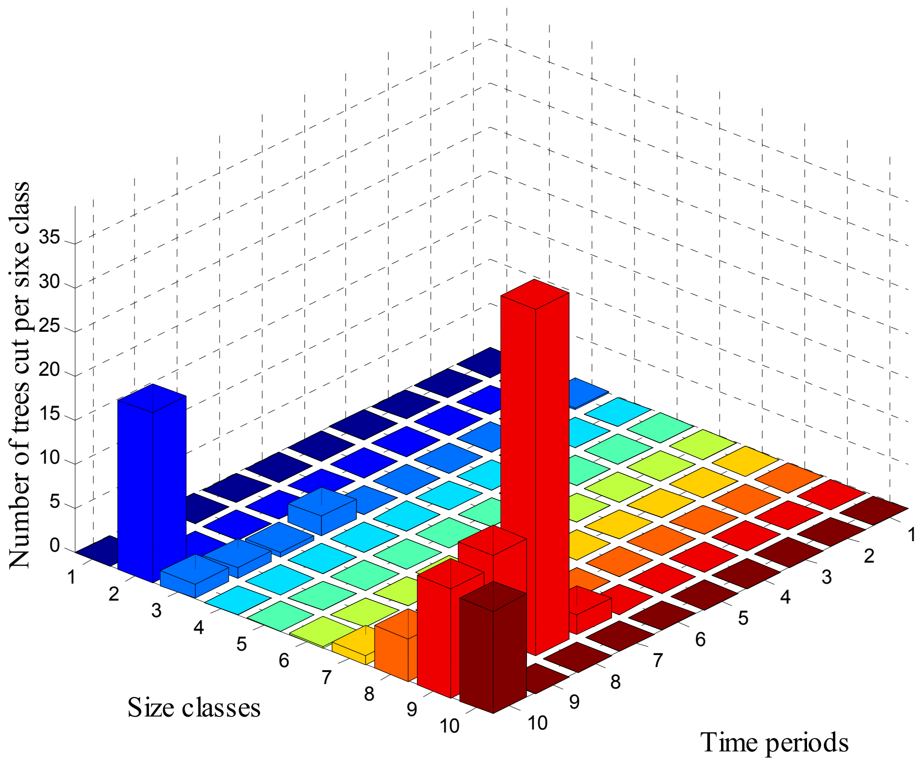


Figure 4d. Even-aged forestry under shading and natural regeneration

Development of cuttings over the rotation, b=0.99

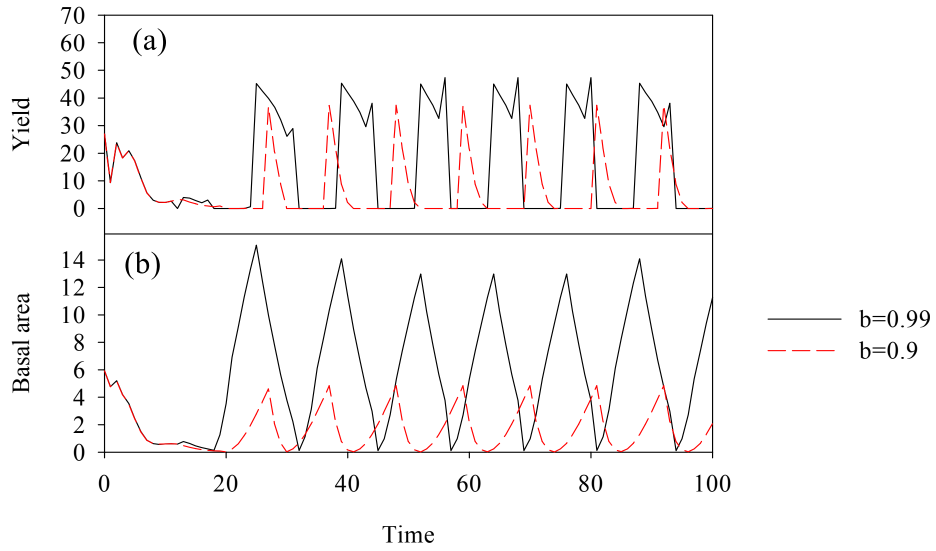


Figure 5a,b. Even-aged forestry under shading and artificial regeneration

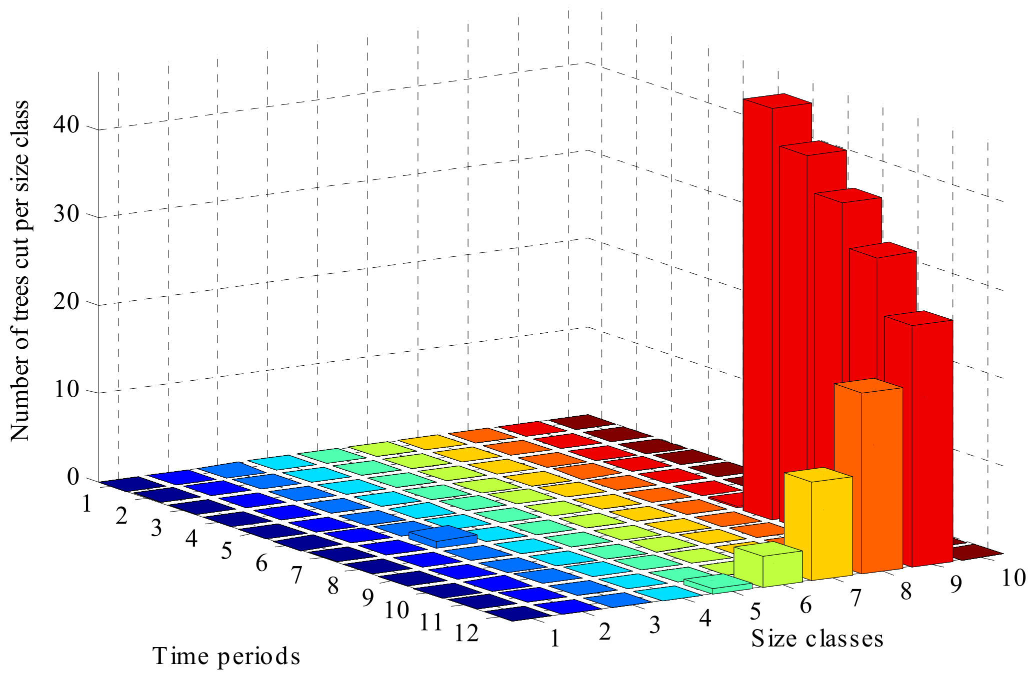


Figure 5c. Even-aged forestry under shading and artificial regeneration

b=0.99

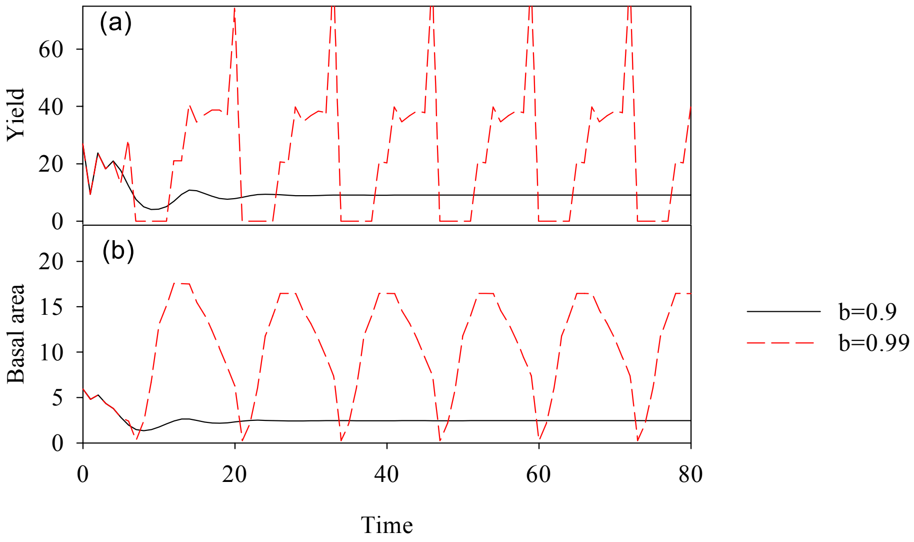


Figure 6a,b. The rate of interest and optimal management system

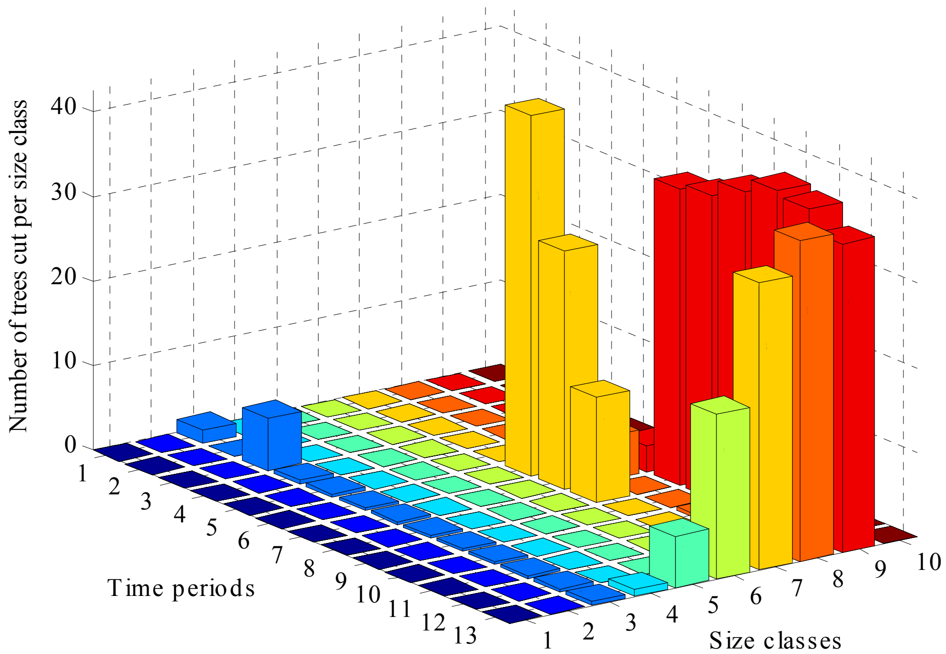


Figure 6c.The rate of interest and the optimal management system.
Optimal solution under low rate of interest. (b=0.99)

3.2.Optimal solutions for shading specifications

Fig. 4a-d show optimal solutions under the strong shading specification. Regeneration is natural and depends on basal area (case b in Table 1). After an initial transition the solution follows a stationary cycle. Since the basal area periodically reaches almost zero level, the trees in each cohort are (approximately) of equal age and thus the solution represents even-aged forestry. Given $b = 0.99$, the length of rotation is 10 periods. Fig. 4c and d show the development of tree distribution and cuttings over the rotation. Fig. 4d shows that before the clearcut at period ten trees are thinned both from above and below. Thinning from below (i.e. harvesting trees from the lower end of the size class distribution) occurs because regeneration is natural and new seedlings emerge over the rotation. Small trees are cut when they reach the dimension of size class three, i.e. the smallest commercially valuable size. In addition it is optimal to apply thinnig from above (i.e. cutting trees from the upper end from the size class distribution). This occurs when trees reach the diameter of size class nine. Due to the size-class transition specification ($\alpha_s < 1$ for $s = 4,\ldots,9$), this occurs gradually over three periods. At the clearcut, a number of harvested trees have reached the dimension of size class ten. This detail is optimal because it leads to production of a higher number of seedlings for the next cohort. At the clearcut all threes are harvested, excluding size-class one. It forms size-class two and part of size-class one of the next rotation. Such a clearcut eliminates the negative shading effect of larger trees on the next cohort but leaves the smallest size class due to its ineligible shading effect (cf. sigmoidic transition). As shown in Fig. 4a and b, decreasing the discount factor shortens the rotation (to nine periods) and lowers the basal area over the rotation but does not change the forest management system.

Fig. 5a-c present optimal solutions under the strong shading specification and when regeneration is artificial and costly (c=12). The optimal solutions again represent even-aged management. Under the higher discount factor, the rotation length is 12 and under the lower one it is ten. In addition, a lower discount factor implies a lower stand density and a lower planting level. When $b = 0.99$ it is optimal to plant 347 seedlings and when $b = 0.9$ the optimal number of seedlings is 82. It is optimal to harvest when trees reach size class nine. This happens over five periods and the first four periods represent thinnings from above. The stand is clearcut at the end of each rotation, but since regeneration is artificial it is not optimal to let any tree reach size class ten (cf. Fig. 4b). In addition to these cuttings, there is one thinning from below from size class three.

The final theoretical example (Fig. 6a-c) shows how the optimal forest management system may depend on economic factors such as the rate of discount. The example is based on the moderate shading specification in Table 1 and assuming that costly artificial regeneration is possible ($c = 6$) simultaneously with natural regeneration that depends on the basal area. If the rate of interest is low ($b = 0.99$), the optimal solution converges toward a stationary fourteen period rotation cycle that represents even-aged management. As shown in Fig. 6c, in addition to thinning from below, trees are mainly harvested in thinnings from above when they reach dimensions of size class 7, 8, and 9. At the beginning of each period it is optimal to plant 826 seedlings in spite of natural regeneration.

Planting represents an investment for obtaining a higher revenue in the future. In addition to biological factors, the profitability of this investment depends on the rate of interest, on planting costs and on timber prices. Fig. 6a,b show that when the rate of interest is increased ($b = 0.9$) it is optimal to apply uneven-aged forestry and rely entirely on natural regeneration. A similar effect follows if planting costs are increased to a level $c \geq 11$ or if timber prices are decreased. These sensitivity analysis results should be compared to the comparative static results obtained by the Faustmann model under an exogenous forest management system. Under this approach, increasing the rate of discount decreases the length of optimal rotation without effects on the choice of forest management systems (Johansson and Löfgren 1985).

The essential question is how the optimal solution transforms from an even-aged to uneven-aged system. Computation shows that given $b \in [0.98, 0.89]$ both even- and uneven-aged solutions represent locally optimal solutions. When $b > 0.915$, the globally optimal solution is the even-aged system and *vice versa* implying that for $b \simeq 0.915$ both forest management systems yield equal present value revenues.

The result that the two management systems may represent locally optimal solutions has several implications. For example, it can be expected that the globally optimal solution depends on the initial size-class structure of the stand. These questions are discussed in the final section but the details are left for future studies.

4. Empirical application

4.1. Data

This study presents empirical results for Norway spruce, a tree species with high economic value throughout the Northern Europe. This tree species is under an even-aged management regime, although possibilities to apply uneven-aged management are frequently discussed (cf. Wickstöm 2001). This study presents transition matrix data calculated from 48 Norway spruce plots in eastern Finland (Kolstöm 1993). The transition matrix data is presented in Table 2. The number of trees per hectare varied between 167 and 3750, the basal area between 10.8m^2 and 52.5m^2, the mean tree diameter between 13.5cm and 39.5cm and the number of seedlings between 100 and 26000. The time period is 5 years.

Table 2. Parameter values for a one hectare Norway spruce stand

Diameter class, $d_s(cm)$	η_s	ζ_{1s}	ζ_2	ω_{s1} (m^3)	ω_{s2} (m^3)
2	0	1)	1)	0	0
6	0	0.33	0.010495	0	0
10	1	0.449	0.010495	0	0
14	3	0.525	0.010495	0	0.0745
18	5	0.578	0.010495	0	0.1742
22	9	0.617	0.010495	0.19	0.1028
26	15	0.648	0.010495	0.399	0.0866
30	22	0.672	0.010495	0.583	0.1189
34	30	0.693	0.010495	0.8872	0.0799
38	40			1.128	0.1064

1) The smallest size class is given by: $x_{1,t+1} = 0.435x_{1t} + \sum_{s=1}^{n} \mu_s h_{s,t-k} - h_{1t}$

The second size class is given by: $x_{2,t+1} = 0.221x_{1t} + x_{2t}(1 - 0.33 + 0.010495y_t) - h_{2t}$

The data is based on the following linear transition functions

$$\alpha_{st} = \zeta_{1s} - \zeta_2 y_t,\ s = 2,\dots,9,$$
$$\beta_{st} = 1 - \alpha_{st},\ s = 3,\dots 9,$$

where $\zeta_{1s},\ s = 2,\dots,9$ and ζ_2 are constants given in Table 2. Tree mortality is zero in size classes 2-10. For the smallest size class, a share equal to 0.221 of the smallest size class will move to the second size class and a share 0.435 will remain in the smallest size class. Thus a share equal to 0.344 of the smallest size class will die. Regeneration is assumed to occur in the gaps of the harvested trees and there is a one-period time-lag between new gaps in the harvested trees and when the new seedlings enter the first size class. Because there is uncertainty in this parameter, the time lag will be varied between 1 and 4 periods. The relationships between tree diameter and log volumes for sawtimber and pulpwood are taken from Laasasenaho (1982). The price of sawlog is €46 and the price of pulpwood €20 (per m^3).

4.2. Empirical results

Fig. 7 shows the optimal solution as a function of time. A dominant feature of the solution is that after an initial adjustment period the solution reaches a stationary equilibrium cycle. This holds for both rates of discount although the cycle amplitude is smaller when the rate of discount is higher. Note that the number of trees varies between 700 and 2200 and the basal area between $15m^2$ and $30m^2$. In the original data from which the transition matrix was estimated these variables vary between 167-3750 and $10.8m^2 - 52.5m^2$ respectively. Thus the optimal solutions remain well within the range of the original empirical data.

As may be expected a higher rate of discount yields lower long run basal area and stocking volume. Under the lower rate of discount, trees are harvested when they reach a diameter of 34 cm, while the under higher rate of discount, trees are cut when they reach the size of 26cm. This explains why under the higher rate of discount the number of trees is periodically higher although the basal area and volume are lower than under the lower rate of discount. Note that increasing the rate of discount decreases the output of sawlogs but increases the output of pulpwood. The average timber output under higher and lower rates of discount equal $35\text{m}^3/5yrs$ and $38\text{m}^3/5yrs$ respectively. These compare well with the maximum mean annual increment which may be around $40m^3/5yrs$ in Norway spruce sites of this type (Pukkala and Kolström 1988, Hyytiäinen and Tahvonen 2002). Average annual revenues are 332€ when $b = 0.99$ and 290€ when $b = 0.863$.

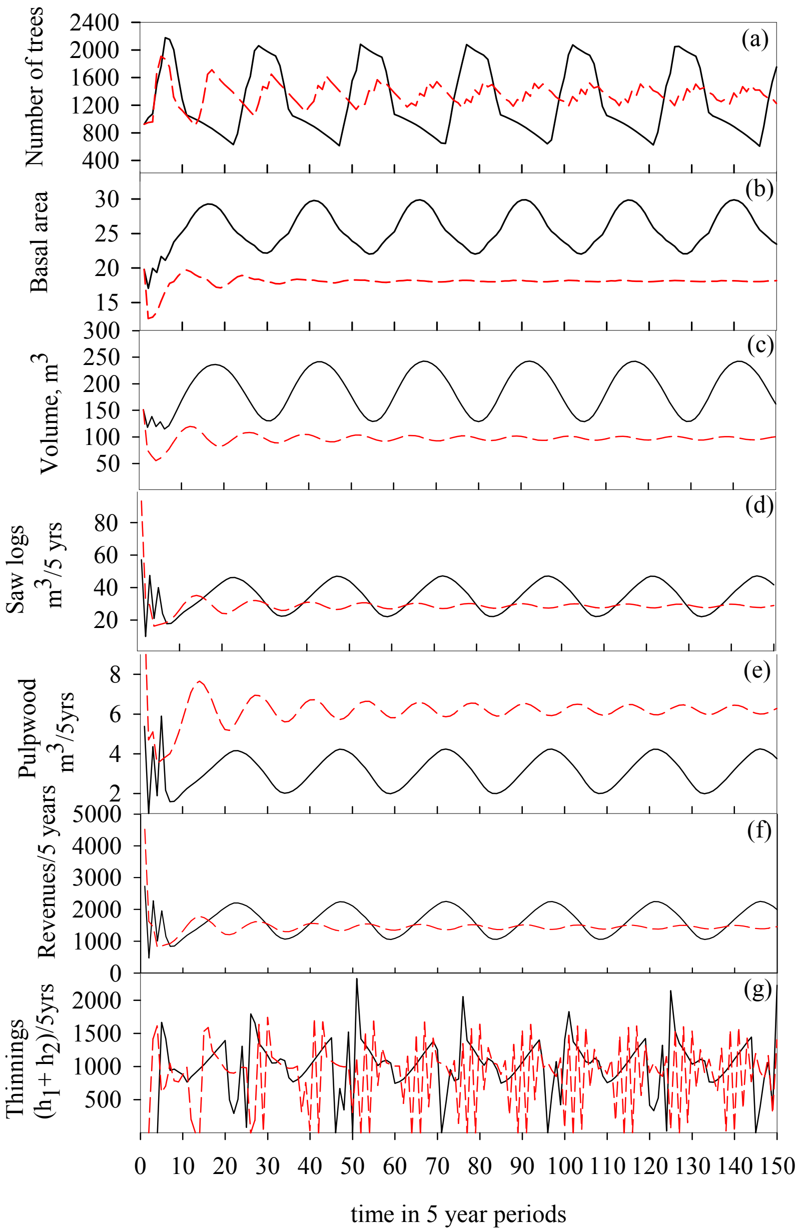


Figure 7. Optimal solution over time

Note: Solid line b = 0.99, dotted line b = 0.863

Inital state: x_0=[682, 322, 188, 123, 86, 64, 48,38,30,24], h_{-1}=0

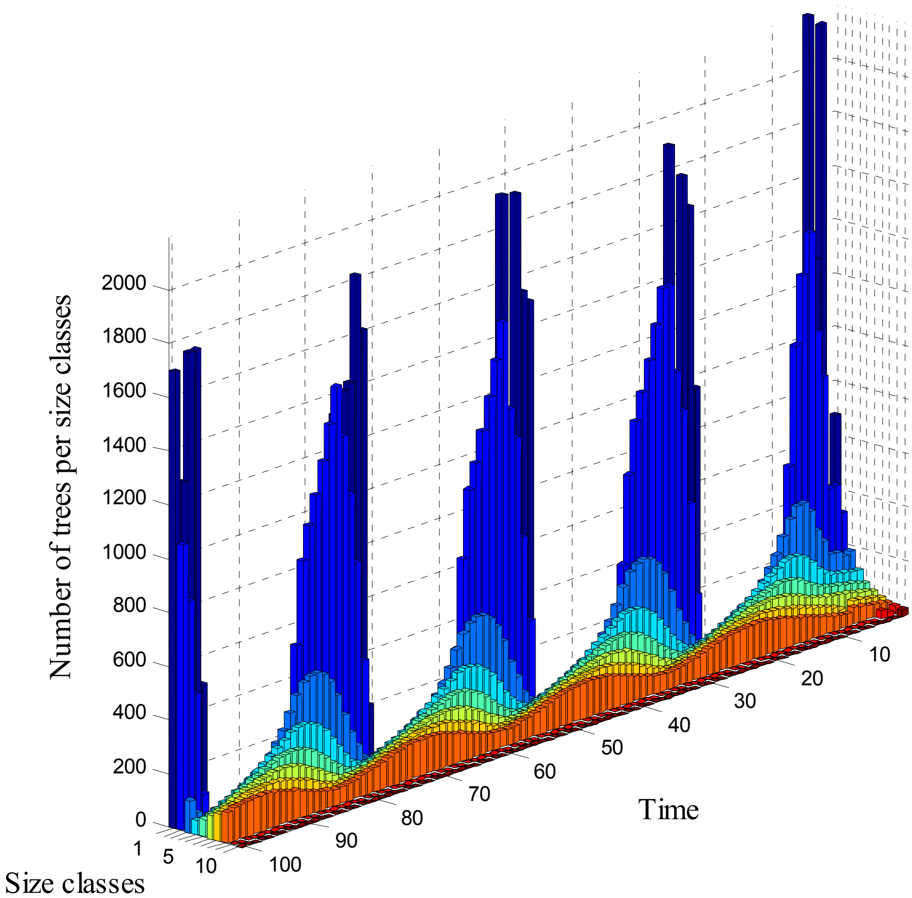


Figure 8. Development of the size classes over time. b = 0.99

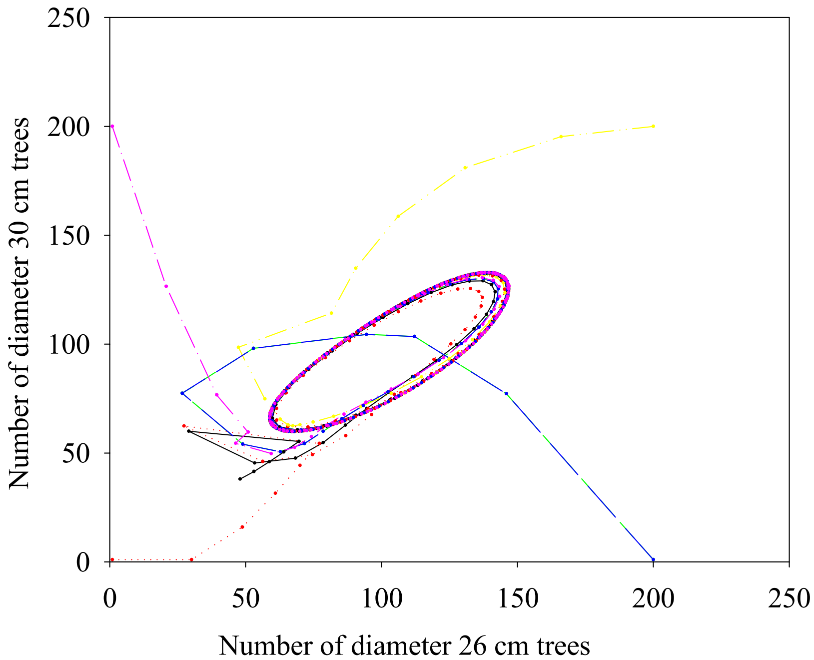
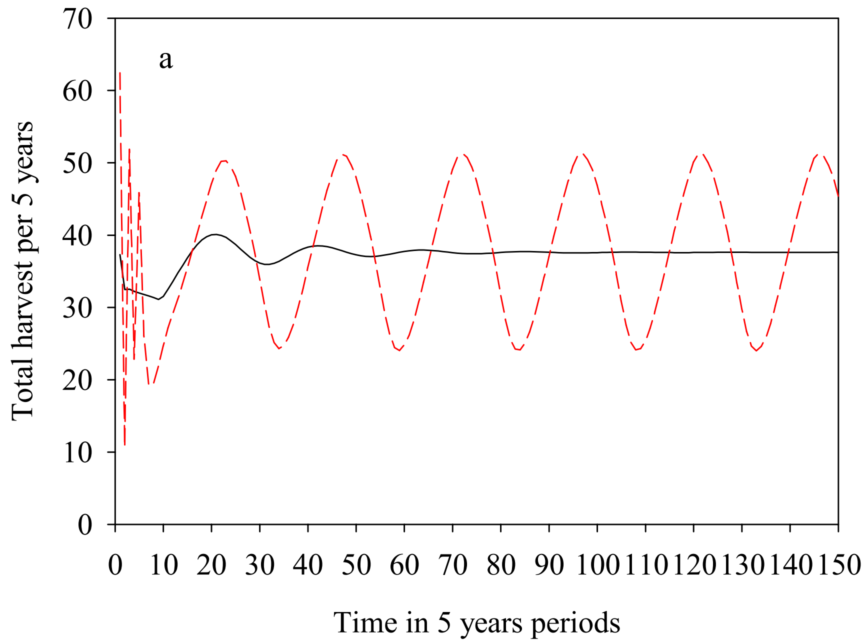


Figure 9. Optimal equilibrium cycle for various initial size class distributions. b = 0.99



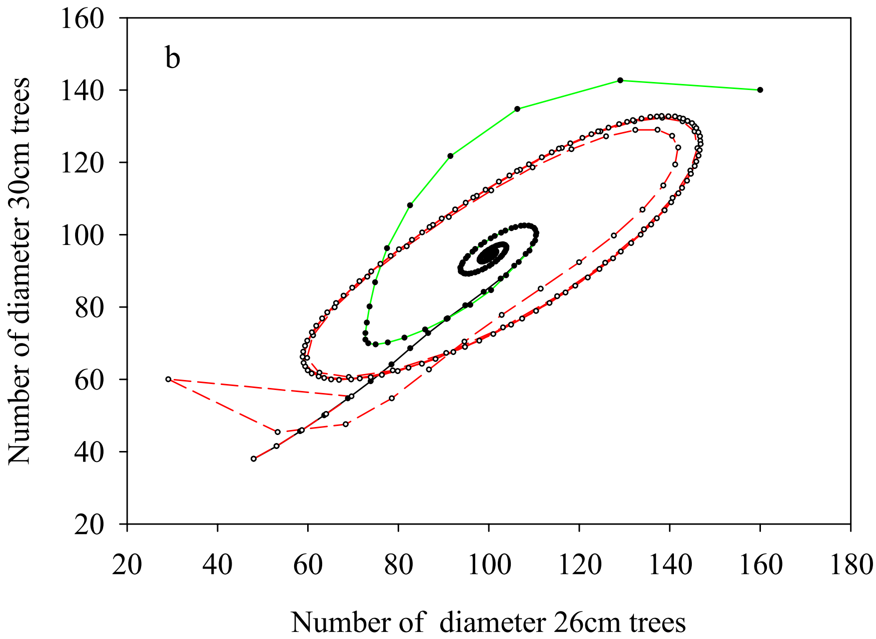


Figure 10a,b. Effects of nonlinear harvesting cost.
Solid lines: nonlinear harvesting cost
Lines with short dash: no harvesting cost
Parameter values, see text

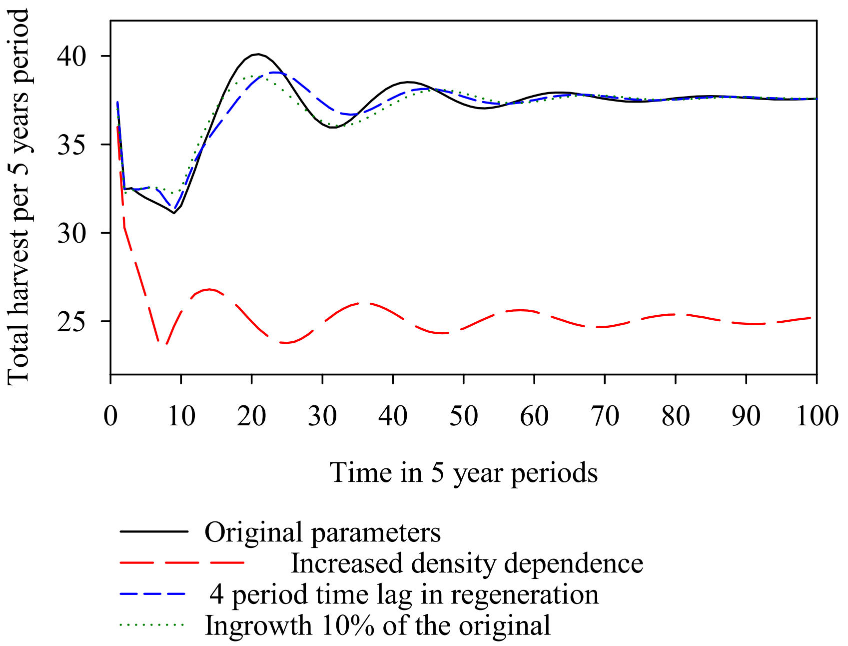


Figure 11. Effects of variations in some ecological parameters

In addition to cuttings from larger size classes, it is optimal to thin trees from the two smallest size classes (Fig. 7g). These thinnings are an implication of excessive ingrowth that, would without thinnings, lead to unoptimally dense stand. Thinnings are made before the trees reach commercially valuable size classes, i.e. thinnings are "noncommercial".

Fig. 8 shows the development of size-classes over time for the solution with the discount factor equal to 0.99. Note that although land is continuously covered with trees, timber is produced in cohorts and a similar size structure is repeated after every 24 periods (120 years). After each cohort is established the stand is thinned from below (i.e. smallest trees are harvested) until there is again more free space after harvesting the main part of the given cohort. The (residual) number of trees in size class 9 is zero since trees are harvested immediately when they reach the dimensions of this size class (recall equation 5).

Given this solution it is pertinent to ask whether the same equilibrium cycle would be reached independently of the initial size class structure. This question is studied in Fig. 9. It shows the optimal development of the diameter class 26 and 30 cm number of trees when $b = 0.99$. The initial size class structure is the same as in Fig. 7 except for the initial number of trees for diameter 26 and 30cm. The optimal solution reaches the same equilibrium cycle from all the initial states suggesting that the cycle represents an optimal long run equilibrium that is independent of the initial size class distribution.

4.3. Nonlinear harvesting cost

In the examples computed this far, the objective function has been linear and the aim has been the maximization of the present value of stumpage revenues. To analyze the effects of nonlinear objective function, one possibility is to include a strictly convex harvesting cost function. An increasing unit harvesting cost may describe the case where logging is done by the forest owner with an increasing opportunity cost for the working hours allocated to forestry. Since empirical estimates for logging costs in uneven-aged management are difficult to obtain, the examples in Fig.s 10a and b are based on assuming that $C = 2Q^{1.6}$, where Q is the number of cubic meters harvested. Fig. 10a shows the total periodic harvest with and without logging costs. As may be expected, strictly convex logging costs remove the equilibrium cycle. The same effect can be observed in Fig. 10b, where optimal solutions approach the steady state instead of the long run equilibrium cycle.

4.4. Effects of varying some ecological parameters

It is useful to analyze how the solution reacts to changes in density dependence effects, to regeneration time lags and to the level of ingrowth. Given the basic parameters in Table 2, $b = 0.99$ and logging cost equal to $C(Q_t) = 2Q^{1.6}$, where Q_t is the harvested volume (m^3), the optimal solution is given by the solid line in Fig. 12. Assume that the density dependence effect is increased by multiplying the parameters ζ_{2s} in Table 2 by 1.5. This decreases the optimal total harvest and it is given by the lowest dashed line in Fig. 12. Thus, the long-run equilibrium level of the total harvest decreases from $38m^3$ to $25m^3$. Compared to this, the effects of increasing the time-lag in regeneration from one period to four periods or decreasing the level of ingrowth to 10% of the original level have only negligible effects on the harvest. This is explained by the fact that, as reflected by optimal thinnings (Fig. 7g), the level of ingrowth is too great to maintain the optimal stand density.

4.5. Comparing the outcomes of even-aged and unrestricted management

As a final experiment, it is instructive to compare the economic outcomes of the unrestricted optimal solution and a solution under the requirement that forest management must follow an even-aged management system and the Faustmann approach. In this case, the problem is to

$$\max_{\{z_{st},\, s=0,\ldots,n,\, t=0,\ldots,T,\, T\}} V_{Faust.} = \frac{\sum_{t=0}^{T} b^t R_t}{1-b^T} \tag{21}$$

subject to the same restrictions as for solving the general unrestricted problem and assuming that the initial size-class structure is sensible initial state for even-aged management. To this end, assume that initially there are 1800 seedlings. Under the unrestricted alternative it is assumed that new seedlings regenerate in the gaps left by harvested trees, as specified in Table 2. Under the Faustmann even-aged solution, assume that 1800 seedlings are obtained with zero cost after each clearcut.

Solving problem (21) yields an optimal rotation of 115 or 105 years for b=0.99 and b=0.863 respectively. These rotation periods are somewhat long but roughly in line with optimal rotations computed with more detailed even-aged models (Hyytiäinen and Tahvonen 2001). It is optimal in both solutions to apply thinnings from above together with some minor thinnings from below. The economic outcomes from optimal (uneven-aged) and even-aged solutions are given in Table 3. It shows that the unrestricted optimal solution will increase the economic outcome about 30% over the outcome obtained when forest management is restricted to follow the even-aged management with clearcutting. Because both regeneration and harvesting costs are ignored this result is only a rough approximation.

Table 3. Comparison between optimal solution and even-aged management

	$V_{opt.}$	$V_{Faust.}$	$\frac{V_{opt.} - V_{Faust.}}{V_{Faust.}}$ x 100%
b = 0.99	€149144	€112432	32%
b = 0.863	€3050	€2360	29%

5. Discussion

The analyses in this paper have shown that a forest economic optimization approach based on matrix transition model of trees is capable to yield different forest management systems endogenously. The essential factors that determine the optimal choice are how density dependence decreases tree growth, tree regeneration alternatives and the level of the rate of discount and other economic parameters, such as timber price and replanting costs. It is also worth noting that such a model produces optimal thinnings from above/below with clear interpretations.

The possibility that even-and uneven-aged forest management may represent locally optimal solutions has several implications. It suggests that the optimal forest management system may depend on the initial state, i.e. on the initial tree size-class distribution. Thus, detailed information on site productivity may not be enough to make a choice between the management systems. In addition, straightforward and general silvicultural instructions may be misleading. The fact that the management alternatives may represent locally optimal solutions makes the on going debate concerning these forest management alternatives understandable but also suggests that it may be useful to apply analytical and general models to view the problem.

Increases in the rate of discount may cause the optimal solution to shift from even-aged management to uneven-aged management. Changes in timber price and regeneration costs may have similar implications. These are very different comparative statics results compared to the properties of the Faustmann model where the forest management system is exogenous. Some studies on the Faustmann model and its extensions (Tahvonen et al. 2001, Tahvonen and Kallio 2006) have shown that stochastic timber price and capital market imperfections imply that the optimal rotation is determined as if forest owners had a higher rate of discount. In addition, these features produce incentives to smooth timber harvesting over time. Adding capital market imperfections to the model studied here, e.g. in the form of a borrowing constraint, would be straightforward and it can be expected to work in favor of uneven-aged management. Adding stochastic timber price and risk aversion, although much more complex, should have a similar effect.

Most forest economic models follow Hartman (1976) when analyzing the implications of environmental preferences within the Faustmann exogenous rotation framework. Since it is likely that uneven-aged management has many favorable characteristics for biodiversity and preserving landscapes, the economically optimal integration of timber production and other values of forests may benefit if the analysis would be done in a framework where the forest management system is endogenously determined.

The result of this study, that uneven-aged management yields about 30% higher economic outcome compared to even-aged management, is surprising in the light of the fact that uneven-aged management has been practically prohibited in Finland (Siiskonen 2007). However, this result is obtained under several simplifications such as stumpage prices that are independent of the management system. On the other hand, regeneration was assumed to be costless, a simplification that favors even-aged management. The result may also be partly determined by the simplifying properties ecological data applied. The transition matrix model includes density dependence in a way that hardly represents all the possible shade intolerance features of Norway spruce. However, taking into account that no economic study has clearly shown the superiority of even-aged management for this species and that the rate of interest, capital market imperfections, risk aversion and environmental preferences also play important role in this choice, it is not possible to support any categorical positions in this debate.

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[1]Textbooks in environmental and resource economics typically describe the Faustmannmodel without any references to uneven-aged management. See e.g. Hanley et al. (2006).Montgomery and Adams (2000) propose the generic biomass model to be used for studieson uneven-aged management. Such a model, however, neglects the internal structure of thestand and cannot reveal the essence of the problem.

[2]The matrix model is closely related to the age-structured model used in fisherymanagement. The question of even vs. uneven-aged management also has a directcounterpart in fishery, i.e. the sustainable vs. pulse fishing harvesting strategy. However,due to differences in harvesting technology, the details behind the optimal choice betweenthese strategies differ.

[3]Basal area is the cross-sectional area of tree stems measured at breast height and summedover all trees in the stand.

[4] Knitro optimization software has been tested extensively together with other similarsolvers, see e.g. Wächter and Biegler (2006).

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