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The paper develops a model in which a forest owner decides an optimal rotation pattern for a forest consisting any number of even-aged stands. The owner has finite lifetime, nonforest income, bequest motive and access to perfect capital markets but may not have access to markets for forest land. The model is motivated by empirical findings suggesting that timber supply depends on variables that are absent from the original Faustmann-Pressler -Ohlin formulation. Given a specific form for the bequest motive, each stand is cut according to the Faustmann-Pressler-Ohlin rotation, but more generally the optimal rotations of different stands are linked together. Imperfect capital markets may shorten the rotation period and cause incentives to smooth forest income over time. The effects of *in situ* valuation on the rotation period depend on forest owner -specific factors like subjective time preference and forest owners age.

Pohjoismaissa ja Yhdysvalloissa huomattava osa raakapuusta tuotetaan yksityismetsänomistajien toimesta. Yksityismetsänomistajien puuntarjontapäätöksiin vaikuttavien tekijöiden ymmärtäminen on entistä tärkeämpää metsälakien uusimisen ja metsien *in situ* -arvostuksen korostumisen seurauksena. Tutkimuksen tavoitteena on kehittää taloustieteellistä kuvausta yksityismetsänomistajan päätöksenteosta siten, että kuvaukseen voitaisiin sisällyttää metsänomistajan muu talous ja metsien *in situ* -arvostus aikaisempia talousteoreettisia malleja realistisemmalla tavalla.

Faustmannin mallin ongelmana ovat empiiriset havainnot, joiden mukaan puun tarjontaan vaikuttavat Faustmannin mallista puuttuvat metsänomistajakohtaiset tekijät. Metsäekonomiassa näiden vaikutusta on tutkittu erityisesti kahden periodin mallilla. Tulosten mukaan puun tarjonta riippuu metsänomistajakohtaisista tekijöistä esimerkiksi, jos pääomamarkkinat ovat epätäydelliset, puun hintaan tai korkoon liittyy epävarmuutta tai, jos metsää arvostetaan *in situ*. Kahden periodin malli perustuu kuitenkin yksinkertaistukseen, jossa metsää kuvataan homogeenisena biomassana ilman ikäluokkia. Malli ei tämän seurauksena voi tuottaa tuloksia puustojen optimaalisista kiertoajoista ja saatujen tulosten empiirinen (käytännön) tulkinta on ongelmallista.

Faustmannin kiertoaikamallia on aikaisemmassa tutkimuksessa laajennettu sisältämään puuston mahdollinen *in situ* -arvostus sekä hintaan, että puuston kasvuun sisältyvä epävarmuus. Nämä laajennukset on kuitenkin tehty ottamatta huomioon metsänomistajan muun talouden vaikutuksia hakkuupäätöksiin. Kahden periodin mallilla saatujen tulosten perusteella, nämä kiertoaikamallin laajennukset ovat puutteellisia ja ehkä harhaanjohtavia.

Tässä tutkimuksessa kehitetään puuntarjontamalli, joka sisältää kuvauksen metsänomistajan taloudesta samalla kun metsää kuvataan todellisuutta vastaavasti ikäluokittain. Faustmannin mallin oletus täydellisistä maamarkkinoista korvataan metsänomistajan mahdollisella perinnönjättömotiivilla. Lisäksi tutkitaan epätäydellisten pääomamarkkinoiden ja *in situ* arvostusten vaikutusta puun tarjontaan. Tutkimuksessa kehitetyssä mallissa metsänomistajan ongelmana on ajoittaa tasaikäisten puustojen hakkuupäätökset elinkaarelleen samalla kun metsänomistajalla on mahdollisuus toimia pääomamarkkinoilla ja jättää perintöä jälkeläisille.

Faustmannin kiertoaika osoittautuu erikoistapaukseksi, joka toteutuu vain, jos "epätäydellisyydet" kuten *in situ* -arvostukset ja epävarmuus eivät vaikuta tarkasteltavan metsänomistajan eiväkä *kenenkään hänen tulevan* jälkeläisensä päätöksentekoon. Lisäksi Faustmannin ratkaisun toteutuminen edellyttää, että metsäperintöön saa liittyä vain rahallisia arvostuksia. Lisäksi perintöverotuksen tulee kohdistua tasapuolisesti metsään ja muuhun omaisuuteen. Jos nämä Faustmannin kiertoajan edellyttämät ehdot eivät ole voimassa, optimaaliset kiertoajat poikkeavat Faustmannin kiertoajasta ja metsänomistajan kaikkien puustojen hakkuupäätökset voivat olla sidoksissa toisiinsa. Toisaalta, jos Faustmannin kiertoaikaan liittyvät ehdot toteutuvat, ei yksittäisen metsänomistajan kohdalla synny taloudellisia kannustimia sopeuttaa puustokokonaisuutta kohti normaalimetsää. Toisin sanoen monissa metsän hakkuupäätöksiä kuvaavissa malleissa oletettu hakkuiden tasaisuuden vaatimus ei ole yksittäisen metsänomistajan kohdalla taloudellisesti perusteltavissa.

Tutkimuksessa tarkastellaan hakkuupäätöstä myös epätäydellisten pääomamarkkinoiden tapauksessa siten, että metsänomistajan lainansaanti on rajoitettua. Optimaalinen kiertoaika poikkeaa Faustmannin kiertoajasta ja on sidoksissa metsänomistajakohtaisiin tekijöihin. Karkea numeerinen tarkastelu osoittaa että, luotonsäännöstelyn seurauksena optimaalinen kiertoaika voi olla merkittävästi Faustmannin kiertoaikaa lyhyempi. Lisäksi metsänomistajalle syntyy taloudellinen kannustin korjata tasaikäinen puusto pienemmissä erissä eli hakkuiden tasaisuuden vaatimus saa taloudellista tukea.

Lopuksi mallia laajennetaan ottamaan huomioon puuston *in situ* -arvostus. Optimaalinen kiertoaika on tässäkin tapauksessa sidoksissa metsänomistajakohtaisiin tekijöihin, jotka ovat aikaisemmissa kiertoaikamalleissa sivuutettu. Lisäksi optimaalisen kiertoajan riippuvuus taloudellisista parametreista kuten puun hinnasta poikkeaa

aikaisemmassa tutkimuksessa saaduista tuloksista. Kiertoajan pituus ja samalla puun tarjonta ovat funktionaalisesti riippuvia esimerkiksi metsänomistajan iästä, varallisuudesta ja subjektiivisesta aikapreferenssistä.

Tutkimuksen tavoitteena oli ensisijaisesti kehittää puun tarjonnan talousteoreettista perustaa ja mallia, johon voidaan myöhemmässä tutkimuksessa sisällyttää metsään liitetyt ei-puuntuotannolliset arvot ja arvostukset. Jos tutkimustuloksia halutaan tulkita käytännönläheisemmästä näkökulmasta, ne osoittavat, että puustojen optimikiertoaikojen määrittäminen vain arvokasvun ja maakoron avulla on mahdollista ainoastaan poikkeustapauksissa. Metsänomistajakohtaisten tekijöiden (subjektiiviset arvot ja arvostukset, tulot, ikä ja varallisuus) huomiotta jättäminen saattaa aiheuttaa merkittäviä taloudellisia hyvinvointitappioita. Kehitetyllä mallilla on lisäksi merkitystä metsämaan arvon määrittelyssä ja suunniteltaessa metsänomistajien hakkuupäätöksiä ohjaavia taloudellisia kannustimia kuten verotusta ja ympäristötukea.

Key words: Timber supply, even-aged management, forest rotation, amenity values

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Content

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1 Introduction

Although the Faustmann-Pressler-Ohlin (FPO) forest rotation model (Johannson and Löfgren 1985, p. 73) has maintained its importance in forest economics, its ability to explain timber harvesting decisions has been questioned. In the Nordic Countries and USA a significant part of timber supply comes from private nonindustrial forest owners. Several studies have found that cutting decisions depend on forest owner -specific variables that are missing from the FPO model such as nontimber incomes and wealth (Binkley 1981, Dennis 1990, Kuuluvainen et al. 1996). Attempts to predict forest harvesting decisions have become increasingly important because of the recent tendency to liberalize public regulation of nonindustrial private forest (NIPF) owners simultaneously with increasing *in situ* value of forests (Hultkranz 1992). This study presents the FPO model in the continuous time life cycle context and offers a new approach for understanding the harvesting decisions of NIPF owners.

As described by Binkley (1981) the first reaction to the above -mentioned empirical findings was that timber supply should be explained by both economic and "noneconomic" factors. However, he shows that a static timber supply can cover forest owner -specific factors as well. More recently, NIPF owners' behavior has been explained by the Fisherian two-period model (Lohmander 1983) augmented by the dynamics of forest growth. Using this formulation timber supply is studied under capital market imperfections, uncertainty as to prices and interest rates (Koskela 1989, Kuuluvainen 1990, Ollikainen 1996) and *in situ* values (Ovaskainen 1992). With these extensions, the Fisherian separation may not hold and timber supply depends e.g. on preferences and nontimber income. Most recently the model has been extended to the OLG framework (Löfgren 1990, Hultkrantz 1992, and Ollikainen 1996).

The two-period model has greatly increased our understanding of timber supply, but the results have not been obtained without costs. The two-period model and its continuous time version (Binkley 1987, Kuuluvainen and Salo 1991) neglects forest age-classes. Instead of analyzing the rotation problem, the model considers the problem of maintaining a biomass at its optimal level over time (cf. the fishery model of Clark 1990). Since the age of stands is an essential factor in forest harvesting, this application has been criticized by Johansson and Löfgren (1985, p. 55) and Binkley (1987). Recently the model has been interpreted to describe uneven-aged forest management (Montgomery and Adams 1995). However, in boreal forests, forestry is largely based on even-aged stands. Binkley (1987) shows that models based on homogeneous biomass do not correspond directly with optimal rotation analysis[1].

Along with the development of the two-period forest model the FPO rotation analysis has been extended toward several directions. Hartman (1976) adds *in situ* valuation. Uncertainty in the form of stochastic price has been studied e.g. by Norström (1975) and Brazee and Mendelsohn (1988). However, referring to results from the two-period tradition, we can hypothesize that when the FPO model is extended to include *in situ* preferences or uncertainty, Fisherian separation may not hold. The model should also include consumption/saving decisions and forest owner -specific features such as subjective time preference and nonforest income. Among other things, these notions may change the results as to how different types of forest taxation affect the rotation period.

The strength of the two-period model is that it includes consumption/saving decisions, but it suffers from neglect of forest age-classes. By contrast the strength of the FPO extensions is the inclusion of age-class structure while the problem is neglect of owner -specific factors. An attempt to include nonforest income in the FPO model is that of Hyberg and Holthausen (1989), but their model also excludes the consumption/savings decision.

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[1]Period t+1 harvestable timber is given by $x_t-h_t+F(x_t-h_t)$, where F is a (strictly) concave growth function and h_t is the harvest in period t. Under constant prices and separability conditions, the first period cuttings satisfy $F'=\rho$, where ρ is the rate of interest. These equations are identical to those of the discrete time version of Clark's (1990) fishery model.

The model presented in this paper observes the FPO approach while including a continuous-time description of life cycle decisions. The owner inherits a forest consisting of $n \geq 1$ stands. The stands may differ in age, growth, species, timber price and harvesting and regeneration costs. The owner's problem is to decide whether and when to harvest the stands within his life time and what amount and form of bequest to leave for his heirs. Along with the cutting decision, the owner decides his consumption and savings, including the pecuniary bequest. Nonforest assets can be consumed or saved, and their amount can be increased by nonforest income, net revenues from harvests and interest on investments. Technically, we include the FPO model in the life cycle model of Yaari (1964). When the original FPO model is applied to NIPF owners, the infinite horizon reflects competitive land markets. However, land markets are seldom competitive and a major share of NIPF owners inherit their forests (Hultkrantz 1992). This raises additional unresolved problems, and both the determination of the rotation period and the role of bequest motives are open. In contrast to Mitra and Wan (1985) we assume that the forest owner has access to perfect capital markets.

This study shows that under perfect capital markets, without *in situ* values and uncertainty, the harvesting and consumption decisions are separable, given the bequest motive has a specific form. As a bequest, the forest land must be a perfect substitute for other assets. If the forest owner or any future generation deviates from such a bequest motive, harvesting of all stands are linked together and depend on all the properties of the present and (perhaps) future forest owners. The owner applies the FPO rotation if the value of land for his heirs is expected to be given by the FPO formula and this holds for all future generations. Thus a specific form of bequest motive yields FPO rotation and "replaces" the land markets. However, an inheritance tax that treats forest and other assets asymmetrically or an expected future *in situ* value, for example, would suffice to produce a deviation from FPO rotation and its comparative statics. These results show that describing forests as homogeneous biomass neglects part of the long-term decisionmaking that makes the forest a unique economic resource. Because of perfect capital markets there may not be any convergence toward a normal forest at a level of a single "tree farm" (cf. Mitra and Wan 1985).

The rotation analysis under imperfect capital markets is virtually nonexistent in forest economics. As a step in this direction, we next consider a simplified version of the model with credit rationing. It is shown that at the moment of harvesting the consumption level jumps up. Imperfect capital markets create incentives to smooth the forest income over time, implying that stands cannot be cut independently and that two perfectly similar stands may not be cut simultaneously. Credit rationing may decrease rotation length and timber supply and using the FPO rotation under imperfect capital markets may lead to serius deviation from the optimal harvesting policy. These results are new and cannot be obtained when forests are described without the age classes.

Finally, we study *in situ* preferences with competitive capital markets and without bequest motives. The rotation exceeds the FPO solution and all the comparative statics results deviate from the existing literature based on Hartman (1976). In addition, the rotation depends on owner-specific factors and e.g. if the rate of interest exceeds the subjective time preference the rotation length increases with the owner's age. The study generalizes the FPO model to include the forest owner-specific factors. Although such a model has obvious demand in empirical studies, it has been absent in the forest economic literature.

The paper is organized as follows. Section 2 contains the life cycle model with n even-aged stands and derives the optimal cutting rule. Section 3 develops the FPO rotation as a special case. Section 4 considers optimal rotation under credit rationing, and section 5 deals with *in situ* preferences. Section 6 concludes the study.

2 Life cycle consumption decisions and even-aged forest harvesting

Assume that any economically relevant rotation period is longer than the period for which a forest owner makes his harvesting decisions. For example, in boreal forests rotation periods typically vary from 50 to 150 years but on average forest owners hold their forests for only about 30-50 years. The owner's problem is now to choose the year for harvesting each of his stands within his life cycle after inheriting a forest consisting of n even-aged stands. We define a stand as the smallest unit of trees that can be harvested and sold one by one. Physically this smallest unit is one tree since it is impossible to harvest half a tree and leave the other half growing. However, we may as well assume that a stand is the number of trees in one hectare since it is normally impossible to sell coniferous trees in smaller units. As for the forest owner's preferences we assume that he has preferences regarding the bequest he will leave for his heirs. During his life cycle he can borrow and lend money from perfect capital markets. The possibility of selling forest land during the life cycle is neglected on the assumption that well-functioning land markets do not exist.

We first specify the optimal consumption schedule for the period following the last harvest. The maximum utility for this period will depend on the moment of last harvest and the level of nonforest and forest assets at the beginning of the period. After solving this problem we can use the obtained value function to solve the problem for the period between the last and second-to-the-last cutting. This chain of interrelated optimization problems can be extended backwards to the beginning of the forest owner's life cycle. Figure 1 depicts the evolution of the forest for the three-stand case. With n stands, there are n+1 periods and a chain of n+1 optimization problems.

Denote the volume of any stand by x_i (m^3), i=1,...,n. For any stand, the annual growth is given by $F_i(x_i)$ (m^3), which is a concave growth function with $F_i(0)=F_i(\bar{x}_i)=0$,

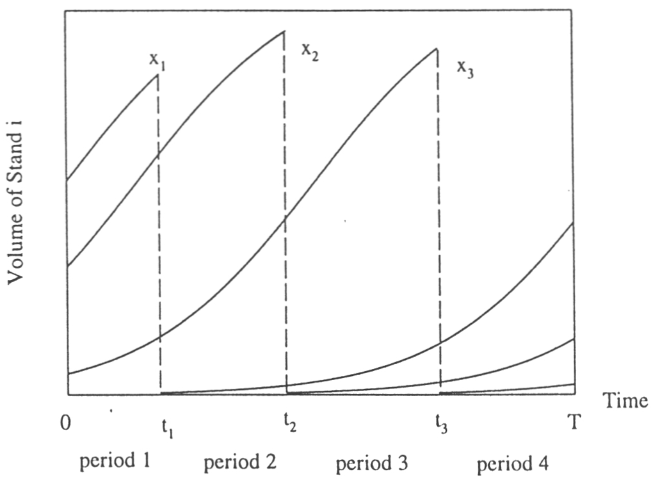

Figure 1. Optimization periods with three stands.

where $\bar{x}_i>0$. We will frequently assume that the growth function satisfies the logistic form. The date for cutting the ith stand is t_i and the length of the forest owner's life is T years (after inheriting the forest). U(c) is an increasing and strictly concave utility function with the inada conditions and $B[a(T),x_1(T),\ldots,x_n(T)]$ an increasing and concave bequest function, where nonforest assets are denoted by a(T). The annual interest rate is ρ and δ is the subjective rate of discount. Let m(t) denote the annual nonforest income, $p_i(t)$ the net price of harvested stand i (i.e. net of cutting costs) and w_i the planting costs. Because some variables of this problem may jump at t_i we apply the notation $t_i=t_i^-$ when t approaches t_i from below.

The problem for the period after the last cutting, i.e. for $t\in[t_n,T]$ takes the form:

$$W_{n+1}=\max_{\{c,a(T)\}} \int_{t_n}^{T} U(c)e^{-\delta t}dt+e^{-\delta T}B\left[a(T),x_1(T),...,x_n(T)\right], \qquad (1)$$

s.t.

$$\dot{a}=\rho a+m-c, \qquad (2)$$

$$a(t_n)=a(t_n^-)+p_n(t_n^-)x_n(t_n^-)-w_n, \qquad (3)$$

$$x_n(t_n)=x_{n_{t_n}}, \qquad (4)$$

$$x_i(t_n)=x_i(t_n^-) \text{ for all } i\neq n, \qquad (5)$$

$$\dot{x}_i=F_i(x_i),\ i=1,...,n. \qquad (6)$$

Note that in (1) utility is given in terms of t=0. At the beginning of the last period ($t=t_n$), nonforest assets equal nonforest assets at the end of the second-to-the-last period $a(t_n^-)$ plus the net benefits from the last cutting $p_n(t_n^-)x_n(t_n^-)-w_n$ (equation 3). Planting costs w_n are assumed to follow the clearcutting[2]. Equation (4) defines the (given) initial level of the new stand after planting. Equation (5) shows that the volume in stands not cut at t_n are continuous. Finally, equation (6) gives growth for the i=1,...,n stands.

Let the costate for nonforest assets be λ and φ_i the costate for the ith stand. The current value Hamiltonian (with i=n+1) and the necessary conditions for optimum are:[3]

$$H_i=U(c)+\lambda(\rho a+m-c)+\sum_{i=1}^{n}\varphi_i F_i(x_i), \qquad (7)$$

$$U'(c)-\lambda=0, \qquad (8)$$

$$\dot{\lambda}=\lambda(\delta-\rho), \qquad (9)$$

$$\dot{\varphi}_i=\varphi_i\left[\delta-F_i'(x_i)\right],\ i=1,...,n, \qquad (10)$$

$$\lambda(T)=\partial B\left[a(T),x_1(T),...,x_n(T)\right]/\partial a(T), \qquad (11)$$

$$\varphi_i(T)=\partial B\left[a(T),x_1(T),...,x_n(T)\right]/\partial x_i(T),\ i=1,...n. \qquad (12)$$

together with conditions (2)-(6). W_{n+1} depends on the length of the optimization period and

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[2]The planting decision can be studied as in the FPO model.

[3]Seierstad and Sydsæter (1987, p. 182), theorem 3. By theorem 4 (p. 182) the conditions are sufficent.

the level of assets at t_n^-. We write $W_{n+1}=W_{n+1}[a(t_n^-),x_1(t_n^-),...,x_n(t_n^-),t_n]$ and assume that this function is differentiable w.r.t. $a(t_n^-)$, $x_i(t_i^-)$, for i=1,...,n and t_n. It is now possible to proceed period by period backwards toward t=0 and for any period i=2,...n, we obtain the following optimization problem:

$$W_i=\max_{\{c,t_i,\ a(t_i^-),\ x_1(t_i^-),\dots,x_n(t_i^-)\}} \int_{t_{i-1}}^{t_i} U(c)e^{-\delta t}dt+W_{i+1}[a(t_i^-),x_1(t_i^-),...,x_n(t_i^-),t_i], \tag{13}$$

$$\text{s.t.} \quad a(t_{i-1})=a(t_{i-1}^-)+p_{i-1}(t_{i-1}^-)x_{i-1}(t_{i-1}^-)-w_{i-1} \tag{14}$$

$$x_{i-1}(t_{i-1})=x_{i-1_{t_{i-1}}}, \tag{15}$$

$$x_j(t_{i-1})=x_j(t_{i-1}^-),\ \text{for all } j\neq i\text{-}1, \tag{16}$$

together with conditions (2) and (6). When i=1 we have the first period. In this case we maximize (13) with i=1 subject to (2), (6), $a(0)=a_0$, and the initial volumes in each stand $x_i(0)=x_{i0}$, i=1,...,n. If there are cuttings at t=0, we include their net benefits in the initial value of nonforest assets a_0.

Again the Hamiltonian is given by (7), and the necessary conditions for optimality are:

$$\lambda(t_i^-)=e^{\delta t_1}\partial W_{i+1}[a(t_i^-),x_1(t_i^-),...,x_n(t_i^-),t_i]/\partial a(t_i^-), \tag{17}$$

$$\varphi_j(t_i^-)=e^{\delta t_1}\partial W_{n+1}[a(t_i^-),x_1(t_i^-),...,x_n(t_i^-),t_i]/\partial x_j(t_i^-),\ j=1,...n, \tag{18}$$

$$H_i(t_i^-)=H_{i+1}(t_i)-\lambda(t_i)\dot{p}_i(t_i^-)x_i(t_i^-). \tag{19}$$

together with (2), (6), and (8)-(10)[4]. Note that in (19) we have taken into account two effects of t_i on W_{i+1}. The first effect is $H_{i+1}(t_i)$, i.e. increasing t_i decreases the length of the period after the last cutting. The second effect follows if $\dot{p}_i(t_i^-)\neq 0$ because increasing t_i changes the initial value of the next period nonforest assets.

A priori, it is possible to obtain cases where interior solutions for some of the

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[4]Seierstad & Sydsæter (1987) theorem 4 and note 2, pages 182, 183 and theorem 9 page 213.

most mature stands do not exist, and it may then be optimal to cut these stands immediately at $t=0$. In addition, there is no need to exclude an option where some of the stands are not cut at all nor the possibility that there are boundary solutions such that for some stands $t_i=T$. The restrictions $T \geq t_n$ and $t_i \geq t_{i-1}$ $i=1,\ldots,n$, can be included but here they are neglected for simplicity.

We study the optimal cutting moments. By (19), (7), (14) and (15) we obtain

$$\left.\begin{aligned}
&U[c(t_i^-)]+\lambda(t_i^-)[\rho a(t_i^-)+m(t_i^-)-c(t_i^-)]+\sum_{j=1}^{n}\varphi_j(t_i^-)F_j[x_j(t_i^-)]+\varphi(t_i^-)F_i[x(t_i^-)]= \\
&U[c(t_i)]+\lambda(t_i)\{\rho[a(t_i^-)+p_i(t_i^-)x_i(t_i^-)-w_i]+m(t_i)-c(t_i)\} \\
&+\sum_{j=1}^{n}\varphi_j(t_i)F_j[x_j(t_i)]+\varphi(t_i)F_i[x_{i_{t_i}}]-\lambda(t_i)\dot{p}_i(t_i^-)x_i(t_i^-), \text{ for } i=1,\ldots,n \text{ and } j\neq i.
\end{aligned}\right\} \quad (20)$$

By theorem 9 in Seierstad and Sydsæter (1987, page 213) we obtain $e^{\delta t_1}\partial W_{i+1}/\partial a(t_i)=\lambda(t_i)$. (14) implies $\partial a(t_i)/\partial a(t_i^-)=1$. Thus by (17) $\lambda(t_i^-)=\lambda(t_i)$ and by (8) $c(t_i^-)=c(t_i)$. Note that the states and costates for $j=1,\ldots,n$, $j\neq i$ are continuous (equations 16 and 18). Now equation (20) simplifies to: $\varphi(t_i^-)F_i[x_i(t_i^-)]= \lambda(t_i)\rho[p(t_i^-)x_i(t_i^-)-w_i]+\varphi_i(t_i)F_i[x_{i_{t_i}}]-\lambda(t_i)\dot{p}(t_i^-)x_i(t_i^-)$, for $i=1,\ldots n$, where it is assumed that m is continuous at t_i. Next $\partial a(t_i)/\partial x(t_i^-)=p_i(t_i)$ and (17) and (18) yield $\varphi_i(t_i^-)=p_i(t_i^-)\lambda(t_i)$. We obtain

$$p_i(t_i^-)\lambda(t_i)\{F_i[x_i(t_i^-)]-x_i(t_i^-)[\rho-\dot{p}_i(t_i^-)/p_i(t_i^-)]+\rho w_i/p_i(t_i^-)\}=\varphi_i(t_i)F_i(x_{i_{t_i}}),\ i=1,\ldots n. \quad (21)$$

For interpretation of the term $\varphi(t_i)F_i[x_{i_{t_i}}]$ in (21) note that after cutting at t_i the volume of stand i is a function of time and the cutting moment. Thus $x_i(t_i,\tau)=\int_{t_i}^{\tau}F_i[x_i(s)]ds+x_{i_{t_i}}$. By equation (10) we obtain a linear first order differential equation with nonconstant coefficient; its solution can be written as

$$\varphi_i(t_i)=\varphi_i(T)e^{-\int_{t_i}^{T}\{\delta-F_i'[x_i(t_i,\tau)]\}d\tau},\ i=1,\ldots n. \quad (22)$$

Integration, applying the logistic growth function (22) and (12) yields

$$\varphi_i(t_i)F_i[x_{i_{t_i}}]=\varphi_i(T)e^{-\int_{t_i}^{T}\{\delta-F_i'[x_i(t_i,\tau)]\}d\tau}F_i[x_{i_{t_i}}]=\varphi_i(T)e^{-\delta(T-t_i)}F_i[x_i(T)]=$$
$$\{\partial B[a(T),x_1(T),...,x_n(T)]/\partial x_i(T)\}F_i[x_i(T)]e^{-\delta(T-t_i)},\ i=1,...n. \qquad (23)$$

Substituting (23) for $\varphi(t_i)F_i(x_{i_{t_i}})$ in (21) gives an equation for the optimal cutting moment. According to (23) the RHS of (21) equals the present value of a decrease in bequest evaluated at t_i caused by a marginal increase in the growing period for the inherited stand. The LHS of (21) is the increase in utility at t_i due to a marginal increase in the growing period for the initial forest stock. This equals the value of additional growth, $p_i(t_i^-)\lambda(t_i)\{F_i[x_i(t_i^-)]$ minus the interest on the value of harvest at t_i, $p_i(t_i^-)\lambda(t_i)x_i(t_i^-)[\rho-\dot{p}_i(t_i^-)/p_i(t_i^-)]$ plus the gain in interest due to a delay in the regeneration costs, $\lambda(t_i)\rho w_i$. All terms are evaluated in current value utility units at t_i. Note that the rate of interest ρ, is "corrected" by the rate of price change at t_i. More briefly (21) specifies the cutting moment as the moment where the net gain due to an incremental increase in the length of the growing period of a stand equals the associated loss in the value of bequest evaluated at t_i.

In equations (21) and (23), the terms a(T) and $\lambda(t_i)$ depend on the forest owner's life-cycle consumption schedule. In addition, the cuttings of the stands may be connected through the bequest function. Thus the cutting moment for any stand depends, in general, on the cutting of all other stands, nonforest income, consumption preferences, time preference, and initial nonforest assets, in addition to forest growth, prices, planting costs and the rate of interest.

In the case of one stand, equations (21) and (23) form one equation with two unknowns, t_1 and $\lambda(t_1)$. By (9) the latter equals $\lambda_0 e^{t_1(\rho-\delta)}$, where λ_0 is unknown. For determining these unknowns, (8) implies consumption as a function of time and λ_0: $c=c(\lambda_0,t)$. Equation (2) and the initial value of nonforest assets implies a(t) for $t\in[0,t_1^-)$.

Next (14) and (2) yields a(T) as a function of t_1 and λ_0. This together with (11) implies λ_0 as a function of t_1, which makes it possible to solve the optimal cutting moment by equation (21). When the number of stands is higher, following the above procedure gives λ_0 as a function of all the cutting moments: $\lambda_0=\lambda_0(t_1,...,t_n)$. Equation system (21) can then be used to solve for the optimal cutting moments. We next specify some examples for the bequest function that separate the consumption and cutting decisions.

3 The FPO rotation

Assume that nonforest assets and forest land are perfect substitutes as a bequest and that the bequest function is: $B=B\left[a(T)+\sum_{i=1}^{n} V_i\right]$, where V_i is the owner's expectation for the monetary value of stand i. Note that this rules out all future capital market imperfections and *in situ* preferences. V_i may depend on the stand volume at t=T, on expected long-run values for timber prices $\bar{p}$, rates of interest $\bar{\rho}$ and planting costs $\bar{w}$. Thus $V_i=V_i\left[x_i(T),\bar{p}_i,\bar{\rho},\bar{w}_i\right]$. In general, V_i may reflect any future rotation program and may be determined by numerous factors such as, inheritage taxation that favors forest bequest, expected changes in prices, interest rates and biotechnology. Assume that any V_i is higher, the higher the stand volume, i.e. $\partial V_i/\partial x_i(T)>0$. Equation (11) yields $\lambda(T)=B'$, which by (9) implies $\lambda_0=e^{(\rho-\delta)T}B'$. By equation (12), $\varphi_i(T)=B'\partial V_i/\partial x_i(T)$. From equations (21) and (23) one obtains for i=1,...n,

$$p_i(t_i-)\{F_i[x_i(t_i-)]-[\rho-\dot{p}_i(t_i-)/p_i(t_i-)]x_i(t_i-)+\rho w_i/p_i(t_i-)\}-$$
$$e^{-\rho(T-t_i)}[\partial V_i/\partial x_i(T)]F_i[x_i(T)]=0. \qquad (24)$$

This is a cutting rule where the optimal harvesting moment depends on stand growth, prices net of cutting costs, rate of price change, regenerating costs, marginal valuation of the stand at T, and the length of the life cycle. Of these, the expected value of the stands and the length of life cycle are forest owner -specific. However, in spite of the bequest motive, the cutting moment is independent of such forest owner -specific features as nonforest income and preferences. Another implication of perfect substitutability is that the different stands are cut independently.[5] We next show that the FPO rotation is a special case of rule (24).

Denote the value of the land area for stand i under the FPO rotation by $\bar{V}_i$. Thus $\bar{V}_i=[px_i(t_i^f)e^{-\rho t_i^f}-w_i](1-e^{-\rho t_i^f})^{-1}$, where t_i^f is the FPO rotation period and p is the constant price for timber. Recall that $\bar{V}_i$ is the maximized value of forest land just after the regeneration. The value $T-t_i$ years after the regeneration equals $e^{\rho(T-t_i)}(\bar{V}_i+w_i)$ (c.f. Johansson and Löfgren 1985, p. 85).

Proposition 1. Given that $\dot{p}=0$, $B\equiv B\{a(T)+\sum_{i=1}^{n}(\bar{V}_i+w_i)e^{\rho(T-t_i)}\}$ and that $F_i(x_i)$, $i=1,...n$, satisfy the logistic form, then t_i , for $i=1,...,n$, equals the FPO rotation period.

Proof: The task is to show that $\varphi(t_i)F_i[x_{i_{t_i}}]=\lambda(t_i)\rho(\bar{V}_i+w_i)\lambda_0e^{(\delta-\rho)t_i}$ because then (21) yields the FPO rotation. For this purpose we compute (23) using the assumptions above. In differentiating the bequest function w.r.t. $x_i(T)$, note that at T the level of $x_i(T)$ is a (decreasing) function of t_i (by the properties of F_i). The inverse of this function gives t_i as a function of $x_i(T)$. Denote this by $t_i=t_i[x_i(T)]$. Using the logistic growth function we obtain $x_i(T)=K_i/[1+c_ie^{-r_i(T-t_i)}]$, where $c_i=e^{r_it_i}[(x_{i}t_i-K_i)/x_{i}t_i]$. This can be solved for t_i, implying $t_i'[x_i(T)]=K_i/\{x_i(T)r_i[x_i(T)-K_i]\}$, where $x_i(T)=K_i/(1+c_ie^{-r_iT})$. Using this, (11) and the assumed form for the bequest function implies

$$\partial B\{a(T)+\sum_{i=1}^{n}(\bar{V}_i+w_i)e^{\rho(T-t_i)}\}/\partial x_i(T)=-\lambda(T)t_i'[x_i(T)]\rho(\bar{V}_i+w_i)e^{\rho\{T-t_i[x_i(T)]\}},$$

where the explicit form of $t_i'[x_i(T)]$ is given above. The equation above, (23) and $\lambda(t)=\lambda_0e^{(\delta-\rho)t}$ yield

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[5]By differentiating (24) it can be shown that the optimal rotation period is longer, the lower the value of $\partial V_i/\partial x_i(T)$, i.e. the effect of stand volume on land value, the higher the planting costs and the greater the price increase. The effects for short-run timber price and interest rate are ambiguos.

$$\varphi_i(t_i)F_i[x_it_i]=-\rho(\bar{V}_i+w_i)t_i'[x_i(T)]F_i[x_i(T)]\lambda_0e^{t_i(\delta-\rho)},$$

where $t_i'[x_i(T)]F_i[x_i(T)]=-1$ completes the proof.■

We have shown that in the absence of "imperfections" like *in situ* preferences or imperfect capital market, the bequest motive may guarantee the FPO rotation and efficient allocation of total assets between forest land and monetary assets. However, the requirements for the FPO rotation are rather restrictive. We hypothesize that the forest owner cannot have *in situ* preferences; he must have access to perfect capital markets; the price, interest rates and forest growth must be known with certainty; and the bequest motive must satisfy the specification in Proposition 1. In addition, these requirements must be expected to hold for all future generations up to infinity.

In comparison with the above results the homogeneous biomass forest models neglect part of the long-run dynamics involved in forestry. In these models the stock level can be adjusted between different singular paths within a relatively short period of time and e.g. neglecting the bequest motive does not typically change the first period harvest in the two period model (e.g. Kuuluvainen and Salo 1991). In our model neglect of the bequest motive always changes the current generations harvesting decisions and also has permanent consequences on the future rotation program.

Mitra and Wan (1985) ask what is the optimal pattern of harvesting trees if a forest owner (with infinite life time) obtains utility in any time period, which is determined by the trees harvested in that period. The answer is complicated but e.g. with strictly concave utility function and zero rate of discount it is optimal to adjust the forest toward a normal forest, where each stand is cut according the FPO rotation. With discounting, there may not occur converge toward the normal forest. Their model has many interesting interpretations, but as a description of NIPF owner's problem it neglects capital markets. Above we have considered a similar question, except that the owner has a finite lifetime, bequest motive, and access to perfect capital markets. Under the conditions in Proposition

1, a forest owner with n≥1 similar stands of the same age will cut them at the same moment and still have a continuous consumption schedule. Due to perfect capital markets there are no incentives to smooth the forest incomes in time and convergence toward the normal forest does not occur.

4 Imperfect capital markets

The above analysis suggests that the FPO solution and related consumption path depend strongly on perfect capital markets. Several authors (e.g. Samuelson 1976) stress the importance of extending the optimal rotation analysis to the case of imperfect capital markets. However, this question has been studied mainly in the context of the two-period model (Koskela 1989, Kuuluvainen 1990, Ollikainen 1996) and optimal rotation analysis with imperfect capital markets is virtually nonexistent. As a step in this direction, we study our model with one stand and without nonforest income. This yields a theoretical formulation and reveals a route for studying the difficult case of imperfect capital markets with multiple stands.

We introduce imperfect capital markets in the form of credit rationing and assume that nonforest assets must be nonnegative. With one stand and without nonforest income, the owner's problem is to

$$\max_{\{c\geq 0,t_1\}} \quad W_1=\int_0^{t_1} U(c)e^{-\delta t}dt+W_2[a(t_1^-),x(t_1^-),t_1] \tag{25}$$

$$\text{s.t.} \quad \dot{a}=\rho a-c,\ a(0)=a_0,\ a(t_1^-)\geq 0, \tag{26}$$

and $\dot{x}=F(x)$, $x(0)=x_0$, where t_1 is the cutting moment and W_2 is the maximized value of a consumption/ rotation program after the first harvest at t_1. Note that the inada conditions imply c>0 and that it is enough to have an endpoint constraint $a(t_1^-)\geq 0$ on the assets to be consumed before the first harvest.

The problem (25)-(27) may take different forms depending on the assumptions as to bequest motives. We concentrate on a theoretical case where $W_2[a(t_1^-),x(t_1^-),t_1]$ is a value function of an optimization problem identical to that stated above and the situation holds forever. This means that we either assume that each human generation has perfectly altruistic bequest motives or that the forest owner lives infinitely.

We obtain the Hamiltonian $H=U(c)+\lambda(\rho a-c)+\varphi F(x)$ and the following necessary conditions: $U'(c)-\lambda=0$, $\dot{\lambda}=\lambda(\delta-\rho)$, $\dot{\varphi}=\varphi[\delta-F'(x)]$ and

$$U[c(t_1^-)]+\lambda(t_1^-)[\rho a(t_1^-)-c(t_1^-)]+\varphi(t_1^-)F[x(t_1^-)]=$$

$$U[c(t_1)]+\lambda(t_1)\{\rho[a(t_1^-)+px(t_1^-)-w]-c(t_1)\}+\varphi(t_1)F(x_{t_1}) \qquad (27)$$

$$[\lambda(t_1^-)-\lambda(t_1)]\geq 0,\ [\lambda(t_1^-)-\lambda(t_1)]a(t_1^-)=0,\ a(t_1^-)\geq 0, \qquad (28a,b,c)$$

$$\varphi(t_1^-)=p\lambda(t_1), \qquad (29)$$

where we assumed that $\dot{p}=0$. When studying a problem where the only source of income is forestry, it is natural to study a stationary solution and assume $a_0=px(t_1^-)-w$ in equation (26). We can study two cases depending on whether the credit rationing is binding.

Case $\delta-\rho\leq 0$: In this case a solution where consumption is continuous and where the rotation length equals FPO satisfies the necessary conditions. To show this we can argue precisely as for equation (23) to obtain $\varphi(t_1)F(x_{t_1})=\varphi(t_2^-)e^{-\delta(t_2-t_1)}F[x(t_2^-)]$, where t_2 is the date for the second cutting moment. In the FPO solution the rotation period is constant, implying $t_2=2t_1$ and $x(t_2^-)=x(t_1^-)$. Thus $\varphi(t_2^-)e^{-\delta(t_2-t_1)}F[x(t_2^-)]=\varphi(t_2^-)e^{-\delta t_1}F[x(t_1^-)]$. Equation (29) implies $\varphi(t_2^-)=p\lambda(t_2)=p\lambda_0 e^{(\delta-\rho)2t_1}$ and yields with $\lambda=\lambda_0 e^{t(\delta-\rho)}$ that $\varphi(t_2^-)e^{-\delta t_1}F[x(t_1^-)]=p\lambda_0 e^{(\delta-\rho)t_1-\rho t_1}F[x(t_1^-)]$. Next, by (27) and (29)

$$pF[x(t_1^-)]=\rho[px(t_1^-)-w]+pe^{-\rho t_1}F[x(t_1^-)], \qquad (30)$$

which is one form of the FPO formula. Note that this result is perfectly in line with Proposition 1 because when $\delta \leq \rho$ and when we consider a stationary solution where the initial assets equal the net income from optimal harvest, the credit rationing constraint cannot be binding.

Case $\delta > \rho$: The case where $\lambda(t_1^-)-\lambda(t_1)=0$ and $a(t_1^-)\geq 0$ in (28) implies that λ is continuous and that $\lambda \to \infty$ and $c \to 0$ as $t \to \infty$. This cannot be optimal in the case of credit rationing and continuously available income from forestry. Thus $a(t_1^-)=0$ and $\lambda(t_1^-)>\lambda(t_1)$, implying that $c(t_1^-)<c(t_1)$. In a stationary solution the cycles are repeated as perfectly similar, implying that $c(t_1)=c(0)$ and $\lambda(t_1)=\lambda_0$.

Thus when the credit rationing constraint is binding the value of λ jumps down and consumption up at each harvesting moment. This follows because due to the constraint $a(t)\geq 0$, consumption possibilities cannot be transferred from the future to the present. A transfer from present to future is possible by saving and thus downward jumps in c are ruled out, as implied by (28a). Between rotations the shadow price of saved forest income increases exponentially, implying that consumption decreases between the moments of forest harvesting.

For studying the optimal cutting moment, note that (29) yields $\varphi(t_1^-)=p\lambda_0$ and next applying (23) $\varphi(t_1)F(x_{t_1})=p\lambda_0 F[x(t_1^-)]e^{-\delta t_1}$. Equation (27) takes the form

$$U[c(t_1^-)]-U'[c(t_1^-)]c(t_1^-)+U'(c_0)pF[x(t_1^-)]=$$
$$U(c_0)+U'(c_0)\{\rho[px(t_1^-)-w]-c_0\}+U'(c_0)pF[x(t_1^-)]e^{-\delta t_1}, \quad (31)$$

where $c_0=c(0)$. The RHS of (31) is the current value Hamiltonian after the first harvest. Appendix 1 shows that Hamiltonian at t_1 equals the value of the optimal solution for $t\in[t_1,\infty)$ multiplied by the rate of time preference, i.e. $\delta W_2=H(t_1)$. Thus equation (31) gets the form $U[c(t_1^-)]-U'[c(t_1^-)]c(t_1^-)+U'(c_0)pF[x(t_1^-)]=\delta W_2$. The term $U'(c_0)pF[x(t_1^-)]$ on the LHS is the marginal increase in stand value if the growing period is marginally longer. Because an increase in stand volume increases the stock of assets to be consumed before the next cut, this term is given in utility units at t_1. Lengthening the rotation also changes the consumption profile before the cut as reflected by the terms $U[c(t_1^-)]-U'[c(t_1^-)]c(t_1^-)$. The term $U[c(t_1^-)]$ is the utility from extending the rotation and $U'[c(t_1^-)]c(t_1^-)$ is the cost of producing this prolongation by marginally decreasing the rate of consumption before the cut. The RHS is the cost of postponing the whole future rotation/consumption program. Thus equation (31) can be compared with the FPO rule given in the form $pF=\rho[px(t_1^-)-w]/(1-e^{-\rho t_1})$, where the RHS is the rate of discount multiplied by the value of the mature stand.

Write equation (31) in the form $U'(c_0)\{pF[x(t_1^-)]-\rho[px(t_1^-)-w]-F[x(t_1^-)]e^{-\delta t_1}\}-G$, where $G=U(c_0)-U'(c_0)c_0-\{U[c(t_1^-)]-U'[c(t_1^-)]c(t_1^-)\}$. A comparison with equation (30) reveals that with FPO rotation and $\delta>\rho$ the term $pF[x(t_1^-)]-\rho[px(t_1^-)-w]-pF[x(t_1^-)]e^{-\delta t_1}$ is positive. Because $G>0$ it is not immediately clear whether credit rationing lengthens or shortens the rotation period.

When credit rationing is binding the optimal cutting moment always depends on owner -specific factors such as the subjective time preference and the utility function. For studying these dependencies we consider a simple case where $\rho=0$ and the utility function takes the form $U(c)=(c^{1-\alpha}-1)/(1-\alpha)$, where $\alpha\in(0,1)$. We obtain $c=c_0e^{-\delta t/\alpha}$. Because the optimal consumption schedule exhausts the harvest net income between rotations we obtain the budget constraint $\int_0^{t_1}c_0e^{-t\delta/\alpha}dt-px(t_1^-)+w=0$ and

$$c_0=p[x(t_1^-)-w/p]\,\delta/\alpha(1-e^{-t_1\delta/\alpha}). \tag{32}$$

Equations (31) and (32) yield the comparative statics for the optimal rotation period. Using the utility function given above and assuming $\rho=0$, equation (31) takes the form

$\alpha e^{-\delta t_1/\alpha} c_0^{(1-\alpha)}(e^{\delta t_1/\alpha} - e^{\delta t_1})/(\alpha-1)+c_0^{-\alpha}pF[x(t_1^-)](1-e^{-\delta t_1})=0$. By (32) we can write $c_0=pS$, where $S=[x(t_1^-)-w/p]\delta/\ [\alpha(1-e^{-\delta t_1/\alpha})]$. This yields $(pS)^{-\alpha}p\{\alpha S(1-e^{-\delta t_1/\alpha+\delta t_1})/(\alpha-1)+ F[x(t_1^-)](1-e^{-\delta t_1})\}=0$. Comparative static results follow by using the fact that $\Gamma\equiv\alpha S(1-e^{-\delta t_1/\alpha+\delta t_1})/(\alpha-1)+F[x(t_1^-)](1-e^{-\delta t_1})=0$ is necessary for optimality. Note that the value of Γ depends only on w/p and not on the absolute values of timber price or planting costs. Differentiating Γ implies

$$\partial\Gamma/\partial p=\alpha\partial S/\partial p(1-e^{-\delta t_1/\alpha+\delta t_1})/(\alpha-1)<0, \tag{33}$$

$$\partial\Gamma/\partial w=\alpha\partial S/\partial w(1-e^{-\delta t_1/\alpha+\delta t_1})/(\alpha-1)>0. \tag{34}$$

By (32) the level of c_0 is higher, the higher the timber price and the lower the planting costs. Thus $\partial S/\partial p>0$ and $\partial S/\partial w<0$. Equations (33) and (34) now imply that the higher the timber price or the lower are planting costs the longer is the rotation period. Thus these comparative statics results are qualitatively similar as in the FPO model. The effects of the subjective time preference and utility parameter α are more complicated and are demonstrated by numerical examples.

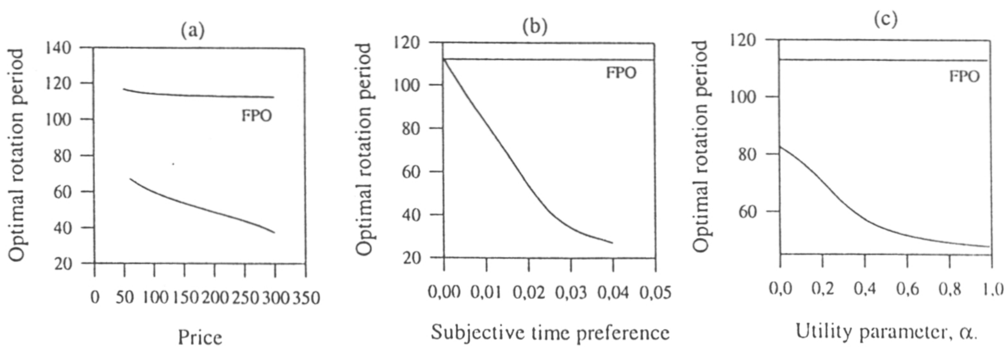

Figure 2a,b,c. Optimal rotation and imperfect capital markets

Note:
The parameter values approximating the conditions in Sounthern Finland are: $K=500\ (m^3)$, $r=0.048/a$, $x_{t_1}=10\ (m^3)$, $w=3000$ (FIM). In addition, $p=170$ (FIM), $\delta=0.02/a$ and $\alpha=0.5$ if not expessed otherwise.

Figure 2 shows examples of the rotation with credit rationing and the FPO rotation. With zero rate of interest the latter maximizes the "average net sustained yield", $[px(t_1^-)-w]/t_1$. In Figure (2a) both periods are shorter, the higher the price. Rotation with credit rationing is considerably shorter than the FPO rotations. When timber price equals 300 the FPO rotation is about three times longer than the rotation with credit rationing. Note, however, that the logistic growth model is able to give only rough numerical appriximations. Figure (2b) shows that rotation period decreases with the subjective time preference δ. The periods converge in length as $\delta \to 0$ because with $\delta = 0$ the credit rationing constraint is not binding. Figure 2c compares rotation periods with different levels of the utility parameter α. With greater curvature (greater α) the forest owner prefers a flatter consumption schedule and more frequent but lower forest income implying shorter rotations. In general, Figures 3a-c demonstrate that using FPO rotation when capital markets are imperfect may lead to rather serious deviations from the optimal harvesting policy.

In section 3 we showed that with perfect capital markets there are no incentives to smooth the forest income over time and thus there cannot be convergence toward a normal forest. Binding credit rationing implies an outcome where consumption jumps up at the harvesting moments. Such a jump in consumption reveals inefficiency caused by the forest owner's inability to borrow. The stand may be cut before the FPO rotation age, implying that after the harvest the assets of the owner earn a lower marginal rate of interest than which would be obtainable if the stand could be harvested in smaller units. This suggests that extending the model to include multiple stands implies that it will not be optimal to cut two perfectly similar stands simultaneously but instead to smooth the forest income over time. The solution with multiple stands may converge toward some long-run steady state solution. If the number of stands is finite the resulting rotation period may still deviate from the FPO rotation and resemble the rotation period with one stand as described by equation (31).

It is shown by Koskela (1989) and Kuuluvainen (1990) that credit rationing increases the first period harvest in the two-period model. In Figure 3 the result is quite the reverse because credit rationing shortens the rotation period, implying a lower level of harvestable timber. However, the comparison of these two models is far from unambiguous.

Our model based on the FPO approach suggests that imperfect capital markets may contain economic motives to smooth the forest income over time and sell timber in smaller units. In addition, this leads to long-term differences in the age structure of forests. Whether these incentives are actually realized depends on whether the imperfections of capital markets are binding. This in turn may depend on various factors like the subjective time preference, rate of interest, nonforest income, initial amount of nonforest assets and the initial age composition of the forest. These problems may be studied e.g. by extending the number of stands but neglecting the bequest motive.

5 *In situ* preferences

We next extend our perspective by taking into account that the forest owner may evaluate forests *in situ* in addition to the pecuniary value of timber. It may be expected that with *in situ* valuation the Fisherian separation theorem may not hold, implying that the cutting decision depends on forest owner -specific factors. Recall that this complication is not taken into account by simply adding the *in situ* valuation in the FPO model (e.g. Hartman 1976). Thus this widely applied extension presents only a partial picture of how *in situ* valuation changes the rotation period. In our framework, analogous extensions may take many different forms depending on capital markets, bequest motive and number of stands. We consider a case with one stand, perfect capital markets and a bequest motive reflected by a lower bound restriction for nonforest assets at the end of the life cycle. To simplify we neglect forest bequest motive but assume (as is the case in Finland) that replanting is required by the law.

Let the *in situ* preferences be given by $A(x)$, $A'(x)>0$, $A''(x)\leq 0$. Thus the *in situ* value of the stand is taken to be an increasing concave function of timber volume. We will focus the analysis on optimal interior solutions instead of cases where the stand is cut at the end of the owner's life cycle. For the period after the stand is cut, the forest owner aims to

$$\max_{\{c\geq 0\}} \quad W_2=\int_{t_1}^{T} [U(c)-A(x)]e^{-\delta t}dt,$$

subject to $\dot{a}=\rho a+m-c$, $a(t_1)=a(t_1^-)+px(t_1^-)-w$, $\dot{x}=F(x)$, $x(t_1)=x_{t_1}$ (given), and $a(T)\geq 0$. Using Seierstad and Sydsæter (1987, p. 85, theorem 2), the Hamiltonian is $H_2=U(c)+A(x)+\lambda(\rho a+m-c)+\varphi F(x)$ and the necessary conditions include $U'(c)-\lambda=0$, $\dot{\lambda}=\lambda(\delta-\rho)$

$$\dot{\varphi}=-A'(x)+\varphi[\delta-F'(x)], \tag{35}$$

$$\lambda(T)a(T)=0,\ \lambda(T)\geq 0,\ a(T)\geq 0, \tag{36}$$

$$\varphi(T)=0. \tag{37}$$

Write the value function as $W_2=W_2[a(t_1^-),x(t_1),t_1]$. The criterion for the period $t\in[0,t_1)$ is $\int_0^{t_1}[U(c)+A(x)]e^{-\delta t}+W_2[a(t_1^-),x(t_1^-),t_1]$. The necessary conditions for interior t_1 include $a(0)=a_0$, $x(0)=x_0$ and

$$\lambda(t_1^-)=e^{\delta t_1}\partial W_2[a(t_1^-),x(t_1),t_1]/\partial a(t_1^-), \tag{38}$$

$$\varphi(t_1^-)=e^{\delta t_1}\partial W_2[a(t_1^-),x(t_1),t_1]/\partial x(t_1^-) \tag{39}$$

$$H_1(t_1^-)=H_2(t_1). \tag{40}$$

In (40) we assumed that $\dot{p}=0$. Condition (38) implies that λ must be continuous and condition (39) with $a(t_1)=a(t_1^-)+px(t_1^-)-w$ that $\varphi(t_1^-)=\lambda(t_1^-)p$. Using these facts condition (40) can be written in the form

$$p\lambda(t_1)\{F[x(t_1^-)]-\rho x(t_1^-)+\rho w/p\}+A[x(t_1^-)]=A(x_{t_1})+\varphi(t_1)F(x_{t_1}). \tag{41}$$

The LHS of (41) is the increase in utility before t_1, given that the cutting moment is marginally postponed. Note that the value growth net of interest costs is given in utility units at t_1. To interpret the RHS recall[6] that $-\partial W_2/\partial t_1=-\partial\left\{\int_{t_1}^{T}\{U[c*(t)]+A[x*(t)]\}e^{-\delta t}dt\right\}/\partial t_1=H_2(t_1)e^{-\delta t_1}$, where $c*(t)$ and $x*(t)$ denote the optimal solutions for the period $t\in[t_1,T]$. Along this period the new stand grows independently on the optimal consumption time path. Thus $-\partial\left\{\int_{t_1}^{T}A[x*(t)]e^{-\delta t}dt\right\}/\partial t_1 = [A(x_{t_1})+\varphi(t_1)F(x_{t_1})]e^{-\delta t}$, i.e. the RHS of (41) denotes the decrease in *in situ* benefits due to a marginal decrease in the length of the period after the stand is cut. Evidently, with any $\delta>0$, $-\partial\left\{\int_{t_1}^{T}A[x*(t)]e^{-\delta t}dt\right\}/dt_1 <-\partial\left\{\int_{t_1}^{T}A[x*(t)]dt\right\}/dt_1= A[x*(T)]e^{-\delta t_1}$. Assuming that the length of the forest owner's life cycle is shorter than the rotation period yields $A(x_{t_1})+\varphi(t_1)F(x_{t_1})<A[x*(T)]<A[x*(t_1^-)]$. Thus by (41) *in situ* preferences increase the rotation period. Because the t_1 solving $F[x(t_1^-)]-\rho x(t_1^-)+\rho w/p$ is longer than the FPO rotation, it follows that the rotation period defined by (41) must be longer than the FPO rotation.

The cutting moment defined by (41) depends on the shadow price of nonforest assets and is thus forest owner -specific. The coefficient of $\lambda(t_1)$ is negative and the rotation period must be shorter the lower the consumption at t_1. For studying the comparative statics of the rotation period, we restrict the analytical investigation to the simple case where the subjective time preference equals the rate of discount implying that $\dot{c}=0$. The equations $\dot{a}=\rho a+m-c$ and $a(T)=0$ imply

$$a(t)=\begin{cases}[a_0+(m-c)/\rho]e^{\rho t}+(c-m)/\rho, & \text{when } t\in[0,t_1)\\ [(m-c)e^{-\rho T}/\rho]e^{\rho t}+(c-m)/\rho, & \text{when } t\in[t_1,T].\end{cases} \tag{42}$$

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[6]Seierstad and Sydsæter (1987 p. 213, theorem 9).

Next using $a(t_1)=a(t_1^-)+px(t_1^-)-w$ yields by (42):

$$c=\rho[px(t_1^-)-w+a_0e^{\rho t_1}]/[e^{\rho t_1}-e^{\rho(t_1-T)}]+m, \tag{43}$$

$$\partial c/\partial p=\rho x(t_1^-)/\mu>0, \tag{44}$$

$$\partial c/\partial w=-\rho/\mu<0, \tag{45}$$

$$\partial c/\partial m=1>0, \tag{46}$$

$$\partial c/\partial a_0=\rho e^{\rho t_1}/\mu>0 \tag{47}$$

$$\partial c/\partial T=-\rho^2e^{\rho(T-t_1)}[a_0e^{\rho t_1}+px(t_1^-)-w]/(e^{\rho T}-1)<0 \tag{48}$$

$$\partial c/\partial \rho=\{\mu-\rho[t_1e^{\rho t_1}-(t_1-T)e^{\rho(t_1-T)}]\}[px(t_1^-)-w+a_0e^{\rho t_1}]/\mu^2+\rho^2a_0e^{\rho t_1}/\mu\gtreqless 0, \tag{49}$$

where $\mu\equiv[e^{\rho t_1}-e^{\rho(t_1-T)}]>0$. Thus consumption is higher, the higher the timber price, nonforest income, and initial nonforest assets, and the lower the planting costs and shorter the life cycle. The dependence on the rate of interest is more complicated. Low initial nonforest assets and cutting moment near T may imply that one is a borrower for most of the life cycle implying that an increase in ρ decreases consumption. High initial nonforest assets and an early cutting moment may imply the reverse. Note that similar outcomes are absent in homogeneous biomass models (Ovaskainen 1992).

Noting that the shadow price for nonforest assets decreases with consumption and denoting $\Gamma\equiv p\lambda(p,w,m,a_0,T,\rho)\{F[x(t_1^-)]-\rho x(t_1^-)+\rho w/p\}+A[x(t_1^-)]-A(x_{t_1})-\varphi(t_1)F(x_{t_1})$ yields

$$\partial\Gamma/\partial p=\partial\lambda/\partial pV+\lambda\{F[x(t_1^-)]-\rho x(t_1^-)\}\gtreqless 0, \tag{50}$$

$$\partial\Gamma/\partial w=\partial\lambda/\partial wV+\lambda\rho\gtreqless 0, \tag{51}$$

$$\partial\Gamma/\partial m=\partial\lambda/\partial mV>0, \tag{52}$$

$$\partial\Gamma/\partial a_0=\partial\lambda/\partial a_0V>0, \tag{53}$$

$$\partial\Gamma/\partial T=\partial\lambda/\partial TV-\partial\varphi(t_1)/\partial T<0, \tag{54}$$

$$\partial\Gamma/\partial\rho=\partial\lambda/\partial\rho V+\lambda[-px(t_1^-)+w]-\partial\varphi(t_1)/\partial\rho\gtreqless 0, \tag{55}$$

where $\Gamma=0$ and $A[x(t_1^-)]-A(x_{t_1})-\varphi(t_1)F(x_{t_1})<0$ imply that $V=p\{F[x(t_1^-)]-\rho x(t_1^-)+\rho w/p\}<0$. Recall that at any interior maximum it is necessary that $\partial\Gamma/\partial t_1<0$. The effects of price and planting costs depends on whether the income or substitution effect dominates. The effect of the rate of interest or the subjective time preference is ambiguous for reasons explained above. Note that all these results deviate from the Hartman (1976) model, where (given that *in situ* value increases the rotation period) an increase in price or interest rate shortens and an increase in planting costs lengthens the rotation period. In addition, here the rotation period depends on parameters that are absent from Hartman (1976). An increase in nonforest income or initial nonforest assets lengthens the growing period because the owner has more resources for taking into account the value of the forest *in situ*. A longer life cycle shortens the rotation period for two reasons: the level of consumption decreases and the *in situ* value of the forest after cutting obtains more weight. In addition, the rotation period depends on the properties of the utility function, U(c).

We may also study the dependence of rotation length on the owner's age. In equation (41) the term $A(x_{t_1})+\varphi(t_1)F(x_{t_1})$ is the marginal value of *in situ* benefits for the period after the stand is cut and it shortens the rotation period. This term is lower, the shorter the remaining period before the end of the life cycle, implying that young forest owners are more "inpatient" than old owners. Note that if $\delta<\rho$, consumption increases and its marginal value decreases during the life cycle. Thus for younger owners it is more costly to postpone the harvest for maintaining the *in situ* value of the stand. This compounds the outcome that older forest owners let the stand grow older before the cut than young owners. However, this result may be the reverse if the subjective time preference exceeds the rate of interest.

In Figures 3a,b it is assumed that $A(x)=Ax^{\beta}$, where $0<\beta<1$, $U(c)=(c^{1-\alpha}-1)/(1-\alpha)$, $0<\alpha<1$, and F(x) is the logistic growth function. Figure 3a shows the rotation period as a function of owner's age and demonstrates two cases where the age of harvested stand increases with owner's age. In addition, the numerical example suggests that this effect may be stronger for high-income forest owners. The dotted line shows the reverse outcome when $\delta > \rho$. Figure 3b shows the rotation period as a function of nonforest income for two different *in situ* parameter levels. By comparison, the FPO rotation period with the given parameter values is shorter and equals 64.3 years. Finally, note that dependencies shown in Figure 3 a,b are absent in the Hartman (1976) formulation.

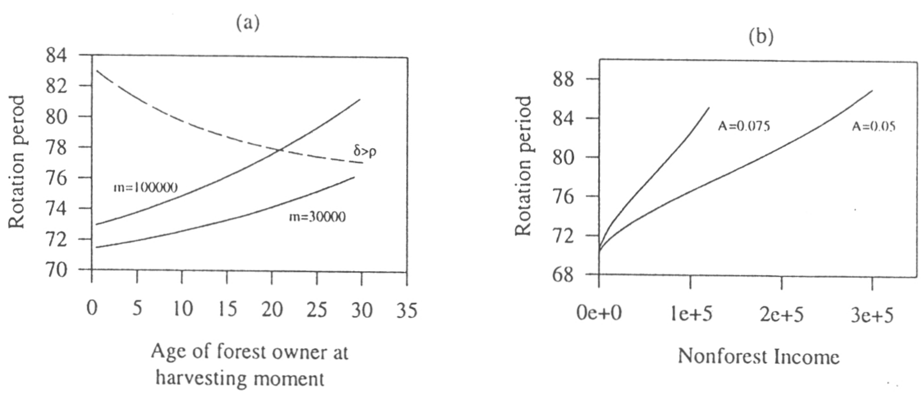

Figure 3 a, b. Optimal rotation and *in situ* values.

Note:

Figure 3a, solid lines: p=170, w=4000, r=0.048, K=500, A=0.05, a_0=15000, ρ=0.035, δ=0.001, α=β=0.5, x_0=10, dotted line: δ=0.1, ρ=0.03, m=30000.

Figure 3b: x_0=133.31 or x_{t_0}=60 years, m=30000, other parameters as in Fig.3a.

6 Conclusions

When the original FPO model is used to predict the harvesting behavior of NIPF owners it is assumed that the Fisherian separation theorem holds and that markets for forest land exist and are perfect. These requirements seldom hold, and they are relaxed in the Fisherian two-period forest model. However, this approach neglects the age classes of stands, which in the FPO approach are an essential factor in forest harvesting decisions. This study aims to relax the restrictive assumptions of the FPO model without neglecting the forest rotation aspect. The study combines the forest owner's life cycle consumption/savings decisionmaking, harvesting decisions for an unlimited number of unequal even-aged stands and a bequest motive for forest and nonforest assets. The FPO rotation follows as a special case under restrictions on preferences, capital markets, the bequest motive and future expectations. Among other things, the results reveal that forest models that exclude the rotation aspect also neglect part of the long-term problems that may be essential in forestry decisions. Imperfect capital markets may shorten the rotation period and cause incentives to smooth forest income by dividing large stands for cutting in smaller units. The results on *in situ* preferences and forest rotation show that including the life cycle decisionmaking completely changes the properties of timber supply.

Price and interest rate uncertainty have been studied extensively in forest economics, but their effects on rotation have been analyzed by implicitly relying on the Fisherian separation theorem. This study suggests that the optimal stopping rule results may change considerably if the unwarranted assumption of complete separability between harvesting and consumption decisions is relaxed.

Appendix 1

Between the cutting moments all variables are continuous and by using the necessary conditions we can write: $-d(e^{-\delta t}H)/dt=-e^{-\delta t}(dH/dt-\delta H)=-e^{-\delta t}[U'(c)\dot{c}+\lambda(\rho a-c)+\lambda\rho\dot{a}-\lambda\dot{c}+\dot{\varphi}F(x)+\varphi F'(x)\dot{x}-\delta U(c)-\delta\lambda(\rho a-c)-\delta\varphi F(x)]=-\delta U(c)$. Integration both sides of $-e^{-\delta t}(dH/dt-\delta H)=-\delta U(c)$ yields $\delta\int_0^{t_1}U(c)e^{-\delta t}dt= H(0)- e^{-\delta t}H(t_1-)$. By condition (27), $H(0)=H(t_1-)$. Thus we obtain $\delta\int_0^{t_1}U(c)e^{-\delta t}dt/(1-e^{-\delta t_1})=H(0)$. The term $\int_0^{t_1}U(c)e^{-\delta t}dt$ is the present value utility of the first rotation period. Recall that this cycle continues forever implying that the present value of these cycles is given by $\int_0^{t_1} U(c)e^{-\delta t}dt/(1-e^{-\delta t_1})$. Thus we obtain $\delta W_1 = H(0)$. The same situation occurs after the first cut and we obtain $\delta W_2 = H(t_1)$ as well.

References

Binkley, C. (1981) Timber supply from private nonindustrial forests, Yale School of Forestry and Environmental Studies, Bulletin No. 92.

Binkley, C. (1987) Economic models of timber supply, in M. Kallio, D. Dykstra and C. Binkley (eds.), The global forest sector, an analytical perspective, 109-136, John Wiley & Sons, N.Y.

Brazee, R. and Mendelsohn (1988) Timber supply with fluctuating prices, Forest Science 34, 359-372.

Clark, C. (1990) Mathematical bioeconomics: the optimal management of renewable resources, John Wiley & Sons, N.Y.

Dennis, D. (1990) A probit analysis of harvesting decisions using pooled time-series and cross-sectional data, J. Environ. Econ. Management 18, 176-187.

Hartman, R. (1976) The harvesting decision when a standing forest has value, Economic Inquiry 14, 52-68.

Hultkrantz, L. (1992) Forestry and the bequest motive, J. Environ. Econom. Management 22, 164-177.

Hyberg, B. and D. Holthausen (1989) The behavior of nonindustrial private forest landowners, Can. J. For. Res. 19, 1014-1023.

Johansson, P-O. and K-G. Löfgren (1985) The economics of forestry & natural resources, Basil Blackwell, Oxford.

Koskela, E. (1989) Forest taxation and timber supply under price uncertainty: credit rationing in capital markets, Forest Science 35, 160-172.

Kuuluvainen, J. (1990) Virtual price approach to short-term timber supply under credit rationing, J. of Environm. Econom. and Management 19, 109-126.

Kuuluvainen, J. and J. Salo (1991) Timber supply and life cycle harvest of nonindustrial private forest owners: an empirical analysis of the Finnish case, Forest Science 37, 1011-1029.

Kuuluvainen, J. H. Karppinen and V. Ovaskainen (1996) Landowners objectives and nonindustrial private timber supply, Forest Science 42, 300-309.

Lohmander, P. (1983) Optimal harvest policy under the influence of imperfections and uncertainty, College of Forestry, Department of Forest Economics, Working Paper 22, Umeå.

Löfgren, K.-G. (1991) Another reconciliation between economists and forestry experts: OLG-arguments, Environmental and Resource Economics 1, 83-96.

Mitra, T. and H. Wan (1985) Some theoretical results on the economics of forestry, Review of Economic Studies LII, 263-282.

Montgomery C. and D. Adams (1995) Optimal timber management policies, in D. Bromley (ed.) The handbook of environmental economics, 379-404, Blackwell, Oxford U.K.

Norström, C. (1975) A stochastic model for the growth period decision in forestry, Swedish Journal of Economics 77, 329-337.

Ollikainen, M. (1996) Essays on timber supply and forest taxation, Ph.D. dissertation, Government Institute for Economic Research, Research Reports 33.

Ollikainen, M. (1996) Sustainable forestry, timber bequests and future generations, Department of Economics, University of Helsinki, manuscript.

Ovaskainen, V. (1992) Forest taxation; timber supply, and economic efficiency, Acta Forestalia Fennica 233.

Samuelson, P. (1976) Economics of forestry in an evolving society, Economic Inquiry 14, 466-492.

Seierstad, A. and K. Sydsæter (1987) Optimal control theory with economic applications, North Holland, Amsterdam.

Yaari, M. (1964) On consumer's lifetime allocation process, International Econ. Review 5, 304-317.

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