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Abstract

Many environmental problems involve the transformation of multiple harmful substances into one or more damage agents much in the same way as a firm transforms inputs into outputs. Yet environmental management differs from a firm's production in one important respect: while a firm seeks efficient input allocation to maximize profit, an environmental planner allocates abatement efforts to render the production of damage agents as inefficient as possible. We characterize a solution to the hmultiple pollutants problem and show that the optimal policy is often a corner solution, in which abatement is focused on a single pollutant. Corner solutions may arise even in well-behaved problems with concave production functions and convex damage and cost functions. Furthermore, even concentrating on a wrong pollutant may yield greater net benefits than setting uniform abatement targets for all harmful substances. Our general theoretical results on the management of flow and stock pollutants are complemented by two numerical examples illustrating the abatement of eutrophying nutrients and greenhouse gases.

Key Words: climate change, cost-benefit analysis, eutrophication, multiple pollutants, optimal environmental policy, pollution control

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1. Introduction

Many of the most serious environmental problems are caused by multiple pollutants that interact with each other.[1] Examples include climate change and eutrophication (increased algal growth in a water ecosystem): in both cases, several pollutants are transformed into damage agents that ultimately cause the environmental problem, much in the same way as a firm transforms its inputs into economic outputs. Optimal pollution control policy in the case of multiple pollutants requires that the tradeoffs in the abatement and damage costs of different pollutants be taken into account. Since economists are particularly good at balancing tradeoffs, von Ungern-Sternberg (1987) proposed that economists take a more active role in the decisions concerning which pollution emissions should be reduced, and by how much. His concern that economists are not consulted enough when setting abatement targets still seems valid today, particularly in the case of multiple pollutants: more often than not, natural scientists set the targets for emission reductions on environmental grounds and economists are then asked to find the ways to achieve the targets at minimum cost. As a consequence, the tradeoffs between multiple pollutants are ignored and emission targets are set at inefficient levels. For example, the Helsinki Convention has set 50% reduction targets for the emissions of both nitrogen and phosphorus to combat eutrophication in the Baltic Sea region. The reduction targets in the Kyoto Protocol are also generally specified as percentage reductions in annual average greenhouse gas (GHG) emissions. Global Warming Potentials (GWPs) are used to convert the other GHGs into carbon equivalents, indicating that perfect substitutability is assumed between the different GHGs. From an economic point of view, there is no reason to expect that such uniform rates of reduction will produce a socially optimal abatement mix.

[1] We use the term “pollutant” in a broad sense to refer to any substance or energy that causes harm to the human or natural environment.

The optimal management of multiple, interacting pollutants has regardless received relatively short shrift in the economics literature. Endres (1985) and von Ungern-Sternberg (1987) provided theoretical analyses of the problem in a static framework. Repetto (1987) considered the case where nitrogen oxides and hydrocarbons react to form atmospheric ozone, and Elofsson (2006) and Sarang et al. (2008) accounted for the interdependence of nitrogen and phosphorus in the production of damage linked to excessive algal growth; these analyses were also confined to a static setting. In a dynamic context, multigas mitigation has received attention primarily in the economic analysis of climate change, but few papers have addressed the optimal abatement mix within a cost-benefit framework that accounts for both environmental damage and abatement costs. The analytical articles by Kandlikar (1995) and Hammitt et al. (1996) focused on determining economically sound indices for describing the relative role of different GHGs in an abatement policy. Hoel and Isaksen (1994) studied a damage-based index for different GHGs within an optimal control model and showed that the weights of the various gases depend on assumptions underlying the economic model applied and that many of the weights change over time. Tol (1999) calculated the global damage potential of a number of GHGs in light of the vulnerability of various sectors of the economy. The interdependence of the problems of tropospheric ozone and acidification has been addressed by Schmieman et al. (2002), who conducted a dynamic cost-benefit analysis to determine the paths for three different pollutants. Most recently, Moslener and Requate (2007) provided a more general analysis of the dynamic properties of optimal joint abatement paths for multiple accumulating pollutants that interact with respect to environmental damage.

Despite the rising interest in multipollutant problems, the question of whether it is more efficient to reduce emissions of all pollutants or to focus efforts on one limiting factor has received little attention since von Ungern-Sternberg's static study. We claim that one reason why economic tradeoffs have remained underrepresented in formulating real-world multipollutant environmental policy may lie in the stylized models often used in theoretical economic analyses of pollution abatement. Much of environmental economics literature has focused on models involving a single pollutant. Where multiple pollutants have been accounted for, dynamic analyses in particular have for the sake of analytical convenience focused on interior solutions..

This paper considers the dynamic properties of efficient multi-pollutant abatement strategies, focusing in particular on whether it is more efficient to abate all of the interacting pollutants, or focus all abatement effort on one pollutant. It contributes to the existing literature as follows. Firstly, it adds to the first dynamic multipollutant analysis by Moslener and Requate (2007) by explicitly accounting for the possibility of corner solutions, and by allowing for non-separability of both damage and abatement costs in the multiple pollutants. Secondly, differing from Endres' (1985) pioneering work, the present paper discusses multiple reasons for corner solutions. It acknowledges that corner solutions may arise because a realistic description of the problem at hand requires employing non-differentiable damage or abatement cost functions. It characterizes conditions under which corner solutions may also arise with perfectly well-behaved convex damage and abatement cost functions. Thirdly, differing from the analyses by Endres (1985) and von Ungern-Sternberg (1987), both the optimal levels of both environmental quality and abatement expenditure are endogenously determined within the model. Finally, the paper provides a step towards applications through empirical examples based on two of today's major environmental concerns: eutrophication and climate change. In particular, we show that it may be optimal to abate only one of the pollutants in each period, and that the pollutant that abatement effort should focus on may change from one period to another along the optimal dynamic abatement path.

This paper approaches the problem of environmental management in the case of multiple, accumulating pollutants from the perspective of the theory of the firm. This field has a long tradition in dealing with multiple inputs (and, increasingly, with multiple outputs) and can provide useful insights for understanding cases involving multiple pollutants. Environmental management differs from production in a firm in one important respect: while a firm seeks *efficient* input allocations that maximize profit, an environmental planner applies abatement measures to render the production of damage agents as *inefficient* as possible and thus minimize environmental damage. Under the standard regularity conditions, profit is maximized when the marginal rates of substitution between different inputs are equal. If the technologies that produce environmental damage obey the same regularity conditions as firms' production technologies, the minimization of damage will lead to a corner solution in which the abatement efforts focus on a single factor that limits production. Thus, corner solutions may arise even in problems satisfying the standard regularity conditions of concave production and convex damage and cost functions. We show that this important lesson applies to many problems in environmental management and is fairly robust to uncertainty about parameter values. The results also indicate that the optimal abatement mix may change over time. Furthermore, concentrating on a wrong pollutant may yield greater net benefits to society than setting uniform abatement targets for all pollutants. In sum, recognition of the possibility of corner solutions is important for policy formulation. While corner solutions may be analytically cumbersome, models that from the outset only allow for interior solutions can be misleading in the case of many environmental problems.

The rest of the paper is organized as follows. Section 2 discusses the parallels between a firm's production decision and an environmental planner's abatement decision in the case of flow pollutants. This analysis is then generalized to the case of stock pollutants in section 3, and some interesting special cases are examined. Section 4 presents two empirical illustrations. The concluding section summarizes the insights gained for the management of multiple pollutants.

2. Efficient multipollutant abatement policy: flow pollutants

We first consider the optimal multipollutant abatement policy in the static case where environmental damage is caused by a combination of flow pollutants and there are no delayed effects over time. This case has been examined previously by Endres (1985) and von Ungern-Sternberg (1987). Here the purpose is to (1) rediscuss the problem in a way that is analogous to a firm's profit maximization problem, and (2) to generalize the previous models so that the optimal level of environmental protection and the amount of abatement expenditures are determined endogenously. Endres (1985) assumed the environmental target and von Ungern-Sternberg (1987) the budget for covering abatement costs to be given exogenously.

For the sake of clarity, we restrict the analysis to the case of two pollutants (extending the analysis to the general setting with *n* pollutants is straightforward). Let x^0, y^0 denote the business-as-usual (BAU) emissions that would occur without abatement, and $x_r \in [0, x^0]$ and $y_r \in [0, y^0]$ the emission reductions achieved through abatement measures; the domain of (x_r, y_r) is denoted by $\Delta = \{[0, x^0], [0, y^0]\}$. Pollution abatement is viewed in a broad sense as including both technical abatement options and changes in the scale and scope of economic activity. Actual emissions x and y equal the difference between the BAU emission levels and abatement: $x = x^0 - x_r$ and $y = y^0 - y_r$. Emissions are inputs in the production of a physical damage agent z according to the relation $z = f(x^0 - x_r, y^0 - y_r)$. Of particular interest here are the properties of the function f, which will be discussed in more detail below. The damage function $D(z)$ measures the damage cost caused by z units of the physical damage agent. D is assumed to be strictly increasing and convex ($dD/dz > 0, d^2D/dz^2 \geq 0 \ \forall z$) as usual. The minimum cost of achieving the emission reductions x_r and y_r is given by the abatement cost function $C(x_r, y_r)$. The abatement cost function is assumed to be increasing in both arguments. The environmental planner's objective is to allocate resources to abatement activities producing emission reductions x_r and y_r so as to minimize the total cost (*TC*), which is the sum of the environmental damage costs and the abatement costs:

$$\min_{(x_r, y_r) \in \Delta} TC\left(x_r, y_r\right) = \min_{(x_r, y_r) \in \Delta} \left(D(f(x^0 - x_r, y^0 - y_r)) + C(x_r, y_r)\right). \tag{1}$$

This planning problem has a compelling interpretation if we view it in terms of profit maximization: the emissions that remain after abatement has taken place (i.e., $x^0 - x_r$ and $y^0 - y_r$) are inputs of production; function *f* is the firm's production function; and *z* is the output of production. Suppose we posit a revenue function that equals the value of the damage avoided:

$$R(x_r, y_r) = D(f(x^0, y^0)) - D(f(x^0 - x_r, y^0 - y_r)). \tag{2}$$

The firm's profit maximization problem can then be stated as

$$\max_{(x_r, y_r) \in \Delta} R(x_r, y_r) - C(x_r, y_r). \tag{3}$$

$$= \max_{(x_r, y_r) \in \Delta} D(f(x^0, y^0)) - D(f(x^0 - x_r, y^0 - y_r)) - C(x_r, y_r) \tag{4}$$

$$= \max_{(x_r, y_r) \in \Delta} D(f(x^0, y^0)) - TC(x_r, y_r). \tag{5}$$

Observe that $D(f(x^0, y^0))$ is a constant. Therefore, solving the environmental planner's problem (1) is equivalent to solving the profit maximization problem (3): the optimal x_r^*, y_r^* from (1) is also the optimal solution to (3). However, the environmental planner's problem differs from the conventional firm's problem in one important respect: as is evident from (4), the environmental planner should minimize production function *f* by allocating inputs $x^0 - x_r$ and $y^0 - y_r$ as *inefficiently* as possible. This proves to be a very useful insight for understanding problem (1).

The first-order conditions of the environmental planner's problem (1) are

$$\frac{\partial TC}{\partial x_r} = -\frac{dD}{dz} \cdot \frac{\partial f}{\partial x} + \frac{\partial C}{\partial x_r} = 0 \tag{6}$$

$$\frac{\partial TC}{\partial y_r} = -\frac{dD}{dz} \cdot \frac{\partial f}{\partial y} + \frac{\partial C}{\partial y_r} = 0 . \tag{7}$$

It is easy to verify that the first-order conditions of problem (3) are equivalent to (6) and (7). A point (x_r, y_r), which satisfies conditions (6) and (7) simultaneously, is referred to as a *critical point*. It is a point at which the marginal reduction of damage is equal to the marginal abatement cost. In terms of production theory, a critical point satisfies the standard marginal-revenue-equals-marginal-cost rule. The problem of the firm is usually well behaved in the sense that the production function is increasing and concave, and hence the optimal solution is achieved at the critical point. However, the problem of the environmental planner is not necessary so well behaved: recall from (4) that we are trying to minimize function *f* rather than to maximize it. Even if conditions (6) and (7) characterize a unique critical point, it could be a local maximum of (1) rather than a minimum. In such cases, the optimum will be a *corner solution,* where $x_r \in \{0, x^0\}$ or $y_r \in \{0, y^0\}$, or both. There may also be cases where no critical point exists. The possibility that the optimum may be a corner solution even where problem (1) is well behaved has also been acknowledged by Endres (1985), who characterized the relationship of the derivatives of *f* and *C* at a corner..

The type of optimum (i.e., a critical-point solution vs. a corner solution) has important implications for the optimal abatement strategy and the environmental policy as a whole. A critical-point solution typically involves simultaneous abatement of all pollutants following the standard marginal-benefit-equals-marginal-cost efficiency rule. By contrast, a corner solution implies that all abatement efforts are focused on a single damage-producing input. Intuitively, a corner solution makes use of decreasing marginal products in that it creates scarcity in one of the input factors needed in damage production.

Whether the critical point is a local minimum or maximum depends on the second-order optimality conditions, which can be presented by means of the Hessian matrix

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} = \begin{bmatrix} \partial^2 TC / \partial x_r^{\ 2} & \partial^2 TC / \partial x_r \partial y_r \\ \partial^2 TC / \partial y_r \partial x_r & \partial^2 TC / \partial y_r^{\ 2} \end{bmatrix} \tag{8}$$

The Hessian of problem (1) and its determinant $|H|$ are expressed in closed form in Appendix 1. A necessary condition for the objective function to attain its minimum at the critical point is that the Hessian is positive semi-definite at this point. Positive semi-definiteness requires that

(A) $H_{11} \geq 0$,

(B) $H_{22} \geq 0$,

(C) $|H| = H_{11} \cdot H_{22} - H_{12} \cdot H_{21} \geq 0$.

If these conditions hold, the critical point is a local minimum, but not necessarily the global minimum. If any one of these three conditions fails, the optimal abatement policy will be a corner solution. In contrast to the analysis presented here, Endres (1985) proceeded from the assumption that the functions f and TC are such that conditions (A) to (C) are satisfied; corner solutions arising from one of conditions (A) to (C) failing were not addressed in his analysis.

As it is difficult to determine from the outset whether a critical-point or corner solution is optimal, it is again illustrative to interpret conditions (A)-(C) from the perspective of the theory of the firm. Condition (A) can be expressed in the form

$$\frac{d^2D}{dz^2}\cdot\left(\frac{\partial f}{\partial x}\right)^2+\frac{dD}{dz}\cdot\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 C}{\partial x_r^{\ 2}}\geq 0\,. \tag{9}$$

The first term in (A) is the product of the scale effect in the damage function and the squared marginal product of input *x*. The second term consists of the returns to scale in the production of the damage agent multiplied by the marginal monetary damage. The third term represents economies of scale in the abatement activities. Summing up the effects, condition (A) will always hold when both *f* and *C* exhibit increasing returns to scale ($\frac{\partial^2 f}{\partial x^2}>0$ and $\frac{\partial^2 C}{\partial x_r^{\ 2}}>0$). Condition (A) may be violated if *f* exhibits (strongly) decreasing returns to scale ($\frac{\partial^2 f}{\partial x^2}<0$) or there are economies of scale in abatement ($\frac{\partial^2 C}{\partial x_r^{\ 2}}<0$). A similar conclusion holds for condition (B).

Condition (C) is most likely to hold when the production function exhibits increasing returns to scale ($\frac{\partial^2 f}{\partial x^2}>0, \frac{\partial^2 f}{\partial y^2}>0$) and negative synergies ($\frac{\partial^2 f}{\partial x\partial y}<0$), and when the abatement activity exhibits diseconomies of scale ($\frac{\partial^2 C}{\partial x_r^{\ 2}}>0, \frac{\partial^2 C}{\partial y_r^{\ 2}}>0$) and economies of scope ($\frac{\partial^2 C}{\partial x_r\partial y_r}<0$) (see Appendix 1 for further details). The optimum is most likely to be a corner solution when the production function exhibits decreasing returns to scale ($\frac{\partial^2 f}{\partial x^2}<0, \frac{\partial^2 f}{\partial y^2}<0$) and positive synergies ($\frac{\partial^2 f}{\partial x\partial y}>0$), and when the cost function exhibits economies of scale ($\frac{\partial^2 C}{\partial x_r^{\ 2}}<0, \frac{\partial^2 C}{\partial y_r^{\ 2}}<0$) and diseconomies of scope ($\frac{\partial^2 C}{\partial x_r\partial y_r}>0$). In sum, whether the optimum is a critical-point or a corner solution depends essentially on the shape of functions f and C, although the damage function D may also play a role.

3. Efficient multipollutant abatement policy: stock pollutants

We next extend the analysis in section 2 to the case of stock pollutants, and then illustrate the types of solutions arising in some instructive examples. The inputs to damage production now consist of accumulated stocks of pollutants, denoted by X^t and Y^t, with t indicating the period. These accumulated inputs are analogous to the capital stocks in a firm's production process. The physical damage agent is produced according to the function $z^t = f(X^t, Y^t)$. The damage function $D(z^t)$ again measures the monetary damage. The business-as-usual flow emissions which would occur in period t without an abatement policy are denoted by x^0 and y^0. For simplicity, we assume that x^0 and y^0 are given and constant over time. The emission reductions produced by abatement activities in period t are denoted by x_r^t and y_r^t. As before, actual flow emissions in period t equal the difference between the BAU emissions and abatement. The change in the pollutant stocks from one period to the next is then governed by

$$X^{t+1} = (1-d_x)X^t + x^0 - x_r^t \tag{10}$$

$$Y^{t+1} = (1-d_y)Y^t + y^0 - y_r^t . \tag{11}$$

The environmental planner's problem now is to minimize the total cost (i.e., the sum of damage and abatement costs) over time through his or her choice of investment levels. The total cost in period t can be expressed as

$$TC(X^t, Y^t, x_r^t, y_r^t) = D(f(X^t, Y^t)) + C(x_r^t, y_r^t) . \tag{12}$$

To analyze the problem within a dynamic programming framework, we formulate it as a maximization problem by positing a reward function that equals the negative of the sum of damage and abatement costs in each period:

$$R(X^t, Y^t, x_r^t, y_r^t) = -TC(X^t, Y^t, x_r^t, y_r^t)\,. \tag{13}$$

Let $\delta = (1+\beta)^{-1}$ denote the discount factor associated with discount rate β. The environmental planner's intertemporal optimization problem is then

$$\max_{(x_r^t, y_r^t)\in\Delta} \sum_{t=0}^{\infty} \delta^t \left\{-TC(X^t, Y^t, x_r^t, y_r^t)\right\}, \tag{14}$$

subject to the stock equations (10) and (11).

We assume that the abatement costs occur in the same period as the corresponding emission reductions and that no costs carry over to future periods. In reality, abatement measures may require investments in abatement technology that reduces emissions over several subsequent periods without additional costs. In this regard, we simply assume that the investment expenditure is allocated proportionally over the time periods in which the emission reductions occur.

The sum of current and future rewards satisfies the Bellman equation

$$V\left(X^t, Y^t\right) = \max_{(x_r^t, y_r^t)\in\Delta} \left\{-TC(X^t, Y^t, x_r^t, y_r^t) + \delta V\left(X^{t+1}, Y^{t+1}\right)\right\}. \tag{15}$$

Consider now the optimal abatement levels. The first-order conditions for the problem in (15) imply that the optimal abatement levels must satisfy

$$-\frac{\partial TC}{\partial x_r^t} + \delta\left[\lambda_X \frac{\partial X^{t+1}}{\partial x_r^t} + \lambda_Y \frac{\partial Y^{t+1}}{\partial x_r^t}\right] = 0\,, \tag{16}$$

$$-\frac{\partial TC}{\partial y_r^t} + \delta\left[\lambda_X \frac{\partial X^{t+1}}{\partial y_r^t} + \lambda_Y \frac{\partial Y^{t+1}}{\partial y_r^t}\right] = 0\,, \tag{17}$$

where $\lambda_X = \dfrac{\partial V}{\partial X}$ and $\lambda_Y = \dfrac{\partial V}{\partial Y}$. Application of the Envelope Theorem to the same problem implies that along the optimal path the states must satisfy

$$-\frac{\partial TC}{\partial X^t} + \delta\left[\lambda_X \frac{\partial X^{t+1}}{\partial X^t} + \lambda_Y \frac{\partial Y^{t+1}}{\partial X^t}\right] = \lambda_X, \tag{18}$$

$$-\frac{\partial TC}{\partial Y^t} + \delta\left[\lambda_X \frac{\partial X^{t+1}}{\partial Y^t} + \lambda_Y \frac{\partial Y^{t+1}}{\partial Y^t}\right] = \lambda_Y, \tag{19}$$

A pair of period *t* emission reductions $(x_r^t, y_r^t) \in \Delta$ that satisfies both (16) and (17), given (X^t, Y^t), is again referred to as a critical point. The optimality conditions set the marginal reward of additional abatement investment in the current period equal to the discounted marginal value of reduced damage in the following period; this is measured by the sum of the marginal reductions in the two pollution stocks, which are valued at the shadow costs of the pollutants, λ_X and λ_Y. However, as in the static case, there may not exist abatement levels that satisfy both conditions (16) and (17), or the optimal policy may be a corner solution where $x_r^t \in \{0, x^0\}$ or $y_r^t \in \{0, y^0\}$.

A sufficient condition for a sequence of abatements satisfying the first-order conditions above to be optimal for maximizing the sum of abatement profits over time is that $TC(X^t, Y^t, x_r^t, y_r^t)$ be convex in its arguments. The conditions for convexity of *TC* are discussed in Appendix 2. As in the static case, it is difficult to determine from the outset whether the sufficient conditions are satisfied or whether corner solutions arise. To illustrate the two types of solutions, we next turn to some interesting examples. The proofs of the propositions discussed below are presented in Appendix 3.

3.1 Linear production function

We start out with the simple but widely used model where pollutants are perfect substitutes in damage production. For example, measuring the relative contribution of different GHGs to climate damage by some fixed coefficients (e.g., the often used global warming potential indices) implies perfect input substitution. Perfect input substitution in turn implies that the production function takes the linear form

$$f(X^t, Y^t) = \alpha X^t + \beta Y^t . \tag{20}$$

In the case of a linear production function, the optimality conditions imply that

$$\frac{\partial C}{\partial x_r^t} \bigg/ \frac{\partial C}{\partial y_r^t} = \lambda_X / \lambda_Y . \tag{21}$$

At a critical point, the ratio of marginal abatement costs should equal the ratio of shadow costs of the two pollutants. The shadow costs are given by $\lambda_X = \frac{D_z \alpha}{\left[1-\delta\left(1-d_X\right)\right]}$ and $\lambda_Y = \frac{D_z \beta}{\left[1-\delta\left(1-d_Y\right)\right]}$, where D_z is the derivative of the damage function with respect to the damage agent. The shadow cost of each pollutant equals the marginal damage caused by the pollutant, adjusted by a discounted effect based on an economic discount factor and the decay rate of the pollutant. We restate the following known result:

Proposition 1: Given a linear production function *f*, if the abatement cost function *C* is *convex* and there exists a sequence $\left\{x_r^{t,*}, y_r^{t,*}, X^{t+1}, Y^{t+1}\right\}_{t=0}^{\infty}$ satisfying the first-order conditions (16) to (19), with $(x_r^*, y_r^*) > (0,0)$, then this sequence is the optimal solution to problem (14).

However, the fact that such a critical point does not always exist has received little attention since the static treatment by Endres (1985). It is possible that

$$\frac{\partial C}{\partial x_r^t} \bigg/ \frac{\partial C}{\partial y_r^t} > \frac{\alpha\left[1-\delta\left(1-d_Y\right)\right]}{\beta\left[1-\delta\left(1-d_X\right)\right]} \;\forall\; (x_r^t, y_r^t) \in \Delta . \tag{22}$$

Equation (22) states the following: if the marginal cost of abating *x* emissions relative to that of abating *y* emissions is greater than the shadow cost of pollutant *X* relative to that of pollutant *Y,* for all $(x_r^t, y_r^t) \in \Delta$, the optimal abatement strategy is a corner solution, where all abatement efforts are focused on pollutant *y*, one which is relatively less expensive to abate. If the sign of the inequality is reversed (<), then all abatement should be focused on pollutant *x*. If a critical point does exist, then the optimal abatement strategy depends on the shape of the abatement cost function.

This result shows that simultaneous abatement of all pollutants is likely to prove beneficial when the inputs to damage production are perfectly substitutable and the abatement cost function is convex. Convexity of the abatement cost function means that the abatement activities exhibit diseconomies of scale and economies of scope. Both are reasonable assumptions in the context of abatement measures. We emphasize again that Proposition 1 assumes the existence of a critical point in all periods *t* that satisfies the first-order conditions. Even if production function *f* is linear and the abatement cost function is convex, the optimal abatement strategy in period *t* may be a corner solution if no critical point exists.

3.2 Leontief production function

We next consider the case where damage production is represented by a Leontief function. Among the canonical production functions in economics, the Leontief function is an extreme special case in that it allows no input substitution. In ecology, however, it is considered to most accurately describe the production of algae from nitrogen and phosphorus (e.g., Redfield et al. 1963; Tyrrell 1999). The Leontief production function takes the form

$$f(X^t, Y^t) = \min\left\{\alpha X^t, \beta Y^t\right\}. \tag{23}$$

The function is not differentiable. In production theory, the standard cost-minimizing solution to producing a given output level in this case implies that the inputs are used in fixed proportions, with $\frac{X}{Y} = \frac{\beta}{\alpha}$. Finding the optimal solution to a dynamic pollution control problem is more complicated in that the inputs to the damage function are stocks and thus change only slowly over time in response to changes in the abatement levels. When the production process and investment decisions concern a "bad", should the emissions of all pollutants be abated in fixed proportions, or should one focus abatement efforts on a single pollutant?

In case of the Leontief function, we obtain a rather strong result:

Proposition 2: If the production function is of the Leontief form and the marginal abatement costs are positive, then the optimal cost-minimizing abatement path consists of the corner solutions $(x_r^{t,*}, 0)$ or $(0, y_r^{t,*})$, where all abatement efforts focus on a single pollutant in each period *t*.

This result is remarkably general in its scope, because it does not depend on the shape of the damage function D or the synergies (convexity, concavity) of the abatement cost function. Indeed, the strategy of a creating shortage in the input supply for f works best when the production activity consumes inputs in fixed proportions. Interestingly, marine ecologists have already applied this strategy for a long time in analyzing which nutrient limits algae production in coastal areas (see, e.g., Elofsson 2006 and references therein).

3.3 Cobb-Douglas production function and linear abatement costs

Finally, consider the canonical textbook case of the Cobb-Douglas production function together with a linear abatement cost function and exponential damage: $z = f(X,Y) = X^{\alpha}Y^{\beta},\ \alpha,\beta > 0$, $C(x_r, y_r) = Ax_r + By_r$, and $D(z) = \exp(z)$. In this setting, the optimal solution will always be a corner solution.

Proposition 3: If the production function is of Cobb-Douglas form, the abatement cost function is linear, and the damage function is exponential, then the optimal cost-minimizing abatement path consists of corner solutions $(x_r^{t,*}, 0)$ or $(0, y_r^{t,*})$, where all abatement efforts are focused on a single pollutant in each period t.

This result depends on specific assumptions about the functional forms, and as such it is not as general in scope as Proposition 1. Still, taken together these special cases show that the optimal solution in a wide variety of environmental problems is likely to be a corner solution.

The conclusion that can be drawn here is that the optimal solution to the dynamic pollution abatement problem may well involve corner solutions, where abatement efforts are focused on a single damage-causing factor that limits pollution. Similar results hold for the static setting. Technically, the corner solutions can arise because either the first-order conditions or the second-order conditions cannot be satisfied. Since the problem cannot be assumed from the outset to be well behaved, it is important to verify that the second-order conditions of an optimization problem also hold. We will proceed to illustrate the above findings in the context of some illustrative empirical examples.

4. Empirical examples: eutrophication and climate change

4.1 Eutrophication

Eutrophication occurs when high concentrations of nitrogen and phosphorus stimulate excessive growth of aquatic plants, primarily algae. Excessive production of algae can affect people's health directly, as in the case of toxic blue-green algae, and detract from the value of recreational activities in the form of decreased water transparency and filamentous algae covering the seabed. Algae contain nitrogen and phosphorus in fixed proportions (Redfield et al. 1963, Tyrrell 1999), and thus algae production can be described by a Leontief production function:

$$z^t = f(X^t, Y^t) = \min\left\{aX^t, bY^t\right\}, \tag{24}$$

where X^t is the stock of accumulated nitrogen, Y^t the stock of accumulated phosphorus, and *a* and *b* parameters of the production function (see Table 1). The essential dynamics of the stocks of nitrogen and phosphorus can in many cases be represented by the simple nutrient carry-over equations

$$X^{t+1} = (1-d_x)X^t + x_0 - x_r^t \tag{25}$$

$$Y^{t+1} = (1-d_y)Y^t + y_0 - y_r^t, \tag{26}$$

where d_x is annual denitrification, d_y net sedimentation of phosphorus, x_0 and y_0 the nitrogen and phosphorus loads corresponding to no abatement, and x_r^t and y_r^t the abatement levels for nitrogen and phosphorus.

We consider an ecosystem receiving external nutrient loading from agricultural sources and municipal wastewater discharges. We assume that in agriculture only nitrogen abatement is possible in the short run. The abatement cost is of quadratic form

$$C(x_r^t, 0) = A(x_r^t)^2. \tag{27}$$

Abatement measures include reducing fertilization rates and establishing buffer zones and wetlands. Both reduced fertilization and agricultural abatement practices generally incur increasing marginal costs.

In the case of wastewater, both nitrogen and phosphorus can be removed at wastewater treatment plants. The cost of biological nitrogen removal is Bx_r^t, and the same process removes $y_r^t = qx_r^t$ units of phosphorus. Nutrient abatement through wastewater treatment primarily involves applying chemicals to pools of collected wastewater; constant marginal cost is hence a reasonable approximation (assuming that treatment facilities are already in place). The capacity of the wastewater treatment facilities sets an upper bound on this abatement measure.

Combining the different abatement measures, the cost function becomes a piecewise quadratic function:

$$C(x_r^t, y_r^t) = \begin{cases} \dfrac{By_r^t}{q} & \text{if } x_r^t \le qy_r^t \le \overline{x} \\ \dfrac{By_r^t}{q} + A(x_r^t - qy_r^t)^2 & \text{if } qy_r^t < x_r^t \le qy_r^t + \dfrac{B}{2A} \text{ and } qy_r^t \le \overline{x} \\ B\left(qy_r^t + \dfrac{B}{2A}\right) + A\left(x_r^t - qy_r^t - \dfrac{B}{2A}\right)^2 & \text{if } x_r^t > qy_r^t + \dfrac{B}{2A} \text{ and } qy_r^t \le \overline{x} \\ B\overline{x} + A\left(x_r^t - \overline{x}\right)^2 & \text{if } x_r^t > qy_r^t + \dfrac{B}{2A},\ x_r^t > \overline{x}, \text{ and } qy_r^t \le \overline{x} \\ \infty & \text{if } qy_r^t > \overline{x} \end{cases} \quad (28)$$

This function is derived by applying the abatement measures that achieve the given abatement target (or better) at the minimum cost; note that wastewater treatment is typically less expensive than agricultural nitrogen abatement. The first line describes a situation where more phosphorus than nitrogen is removed (in relative terms), and thus abatement costs depend on the amount of phosphorus abatement; removal of nitrogen is a side benefit. The second line relates to a situation where phosphorus is reduced through wastewater treatment, and a small additional amount of nitrogen is reduced in agriculture. The third line differs from the second in that nitrogen abatement is relatively more extensive, and phosphorus abatement is achieved as a side benefit of wastewater treatment. The fourth line refers to a situation where nitrogen is reduced in both wastewater treatment facilities and agriculture, while phosphorus abatement remains within the capacity of the wastewater treatment facility. Finally, the fifth line describes a case where the abatement target for phosphorus exceeds the capacity of the wastewater treatment facility.

An exponential damage function is assumed that is similar to the one in Laukkanen and Huhtala (2008):

$$D\left(z^t\right) = E + \exp\left[F/(z^t - G)\right]. \quad (29)$$

Table 1 reports the values of the parameters in functions (24) to (29) and the sources of the values. The parameterization is based on the situation in the Finnish coastal waters of the Gulf of Finland. However, since we consider a stylized model of the ecosystem, the application is intended to provide an illustration of the model presented in the preceding sections rather than policy prescriptions for a specific coastal zone.

Consider first the case of no abatement. In the absence of any abatement measures, the nitrogen stock is projected to increase from 40 to nearly 70 thousand tons in the next ten years. An increase in the phosphorus stock is also anticipated, albeit a more moderate one, from 6 to 7.5 thousand tons. The damage costs are projected to increase from 8.8 million euro to nearly 8 billion euro in the next 40 years. Clearly, abatement measures can yield great savings in damage costs. It is noteworthy that phosphorus would replace nitrogen as the limiting factor of algae growth from the first year onwards. Many economic and ecological models of combating eutrophication have been constructed to account only for the nutrient that is the limiting factor at the initial state. Given that both nitrogen and phosphorus accumulate over time, according to different dynamic processes, the factor limiting algae growth may well change as the relative concentrations of the two nutrients change. Focusing on one nutrient only may yield results that should be interpreted with considerable caution.

[Table 1 around here]

The optimal abatement strategy was found by means of a numerical search. Focusing abatement efforts on phosphorus removal in wastewater treatment facilities was the profit-maximizing solution. The optimal abatement strategy involves maximum reduction of phosphorus in the wastewater treatment facilities in the first two years to achieve the steady state, and a subsequent reduction of 584 tons of phosphorus yearly from the third year onwards. As a side benefit, nitrogen loads are also reduced. The damage and abatement costs of the optimal phosphorus abatement strategy and the total cost saving are summarized in Table 2.

For comparison, we also computed the corresponding figures for the optimal nitrogen abatement strategy, as well as for a policy that would reduce the emissions of both nutrients to fifty percent of their current levels, as the strategy of the Helsinki Commission recommends. The results clearly illustrate the economies of specialization. The optimal abatement strategy that focuses resources on phosphorus abatement reduces total cost by over 6 billion euro compared to the optimal nitrogen abatement strategy, and almost 10 billion euro compared to a fifty-percent reduction of each nutrient. The main advantage of the phosphorus abatement strategy is its low abatement cost compared to the other options.

[Table 2 around here]

Figure 1 illustrates the development of the total cost savings of the different abatement strategies (in net present value terms) during the first 40 years. In all strategies, the highest cost savings materialize about 15 years after the initial abatement measures are implemented. Note that the optimal phosphorus abatement strategy saves costs immediately, from the first year onwards, whereas the nitrogen abatement and the fifty-percent uniform reduction strategies incur higher costs than the strategy of no abatement during the first five or six years.

[Figure 1 around here]

These findings indicate that focusing resources on phosphorus abatement is the optimal strategy for reducing damage caused by the excessive growth of algae. Additional support for adopting the phosphorus limitation strategy is the fact that blooms of nitrogen-fixing cyanobacteria, some of which are toxic, are governed solely by phosphorus concentrations; this has not been taken into account in the above analysis. That nitrogen abatement alone yields higher savings than the fifty-percent joint reduction strategy suggests that specialization is a relatively robust choice under uncertainty about the parameter values: even a focus on the wrong nutrient yields higher benefits than a strategy that applies equal abatement rates for both substances.

4.2 Climate change

The greenhouse effect is enhanced by many different gases that influence the climate in various ways. Today there are still many uncertainties in the causal chain connecting emissions and abatement measures to impacts. As new information has been obtained, the focus of the efforts to reduce the threat of climate change has shifted from stabilizing carbon dioxide emissions to stabilizing atmospheric greenhouse gas (GHG) concentrations. Moreover, scientists are increasingly looking beyond GHG concentrations to radiative forcing. The need to consider multigas abatement polices is widely recognized today. Different GHGs contribute to total radiative forcing and hence to temperature in different ways. The dynamics of the different GHGs also differ. For example, nitrous oxide (N_2O) has a lifetime of 120 years, whereas methane (CH_4) degrades in just 12 years. The following example examines how the tradeoffs between the different GHGs should be balanced given the gases' different impacts and stock dynamics. The optimal abatement strategy in the presence of multiple GHGs is analyzed within a dynamic cost-benefit framework. Again, the empirical model serves to illustrate the different types of solutions and the importance of accounting for the possibility of corner solutions. The underlying climate model is a simplified one, and the results should be used as modeling rather than policy guidelines. The functional forms and the parameter values of the example have been adapted from Aaheim et al. (2006), Tol (2006), and Fisher and Narain (2003).

The process that causes the greenhouse effect has the following nested structure. Firstly, the concentrations of GHGs contribute to total radiative forcing according to equation (IPCC 2001; Aaheim et al. 2006)

$$\phi^t = \alpha_1 \ln(X_1^t / X_1^0) + \alpha_2(\sqrt{X_2^t} - \sqrt{X_2^0}) + \alpha_3(\sqrt{X_3^t} - \sqrt{X_3^0}) + \alpha_4(X_4^t - X_4^0)\,, \tag{30}$$

where X_1^t is the concentration of carbon dioxide (CO_2), X_2^t the concentration of CH_4, X_3^t the concentration of N_2O, and X_4^t the concentration of sulfur hexafluoride (SF_6) in period *t*; values of *X* with superscript 0 refer to the initial concentrations in the year 2000, and parameters α are scaling factors that transform the concentration into radiative forcing (indirect effects of methane have been taken into account in the calibration of parameter α_2). The increase in the global mean temperature is governed by radiative forcing according to (Tol 2006)

$$T^t = \varphi T^{t-1} + \gamma \phi^t\,. \tag{31}$$

Finally, the damage from increased temperature is given by the quadratic damage function (Fisher and Narain 2003)

$$D(T^t) = \theta_1 T^t + \theta_2 (T^t)^2\,. \tag{32}$$

For CH_4, N_2O and SF_6, the dynamics of the GHG concentrations are represented by the difference equation

$$X_j^{t+1} = (1 - 1/\tau_j) X_j^t + x_j^t - a_j^t\,, \tag{33}$$

where the subscript *j* =2,3,4 refers to the GHG, τ_j is the lifetime of gas *j*, x_j^t is the emission of gas *j* in period *t*, and a_j^t is the reduction of gas *j* in period *t*. The baseline emissions x_j^t are exogenously given; in the baseline scenario, the emissions are assumed to remain constant at their year 2000 levels. For CO_2, the dynamics are somewhat more complicated. Following Aaheim et al. (2006), only a fraction $\psi \in (0,1)$ of the CO_2 stock degrades naturally, the remaining part, $(1-\psi)$, accumulating in the atmosphere. Thus, the dynamics of the CO_2 stock can be represented by

$$X_1^{t+1} = (1 - \psi / \tau_j) X_1^t + x_1^t - a_1^t\,. \tag{34}$$

The abatement cost functions have been estimated by Aaheim et al. (2006) based on data of the Energy Modeling Forum (EMF) and the US Environmental Protection Agency (EPA). The abatement cost functions take the log-linear form

$$C(t) = \sum_{j=1}^{4} A_j (a_j^t)^{\beta_j} , \tag{35}$$

where the parameters A_j, β_j are gas-specific constants. Note that the marginal abatement costs are given by

$$MC_j(a_j^t) = \frac{\partial C(t)}{\partial a_j^t} = \beta_j A_j (a_j^t)^{\beta_j - 1} . \tag{36}$$

According to the parameter values estimated by Aaheim et al. (2006), the marginal costs are positive for all gases, as expected. The abatement activities are said to exhibit economies of scale if

$$\frac{dMC(a_j^t)}{da_j^t} = (\beta_j^2 - \beta_j) A_j (a_j^t)^{\beta_j - 2} < 0 . \tag{37}$$

Parameter estimates of β_j by Aaheim et al. range between 0.12 and 0.35, which implies that the abatement cost function exhibits strong economies of scale. Finally, to guarantee that the model achieves meaningful abatement levels, we impose the following inequality constraints: $0 \le a_i^t \le x_i^t \quad \forall i = 1,2,3,4; t = 1,...,\infty$. The parameter values are calibrated for the global region. Table 3 reports the parameter values of the functions together with their sources.

[Table 3 around here]

As solving the optimal abatement paths by analytical methods proved too complex, we resorted to the "brute force" strategy of numerical optimization. We first solved the optimal abatement levels under the simplifying assumption $a_i^t = a_i^s \quad \forall s,t = 1,...,\infty; \quad i = 1,2,3,4$. These levels were then used as an initial guess for subsequent iterations. Given the initial guess, we optimized the abatement levels of period *t* using the Newton's algorithm. We then updated our guess for the remaining periods and moved on to the next period. We enumerated the first 500 periods and then returned to period 1. The algorithm was run several times to achieve full convergence.

This iterative procedure converged to a corner solution where CH_4 and NO_2 were not reduced at all, whereas all emissions of SF_6 were. Interestingly, the abatement of CO_2 exhibited a periodic pattern in which periods of one-hundred-percent and zero abatement alternate. The development of the GHG stocks is illustrated in Figure 2, and the increase in global temperature (in degrees Celsius) in Figure 3. The CH_4 and NO_2 stocks remain at a constant level without abatement. The CO_2 stock more than doubles in size in the first eight years, as there is no abatement during that period. From the year 2009 onwards, all CO_2 emissions are reduced for a period of 40 years, which decreases the CO_2 stock back towards its starting level. This is followed by a one-period break in abatement activities. Subsequently, the periods of eight to nine years of intensive abatement are followed by a one-year break. As a result, the CO_2 stock fluctuates heavily, which also shows up as some variation in the temperature path over time. Despite the fluctuations, according to this model the periodic abatement of CO_2 and total abatement of SF_6 suffice to halt the rise of the global temperature. Such an abatement strategy yields $446.2 billion in cost savings in net present value terms (with a five-percent discount rate). Compared to the BAU scenario, the reduction of damage costs amounts to $480 billion; the abatement costs amount to only $34 billion.

[Figures 2 and 3 around here]

In this example, the corner solution is driven by the economies of scale in abatement, which makes the cost function non-convex. A well-behaved abatement problem would require a convex cost function. A concave cost function makes it advantageous to alternate periodically between zero and one-hundred-percent abatement levels. Although there is a great deal of uncertainty about the parameters of the abatement cost function, the corner solution outcome is robust to changes in the parameter values: the estimates of β_j by Aaheim et al. range between 0.12 and 0.35. Convexity of the cost function would require parameter values greater than one.

To conclude, it must be emphasized that this stylized example is strictly for illustrative purposes. The model presents a drastic simplification of reality. Besides uncertainty about the model parameters, there is considerable uncertainty about the model structure, appropriate functional forms, and the future flows of greenhouse gases. Thus, the analysis presented in this example should not be used for drawing conclusions regarding environmental policy. Nevertheless, we believe that this example convincingly demonstrates the potential importance of corner solutions in the context of climate policy.

5. Conclusions and discussion

We have examined the problem of environmental management in the case of multiple pollutants using the perspective of production theory. Production theory is accustomed to dealing with multiple inputs and provides powerful insights for the analysis of environmental policy where multiple pollutants are involved. The objective of environmental management of course differs from that of a firm in that an environmental planner seeks to minimize the output of a damage agent, subject to limited abatement resources, whereas a firm seeks to maximize output. Standard regularity conditions lead to an interior solution in the case of a firm's maximization problem; the same conditions result in a corner solution when it comes to minimization. Indeed, our results indicate that many common environmental problems give rise to corner solutions, where the optimal policy consists of focusing all abatement effort on one pollutant. While corner solutions are analytically inconvenient, in light of the results it is important to build models and analyses in such a way that such solutions are appropriately accommodated.

Two key environmental problems of today - eutrophication and climate change - were analyzed as empirical examples. In the case of eutrophication, the optimal policy consisted of phosphorus abatement. Significant cost savings were obtained relative to the uniform reduction target suggested, for example, by the Helsinki Commission's guidelines for water protection. Even focusing on the wrong pollutant resulted in cost savings when compared to following uniform abatement targets. The case of eutrophication is special in that the production of algae is characterized by a Leontief production function. In the case of climate change, the functional forms pertaining to damage production and costs are smooth. Nevertheless, the optimal policy was again a corner solution, where only the emissions of SF_6 and CO_2 were reduced, with CO_2 abatement alternating between periods of 0% and 100% reductions.

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Appendix 1: The determinant of the Hessian matrix

The Hessian matrix of problem (1) can be stated as

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} = \begin{bmatrix} \partial^2 TC / \partial x_r^{\,2} & \partial^2 TC / \partial x_r \partial y_r \\ \partial^2 TC / \partial y_r \partial x_r & \partial^2 TC / \partial y_r^{\,2} \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{d^2 D}{dz^2} \cdot \left(\dfrac{\partial f}{\partial x} \right)^2 - \dfrac{dD}{dz} \cdot \dfrac{\partial^2 f}{\partial x^2} - \dfrac{\partial^2 C}{\partial x_r^{\,2}} & -\dfrac{d^2 D}{dz^2} \cdot \left(\dfrac{\partial f}{\partial y} \right)^2 - \dfrac{dD}{dz} \cdot \dfrac{\partial^2 f}{\partial x \partial y} - \dfrac{\partial^2 C}{\partial x_r \partial y_r} \\ -\dfrac{d^2 D}{dz^2} \cdot \left(\dfrac{\partial f}{\partial x} \right)^2 - \dfrac{dD}{dz} \cdot \dfrac{\partial^2 f}{\partial x \partial y} - \dfrac{\partial^2 C}{\partial x_r \partial y_r} & -\dfrac{d^2 D}{dz^2} \cdot \left(\dfrac{\partial f}{\partial y} \right)^2 - \dfrac{dD}{dz} \cdot \dfrac{\partial^2 f}{\partial y^2} - \dfrac{\partial^2 C}{\partial y_r^{\,2}} \end{bmatrix}. \tag{A1.1}$$

Its determinant can be written as

$$\begin{aligned} |H| = & \frac{d^2 D}{dz^2} \cdot \frac{dD}{dz} \cdot \left[\left(\frac{\partial f}{\partial x} \right)^2 \cdot \frac{\partial^2 f}{\partial y^2} + \left(\frac{\partial f}{\partial y} \right)^2 \cdot \frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial x \partial y} \cdot \left(\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right) \right] \\ & + \left(\frac{dD}{dz} \right)^2 \cdot \left[\frac{\partial^2 f}{\partial y^2} \cdot \frac{\partial^2 f}{\partial x^2} - \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2 \right] \\ & + \frac{d^2 D}{dz^2} \cdot \left[\left(\frac{\partial f}{\partial x} \right)^2 \cdot \frac{\partial^2 C}{\partial y_r^{\,2}} + \left(\frac{\partial f}{\partial y} \right)^2 \cdot \frac{\partial^2 C}{\partial x_r^{\,2}} - \frac{\partial^2 C}{\partial x_r \partial y_r} \cdot \left(\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right) \right] \\ & + \frac{\partial^2 C}{\partial x_r^{\,2}} \cdot \frac{\partial^2 C}{\partial y_r^{\,2}} - \left(\frac{\partial^2 C}{\partial x_r \partial y_r} \right)^2 \\ & + \frac{dD}{dz} \cdot \left[\frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 C}{\partial y_r^{\,2}} + \frac{\partial^2 f}{\partial y^2} \cdot \frac{\partial^2 C}{\partial x_r^{\,2}} - 2 \cdot \frac{\partial^2 f}{\partial x \partial y} \cdot \frac{\partial^2 C}{\partial x_r \partial y_r} \right] \geq 0. \end{aligned} \tag{A1.2}$$

The terms on the first and the second lines of the determinant can be positive or negative depending on the economies of scale and scope of the production function *f*. Similarly, the terms on the third and fourth lines can be positive or negative depending on the economies of scale and scope of the abatement cost function *C*. Finally, the term on the fifth line depends on the economies of scale and scope of both *f* and *C*. In general, condition (C) is most likely to hold when the production function exhibits increasing returns to scale $\left(\dfrac{\partial^2 f}{\partial x^2} > 0, \dfrac{\partial^2 f}{\partial y^2} > 0 \right)$ and negative synergies $\left(\frac{\partial^2 f}{\partial x \partial y} < 0\right)$, and when the abatement activity exhibits diseconomies of scale $\left(\frac{\partial^2 C}{\partial x_r^{\ 2}} > 0, \frac{\partial^2 C}{\partial y_r^{\ 2}} > 0\right)$ and economies of scope $\left(\frac{\partial^2 C}{\partial x_r \partial y_r} < 0\right)$. The optimum is most likely to be a corner solution when the production function exhibits decreasing returns to scale $\left(\frac{\partial^2 f}{\partial x^2} < 0, \frac{\partial^2 f}{\partial y^2} < 0\right)$ and positive synergies $\left(\frac{\partial^2 f}{\partial x \partial y} > 0\right)$, and when the cost function exhibits economies of scale $\left(\frac{\partial^2 C}{\partial x_r^{\ 2}} < 0, \frac{\partial^2 C}{\partial y_r^{\ 2}} < 0\right)$ and diseconomies of scope $\left(\frac{\partial^2 C}{\partial x_r \partial y_r} > 0\right)$.

Appendix 2. Sufficient conditions for a maximum: dynamic case

A sufficient condition for a sequence of abatements satisfying the first-order conditions (16) to (19) to be optimal for maximizing the sum of abatement profits over time is that $TC(X^t, Y^t, x_r^t, y_r^t)$ be convex in its arguments. Convexity holds if and only if the Hessian matrix

$$H = \begin{bmatrix} \frac{\partial^2 TC}{\partial X^2} & \frac{\partial^2 TC}{\partial X \partial Y} & \frac{\partial^2 TC}{\partial X \partial x_r} & \frac{\partial^2 TC}{\partial X \partial y_r} \\ \frac{\partial^2 TC}{\partial Y \partial X} & \frac{\partial^2 TC}{\partial Y^2} & \frac{\partial^2 TC}{\partial Y \partial x_r} & \frac{\partial^2 TC}{\partial Y \partial y_r} \\ \frac{\partial^2 TC}{\partial x_r \partial X} & \frac{\partial^2 TC}{\partial x_r \partial Y} & \frac{\partial^2 TC}{\partial x_r^{\ 2}} & \frac{\partial^2 TC}{\partial x_r \partial y_r} \\ \frac{\partial^2 TC}{\partial y_r \partial X} & \frac{\partial^2 TC}{\partial y_r \partial Y} & \frac{\partial^2 TC}{\partial y_r \partial x_r} & \frac{\partial^2 TC}{\partial y_r^{\ 2}} \end{bmatrix} \tag{A2.1}$$

is positive semi-definite. Strict convexity would requires the Hessian to be positive definite. The Hessian can be written as

$$H = \begin{bmatrix} \frac{\partial^2 TC}{\partial X^2} & \frac{\partial^2 TC}{\partial X \partial Y} & 0 & 0 \\ \frac{\partial^2 TC}{\partial Y \partial X} & \frac{\partial^2 TC}{\partial Y^2} & 0 & 0 \\ 0 & 0 & \frac{\partial^2 TC}{\partial x_r^{\ 2}} & \frac{\partial^2 TC}{\partial x_r \partial y_r} \\ 0 & 0 & \frac{\partial^2 TC}{\partial y_r \partial x_r} & \frac{\partial^2 TC}{\partial y_r^{\ 2}} \end{bmatrix} \tag{A2.2}$$

Positive semi-definitiveness requires that the principal minors of H are all positive. We have

$$\frac{\partial^2 TC}{\partial X^2} \geq 0 \quad \Leftrightarrow \quad \left[\frac{d^2 D}{dz^2} \left(\frac{\partial f}{\partial X} \right)^2 + \frac{dD}{dz} \frac{\partial^2 f}{\partial X^2} \right] \geq 0 \,. \tag{A2.3}$$

Condition (A2.3) is similar to condition (A) in the static case, with the exception that the second derivative of the cost function is absent. The production function now has pollutant stocks rather than flows as arguments, whereas the costs remain a function of the pollutant flows. Again, the first two components in (A2.3) can be interpreted as the product of the scale effect in the damage function and the squared marginal product of input *X,* both of which are generally positive. The second term depends on the returns to scale in damage production. As marginal damages are increasing by assumption, condition (A2.3) will always hold when f exhibits increasing or constant returns to scale $\left(\frac{\partial^2 f}{\partial X^2} \geq 0 \right)$. If f instead exhibits decreasing returns to scale, condition (A2.3) may be violated.

$$\begin{aligned} &\frac{\partial^2 TC}{\partial X^2}\frac{\partial^2 TC}{\partial Y^2} - \frac{\partial^2 TC}{\partial Y \partial X}\frac{\partial^2 TC}{\partial X \partial Y} \geq 0 \quad \Leftrightarrow \\ &\left[\frac{d^2 D}{dz^2} \left(\frac{\partial f}{\partial X} \right)^2 + \frac{dD}{dz} \frac{\partial^2 f}{\partial X^2} \right] \left[\frac{d^2 D}{dz^2} \left(\frac{\partial f}{\partial Y} \right)^2 + \frac{dD}{dz} \frac{\partial^2 f}{\partial Y^2} \right] - \left(\frac{d^2 D}{dz^2} \frac{\partial f}{\partial X} \frac{\partial f}{\partial Y} + \frac{dD}{dz} \frac{\partial^2 f}{\partial X \partial Y} \right)^2 \geq 0. \end{aligned} \tag{A2.4}$$

As with condition (C) in the static case, it is difficult to determine from the outset whether condition (A2.4) holds. Here condition (A2.4) is most likely to hold when the production function

f exhibits negative synergies ($\frac{\partial^2 f}{\partial X \partial Y}$<0) or the synergies are small relative to the marginal product of each input.

$$\frac{\partial^2 TC}{\partial X^2}\frac{\partial^2 TC}{\partial Y^2}\frac{\partial^2 TC}{\partial x_r^{\ 2}} - \frac{\partial^2 TC}{\partial X \partial Y}\frac{\partial^2 TC}{\partial Y \partial X}\frac{\partial^2 TC}{\partial x_r^{\ 2}} \geq 0 \quad \Leftrightarrow \qquad \text{(A2.5)}$$

$$\frac{\partial^2 C}{\partial x_r^2}\left\{\left[\frac{d^2 D}{dz^2}\left(\frac{\partial f}{\partial X}\right)^2 + \frac{dD}{dz}\frac{\partial^2 f}{\partial X^2}\right]\left[\frac{d^2 D}{dz^2}\left(\frac{\partial f}{\partial Y}\right)^2 + \frac{dD}{dz}\frac{\partial^2 f}{\partial Y^2}\right] - \left(\frac{d^2 D}{dz^2}\frac{\partial f}{\partial X}\frac{\partial f}{\partial Y} + \frac{dD}{dz}\frac{\partial^2 f}{\partial X \partial Y}\right)^2\right\} \geq 0.$$

Assuming that $\frac{\partial^2 C}{\partial x_r^2}$>0, condition (A2.5) will hold if and only if condition (A2.4) is satisfied.

$$\left(\frac{\partial^2 TC}{\partial X \partial Y}\frac{\partial^2 TC}{\partial Y \partial X} - \frac{\partial^2 TC}{\partial X^2}\frac{\partial^2 TC}{\partial Y^2}\right)\left(\frac{\partial^2 TC}{\partial x_r \partial y_r}\frac{\partial^2 TC}{\partial y_r \partial x_r} - \frac{\partial^2 TC}{\partial x_r^{\ 2}}\frac{\partial^2 TC}{\partial y_r^{\ 2}}\right) \geq 0 \quad \Leftrightarrow \qquad \text{(A2.6)}$$

$$\left\{-\left[\frac{d^2 D}{dz^2}\left(\frac{\partial f}{\partial X}\right)^2 + \frac{dD}{dz}\frac{\partial^2 f}{\partial X^2}\right]\left[\frac{d^2 D}{dz^2}\left(\frac{\partial f}{\partial Y}\right)^2 + \frac{dD}{dz}\frac{\partial^2 f}{\partial Y^2}\right] + \left(\frac{d^2 D}{dz^2}\frac{\partial f}{\partial X}\frac{\partial f}{\partial Y} + \frac{dD}{dz}\frac{\partial^2 f}{\partial X \partial Y}\right)^2\right\}$$
$$\cdot\left(\left(\frac{\partial^2 C}{\partial x_r \partial y_r}\right)^2 - \frac{\partial^2 C}{\partial x_r^{\ 2}}\frac{\partial^2 C}{\partial y_r^{\ 2}}\right) \geq 0.$$

Assuming that condition (A2.4) is satisfied, condition (A2.6) will hold when $\left(\left(\frac{\partial^2 C}{\partial x_r \partial y_r}\right)^2 - \frac{\partial^2 C}{\partial x_r^{\ 2}}\frac{\partial^2 C}{\partial y_r^{\ 2}}\right) \leq 0$. The condition is likely to be satisfied when the abatement activity exhibits diseconomies of scale $\left(\frac{\partial^2 C}{\partial x_r^{\ 2}} > 0,\ \frac{\partial^2 C}{\partial y_r^{\ 2}} > 0\right)$ that are large relative to economies or diseconomies of scope.

Appendix 3. Proofs of propositions

Proposition 1

Suppose that the optimal abatement vector in period t is $\left(x_r^{t,*}, y_r^{t,*}\right) > (0,0)$; i.e., both emissions are abated. We will show that this statement is a contradiction.

Let us move along the isocost line $\bar{C} = C(x_r^*, y_r^*)$. Totally differentiating the expression gives $\frac{\partial C}{\partial x_r} dx_r + \frac{\partial C}{\partial y_r} dy_r = 0$, from which the slope of the isocost line is $\frac{dx_r}{dy_r} = -\frac{\partial C}{\partial y_r} \Big/ \frac{\partial C}{\partial x_r}$. Since the marginal abatement costs are positive by assumption, the isocost line is downward sloping.

Under the maintained assumptions about *C, D,* and *f,* the value function V is strictly decreasing in z. Without loss of generality, we may assume that given $\left(x_r^{t,*}, y_r^{t,*}\right)$, input X is the limiting factor in period $t+1$ in the sense that $z^{t+1} = \min\left\{\alpha X^{t+1}, \beta Y^{t+1}\right\} = \alpha X^{t+1}$. Now, since the isocost line is downward sloping, there exists another point $(\tilde{x}_r^t, \tilde{y}_r^t) : \tilde{x}_r^t > x_r^{t,*}, \tilde{y}_r^t < y_r^{t,*}$ such that $C(\tilde{x}_r^t, \tilde{y}_r^t) = C(x_r^{t,*}, y_r^{t,*})$. But clearly,

$$\begin{aligned} &\min\left\{\alpha\left[(1-d_x)X^t + x_0 - \tilde{x}_r^t\right], \beta\left[(1-d_y)Y^t + y_0 - \tilde{y}_r^t\right]\right\} \\ &< \min\left\{\alpha\left[(1-d_x)X^t + x_0 - x_r^{t,*}\right], \beta\left[(1-d_y)Y^t + y_0 - y_r^{t,*}\right]\right\} \end{aligned} \quad \text{(A3.1)}$$

Since there exists another point that yields a lower value for the damage agent in the subsequent period, and thus a higher value for $V\left(z^{t+1}\right)$, with the same abatement cost, the assumption that $\left(x_r^{t,*}, y_r^{t,*}\right) > (0,0)$ is the optimal solution does not hold. Therefore, the optimal solution is always a corner solution $(x_r^*, 0)$ or $(0, y_r^*)$, where it is not possible to move along the isoquant to preferable points. □

Proposition 2

Since f is linear and the damage function D is increasing and convex, conditions (A2.1) and (A2.2) (see Appendix 2) hold. Abatement cost function C is convex by assumption. Given that (A2.2) is satisfied, (A2.3) then also holds. Condition (A2.4) always holds for a linear production function. Thus, the per-period total cost $TC(X^t, Y^t, x_r^t, y_r^t) = D(f(X^t, Y^t)) + C(x_r^t, y_r^t)$ is convex in its arguments. If a sequence $\left\{x_r^t*, y_r^t*, X^{t+1}, Y^{t+1}\right\}_{t=0}^{\infty}$ that satisfies the first-order conditions (16) to (19) and $(x_r^*, y_r^*) > (0,0)$ exists, then it is the optimal solution to problem (14). □

Proposition 3

The first-order conditions for the steady state can be expressed as

$$-A - \delta\lambda_X = 0 \tag{A3.2}$$

$$-B - \delta\lambda_Y = 0 \tag{A3.3}$$

$$-\exp(z)\alpha X^{\alpha-1}Y^{\beta} + \delta\lambda_X(1-d_X) = \lambda_X \tag{A3.4}$$

$$-\exp(z)\beta X^{\alpha}Y^{\beta-1} + \delta\lambda_Y(1-d_Y) = \lambda_Y . \tag{A3.5}$$

These first-order conditions imply that

$$\frac{Y}{X} = \frac{A\beta\left[1-\delta(1-d_X)\right]}{B\alpha\left[1-\delta(1-d_Y)\right]} . \tag{A3.6}$$

It is clear, however, from the equimarginality conditions (A3.2) and (A3.3) that the optimal solution will always be a corner solution in this setting. Whenever

$$\delta[1-\delta(1-d_X)]\exp(z)\alpha X^{\alpha-1}Y^{\beta} \geq A , \tag{A3.7}$$

the optimal x_r is the maximum admissible rate. Whenever the inequality is reversed, the optimal abatement rate equals zero. A similar reasoning holds for pollutant Y. □

Table 1: Parameter values used in the eutrophication example

Symbol	*Value*	*Sources*
a	52	Redfield et al. (1963)
b	373	Redfield et al. (1963)
d_x	0.5	Neumann (2000); Savchuk and Wulff (1999)
d_y	0.2	Kiirikki et al. (2006)
x_0	34 586 t	Helin et al. (2006), Kiirikki et al. (2003), and personal communication, Heikki Pitkänen, Finnish Environment Institute.
y_0	1 514 t	Helin et al. (2006), Kiirikki et al. (2003), and personal communication, Heikki Pitkänen, Finnish Environment Institute.
X^0	40 000 t	Laukkanen and Huhtala (2008), and personal communication, Heikki Pitkänen and Pirjo Rantanen, Finnish Environment Institute.
Y^0	6 000 t	Laukkanen and Huhtala (2008), and personal communication, Heikki Pitkänen and Pirjo Rantanen, Finnish Environment Institute.
A	1.86 €/t^2	Helin et al. (2006)
B	4460 €/t	Vodokanal (2005)
q	0.45	Vodokanal (2005)
$\bar{x}$	4 518 t	Vodokanal (2005)
E	-170190	Laukkanen and Huhtala (2008) (recalibrated)
F	-1536900	Laukkanen and Huhtala (2008) (recalibrated)
G	179200	Laukkanen and Huhtala (2008) (recalibrated)

Table 2: Total cost saving, damage costs and abatement costs of alternative abatement strategies in the eutrophication example

Abatement strategy	Total cost saving (NPV, billion €)	Damage costs (NPV, quadrillion €)	Abatement costs (NPV, quadrillion €)
Optimal P abatement	101.8	877.4	4 552.1
Optimal N abatement	95.2	44 574.7	214 290.5
50% reduction of both	92.3	111.0	362 395.9
Business as usual	0	6 286 079.5	0

Table 3: Parameter values used in the climate change example

Symbol	*Value*	*Sources*	*Symbol*	*Value*	*Sources*
α_1	5.35	Aaheim et al. (2006)	A_1	0.60	Aaheim et al. (2006)
α_2	1.69	Aaheim et al. (2006)	A_2	61.18	Aaheim et al. (2006)
α_3	0.12	Aaheim et al. (2006)	A_3	0.46	Aaheim et al. (2006)
α_4	0.52	Aaheim et al. (2006)	A_4	0.0038	Aaheim et al. (2006)
φ	0.980	Tol (2006)	β_1	0.35	Aaheim et al. (2006)
γ	0.00458	Tol (2006) recalibrated	β_2	0.23	Aaheim et al. (2006)
τ_1	20	Aaheim et al. (2006)	β_3	0.12	Aaheim et al. (2006)
τ_2	12	Aaheim et al. (2006)	β_4	0.17	Aaheim et al. (2006)
τ_3	120	Aaheim et al. (2006)	θ_1	-0.167	Fisher and Narain (2003)
τ_4	3200	Aaheim et al. (2006)	θ_2	0.467	Fisher and Narain (2003)
ψ	0.3	Aaheim et al. (2006)			

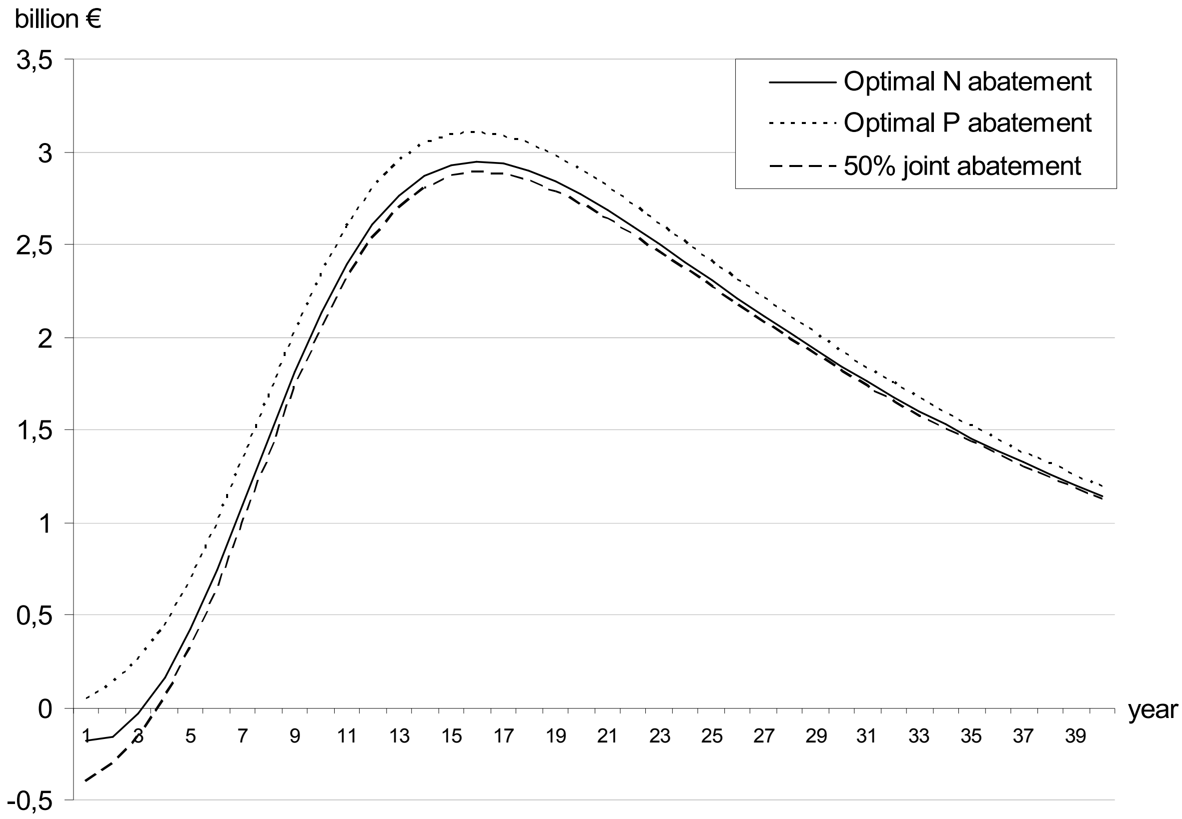

Figure 1: Total cost saving of alternative abatement strategies during the first 40 years (in net present value terms).

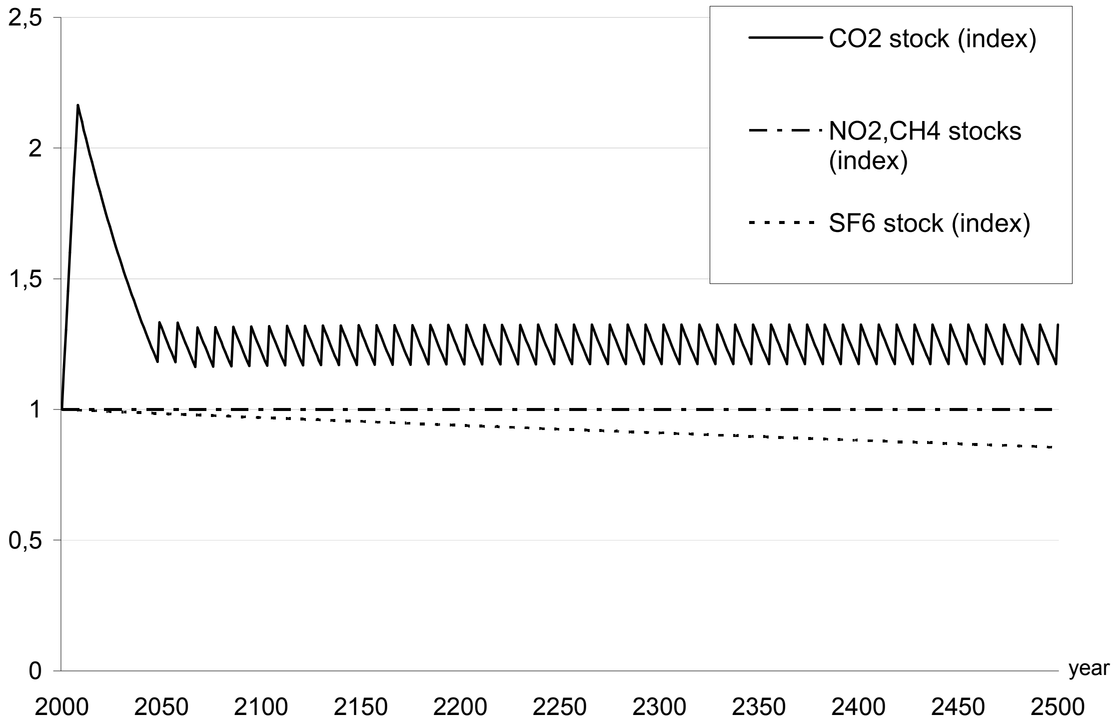

Figure 2: Changes in the stocks of greenhouse gases under the optimal abatement policy

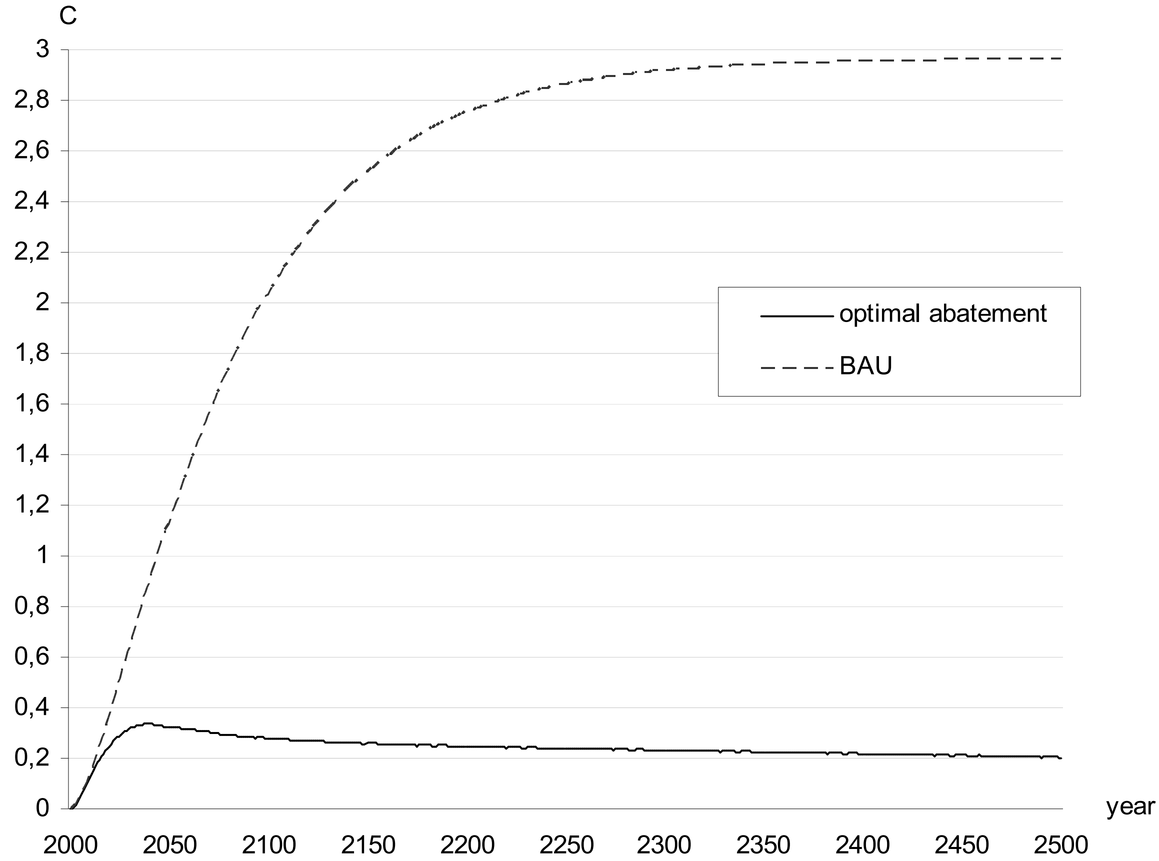

Figure 3: Increase in global temperature (in degrees Celsius) relative to the temperature level in the year 2000 in the "business as usual" (BAU) and optimal abatement scenarios

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