Improving the Accuracy of Predicted Basal-Area Diameter Distribution in Advanced Stands by Determining Stem Number

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The objective of this paper was to study to what extent the accuracy of predicted basal-area diameter distributions (DD_G) could be improved by means of stem number observations in advanced (H > 10 m) stands. In the Finnish forest management planning (FMP) inventory practice, stem number is determined only in young stands; in older stands stand basal area is used. The study material consisted of sixty stands of Norway spruce (*Picea abies* Karst.) and ninety-one stands of Scots pine (*Pinus sylvestris* L.) with birch (*Betula pendula* Roth and *B. pubescens* Ehrh.) admixtures in southern and eastern Finland. For test data, 167–292 independent, National Forest Inventory-based, permanent sample plots were used. DD_Gs were estimated with the maximum likelihood method. Species-specific models for predicting the distribution parameters were derived using regression analysis. The two-parameter Weibull distribution was compared to the three-parameter Johnson's SB distributions in predicting DD_Gs. The models were based on either predictors that are consistent with current FMP (model G), or assuming an additional stem number observation (model G+N). The predicted distributions were compared in terms of the derived stand variables: stem number, total and timber volumes. The results were similar in modelling and test data sets. Methods, based on the SB distribution obtained with model (G+N), proved to give the most accurate description of the stand structure. Differences were marginal in stand total volumes. However, the error variation in stem number was 20 % to 80 % lower than when applying model (G). SB and Weibull distributions gave very much the same results if model (G) was applied.

Keywords parameter prediction, dbh distribution, Johnson's SB distribution, Weibull distribution
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List of Symbols

a, b, c	Parameters of the Weibull fuction
$\xi, \gamma, \delta, \lambda$	Parameters of Johnson's SB function
β_0, β_1	Parameters of Näslund's height curve
d_{gM}	Basal-area-weighted median diameter at breast height (cm)
F	Slenderness of basal area median tree (h_{gM} / d_{gM})
G	Tree species specific basal area of the stand (m^2 ha^{-1})
h_{gM}	Basal-area-weighted median height (m)
n	number of observations in the sample
N	Tree species specific number of trees (stems ha^{-1})
s_b	Standard deviation of the prediction errors (%)
s_e	Mean square error of model
T	Tree species specific mean age at breast height (years)
V	Tree species specific stand volume (m^3 ha^{-1})

1 Introduction

The empirical tree-diameter distribution is not usually determined in standwise forest inventories. The use of tree-specific models in growth simulators requires that the diameter distribution is known or can be predicted using stand characteristics (Bailey and Dell 1973, Päivinen 1980). Forest management planning (FMP) as applied on non-industrial, private estates in Finland is currently in the process of changing. Stands are characterised in more and more detail. Until recently, stand variables (mean age, diameter and height, total stem number and basal area) were considered adequate to characterise the entire growing stock. Tree species were characterised by their proportion of the stand basal area. Today, measurements are recommended to be carried out separately by tree species. Also, in two-storeyed stands, stand characteristics can be described separately for both storeys. Determining the stem number or basal area are alternatives. In practice, stem number is determined in the young stands up to the first-thinning stage. In older stands, this is replaced by stand basal area.

In Finland, diameter-distribution predicting models have been developed since 1980. Päivinen (1980) and Siipilehto (1988) used the beta function to predict basal area-dbh distribution. The computational approach for the beta function has been presented by Loetsch et al. (1973). Later, studies have been concentrated mostly on the use of the Weibull function fitted to angle-count (relascope) sample plots (Mykkänen 1986, Kilkki et al. 1989, Maltamo et al. 1995, Maltamo 1997). Studies on the non-parametric k-nearest neighbor method to select the appropriate stand plots from data-base have been presented by Haara et al. (1997) and Maltamo and Kangas (1998). Bivariate Johnson's SBB distribution has been applied by Siipilehto (1996).

In probability density functions (pdf) the only random variable is diameter at breast height (dbh). In applications, dbh-distributions are presented for either stem frequency (dbh-frequency distribution, DD_N) or for basal area (basal-area-dbh distribution, DD_G) (see Gove and Patil 1998). The basal-area-dbh distribution is most commonly used in Finland, due to its ability to emphasize the large and the most valuable trees (Päivinen 1980). Also, it is in accord with inventory practice, which produces estimates for median diameter and sum of DD_G. Previous studies have shown close connection between DD_G Weibull parameters, especially parameter b, and basal area median diameter (e.g. Kilkki et al. 1989, Hökkä et al. 1991, Maltamo et al. 1995). Such connection was not found for Weibull function applied as DD_N (Hökkä et al. 1991). Usually, DD_G is obtained by weighting the original distribution with the basal area using angle-count (relascope) sampling. Päivinen (1980) used weighting factor for trees tallied on the fixed area sample plots of the third National Forest Inventory (NFI3). The most widely used prediction models in Finland for pine- (Mykkänen 1986) and spruce-dominated stands (Kilkki et al. 1989) are based on smaller NFI7 sample plots. Using the relascope factor 2 (each tallied tree representing 2 m^2 ha^{-1}) resulted in an average of only eight trees per sample plot. Siipilehto (1988) and Maltamo (1997) based their DD_G models on six to thirteen systematic relascope sample plots per stand to avoid imprecision in the description of the smallest diameter classes (see Vuokila 1959, Maltamo and Uuttera 1998). Even though DD_G distribution is better than unweighted DD_N in static situation, its advantages are not so obvious when a change over time is concerned (Saramäki 1992).

Common to all these models is that the prediction is relying on the stand basal area instead of stem number. So, in practise, these distribution models are relevant only in advanced stands. Nevertheless, if the DD_G is described using only the mean diameter and stand basal area, great variation in the shape of the distribution still remains unaccounted for. The same basal area, even with the same median diameter, could be obtained with a greater number of smaller trees or smaller number of larger trees.

The objective of this paper was to study, to what extent the accuracy of the predicted DD_Gs could be improved by means of additional stem number observation in advanced (*hgM* > 10 m) stands. The widely used Weibull distribution was compared with the less used Johnson's SB distributions in predicting DD_Gs. The lower bound of both distributions was excluded (fixed to zero) from the estimation. This was partly due to simplified modelling including parameter estimation (see Hafley and Schreuder 1977) and parameter prediction, but also because similar distributions in terms of shape and peakedness could be resulting from different set of parameters, if not fixed. As a result, stands with similar conditions and diameter distributions can produce highly variable parameter estimates (Knoebel and Burkhart 1991). Therefore, fixing one parameter to reasonable value, the remaining variation in other parameters was assumed to be more correlated with the variation in stand characteristics. If the lower bound would be estimated, the reasonable range for the parameter from zero to lowest observed diameter would be relatively narrow, especially in naturally regenerated stands. Regression models were developed to predict distribution parameters. The models were tested in terms of the derived stand variables: stem number, total and timber volumes derived from the predicted distributions were compared to values of the corresponding variables computed from the original data and independent test data sets. Estimated and predicted distributions were also evaluated using Kolmogorov-Smirnov (K-S) one-sample goodness-of-fit test.

2 Material and Methods

2.1 Study Material

The study material, used in model development, consisted of sixty stands of Norway spruce (*Picea abies* (L.) Karst.) and ninety-one stands of Scots pine (*Pinus sylvestris* L.) with birch (*Betula pendula* Roth and *B. pubescens* Ehrh.) admixtures in southern and eastern Finland (Mielikäinen 1980, 1985). Stands with median heights exceeding 10 m were deemed to represent advanced stands (Table 1). Some stands were excluded due to their bimodal DD_Gs. The pine and spruce data sets were combined (148 stands) to facilitate the modelling of the distributions of birch admixtures. The mean age at breast height was given for conifers only. The age of the birch admixtures was very much the same in pine stands but four years higher in spruce stands.

The size of the circular sample plots in the mixed stands of spruce and birch was adjusted so that there were about 120 trees per plot. The diameters and heights of all trees in the plots were measured. In the case of the mixed stands of pine and birch, a stand plot consisted of a cluster of three circular plots. The size of these plots was such that they contained about thirty stems. The plots were placed subjectively within the stands to represent a pine-dominated plot, a birch-dominated plot, and a plot with a birch admixture of about 50 %. In the present study, the whole cluster represented a stand in order to

Table 1. Mean characteristics of spruce and pine stands with birch admixtures as shown by the modelling data.

	N, n ha^{-1}	*G*, m^2 ha^{-1}	d_{gM}, cm	*V*, m^3 ha^{-1}	*T*, years	*N*, n ha^{-1}	*G*, m^2 ha^{-1}	d_{gM}, cm	*V*, m^3 ha^{-1}	Birch %
			Spruce					Birch		
Mean	834	13.9	18.5	142.3	49	472	10.2	20.8	96.2	36.3
Std	554	8.0	5.0	54.4	16	379	4.0	5.2	44.7	12.6
Min	60	0.8	8.4	34.8	17	86	3.3	9.6	20.3	15.3
Max	2977	30.9	36.1	260.1	86	1895	23.6	30.3	216.3	78.4
			Pine					Birch		
Mean	359	13.7	25.0	138.2	55	382	11.0	21.8	110.9	44.1
Std	182	3.3	4.1	37.4	14	200	2.2	3.6	28.8	8.2
Min	104	6.6	14.6	65.0	14	133	5.7	10.3	49.3	29.5
Max	1100	21.9	36.1	256.3	91	1602	16.3	29.5	190.4	65.9

Table 2. Mean stand characteristics of spruce, pine and birch test distributions in the INKA test data. The values of the birch proportion in stands, where spruce or pine test distributions were formulated, are the values when birch was present (number of observations given in birch % column).

	Southern Finland				Northern Finland				Lapland			
	N, n ha^{-1}	*G*, m^2 ha^{-1}	d_{gM}, cm	Birch %	*N*, n ha^{-1}	*G*, m^2 ha^{-1}	d_{gM}, cm	Birch %	*N*, n ha^{-1}	*G*, m^2 ha^{-1}	d_{gM}, cm	Birch %
Spruce	n=136			n=107	n=97			n=88	n=25			n=24
Mean	939	17.2	20.4	10.9	776	11.1	16.0	9.9	612	9.0	17.9	20.1
Std	604	7.4	5.4	15.1	487	7.3	3.4	9.1	387	5.4	2.7	16.4
Min	60	1.0	8.4	0.2	119	1.1	9.1	0.2	94	1.1	14.1	0.9
Max	2860	30.3	36.1	76.6	2976	30.9	25.9	47.4	1475	18.8	24.6	68.7
Pine	n=128			n=51	n=113			n=88	n=51			n=34
Mean	392	9.4	21.2	12.4	789	14.1	19.4	8.5	653	11.2	19.8	13.1
Std	396	6.2	5.4	20.2	567	6.0	4.2	8.7	425	4.7	4.6	12.0
Min	39	2.0	11.0	0.3	94	1.3	11.9	0.3	109	2.1	11.8	0.7
Max	3079	27.9	34.7	86.0	3351	29.9	29.1	38.4	2008	23.9	33.5	37.4
Birch	n=64				n=71				n=32			
Mean	626	6.2	16.4	36.1	417	3.3	11.5	16.4	318	3.2	13.8	21.4
Std	1050	5.6	6.3	34.9	261	2.5	3.8	12.9	297	2.7	3.7	13.4
Min	45	0.8	6.2	5.0	90	0.7	4.6	5.1	69	0.5	6.5	5.4
Max	6876	24.8	31.3	100.0	1245	11.8	23.9	94.0	1476	14.2	20.6	68.7

yield enough observations for fitting the distributions. However, combining the plots had the disadvantage of diminishing the variation in the proportion of the birch admixture (30 %–65 %).

Models were tested using independent test data. Each NFI6- and NFI7-based permanent INKA sample plot consisted of a cluster of three circular plots within a stand. The total number of tallied trees was about 120. The smallest trees above breast height (*dbh* < 5 cm), with inadequate growing space, have not been measured. A smaller radius has been applied within each circular plot to select one-third of the tallied trees for height (and other more detailed) measurements (see Gustavsen et al. 1988). For the purposes of the present study, missing heights of tallied trees were predicted by Näslund's (1936) height curve, which was fitted by stand and tree species. Sometimes, due to lack of height observations of birch, the fitted height curve for pine or spruce was used for birch. The median height (h_{gM}), corresponding to the basal area median diameter (d_{gM}), was obtained from the fitted height curve. Tree volumes were calculated with models using tree diameter and height as the predictors of the stem volume (Laasasenaho 1982). Diameters in each data sets were measured to accuracy of 1 mm.

The proportion of species admixture was typically low in the test material compared to that in the modelling data. Test distributions for tree species specific volume and stem number were formed if a minimum of ten observations were found in the sample plot. In pure stands, the test distribution contains about 120 observations. Tests were made separately for southern Finland (location of modelling data) and for northern Finland (Province of Oulu) and for Finnish Lapland. The test data within these districts consisted of 136, 97 and 25 spruce dbh-distributions; 128, 113 and 51 pine dbh-distributions; and 64, 71 and 32 birch dbh-distributions, respectively. The mean stand characteristics of these districts are given in Table 2. The minimum value of the birch proportion in Table 2 is the value when birch was present. The dbh-distributions for pine in southern Finland were obtained mostly from pure stands. On the other hand, only 25 out of 167 birch dbh-distributions were from birch-dominated stands. Spruce-dominated stands had typically low proportions of pine and birch admixtures. On the average, the mean age at breast height increased from a 55 years in southern Finland to a 95 years in Lapland.

2.2 Johnson's SB Distribution

Johnson's SB distribution is, together with the beta function, the most flexible parametric distribution (Hafley and Schreuder 1977). In addition, the SBB distribution has been the most promising bivariate distribution in describing stand structure in terms of tree diameters and heights (Schreuder and Hafley 1977, Hafley and Buford 1985). In the Nordic countries, Mønnes (1982), Tham (1988) and Holte (1993) have used SB distribution, predicting DD_Ns with the percentile method.

Johnson's SB distribution (1) is based on transformation (2) to standard normality (Johnson 1949).

$$f(d) = \frac{\delta}{\sqrt{2\pi}} \frac{\lambda}{(d-\xi)(\xi+\lambda-d)} exp\left(-0,5z^2\right) \qquad (1)$$

where

$$z = \gamma + \delta\, ln\left[\frac{d_i - \xi}{\lambda + \xi - d}\right] \qquad (2)$$

γ and δ are shape parameters,
ξ and λ are location and scale parameters,
d is diameter observed in a stand plot.

The parameters were solved as in the study by (Schreuder and Hafley 1977) with the exceptions of basal-area-weighting. Species-specific dbh-distributions were fitted using the method of maximum likelihood (ML), conditional to fixed lower bound ($\xi = 0$). The maximized log-likelihood function $\ln L$ (3), for solving the SB distribution parameters of the basal-area-dbh distribution on a fixed-area plot was as follows (see Mønnes 1982):

$$\begin{aligned} \ln L = & -\frac{G}{2}\ln(2\pi) + G\ln\delta + G\ln\lambda \\ & -\sum_{i=1}^{n} g_i \ln(d_i - \xi) \\ & -\sum_{i=1}^{n} g_i \ln(\lambda + \xi - d_i) \\ & -\frac{1}{2}\sum_{i=1}^{n} g_i\left[\gamma + \delta\ln\frac{d_i-\xi}{\lambda+\xi-d_i}\right]^2 \end{aligned} \qquad (3)$$

where

$$g_i = \tfrac{\pi}{4}(d_i/100)^2$$

and

$$G = \sum_{i=1}^{n} g_i$$

i=1,..,n; n is the number of observed diameters (or dbh-classes) in a stand plot,
d_i is the observed diameter (cm) and
g_i is the corresponding basal area (m^2 ha^{-1}) of a tree.

The value of parameter λ was iteratively searched using conditional closed solution ML estimators of δ (4) and γ (5), such that upper bound $(\xi + \hat{\lambda})$ was greater than greatest observation in the stand plot. Convergence criterion was set to 0.0001.

$$\hat{\delta} = 1/s \quad (4)$$

and

$$\hat{\gamma} = -\bar{f}/s \quad (5)$$

where

$$s = \sqrt{\sum_{i=1}^{n} g_i \left(f_i - \bar{f} \right)^2 \Big/ G}$$

$$f_i = \ln\left[\frac{d_i - \xi}{\lambda + \xi - d_i} \right]$$

and

$$\bar{f} = \sum_{i=1}^{n} g_i f_i \Big/ G$$

For practical solution of predicting distributions, the observed basal area median diameter (d_{gM}) was set for the median of the predicted basal area-dbh distribution. As the values of parameter ξ and median d_{gM} were known and the values of δ and λ were predicted, the parameter γ was solved using the formula 6.

$$\gamma = \delta \ln\left(\lambda + \xi - d_{gM} \right) - \delta \ln\left(d_{gM} - \xi \right) \quad (6)$$

2.3 The Weibull Distribution

The Weibull distribution has been widely used to describe and predict diameter distributions (eg. Bailey and Dell 1973, Rennols et al. 1985, Magnussen 1986, Hökkä et al. 1991 and Holte 1993). Its advantages include simplicity of mathematical derivation, the fewness of the parameters to be estimated, the known analytic cumulative function, and its flexibility in describing different shapes of unimodal distributions (Bailey and Dell 1973). The two-parameter Weibull probability density function is shown in formula 7.

$$f(d) = c/b(d/b)^{c-1} exp\left\{ -(d/b)^c \right\} \quad (7)$$

where
d is the observed diameter in stand plot,
b and c are the parameters of the Weibull function.

The two-parameter Weibull distribution was fitted using the method of ML. The parameters were solved iteratively by maximizing the basal-area-weighted log-likelihood function (8). Convergence criterion was set to 0.0001.

$$\ln L = (c-1)\sum_{i=1}^{n} g_i \ln(d_i) - G(c-1)\ln b + G\ln(c/b) - \sum_{i=1}^{n} g_i (d_i/b)^c \quad (8)$$

In the present study, the location parameter was excluded. In some previous studies, the two-parameter Weibull function has proved to be better than the three-parameter function (Laar and Mosandl 1989, Maltamo 1995). Actually, if the location parameter has been estimated, the given constraints have left very narrow range for the parameter to vary (e.g. Rennols et al. 1985, Kilkki and Päivinen 1986, Maltamo 1995, 1997). In addition, excluding the location parameter simplified the iterative approach. The value of parameter b ($b > d_{gM}$) was iteratively searched using closed solution ML estimator, conditional to parameter c, such that basal area medians of the Weibull distribution and empirical distribution were equal. Prediction model was formulated to parameter c. As the parameter c was predicted and basal area median (d_{gM}) was known, the parameter b was solved by the formula 9 (Kilkki and Päivinen 1986).

$$b = \frac{d_{gM}}{\left(-\ln(0.5) \right)^{1/c}} \quad (9)$$

2.4 Height Curve

The heights were predicted using Näslund's height curve (Näslund 1936) (10).

$$h = \frac{d^i}{\left(\beta_0 + \beta_1 d\right)^i} + 1.3 \tag{10}$$

where
$i = 2$ for pine and birch and $i = 3$ for spruce

The second power was used for pine and birch. The third power made the height curve more flexible and the fit for spruce was considerably better than when using the second power. Height curves were fitted with linear regression estimation using the transformation shown in the formula 11. Linearization homogenized the variation of random error ε.

$$\frac{d}{(h - 1.3)^{i-1}} = \beta_0 + \beta_1 d + \varepsilon \tag{11}$$

The prediction model was formulated for parameter β_1. The predicted height curve was forced to pass through the known point of d_{gM}, h_{gM} by using the value of parameter β_0 given by equation 12.

$$\beta_0 = \frac{d_{gM}}{\left(h_{gM} - 1{,}3\right)^{i-1}} - \beta_1 d_{gM} \tag{12}$$

2.5 The Shape Index

The three stand characteristics (d_{gM}, G, N) were linked together to describe the shape of the diameter distribution. The basal area of the median tree (g_M) was multiplied by the observed stem number (N), resulting in the 'calculated stand basal area'. The observed stand basal area (G) was divided by the 'calculated stand basal area', resulting in a new variable, which was given the name of shape index (13). The shape index was calculated by tree species.

$$Shape\ index = \frac{G}{g_M N} \tag{13}$$

where

$$g_M = \frac{\pi}{4}\left(d_{gM}/100\right)^2$$

The behaviour of the shape index was studied using schematic DD_N and corresponding DD_G (Fig. 1). The shape index values were calculated with numerical integration of Mathcad program (Mathcad user's guide 1995). If the DD_N resembled a peaked unimodal distribution, the value of the index was about one. Unimodal distributions resulted in shape index values below one but greater than 0.54. Values were decreasing with increasing deviation in diameters and with increasing skewness to right. When DD_N was uniform in shape or resembled an inverted letter J, the index value decreased to about 0.54 and 0.48, respectively. The lowest shape index values was found with bimodal DD_Ns. The corresponding DD_Gs were left-skewed except for the inverse J-shaped DD_N (Fig. 1).

2.6 Model Construction and Evaluation

Multiple regression models were constructed to predict the parameters of the height curve and DD_G. Estimations were made using the REG procedure in SAS (SAS User's Guide 1985). Using the method described in this study, both basal area median diameter (d_{gM}) and basal area (G) are given unbiased without residual variation. Instead, the residual variation is retained in the volume and stem-number estimates. The total volume and number of stems were compared by tree species between the prediction models. As the stand total volume is dominated by the greatest diameters, also accuracy in the smaller diameter classes was studied. In order to compare prediction models in practice, a sample of twelve mixed stands of spruce and birch and twelve mixed stands of pine and birch, with the observed and predicted distributions, were simulated using MELA (Siitonen et al. 1996) for a 15–30-year period (including one thinning and a 15-year growing period after thinning). The vol-

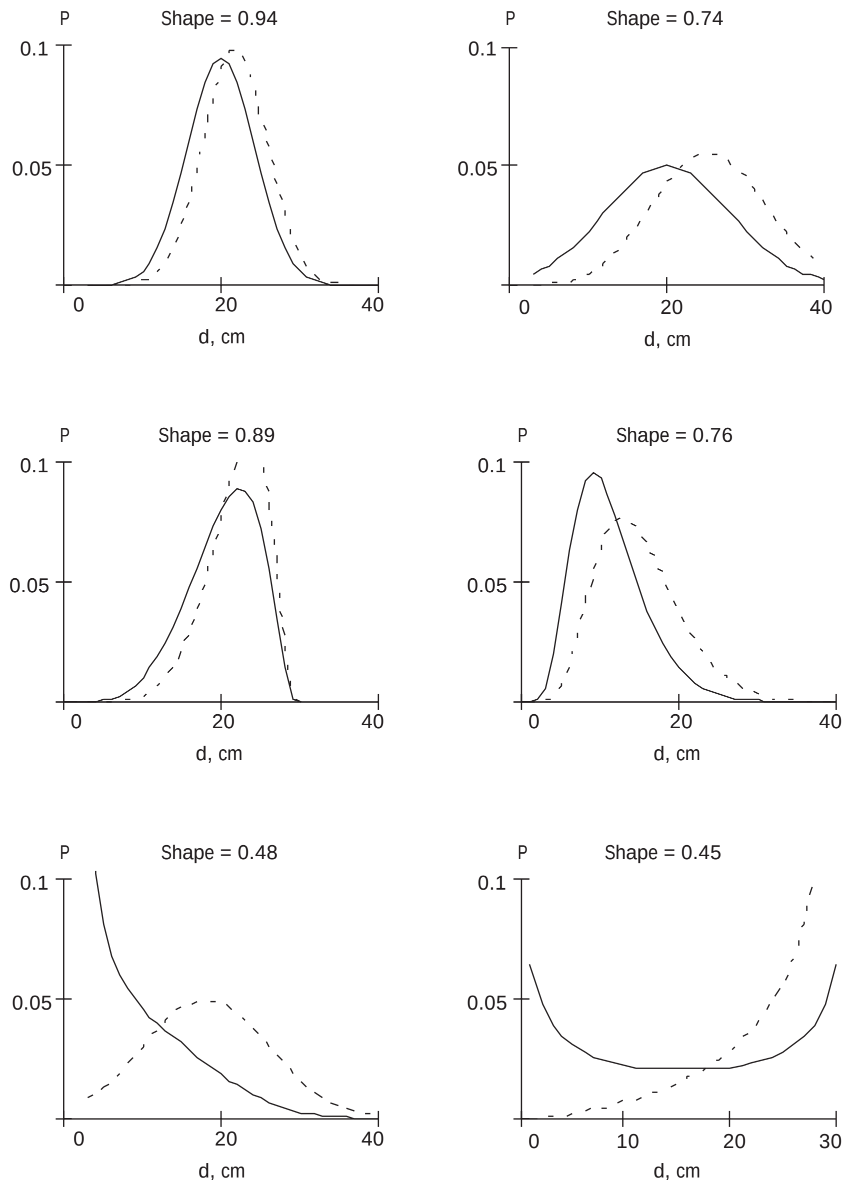

Fig. 1. The shape index behaviour with respect to different shapes of the basal area (- - -) and corresponding stem frequency (—) diameter distributions.

umes of waste wood, pulp wood and saw/veneer logs were compared in initial stands and in the thinning removals. The commercial timber volumes were compared at the end of simulation period. The models were also tested separately in southern and northern Finland, and in Finnish Lapland with an independent test material. The test criteria, relative bias (%) and standard deviation of the prediction errors (s_b, %), were calculated as shown in formulas 14 and 15. Formula 15 shows the residual variation excluding the bias, which is given by formula 14.

$$bias = 100\frac{1}{n}\sum_{i=1}^{n}\left[\left(Y_i - \hat{Y}_i\right)/\hat{Y}_i\right] \quad (14)$$

$$s_b = \frac{1}{n}\sum_{i=1}^{n}\sqrt{\left(e_i - bias\right)^2} \quad (15)$$

where
Y_i is the observed and $\hat{Y}_i$ is the predicted stand characteristic and e_i is the relative prediction error (%) in stand *i*.

The prediction models' behaviour and theorethical bias in terms of stem number, if *N* was the known predictor, were studied using Mathcad (Mathcad User's Guide 1995).

3 Results

The models for heights were constructed using stand variables currently recorded in FMP as the predictors (Table 3). The same predicted height curves were used for all the compared distribution-predicting models. The predicted heights appeared to be unbiased and studied scatter plots with predicted height curves showed a good fit. However, the present study used heights only to make comparisons between the volumes, generated using different distribution-predicting models, and they are no further discussed.

Firstly, stand variables consistent with the current FMP inventory, were selected to predict the parameters of the Weibull and SB distributions (**G** models) (Table 4). In addition to conventional stand characteristics, also the form (slenderness) of the basal area median tree ($F = h_{gM}/d_{gM}$) was used to describe the variation in the distribution parameters. *F* was found to be a significant predictor only in the case of parameter δ for conifers. A considerable part of the variation could be accounted for by the models. The degree of determination was very low only with models predicting the parameter δ for pine and birch, 13 % and 5 %, respectively. The **G** models were unbiased with respect to d_{gM}, *N* or *G*. Thus, stem number alone as a new predictor did not improve the accuracy of these models. As the residuals of SB distribution parameter δ and Weibull parameter *c* were plotted against the shape index, clear linear trends were found (Fig. 2).

Secondly, models for all the parameters were constructed as functions of the shape index (**G+N** models) (Table 6). Variation in the shape index in both data sets was large indicating variation from the inverse J-shaped DD_N to the slightly left skewed DD_N (Table 5). The two lowest values of the index (0.37) were generated from bimodal spruce and birch DD_Ns with great number of smaller trees. For birch, this value was extreme and the second least observation was increased to have the value of 0.59.

The shape index improved the accuracy of the predicted SB parameter δ greatly, but slightly decreased the accuracy of parameter λ. Also, the accuracy of the Weibull parameter *c* was slightly increased for spruce and birch, but clearly decreased for pine. The relationship between the shape index and parameter δ was different by tree species. All the residuals, studied against stand variables, appeared to be unbiased. When

Table 3. The models for Näslund's (1936) height curve. The estimates (with standard deviations) are presented for parameter β_1 for spruce, pine and birch.

	Spruce: β_1		Pine: β_1		Birch: β_1	
Constant	0.3834	(0.0067)	0.2908	(0.0091)	0.2754	(0.0057)
d_{gM}	0.002992	(0.00080)	0.001341	(0.00053)		
h_{gM}	–0.00807	(0.0010)	–0.006337	(0.00077)	–0.004176	(0.00026)
r^2	0.75		0.61		0.64	
s_e	0.012		0.011		0.011	

Table 4. The models with predictors consistent with the current forest management planning field data (**G** models). The estimates (and standard deviations) are presented for the SB distribution parameters λ and δ and for the Weibull distribution parameter *c*. The predictors are specific to tree species.

	λ		$\ln \delta$		c	
Spruce						
Constant	14.658	(4.114)	1.5046	(0.980)	1.7371	(0.433)
d_{gM}	0.6519	(0.246)			0.1226	(0.020)
F			0.9989	(0.371)		
T	0.2089	(0.084)	0.0227	(0.007)		
$\ln T$			–0.8177	(0.326)		
r^2	0.40		0.28		0.39	
s_e	8.120		0.211		0.875	
Pine						
Constant	–17.5244	(11.724)	–1.4617	(0.586)	1.5302	(1.007)
d_{gM}					0.2017	(0.040)
F			0.9928	(0.404)		
$\ln T$	14.667	(2.948)	0.3063	(0.108)		
r^2	0.22		0.13		0.22	
s_e	8.120		0.296		1.530	
Birch						
Constant	13.1531	(3.890)	0.03029	(0.192)	3.7062	(0.748)
d_{gM}	1.1977	(0.178)			0.08632	(0.037)
T					0.02185	(0.010)
$\ln T$			0.1482	(0.050)		
G					–0.1126	(0.042)
r^2	0.54		0.05		0.18	
s_e	8.824		0.370		1.483	

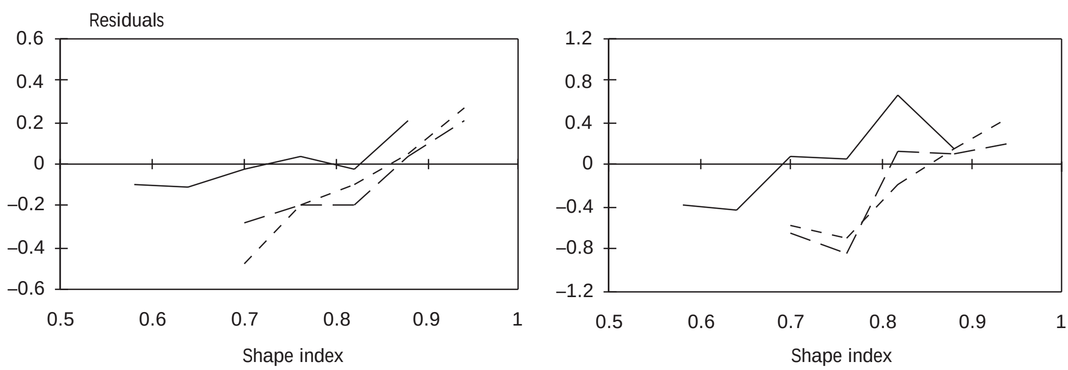

Fig. 2. The mean residual of SB distribution parameter δ (left) and the Weibull distribution parameter *c* (right) with respect to the shape index of spruce (—), pine (— — —), and birch (– – –). Parameters were predicted using the **G** models.

Table 5. Variation of the shape index in the modelling data and in the test data sets.

	Modelling data				INKA test data			
	Spruce	Birch	Pine	Birch	Spruce	Birch	Pine	Birch
Mean	0.68	0.80	0.86	0.85	0.59	0.75	0.87	0.82
Min	0.36	0.36	0.67	0.54	0.42	0.46	0.66	0.62
Max	0.93	1.05	0.99	1.04	0.96	1.01	1.05	1.02

Table 6. The models with additional stem number measurement (**G+N** models). The estimates (and standard deviations) are presented for the SB distribution parameters λ and δ and for the Weibull distribution parameter *c*. The predictors are specific to tree species.

	λ		δ		c	
Spruce						
Constant	5.3617	(7.468)	–0.4230	(0.325)	0.2895	(0.333)
d_{gM}	0.9540	(0.198)				
ln d_{gM}					0.5168	(0.0860)
Shape	18.5939	(10.230)				
1/Shape					–0.2877	(0.1028)
ln(1+Shape)			3.5742	(0.602)		
r^2	0.37		0.39		0.51	
s_e	8.328		0.190		0.181	
Pine						
Constant	–15.4051	(11.070)	–1.8339	(0.337)	11.265	(2.149)
d_{gM}	0.8758	(0.208)				
Shape	39.4779	(12.432)	2.8417	(0.387)		
1/Shape					–4.0618	(1.846)
r^2	0.28		0.38		0.05	
s_e	7.837		0.249		1.690	
Birch						
Constant	–0.7630	(6.958)	–5.0019	(2.071)	0.5213	(1.154)
d_{gM}	1.1181	(0.178)	–0.01897	(0.0053)		
G					–0.09194	(0.0398)
T					0.03096	(0.0080)
Shape	18.3167	(7.648)	5.0148	(1.306)	5.0761	(1.212)
1/ Shape			1.4746	(0.813)		
r^2	0.27		0.50		0.24	
s_e	9.008		0.270		1.426	

applying the **G+N** models, the shape index values were restricted to be greater or equal to 0.5 for spruce, 0.59 for pine, and 0.55 for birch distributions. In any case, it was necessary to restrict the index value for pine to be greater than 0.5 to avoid maximum and median diameters from being equal. Note, the bias correcting factor $(s_e^2/2)$ should be used when applying any models for ln(δ).

All the estimated SB and Weibull distributions and predicted distributios for pine and birch passed the K-S test at 0.1 level. Four and three SB distributions and three Weibull distributions for spruce failed to pass K-S test, if predicted with **G** or **G+N** models, respectively. The predicted distributions for dominant tree species were tested similarly in INKA test data. In spruce dominated stands, five and six SB distributions failed to pass the K-S test, but ten to twelve Weibull distributions failed to pass the K-S test out of 173 distributions with **G+N** or **G** models, respectively. In pine dominated stands, 17 and 18 SB distributions, and 20 and 18 Weibull distributions out of 273 distributions failed to pass the K-S test with **G+N** or **G** models, respectively. In birch dominated stands one predicted distribution out of 17 distributions failed to pass the K-S test regardless of the model. **G+N** models were slightly better than **G** models according to the K-S tests.

The bias in the total stand volume was less than 1 % (Table 7). The stem number was usually overestimated by about 4 % when using the **G+N** models (Table 7). The smallest error variation (s_b) was found when using the **G+N** models, the great majority being found when using the SB distribution (Tables 7–11). The accurary of pine characteristics was very much the same with the Weibull distribution despite of the model, even though the degree of determination with **G** model was considerably greater.

If the stem number was known, the improvement in the accuracy of the volume estimates in timber assortments was noticeable (Table 8). The increase in accuracy was usually at its greatest in the fractions involving the smallest diameters (pulpwood, waste wood). Improved accuracy was most evident in the initial state of the stands and in thinning removals. The error variation in the case of the SB distribution, predicted using the **G+N** models, was usually about 50 % smaller than that achieved with **G** models. For some reason (perhaps the small sample size) this was not true for the initial state of the pine-dominated stands. At the end of 15 to 30 years of simulation, the remaining timber assortments were very much the same regardless of the distribution or model used.

Bias greater than 10 % in the stem number, was not found in the independent test data when applying the SB distribution with **G+N** models, but it did occur twice with the **G** models (Tables 9–11). When predicting the Weibull distribution with the **G+N** models, the 10 % bias was again exceeded twice, and with **G** models this occured five times. On the contrary, a bias of 4 % in volume was exceeded more often when applying the SB distribution (four times) than when applying the Weibull distribution (two times). Note: The greatest biases in both stem number and volume rarely occured in the same model. In addition, because the known basal area median diameter and stand basal area were set for predicted distribution, the overestimate in stem number (too many small trees) appeared to result in underestimate in volume and vice versa.

4 Discussion

The accuracy of the presented models was difficult to compare with previous models due to some methodological differences. The predicted stand volumes had not usually been derived in these studies. Instead, the predicted distributions were tested using sums of diameters with different powers: the first power for the stem number, the second power for characterising volume, and the fourth power for characterising stand value (eg. Kilkki and Päivinen 1986, Kilkki et al. 1989, Maltamo et al. 1995, Maltamo 1997). This enabled the effect of prediction error in tree height to be avoided. Also, independent test material had been rarely used. However, the stem number predicted with the **G+N** models seemed to be far more accurate than with the earlier models presented in the introductory chapter; this was the case even with the test data.

As the models by Mykkänen (1986) and Kilk-

Table 7. The relative bias (and s_b) in the volume (V) and stem number (N) estimates by tree species, the used distribution, and the prediction model (**G, G+N**). The smallest biases and deviations are highlighted in **bold**.

Distribution		Weibull				SB			
Model		**G**		**G+N**		**G**		**G+N**	
		bias	sd	bias	sd	bias	sd	bias	sd
		%							
Spruce	N	2.0	(20.4)	**–0.6**	(14.1)	**–0.6**	(17.2)	–6.2	**(11.0)**
	V	**1.08**	(2.6)	1.17	**(2.5)**	1.14	**(2.5)**	1.13	(2.6)
Birch	N	**0.2**	(22.8)	–2.6	**(11.2)**	3.0	(22.5)	–3.7	(12.2)
	V	**0.82**	(2.6)	1.20	**(2.5)**	0.91	(2.7)	1.09	(2.6)
Total	N	–0.8	(16.7)	–2.6	(11.1)	**–0.3**	(13.6)	–6.4	**(7.5)**
	V	0.79	(1.8)	0.96	**(1.5)**	**0.78**	(1.7)	0.82	(1.6)
Pine	N	–0.1	(8.2)	0.2	(6.4)	**0.03**	(8.4)	–1.6	**(4.5)**
	V	**0.01**	(1.2)	0.08	(1.3)	–0.07	(1.2)	**0.01**	**(1.1)**
Birch	N	–4.2	(11.6)	–4.9	(7.7)	**–1.3**	(12.1)	–5.4	**(1.9)**
	V	0.11	(2.1)	0.23	(1.7)	**0.04**	(2.0)	0.27	**(1.4)**
Total	N	–2.7	(7.6)	–2.7	(5.0)	**–1.0**	(8.3)	–3.8	**(2.0)**
	V	0.05	(1.3)	0.14	(1.1)	**–0.03**	(1.3)	0.11	**(1.0)**

Table 8. The relative bias (and s_b) in the timber volume of the initial stand, in thinning removals, and in the final stand after 15–30 years of simulation in mixed spruce-birch (n = 12) and pine-birch stands (n = 12). The samples are taken from modelling data. The smallest biases and deviations are highlighted in **bold**.

Distribution		Weibull				SB			
Model		**G**		**G+N**		**G**		**G+N**	
		%							
Spruce-birch stands									
Initial	Log	18.6	(53.0)	11.0	(28.6)	17.8	(53.2)	**6.9**	**(21.2)**
	Pulp	–3.4	(10.5)	**–2.4**	(8.5)	–3.7	(10.6)	–2.7	**(6.9)**
	Waste	–5.5	(20.5)	**–5.4**	(14.7)	–8.1	(19.3)	–7.3	**(10.1)**
Remov.	Log	–7.4	(19.8)	–10.5	(16.9)	**–1.4**	(21.9)	–3.6	**(16.7)**
	Pulp	**–3.8**	(13.0)	–4.0	(10.6)	–6.6	(11.9)	–4.4	**(7.0)**
	Waste	3.6	(41.7)	2.2	(30.1)	**–1.4**	(37.1)	–5.8	**(13.5)**
Final	Log	–12.8	**(8.1)**	–15.7	(10.0)	**–12.0**	(9.4)	–12.3	(9.1)
	Pulp	2.1	(6.0)	–1.9	(7.3)	–1.8	(6.1)	**–1.2**	**(5.6)**
Pine-birch stands									
Initial	Log	7.6	(12.5)	8.6	(14.5)	**6.3**	**(10.4)**	7.6	(11.9)
	Pulp	**–2.3**	(3.6)	–2.9	(3.9)	**–2.3**	**(3.5)**	–3.0	(4.0)
	Waste	–6.9	(16.0)	–9.9	(9.2)	**–2.9**	(19.7)	–10.4	**(7.2)**
Remov.	Log	4.1	(20.9)	**2.4**	(16.4)	5.4	(20.2)	5.3	**(13.3)**
	Pulp	**–2.5**	(6.8)	–3.0	(4.8)	–3.1	(6.7)	–3.5	**(3.5)**
	Waste	–4.4	(24.9)	–10.6	(15.1)	**1.2**	(31.4)	–11.8	**(11.7)**
Final	Log	–2.2	(9.0)	–2.3	(8.6)	–2.1	**(8.4)**	**–1.0**	(9.7)
	Pulp	–5.3	(5.6)	–5.7	(5.8)	**–5.2**	**(5.3)**	–5.5	(6.1)

Table 9. The relative bias (and s_b) of total volume and number of stems in spruce (n = 136), pine (n = 128), and birch (n = 64) distributions in southern Finland as predicted by different models. The smallest biases and deviations are highlighted in **bold**. The models applied in practice (Kilkki et al. 1989, Mykkänen 1986) were tested with the same test data sets.

Distribution	Weibull				SB			
Model	G		G+N		G		G+N	
				%				
Spruce distributions								
N	12.2	(26.9)	7.6	(20.9)	8.67	(25.0)	**–6.0**	**(12.3)**
V	1.75	(5.8)	2.14	(5.7)	**1.67**	(6.0)	2.19	**(5.4)**
Weibull distribution for spruce-dominated stands, by Kilkki et al. (1989)[1]								
N	40.1	(29.8)						
V	–4.18	(11.3)						
Pine distributions								
N	–2.9	(11.9)	**1.9**	(9.7)	–4.8	(12.6)	–4.4	**(6.1)**
V	2.43	(5.0)	**1.98**	**(4.9)**	3.01	(5.1)	2.42	**(4.9)**
Weibull distribution for pine-dominated stands, by Mykkänen (1986)[2]								
N	5.1	(12.4)						
V	2.49	(5.0)						
Birch								
N	9.9	(24.2)	7.2	(18.9)	9.5	(23.2)	**–4.2**	**(3.7)**
V	**3.15**	(6.2)	3.66	(6.3)	4.05	(6.3)	4.76	**(6.1)**

[1] Models presented by Kilkki et al. (1989):
$a = 0.001389 + 0.517444\ dg_M$
$\ln(b) = -0.346223 + 0.934993 \ln(dg_M) - 0.000925G$

[2] Models presented by Mykkänen (1986):
$\ln(a) = -1.306454 + 1.154433 \ln(dg_M)$
$\ln(c) = 0.647888 + 0.025530\ dg_M - 0.005558\ G$

Table 10. The relative bias (and s_b) of the total volume and number of stems in spruce (n = 97), pine (n = 113) and birch (n = 71) distributions in northern Finland (test data were beyond the geographical variation of the modelling data). The smallest biases and deviations are highlighted in **bold**.

Distribution	Weibull				SB			
Model	G		G+N		G		G+N	
				%				
Spruce								
N	–11.1	(18.5)	–8.1	(16.2)	**–2.5**	(19.2)	–9.4	**(6.5)**
V	4.46	(6.8)	4.22	(6.6)	**4.01**	(6.8)	4.31	**(6.5)**
Weibull distribution for spruce-dominated stands, by Kilkki et al. (1989)								
N	22.1	(20.3)						
V	**–3.20**	(7.8)						
Pine distributions								
N	12.2	(17.0)	15.4	(13.8)	7.5	(14.3)	**–4.3**	**(10.3)**
V	**–0.22**	(4.1)	–0.34	**(4.0)**	0.71	(4.3)	0.44	**(4.0)**
Weibull distribution for pine-dominated stands, by Mykkänen (1986)								
N	22.3	(17.8)						
V	–0.38	(4.1)						
Birch								
N	6.1	(20.4)	8.6	(16.3)	**2.0**	(21.4)	–3.7	**(3.5)**
V	2.10	(5.5)	**1.99**	(5.4)	2.74	(5.5)	3.06	**(5.2)**

Table 11. The relative bias (and s_b) of the total volume and number of stems in spruce (n = 25), pine (n = 51), and birch (n=32) distributions in Finnish Lapland (the test data were beyond the geographical variation of the modelling data). The smallest biases and deviations are highlighted in **bold**.

Distribution	Weibull				SB			
Model	G		G+N		G		G+N	
				%				
Spruce								
N	**–5.1**	(23.8)	6.5	(16.2)	25.5	(31.9)	–7.6	**(7.1)**
V	6.14	(7.4)	4.87	(6.5)	**4.36**	(8.8)	4.84	**(6.3)**
Weibull distribution for spruce-dominated stands, by Kilkki et al. (1989)								
N	42.6	(26.4)						
V	**2.39**	(13.2)						
Pine distributions								
N	26.3	(21.1)	26.8	(17.1)	19.1	(18.9)	**2.0**	**(14.4)**
V	–0.75	**(4.7)**	–0.57	(4.8)	0.99	(5.3)	**0.18**	**(4.7)**
Weibull distribution for pine-dominated stands, by Mykkänen (1986)								
N	37.0	(21.7)						
V	–0.78	(4.8)						
Birch								
N	17.0	(18.3)	17.6	(15.8)	12.7	(18.1)	**–3.4**	**(2.7)**
V	**2.58**	(6.2)	2.71	**(6.0)**	4.08	(7.2)	4.86	(6.4)

ki et al. (1989) are commonly used to predict DD_G for pine- and spruce-dominated stands, respectively, they were tested against the same test data set. These models were applied by tree species as recommended by Maltamo (1997). The results obtained when using models presented by Kilkki et al. (1989) were accurate in volume estimates (the smallest biases in northern part of Finland) but the total number of stems was underestimated by 20 %–40 % (Tables 9–11). Underestimation was to be expected because these models are based on angle-count (relascope) sample plots. However, the underestimate was disconcertingly high. While the volume was biased to the degree of just 2 %–4 %, the greatly underestimated stem number was probably related to the high number of the smallest trees in the spruce-dominated stands. The increased accuracy in the volume and stem-number estimates when using the **G** models, as compared with the models presented by Kilkki et al. (1989), was most probably due to considerably larger fixed area sample plots instead of using the angle-count (relascope) method.

The results obtained with the models presented by Mykkänen (1986) were as good as those obtained with the **G** models for southern Finland (Table 9), producing greater underestimates in stem number the further north the models were applied (Tables 10, 11). Still, the accuracy achieved in volume was comparable to that obtained with both the **G** and the **G+N** models. The bias in stem number increased the further north the **G** and **G+N** models were applied. This was particularly true with the models for pine, with the exception of the **G+N** models applied with the SB distribution. The results obtained with the SB distribution, together with the **G+N** models, were very much the same regardless of the tree species and the geographical location: the stem number was slightly overestimated and the volume was slightly underestimated.

Johnson's SB distribution is found to be more flexible than the Weibull distribution (Hafley and Schreuder 1977). In the present study, the log-likelihood of the fitted SB distribution was usually a little greater than the log-likelihood with the Weibull distribution, indicating slightly better fit. If the prediction was made using the current FMP stand characteristics, the difference in flexibility had hardly any practical meaning. The greater variation in the shape of the SB distributions could not be fully utilized without resorting to additional stem number observations and the formulated shape-index. In fact, the **G** models for the SB distributions often appeared to have the greatest error variation when processing the test material.

If the shape-index could be utilized, the accuracy of the stand characteristics could be considerably increased. The improvement in the accuracy of the stem number estimate was great. The error variation decreased by as much as 20 % compared to that achieved with the **G** models in the modelling data and by 50 % with the conifers in the test data. With birch distributions, this error variation was decreased by about 80 % compared to **G** models.

Some examples (Figs. 3,4, and 5) of the DD_G, predicted using the **G** and **G+N** models are given. The effect of the slenderness (form) of the basal area median tree and the shape index were focused. The slimmer the median tree, the wider the diameter distribution and the greater the stem number were (Fig. 3). The median tree form was useful when predicting DD_Gs due to fact that slenderness is dependent on the history of the stand density (Hynynen and Arola 1998, Niemistö 1994). However, form did not immediately follow rapid changes in distributions (i.e. thinnings). Thus, the predicted DD_G for recently thinned stands would be too wide, resulting in overestimated stem number. On the other hand, thinning would have an effect on all the factors of the shape index making the SB distribution with the **G+N** models very flexible to changes. Decreas-

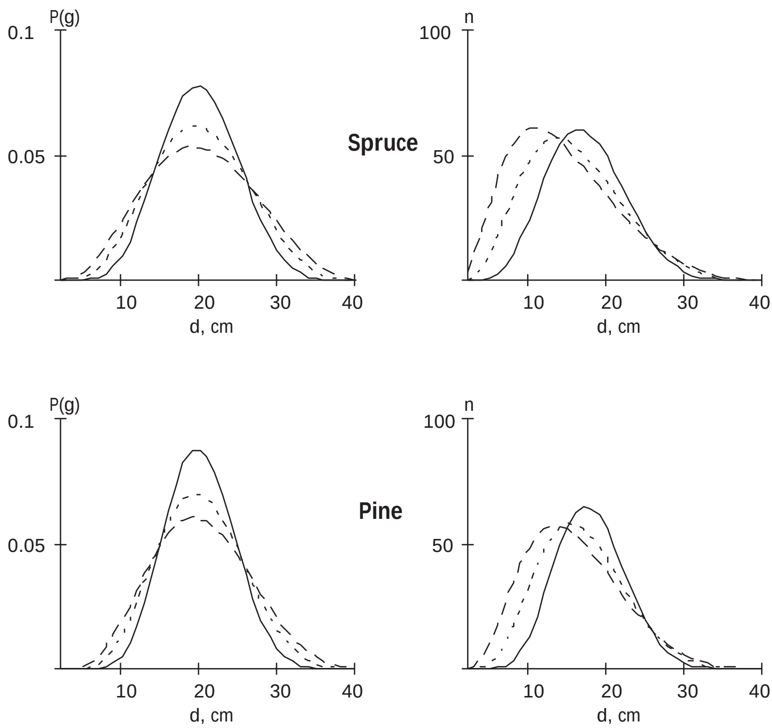

Fig. 3. The predicted SB basal-area diameter distributions (DD_Gs) (left) and the derived stem frequency distributions (DD_Ns) (right) for spruce and pine. The stand characteristics were: d_{gM} = 20 cm, G = 20 m^2 ha^{-1}, T = 80 years. The variation in the slenderness (F = 1.0 —, 0.77 - - -, 0.63 – –) resulted in stem number variation from 780 to 1010 trees/ha for spruce and from 740 to 890 trees/ha for pine.

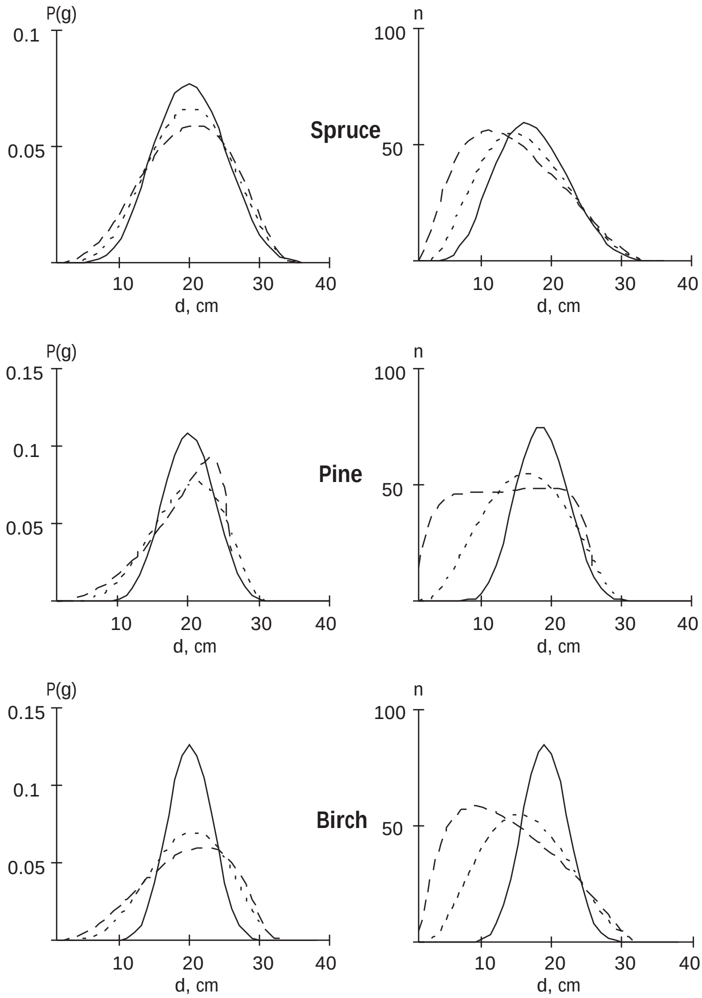

Fig. 4. The predicted SB DD_Gs (left) and the derived DD_Ns (right) for spruce, pine and birch. The stand characteristics were: d_{gM} = 20 cm, G = 20 m^2 ha^{-1}. The shape index variation (1.0 —, 0.77 - - - , 0.63 – –) resulted in stem number variation from 705 to 1100 trees/ha for pine, from 790 to 1020 trees/ha for spruce and from 690 to 1110 trees/ha for birch. Note: The unbiased stem numbers were 640 to 1020 stems/ha. The shape index value 0.63 was beyond the modelling data for pine.

ing the shape index enlarged the distribution of spruce, but the symmetry of the DD_G was not changed in given example (Fig. 4). The distributions of pine and birch achieved differently as DD_G became more and more skewed to the left with decreasing shape index. The corresponding DD_N of spruce and birch were more skewed to the right than those of pine. This may be related to the greater shade tolerance of spruce. The basal area median diameter was fixed to 20 cm and basal area to 20 m^2 ha^{-1} in these examples. The shape index values used were 1.0, 0.77 and 0.63. Thus, the unbiased stem numbers would be 640, 830 and 1020 stems per hectare, respectively. The predicted densities with SB distributions for spruce were 790, 870 and 1020 stems ha^{-1}, for pine 705, 860 and 1100 stems ha^{-1}, and for birch 690, 870 and 1110 stems ha^{-1}. The biases

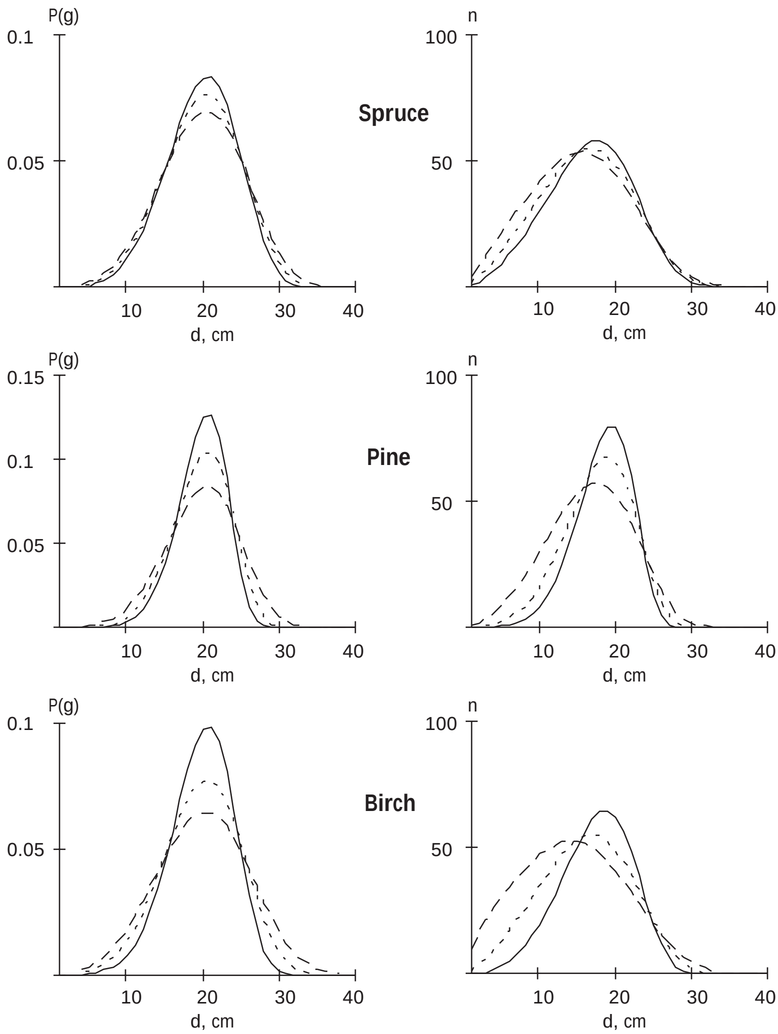

Fig. 5. The predicted Weibull DD_Gs (left) and derived DD_Ns (right) for spruce, pine and birch. The stand characteristics were: d_{gM} = 20 cm, G = 20 m^2 ha^{-1}, T = 60 years for birch. Shape index variation (1.0 —, 0.77 - - -, 0.63 – –) resulted in stem number variation from 730 to 840 trees/ha with pine, from 840 to 930 trees/ha with spruce and from 780 to 1000 trees/ha with birch.

were less than 10 %, except for spruce in the case of index value of 1.0, when the overestimate was 19 %. Changes in the outline of the predicted Weibull distributions were inadequate resulting in greater biases in stem number with respect to extreme shape index values (Fig. 5).

The lowest shape index values, recommended to be used when applicating **G+N** models, would be 0.5 for spruce, 0.59 for pine and 0.55 for birch. The SB distribution's behaviour with these extreme index values was studied with varying median diameters, i.e. 15–25 cm (Fig. 6). The stem numbers were considerably overestimated with the lowest d_{gM} for spruce (30 %) and pine (27 %). All the other biases were below 10 %. The bias in stem number increased if even lower shape index values was used. The lowest shape index values indicated more or less unmanaged

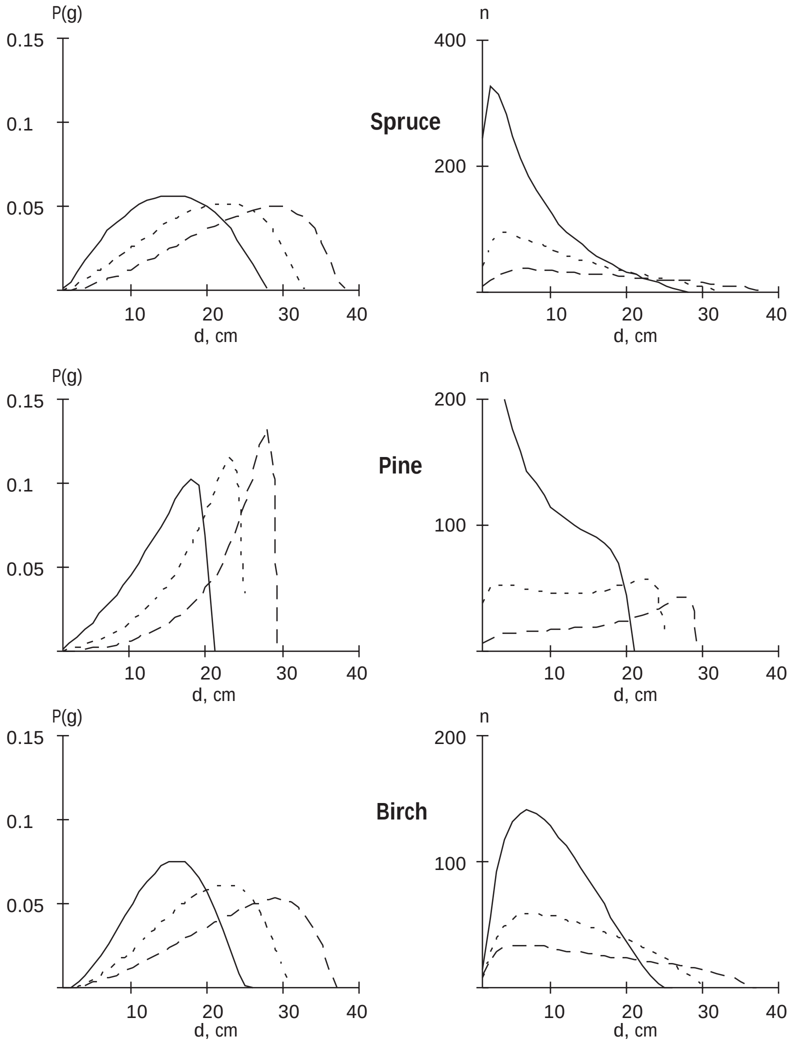

Fig. 6. The predicted SB DD_Gs and derived DD_Ns with the recommended lowest shape indices: 0.5 for spruce, 0.59 for pine and 0.55 for birch. The basal area median diameters were 15 cm (—), 20 cm (- - -) and 25 cm (– –). Note: The stem number was overestimated with the lowest d_{gM} for spruce (30 %) and pine (27 %). All the other biases were below 10 %.

stands; the suppressed trees had not been removed in thinnings.

5 Conclusions

The additional stem number recording, together with mean diameter and basal area measurements, could be utilized in predicting diameter distributions considerably more accurately than without the stem number data. In practical FMP inventory, these measurements could be done by counting the stems within a fixed radius. If the radius of 4 m or 5.6 m is used, one stem represents 200 or 100 stems per ha, respectively. If stem number determination is done together with the recommended 4–8 angle-count (relascope) sample plots per stand, the theoretical accuracy would be 50 to 12.5 stems per ha. Sampling errors should be studied before the final recommendation for the plot size is made. The SB distribution proved to give better description of the varying stand structures than Weibull distribution, if stem number was utilized. However, the possibility of utilizing minimum and/or maximum diameter observations for the same purpose should be studied before formulating final recommendations for practice. In addition, one should pay attention to the effect of sampling errors in predictors (stand caharacteristics) on the precision of the diameter distribution prediction models.

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